

# Computer algebra independent integration tests

4-Trig-functions/4.4-Cotangent/4.4.2.1-a+b-cot-<sup>m</sup>-c+d-cot-<sup>n</sup>

Nasser M. Abbasi

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3.81	$\int \frac{(e \cot(c+dx))^{9/2}}{(a+b \cot(c+dx))^3} dx$	576
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3.83	$\int \frac{(e \cot(c+dx))^{5/2}}{(a+b \cot(c+dx))^3} dx$	614
3.84	$\int \frac{(e \cot(c+dx))^{3/2}}{(a+b \cot(c+dx))^3} dx$	632
3.85	$\int \frac{\sqrt{e \cot(c+dx)}}{(a+b \cot(c+dx))^3} dx$	650
3.86	$\int \frac{1}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))^3} dx$	668
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3.90	$\int \frac{1+i \cot(c+dx)}{\sqrt{a+b \cot(c+dx)}} dx$	715
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3.92	$\int \frac{A+B \cot(c+dx)}{a+b \cot(c+dx)} dx$	725
3.93	$\int \frac{A+B \cot(c+dx)}{(a+b \cot(c+dx))^2} dx$	729
3.94	$\int \frac{A+B \cot(c+dx)}{(a+b \cot(c+dx))^3} dx$	735
3.95	$\int (a + b \cot(c + dx))^{5/2} (A + B \cot(c + dx)) dx$	740
3.96	$\int (a + b \cot(c + dx))^{3/2} (A + B \cot(c + dx)) dx$	747
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3.99	$\int (-a + b \cot(c + dx))(a + b \cot(c + dx))^{3/2} dx$	765
3.100	$\int (-a + b \cot(c + dx)) \sqrt{a + b \cot(c + dx)} dx$	773
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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 106 ]. This is test number [ 112 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 106 )	% 0.00 ( 0 )
Mathematica	% 99.06 ( 105 )	% 0.94 ( 1 )
Maple	% 97.17 ( 103 )	% 2.83 ( 3 )
Maxima	% 74.53 ( 79 )	% 25.47 ( 27 )
Fricas	% 29.25 ( 31 )	% 70.75 ( 75 )
Sympy	% 1.89 ( 2 )	% 98.11 ( 104 )
Giac	% 2.83 ( 3 )	% 97.17 ( 103 )
Mupad	% 97.17 ( 103 )	% 2.83 ( 3 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.



grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

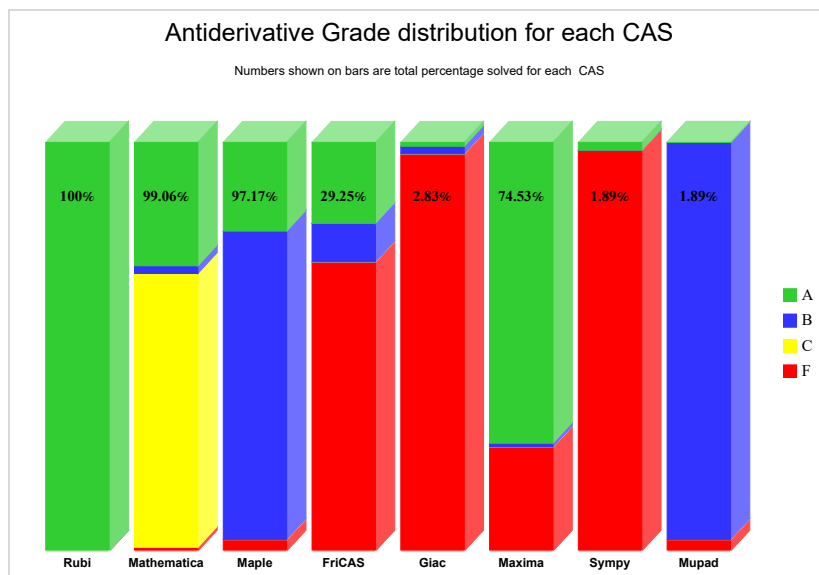
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

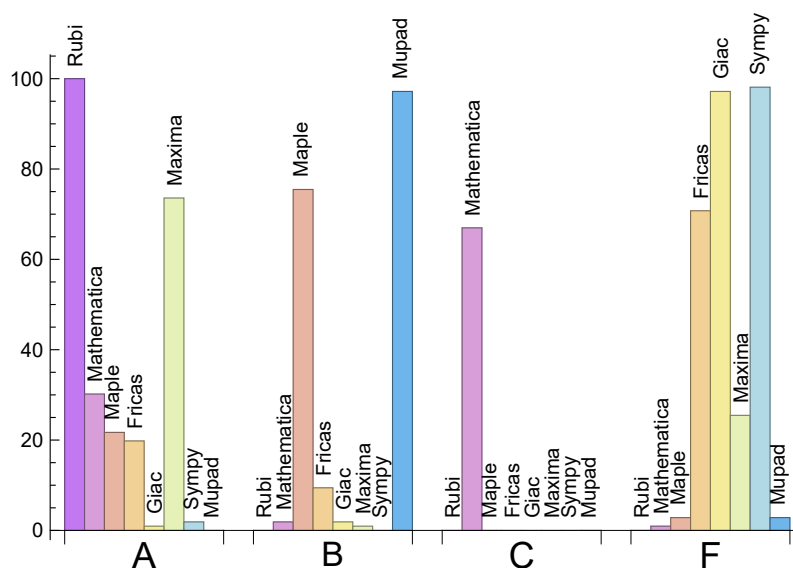
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	30.19	1.89	66.98	0.94
Maple	21.70	75.47	0.00	2.83
Maxima	73.58	0.94	0.00	25.47
Fricas	19.81	9.43	0.00	70.75
Sympy	1.89	0.00	0.00	98.11
Giac	0.94	1.89	0.00	97.17
Mupad	0.00	97.17	0.00	2.83

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	3	100.00 %	0.00 %	0.00 %
Maxima	27	88.89 %	11.11 %	0.00 %
Fricas	75	4.00 %	78.67 %	17.33 %
Sympy	104	95.19 %	3.85 %	0.96 %
Giac	103	100.00 %	0.00 %	0.00 %
Mupad	3	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

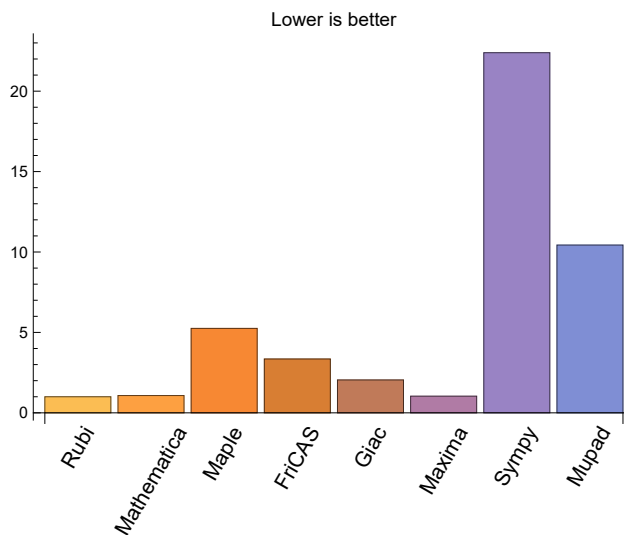
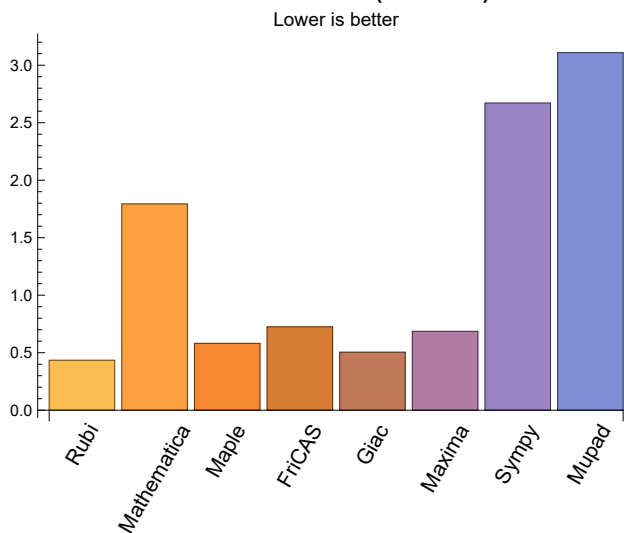
## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	232.98	1.00	221.50	1.00
Mathematica	1.79	212.17	1.07	193.00	0.86
Maple	0.58	863.94	5.25	434.00	2.32
Maxima	0.69	247.84	1.04	231.00	0.92
Fricas	0.73	402.06	3.35	377.00	3.34
Sympy	2.67	2250.00	22.39	2250.00	22.39
Giac	0.50	249.33	2.05	241.00	2.17
Mupad	3.11	3189.42	10.43	366.00	1.69

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.

**Normalized mean size of antiderivative****Mean time used (seconds)**

## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {81}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>



[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

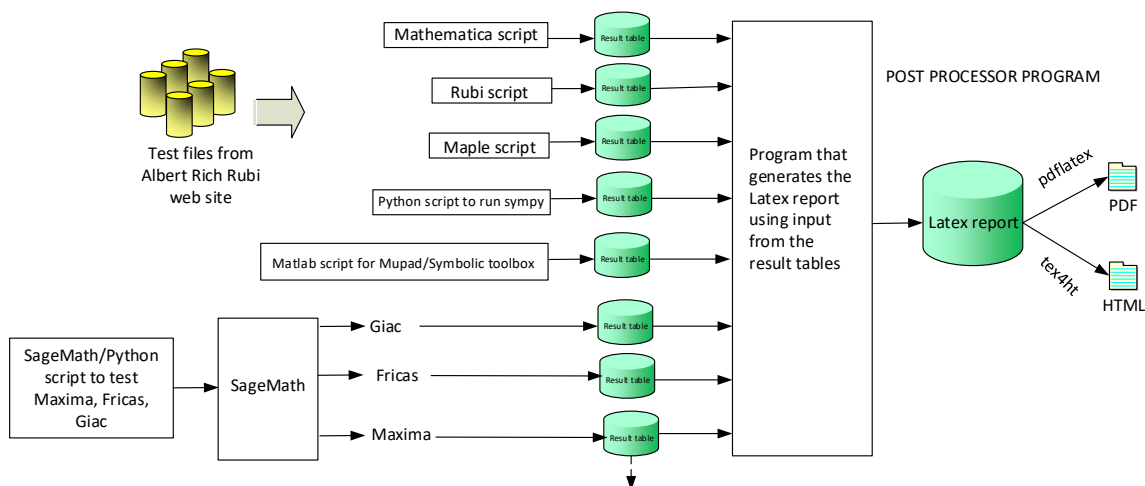
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 8, 10, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 90, 91, 92, 96, 97, 98, 101, 102, 103, 104, 105, 106 }

B grade: { 1, 95 }

C grade: { 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 99, 100 }

F grade: { 89 }

#### 2.1.3 Maple

A grade: { 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 51, 52, 54, 55, 69, 70, 71, 72, 73, 74 }

B grade: { 2, 3, 4, 5, 6, 7, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }  
}

C grade: { }

F grade: { 1, 88, 89 }

## 2.1.4 Maxima

A grade: { 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94 }

B grade: { 5 }

C grade: { }

F grade: { 1, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

## 2.1.5 FriCAS

A grade: { 2, 4, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 35, 36, 37, 38, 40, 92 }

B grade: { 3, 5, 6, 7, 27, 39, 90, 91, 93, 94 }

C grade: { }

F grade: { 1, 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

## 2.1.6 SymPy

A grade: { 92, 93 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

## 2.1.7 Giac

A grade: { 92 }

B grade: { 93, 94 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

## 2.1.8 Mupad

A grade: { }

B grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade: { }

F grade: { 1, 88, 89 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	117	0	0	0	0	0	-1
normalized size	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.322	2.502	0.000	0.861	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	68	388	149	377	0	0	144
normalized size	1	1.00	0.59	3.34	1.28	3.25	0.00	0.00	1.24
time (sec)	N/A	0.157	0.187	0.495	0.760	0.902	0.000	0.000	1.978
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	67	363	124	334	0	0	98
normalized size	1	1.00	0.71	3.86	1.32	3.55	0.00	0.00	1.04
time (sec)	N/A	0.116	0.112	0.427	0.616	0.750	0.000	0.000	1.161

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	154	337	108	236	0	0	128
normalized size	1	1.00	2.17	4.75	1.52	3.32	0.00	0.00	1.80
time (sec)	N/A	0.078	0.289	0.431	0.896	0.650	0.000	0.000	0.777

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	165	327	83	172	0	0	65
normalized size	1	1.00	3.37	6.67	1.69	3.51	0.00	0.00	1.33
time (sec)	N/A	0.044	0.217	0.430	0.692	0.601	0.000	0.000	0.728

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	191	355	111	321	0	0	84
normalized size	1	1.00	2.55	4.73	1.48	4.28	0.00	0.00	1.12
time (sec)	N/A	0.086	0.259	0.359	0.719	0.533	0.000	0.000	0.960

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	203	374	123	358	0	0	103
normalized size	1	1.00	2.05	3.78	1.24	3.62	0.00	0.00	1.04
time (sec)	N/A	0.135	0.419	0.356	0.599	0.815	0.000	0.000	1.483

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	187	234	232	0	0	0	125
normalized size	1	1.00	0.70	0.87	0.86	0.00	0.00	0.00	0.46
time (sec)	N/A	0.288	1.188	0.606	0.595	0.000	0.000	0.000	1.728

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	52	213	213	0	0	0	104
normalized size	1	1.00	0.21	0.87	0.87	0.00	0.00	0.00	0.42
time (sec)	N/A	0.235	0.386	0.613	0.601	0.000	0.000	0.000	0.948

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	175	204	211	0	0	0	104
normalized size	1	1.00	0.72	0.84	0.86	0.00	0.00	0.00	0.43
time (sec)	N/A	0.227	0.432	0.592	0.705	0.000	0.000	0.000	0.697

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	53	186	193	0	0	0	86
normalized size	1	1.00	0.24	0.84	0.87	0.00	0.00	0.00	0.39
time (sec)	N/A	0.200	0.257	0.535	0.787	0.000	0.000	0.000	0.445

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	236	195	202	0	0	0	86
normalized size	1	1.00	1.06	0.88	0.91	0.00	0.00	0.00	0.39
time (sec)	N/A	0.208	1.808	0.484	0.461	0.000	0.000	0.000	0.593

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	233	216	211	0	0	0	99
normalized size	1	1.00	0.94	0.87	0.85	0.00	0.00	0.00	0.40
time (sec)	N/A	0.237	1.262	0.467	0.540	0.000	0.000	0.000	0.701



Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	141	216	221	0	0	0	99
normalized size	1	1.00	0.57	0.87	0.89	0.00	0.00	0.00	0.40
time (sec)	N/A	0.237	0.414	0.460	0.690	0.000	0.000	0.000	1.292

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	729	446	193	535	0	0	177
normalized size	1	1.00	3.92	2.40	1.04	2.88	0.00	0.00	0.95
time (sec)	N/A	0.300	6.104	0.791	0.500	0.951	0.000	0.000	2.440

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	332	419	175	487	0	0	143
normalized size	1	1.00	2.08	2.62	1.09	3.04	0.00	0.00	0.89
time (sec)	N/A	0.259	2.810	0.869	0.466	0.723	0.000	0.000	1.617

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	315	391	149	366	0	0	136
normalized size	1	1.00	2.28	2.83	1.08	2.65	0.00	0.00	0.99
time (sec)	N/A	0.203	1.553	0.960	0.760	0.718	0.000	0.000	0.990

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	292	379	136	349	0	0	100
normalized size	1	1.00	2.50	3.24	1.16	2.98	0.00	0.00	0.85
time (sec)	N/A	0.169	5.171	0.568	0.551	0.841	0.000	0.000	0.621

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	311	388	130	372	0	0	119
normalized size	1	1.00	2.73	3.40	1.14	3.26	0.00	0.00	1.04
time (sec)	N/A	0.175	2.894	0.479	0.680	0.665	0.000	0.000	0.576

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	417	388	133	378	0	0	101
normalized size	1	1.00	3.56	3.32	1.14	3.23	0.00	0.00	0.86
time (sec)	N/A	0.188	6.109	0.498	0.791	1.540	0.000	0.000	0.705

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	269	409	148	485	0	0	126
normalized size	1	1.00	1.91	2.90	1.05	3.44	0.00	0.00	0.89
time (sec)	N/A	0.229	3.345	0.526	0.773	0.454	0.000	0.000	1.261

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	174	430	170	514	0	0	129
normalized size	1	1.00	1.05	2.61	1.03	3.12	0.00	0.00	0.78
time (sec)	N/A	0.296	1.998	0.536	0.855	0.865	0.000	0.000	1.924

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	110	394	134	400	0	0	123
normalized size	1	1.00	0.99	3.55	1.21	3.60	0.00	0.00	1.11
time (sec)	N/A	0.451	0.884	0.730	1.567	0.899	0.000	0.000	0.682

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	107	368	118	333	0	0	79
normalized size	1	1.00	1.23	4.23	1.36	3.83	0.00	0.00	0.91
time (sec)	N/A	0.239	4.052	0.732	0.703	0.792	0.000	0.000	0.492

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	98	358	113	331	0	0	102
normalized size	1	1.00	1.13	4.11	1.30	3.80	0.00	0.00	1.17
time (sec)	N/A	0.218	0.251	0.812	0.809	0.841	0.000	0.000	0.368

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	107	365	120	321	0	0	79
normalized size	1	1.00	1.29	4.40	1.45	3.87	0.00	0.00	0.95
time (sec)	N/A	0.219	0.516	0.877	0.632	0.624	0.000	0.000	0.523

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	176	394	136	472	0	0	123
normalized size	1	1.00	1.59	3.55	1.23	4.25	0.00	0.00	1.11
time (sec)	N/A	0.452	2.034	0.810	1.645	1.134	0.000	0.000	0.644

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	131	416	154	500	0	0	132
normalized size	1	1.00	0.97	3.08	1.14	3.70	0.00	0.00	0.98
time (sec)	N/A	0.538	1.333	0.758	0.444	0.935	0.000	0.000	0.929

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	224	234	233	0	0	0	375
normalized size	1	1.00	0.80	0.83	0.83	0.00	0.00	0.00	1.33
time (sec)	N/A	0.544	2.019	0.829	0.438	0.000	0.000	0.000	0.898

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	312	234	232	0	0	0	376
normalized size	1	1.00	1.12	0.84	0.83	0.00	0.00	0.00	1.35
time (sec)	N/A	0.564	2.829	0.745	0.491	0.000	0.000	0.000	0.823

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	207	223	231	0	0	0	366
normalized size	1	1.00	0.74	0.80	0.83	0.00	0.00	0.00	1.32
time (sec)	N/A	0.531	1.299	0.792	0.585	0.000	0.000	0.000	0.722

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	337	222	238	0	0	0	366
normalized size	1	1.00	1.20	0.79	0.85	0.00	0.00	0.00	1.30
time (sec)	N/A	0.566	0.839	0.714	0.545	0.000	0.000	0.000	0.813

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	203	255	256	0	0	0	414
normalized size	1	1.00	0.66	0.83	0.84	0.00	0.00	0.00	1.35
time (sec)	N/A	0.796	1.327	0.654	0.870	0.000	0.000	0.000	0.923

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	467	276	274	0	0	0	425
normalized size	1	1.00	1.41	0.83	0.83	0.00	0.00	0.00	1.28
time (sec)	N/A	1.075	6.341	0.666	1.013	0.000	0.000	0.000	1.226

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	192	440	197	567	0	0	154
normalized size	1	1.00	1.17	2.68	1.20	3.46	0.00	0.00	0.94
time (sec)	N/A	0.617	2.134	0.831	0.456	0.465	0.000	0.000	1.037

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	131	434	189	533	0	0	178
normalized size	1	1.00	0.80	2.65	1.15	3.25	0.00	0.00	1.09
time (sec)	N/A	0.663	2.023	0.845	0.918	0.543	0.000	0.000	0.938

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	181	423	190	518	0	0	151
normalized size	1	1.00	1.12	2.63	1.18	3.22	0.00	0.00	0.94
time (sec)	N/A	0.592	0.818	0.856	0.601	0.534	0.000	0.000	0.898

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	217	426	189	504	0	0	173
normalized size	1	1.00	1.32	2.58	1.15	3.05	0.00	0.00	1.05
time (sec)	N/A	0.648	1.271	0.833	0.512	0.451	0.000	0.000	0.944

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	156	458	214	697	0	0	175
normalized size	1	1.00	0.83	2.42	1.13	3.69	0.00	0.00	0.93
time (sec)	N/A	0.863	1.211	0.795	0.713	0.662	0.000	0.000	1.141

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	167	482	224	718	0	0	193
normalized size	1	1.00	0.78	2.24	1.04	3.34	0.00	0.00	0.90
time (sec)	N/A	1.105	3.190	0.855	0.778	0.480	0.000	0.000	1.385

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	69	356	0	0	0	0	119
normalized size	1	1.00	0.31	1.60	0.00	0.00	0.00	0.00	0.53
time (sec)	N/A	0.266	0.166	0.269	0.000	0.000	0.000	0.000	0.631

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	61	249	0	0	0	0	210
normalized size	1	1.00	0.45	1.84	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.240	0.091	0.132	0.000	0.000	0.000	0.000	0.483

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	96	265	0	0	0	0	254
normalized size	1	1.00	0.69	1.91	0.00	0.00	0.00	0.00	1.83
time (sec)	N/A	0.235	0.318	0.175	0.000	0.000	0.000	0.000	0.996

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	98	452	0	0	0	0	254
normalized size	1	1.00	0.44	2.05	0.00	0.00	0.00	0.00	1.15
time (sec)	N/A	0.209	0.260	0.117	0.000	0.000	0.000	0.000	0.674

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	67	442	0	0	0	0	238
normalized size	1	1.00	0.31	2.07	0.00	0.00	0.00	0.00	1.11
time (sec)	N/A	0.185	0.168	0.224	0.000	0.000	0.000	0.000	0.438

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	51	249	0	0	0	0	230
normalized size	1	1.00	0.42	2.06	0.00	0.00	0.00	0.00	1.90
time (sec)	N/A	0.127	0.076	0.161	0.000	0.000	0.000	0.000	0.410

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	65	249	0	0	0	0	208
normalized size	1	1.00	0.47	1.79	0.00	0.00	0.00	0.00	1.50
time (sec)	N/A	0.193	0.132	0.186	0.000	0.000	0.000	0.000	0.501

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	71	356	0	0	0	0	121
normalized size	1	1.00	0.31	1.58	0.00	0.00	0.00	0.00	0.54
time (sec)	N/A	0.193	0.143	0.126	0.000	0.000	0.000	0.000	0.400

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	75	265	0	0	0	0	242
normalized size	1	1.00	0.52	1.85	0.00	0.00	0.00	0.00	1.69
time (sec)	N/A	0.205	0.406	0.193	0.000	0.000	0.000	0.000	0.800

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	69	444	0	0	0	0	238
normalized size	1	1.00	0.32	2.06	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.176	0.242	0.134	0.000	0.000	0.000	0.000	0.709

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	68	363	221	0	0	0	153
normalized size	1	1.00	0.28	1.47	0.89	0.00	0.00	0.00	0.62
time (sec)	N/A	0.207	0.131	0.367	0.852	0.000	0.000	0.000	1.397

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	155	337	199	0	0	0	128
normalized size	1	1.00	0.69	1.49	0.88	0.00	0.00	0.00	0.57
time (sec)	N/A	0.172	0.282	0.355	0.728	0.000	0.000	0.000	0.726

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	166	327	180	0	0	0	118
normalized size	1	1.00	0.80	1.57	0.87	0.00	0.00	0.00	0.57
time (sec)	N/A	0.142	0.216	0.406	0.750	0.000	0.000	0.000	0.647



Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	196	355	204	0	0	0	137
normalized size	1	1.00	0.86	1.55	0.89	0.00	0.00	0.00	0.60
time (sec)	N/A	0.194	0.359	0.372	0.692	0.000	0.000	0.000	0.805

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	196	374	220	0	0	0	158
normalized size	1	1.00	0.78	1.48	0.87	0.00	0.00	0.00	0.63
time (sec)	N/A	0.258	0.758	0.482	0.704	0.000	0.000	0.000	1.245

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	224	581	287	0	0	0	1274
normalized size	1	1.00	0.71	1.83	0.91	0.00	0.00	0.00	4.02
time (sec)	N/A	0.334	1.982	0.650	0.585	0.000	0.000	0.000	2.471

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	220	534	257	0	0	0	1157
normalized size	1	1.00	0.76	1.85	0.89	0.00	0.00	0.00	4.02
time (sec)	N/A	0.275	0.569	0.539	0.451	0.000	0.000	0.000	1.212

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	192	529	242	0	0	0	1234
normalized size	1	1.00	0.72	1.98	0.91	0.00	0.00	0.00	4.62
time (sec)	N/A	0.249	0.890	0.520	0.797	0.000	0.000	0.000	1.013

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	218	538	242	0	0	0	1196
normalized size	1	1.00	0.82	2.01	0.91	0.00	0.00	0.00	4.48
time (sec)	N/A	0.258	0.330	0.456	0.900	0.000	0.000	0.000	0.936

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	82	558	259	0	0	0	1214
normalized size	1	1.00	0.28	1.92	0.89	0.00	0.00	0.00	4.17
time (sec)	N/A	0.334	0.301	0.466	0.626	0.000	0.000	0.000	1.512

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	85	600	286	0	0	0	1227
normalized size	1	1.00	0.26	1.86	0.89	0.00	0.00	0.00	3.81
time (sec)	N/A	0.428	0.358	0.449	0.530	0.000	0.000	0.000	2.320

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	251	807	347	0	0	0	2317
normalized size	1	1.00	0.67	2.17	0.93	0.00	0.00	0.00	6.23
time (sec)	N/A	0.564	3.061	0.753	0.440	0.000	0.000	0.000	5.474

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	247	750	316	0	0	0	2071
normalized size	1	1.00	0.72	2.19	0.92	0.00	0.00	0.00	6.06
time (sec)	N/A	0.477	2.593	0.722	0.437	0.000	0.000	0.000	2.546

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	216	725	292	0	0	0	1896
normalized size	1	1.00	0.69	2.32	0.93	0.00	0.00	0.00	6.06
time (sec)	N/A	0.425	1.034	0.607	0.517	0.000	0.000	0.000	1.413

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	193	742	290	0	0	0	1951
normalized size	1	1.00	0.62	2.37	0.93	0.00	0.00	0.00	6.23
time (sec)	N/A	0.420	3.390	0.467	0.653	0.000	0.000	0.000	1.204

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	104	743	289	0	0	0	1946
normalized size	1	1.00	0.33	2.37	0.92	0.00	0.00	0.00	6.22
time (sec)	N/A	0.458	0.376	0.459	0.693	0.000	0.000	0.000	1.671

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	108	786	316	0	0	0	1969
normalized size	1	1.00	0.31	2.29	0.92	0.00	0.00	0.00	5.74
time (sec)	N/A	0.561	0.612	0.463	0.711	0.000	0.000	0.000	3.063

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	116	829	342	0	0	0	1992
normalized size	1	1.00	0.31	2.20	0.91	0.00	0.00	0.00	5.28
time (sec)	N/A	0.662	0.677	0.451	0.766	0.000	0.000	0.000	5.257

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	286	459	259	0	0	0	5579
normalized size	1	1.00	0.88	1.41	0.80	0.00	0.00	0.00	17.17
time (sec)	N/A	0.660	0.874	0.673	0.538	0.000	0.000	0.000	1.973

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	249	429	236	0	0	0	5129
normalized size	1	1.00	0.82	1.42	0.78	0.00	0.00	0.00	16.98
time (sec)	N/A	0.379	0.567	0.687	0.494	0.000	0.000	0.000	1.627

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	226	417	233	0	0	0	4808
normalized size	1	1.00	0.75	1.38	0.77	0.00	0.00	0.00	15.92
time (sec)	N/A	0.376	0.283	0.790	0.484	0.000	0.000	0.000	1.386

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	248	423	239	0	0	0	4871
normalized size	1	1.00	0.82	1.40	0.79	0.00	0.00	0.00	16.13
time (sec)	N/A	0.370	0.254	0.712	0.526	0.000	0.000	0.000	1.768

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	198	459	261	0	0	0	4899
normalized size	1	1.00	0.61	1.41	0.80	0.00	0.00	0.00	15.07
time (sec)	N/A	0.660	0.446	0.614	0.910	0.000	0.000	0.000	1.860

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	109	481	280	0	0	0	6042
normalized size	1	1.00	0.31	1.37	0.80	0.00	0.00	0.00	17.21
time (sec)	N/A	0.962	0.283	0.629	0.662	0.000	0.000	0.000	2.807

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	445	805	385	0	0	0	13244
normalized size	1	1.00	1.02	1.84	0.88	0.00	0.00	0.00	30.31
time (sec)	N/A	1.108	6.158	0.831	0.462	0.000	0.000	0.000	3.970

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	390	784	359	0	0	0	12617
normalized size	1	1.00	0.99	1.99	0.91	0.00	0.00	0.00	32.10
time (sec)	N/A	0.740	2.795	0.781	0.449	0.000	0.000	0.000	3.121

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	322	768	344	0	0	0	11953
normalized size	1	1.00	0.83	1.98	0.89	0.00	0.00	0.00	30.89
time (sec)	N/A	0.677	3.326	0.757	0.592	0.000	0.000	0.000	3.366

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	401	749	343	0	0	0	11731
normalized size	1	1.00	1.04	1.94	0.89	0.00	0.00	0.00	30.39
time (sec)	N/A	0.646	6.099	0.823	0.668	0.000	0.000	0.000	3.084

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	300	765	360	0	0	0	9400
normalized size	1	1.00	0.76	1.94	0.91	0.00	0.00	0.00	23.86
time (sec)	N/A	0.738	2.865	0.846	0.535	0.000	0.000	0.000	8.163

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	244	803	402	0	0	0	15251
normalized size	1	1.00	0.56	1.84	0.92	0.00	0.00	0.00	34.90
time (sec)	N/A	1.095	0.630	0.763	0.718	0.000	0.000	0.000	4.335

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	556	1254	537	0	0	0	20651
normalized size	1	1.00	1.05	2.37	1.02	0.00	0.00	0.00	39.04
time (sec)	N/A	1.631	6.261	0.853	0.615	0.000	0.000	0.000	10.399

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	525	1232	516	0	0	0	20089
normalized size	1	1.00	1.10	2.59	1.08	0.00	0.00	0.00	42.20
time (sec)	N/A	1.230	6.189	0.880	0.755	0.000	0.000	0.000	7.262

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	488	1229	505	0	0	0	19256
normalized size	1	1.00	1.04	2.61	1.07	0.00	0.00	0.00	40.97
time (sec)	N/A	1.295	6.199	0.995	0.558	0.000	0.000	0.000	6.515

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	518	1212	492	0	0	0	19000
normalized size	1	1.00	1.12	2.63	1.07	0.00	0.00	0.00	41.21
time (sec)	N/A	1.234	6.156	0.870	0.778	0.000	0.000	0.000	6.213

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	483	1187	496	0	0	0	19534
normalized size	1	1.00	1.04	2.56	1.07	0.00	0.00	0.00	42.19
time (sec)	N/A	1.147	6.186	0.972	0.842	0.000	0.000	0.000	6.127

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	411	1190	510	0	0	0	20155
normalized size	1	1.00	0.86	2.50	1.07	0.00	0.00	0.00	42.34
time (sec)	N/A	1.242	6.134	0.845	0.475	0.000	0.000	0.000	6.789

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	303	1245	565	0	0	0	21158
normalized size	1	1.00	0.57	2.35	1.07	0.00	0.00	0.00	40.00
time (sec)	N/A	1.658	1.792	0.795	0.453	0.000	0.000	0.000	9.999

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	118	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.302	1.235	0.000	1.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	3.252	2.421	0.000	1.218	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	1622	0	159	0	0	1410
normalized size	1	1.00	1.00	36.04	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.065	0.081	0.457	0.000	0.710	0.000	0.000	2.536

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	1622	0	159	0	0	1410
normalized size	1	1.00	1.56	36.04	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.059	1.721	0.538	0.000	0.590	0.000	0.000	1.403

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	67	187	89	79	534	95	155
normalized size	1	1.00	1.14	3.17	1.51	1.34	9.05	1.61	2.63
time (sec)	N/A	0.078	0.130	0.419	1.690	0.546	1.109	0.534	0.995

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	144	356	185	340	3966	241	268
normalized size	1	1.00	1.30	3.21	1.67	3.06	35.73	2.17	2.41
time (sec)	N/A	0.149	1.929	0.370	0.530	0.559	4.232	0.490	1.460



Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	202	559	337	549	0	412	481
normalized size	1	1.00	1.15	3.19	1.93	3.14	0.00	2.35	2.75
time (sec)	N/A	0.276	5.003	0.382	0.866	0.708	0.000	0.489	2.602

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	379	2405	0	0	0	0	3864
normalized size	1	1.00	2.02	12.79	0.00	0.00	0.00	0.00	20.55
time (sec)	N/A	0.453	1.795	0.547	0.000	0.000	0.000	0.000	31.594

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	294	1665	0	0	0	0	2823
normalized size	1	1.00	1.96	11.10	0.00	0.00	0.00	0.00	18.82
time (sec)	N/A	0.333	0.974	0.539	0.000	0.000	0.000	0.000	13.758

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	212	968	0	0	0	0	843
normalized size	1	1.00	1.74	7.93	0.00	0.00	0.00	0.00	6.91
time (sec)	N/A	0.262	0.567	0.531	0.000	0.000	0.000	0.000	3.029

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	253	1375	0	0	0	0	3442
normalized size	1	1.00	1.68	9.11	0.00	0.00	0.00	0.00	22.79
time (sec)	N/A	0.278	3.959	0.555	0.000	0.000	0.000	0.000	26.556

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	178	972	0	0	0	0	2529
normalized size	1	1.00	0.44	2.38	0.00	0.00	0.00	0.00	6.20
time (sec)	N/A	0.511	1.938	0.510	0.000	0.000	0.000	0.000	11.956

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	158	2285	0	0	0	0	583
normalized size	1	1.00	0.37	5.41	0.00	0.00	0.00	0.00	1.38
time (sec)	N/A	0.418	1.055	0.540	0.000	0.000	0.000	0.000	2.571

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	154	3976	0	0	0	0	2909
normalized size	1	1.00	1.51	38.98	0.00	0.00	0.00	0.00	28.52
time (sec)	N/A	0.160	0.611	0.490	0.000	0.000	0.000	0.000	2.290

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	226	7951	0	0	0	0	5737
normalized size	1	1.00	1.64	57.62	0.00	0.00	0.00	0.00	41.57
time (sec)	N/A	0.263	1.674	0.459	0.000	0.000	0.000	0.000	6.466

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	319	12836	0	0	0	0	9453
normalized size	1	1.00	1.72	69.38	0.00	0.00	0.00	0.00	51.10
time (sec)	N/A	0.402	3.530	0.440	0.000	0.000	0.000	0.000	17.933

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	146	1905	0	0	0	0	2731
normalized size	1	1.00	1.43	18.68	0.00	0.00	0.00	0.00	26.77
time (sec)	N/A	0.162	0.331	0.543	0.000	0.000	0.000	0.000	2.202

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	216	2291	0	0	0	0	5475
normalized size	1	1.00	1.64	17.36	0.00	0.00	0.00	0.00	41.48
time (sec)	N/A	0.248	1.473	0.563	0.000	0.000	0.000	0.000	5.961

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	232	3055	0	0	0	0	8438
normalized size	1	1.00	1.33	17.56	0.00	0.00	0.00	0.00	48.49
time (sec)	N/A	0.382	5.938	0.556	0.000	0.000	0.000	0.000	16.173

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [48] had the largest ratio of [.9091]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	5	3	1.00	23	0.130
3	A	4	3	1.00	23	0.130
4	A	3	3	1.00	23	0.130
5	A	2	2	1.00	23	0.087
6	A	3	3	1.00	23	0.130
7	A	4	3	1.00	23	0.130
8	A	16	12	1.00	25	0.480
9	A	15	12	1.00	25	0.480
10	A	15	12	1.00	25	0.480
11	A	14	11	1.00	25	0.440
12	A	13	10	1.00	25	0.400
13	A	14	11	1.00	25	0.440
14	A	14	11	1.00	25	0.440
15	A	7	5	1.00	25	0.200
16	A	6	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	4	4	1.00	25	0.160
19	A	4	4	1.00	25	0.160
20	A	4	4	1.00	25	0.160
21	A	5	5	1.00	25	0.200
22	A	6	5	1.00	25	0.200
23	A	7	6	1.00	25	0.240
24	A	6	6	1.00	25	0.240
25	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	10	10	1.00	25	0.400
29	A	17	14	1.00	25	0.560
30	A	18	15	1.00	25	0.600
31	A	17	14	1.00	25	0.560
32	A	18	15	1.00	25	0.600
33	A	18	15	1.00	25	0.600
34	A	20	16	1.00	25	0.640
35	A	8	8	1.00	25	0.320
36	A	8	7	1.00	25	0.280
37	A	8	8	1.00	25	0.320
38	A	8	7	1.00	25	0.280
39	A	9	8	1.00	25	0.320
40	A	10	8	1.00	25	0.320
41	A	12	9	1.00	13	0.692
42	A	6	5	1.00	11	0.454
43	A	8	7	1.00	13	0.538
44	A	14	9	1.00	11	0.818
45	A	12	8	1.00	13	0.615
46	A	5	4	1.00	11	0.364
47	A	6	5	1.00	13	0.385
48	A	13	10	1.00	11	0.909
49	A	8	7	1.00	13	0.538
50	A	13	9	1.00	11	0.818
51	A	12	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	11	8	1.00	23	0.348
53	A	10	7	1.00	23	0.304
54	A	11	8	1.00	23	0.348
55	A	12	8	1.00	23	0.348
56	A	13	9	1.00	25	0.360
57	A	12	9	1.00	25	0.360
58	A	11	8	1.00	25	0.320
59	A	11	8	1.00	25	0.320
60	A	12	9	1.00	25	0.360
61	A	13	9	1.00	25	0.360
62	A	14	10	1.00	25	0.400
63	A	13	10	1.00	25	0.400
64	A	12	9	1.00	25	0.360
65	A	12	9	1.00	25	0.360
66	A	12	9	1.00	25	0.360
67	A	13	10	1.00	25	0.400
68	A	14	10	1.00	25	0.400
69	A	15	12	1.00	25	0.480
70	A	14	11	1.00	25	0.440
71	A	14	11	1.00	25	0.440
72	A	14	11	1.00	25	0.440
73	A	15	12	1.00	25	0.480
74	A	16	13	1.00	25	0.520
75	A	16	13	1.00	25	0.520
76	A	15	12	1.00	25	0.480
77	A	15	12	1.00	25	0.480

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	15	12	1.00	25	0.480
79	A	15	12	1.00	25	0.480
80	A	16	13	1.00	25	0.520
81	A	17	14	1.00	25	0.560
82	A	16	13	1.00	25	0.520
83	A	16	13	1.00	25	0.520
84	A	16	13	1.00	25	0.520
85	A	16	13	1.00	25	0.520
86	A	16	13	1.00	25	0.520
87	A	17	13	1.00	25	0.520
88	A	5	3	1.00	12	0.250
89	A	8	5	1.00	23	0.217
90	A	3	3	1.00	27	0.111
91	A	3	3	1.00	27	0.111
92	A	2	2	1.00	23	0.087
93	A	3	3	1.00	23	0.130
94	A	4	3	1.00	23	0.130
95	A	10	5	1.00	25	0.200
96	A	9	5	1.00	25	0.200
97	A	8	5	1.00	25	0.200
98	A	10	7	1.00	27	0.259
99	A	13	9	1.00	27	0.333
100	A	13	9	1.00	27	0.333
101	A	7	4	1.00	25	0.160
102	A	8	5	1.00	25	0.200
103	A	9	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	7	4	1.00	27	0.148
105	A	8	5	1.00	27	0.185
106	A	9	5	1.00	27	0.185



# Chapter 3

## Listing of integrals

### 3.1 $\int (a + ia \cot(c + dx))^n dx$

Optimal. Leaf size=49

$$\frac{i(a + ia \cot(c + dx))^n {}_2F_1\left(1, n; n + 1; \frac{1}{2}(i \cot(c + dx) + 1)\right)}{2dn}$$

[Out]  $1/2*I*(a+I*a*\cot(d*x+c))^n*\text{hypergeom}([1, n], [1+n], 1/2+1/2*I*\cot(d*x+c))/d/n$

**Rubi [A]** time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3481, 68}

$$\frac{i(a + ia \cot(c + dx))^n {}_2F_1\left(1, n; n + 1; \frac{1}{2}(i \cot(c + dx) + 1)\right)}{2dn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Cot}[c + d*x])^n, x]$

[Out]  $((I/2)*(a + I*a*\text{Cot}[c + d*x])^n*\text{Hypergeometric2F1}[1, n, 1 + n, (1 + I*\text{Cot}[c + d*x])/2])/d*n$

#### Rule 68

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x\_Symbol] := \text{Simp}[( (b*c - a*d)^n*(a + b*x)^{(m + 1)}*\text{Hypergeometric2F1}[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b^{(n + 1)}*(m + 1)), x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3481

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Dist[b/d, Subst[Int[(a + x)^(n - 1)/(a - x), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int (a + ia \cot(c + dx))^n dx = \frac{(ia) \text{Subst} \left( \int \frac{(a+x)^{-1+n}}{a-x} dx, x, ia \cot(c + dx) \right)}{d}$$

$$= \frac{i(a + ia \cot(c + dx))^n {}_2F_1 \left( 1, n; 1 + n; \frac{1}{2}(1 + i \cot(c + dx)) \right)}{2dn}$$

**Mathematica [B]** time = 0.32, size = 117, normalized size = 2.39

$$\frac{i(a + ia \cot(c + dx))^n \left( 2(n + 1) {}_2F_1(1, n; n + 1; i \cot(c + dx) + 1) + (n + in \cot(c + dx)) \left( {}_2F_1 \left( 1, n + 1; n + 2; \frac{1}{2}(i \cot(c + dx) + 1) \right) \right) \right)}{4dn(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Cot[c + d\*x])^n, x]

[Out] ((I/4)\*(a + I\*a\*Cot[c + d\*x])^n\*(2\*(1 + n)\*Hypergeometric2F1[1, n, 1 + n, 1 + I\*Cot[c + d\*x]] + (n + I\*n\*Cot[c + d\*x])\*(Hypergeometric2F1[1, 1 + n, 2 + n, (1 + I\*Cot[c + d\*x])/2] - 2\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + I\*Cot[c + d\*x]]))/d\*n\*(1 + n))

**fricas [F]** time = 0.86, size = 0, normalized size = 0.00

$$\text{integral} \left( \left( -\frac{2a}{e^{(2i dx + 2ic)} - 1} \right)^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*cot(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((-2\*a/(e^(2\*I\*d\*x + 2\*I\*c) - 1))^n, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ia \cot(dx + c) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*cot(d\*x+c))^n,x, algorithm="giac")

[Out] integrate((I\*a\*cot(d\*x + c) + a)^n, x)

maple [F] time = 2.50, size = 0, normalized size = 0.00

$$\int (a + ia \cot(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*cot(d\*x+c))^n,x)

[Out] int((a+I\*a\*cot(d\*x+c))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia \cot(dx + c) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*cot(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((I\*a\*cot(d\*x + c) + a)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a \cot(c + dx) 1i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cot(c + d\*x)\*1i)^n,x)

[Out] int((a + a\*cot(c + d\*x)\*1i)^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia \cot(c + dx) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*cot(d\*x+c))\*\*n,x)

[Out] Integral((I\*a\*cot(c + d\*x) + a)\*\*n, x)

### 3.2 $\int (e \cot(c + dx))^{5/2} (a + a \cot(c + dx)) dx$

**Optimal.** Leaf size=116

$$-\frac{\sqrt{2} a e^{5/2} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} + \frac{2 a e^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2 a e (e \cot(c+dx))^{3/2}}{3d} - \frac{2 a (e \cot(c+dx))^{5/2}}{5d}$$

[Out]  $-2/3*a*e*(e*\cot(d*x+c))^{(3/2)}/d-2/5*a*(e*\cot(d*x+c))^{(5/2)}/d-a*e^{(5/2)}*\arctanh(1/2*(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)})*2^{(1/2)}/d+2*a*e^2*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3528, 3532, 208}

$$\frac{2 a e^2 \sqrt{e \cot(c+dx)}}{d} - \frac{\sqrt{2} a e^{5/2} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} - \frac{2 a e (e \cot(c+dx))^{3/2}}{3d} - \frac{2 a (e \cot(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x]),x]

[Out]  $-((\text{Sqrt}[2]*a*e^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])])/d) + (2*a*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (2*a*e*(e*\text{Cot}[c + d*x])^{(3/2)})/(3*d) - (2*a*(e*\text{Cot}[c + d*x])^{(5/2)})/(5*d)$

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 3528

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(d\*(a + b\*Tan[e + f\*x])^m)/(f\*m), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3532

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&

EqQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{5/2} (a + a \cot(c + dx)) dx &= -\frac{2a(e \cot(c + dx))^{5/2}}{5d} + \int (e \cot(c + dx))^{3/2} (-ae + ae \cot(c + dx)) dx \\
&= -\frac{2ae(e \cot(c + dx))^{3/2}}{3d} - \frac{2a(e \cot(c + dx))^{5/2}}{5d} + \int \sqrt{e \cot(c + dx)} (-ae + ae \cot(c + dx)) dx \\
&= \frac{2ae^2 \sqrt{e \cot(c + dx)}}{d} - \frac{2ae(e \cot(c + dx))^{3/2}}{3d} - \frac{2a(e \cot(c + dx))^{5/2}}{5d} + \int \sqrt{e \cot(c + dx)} (-ae + ae \cot(c + dx)) dx \\
&= \frac{2ae^2 \sqrt{e \cot(c + dx)}}{d} - \frac{2ae(e \cot(c + dx))^{3/2}}{3d} - \frac{2a(e \cot(c + dx))^{5/2}}{5d} + \int \sqrt{e \cot(c + dx)} (-ae + ae \cot(c + dx)) dx \\
&= -\frac{\sqrt{2} ae^{5/2} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e \cot(c + dx)}}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{d} + \frac{2ae^2 \sqrt{e \cot(c + dx)}}{d} - \frac{2ae(e \cot(c + dx))^{3/2}}{3d} - \frac{2a(e \cot(c + dx))^{5/2}}{5d}
\end{aligned}$$

**Mathematica** [C] time = 0.19, size = 68, normalized size = 0.59

$$\frac{2ae(e \cot(c + dx))^{3/2} \left( 5 {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)\right) + 3 \cot(c + dx) {}_2F_1\left(-\frac{5}{4}, 1; -\frac{1}{4}; -\tan^2(c + dx)\right) \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x]),x]

[Out]  $(-2*a*e*(e*\text{Cot}[c + d*x])^{3/2}*(3*\text{Cot}[c + d*x]*\text{Hypergeometric2F1}[-5/4, 1, -1/4, -\text{Tan}[c + d*x]^2] + 5*\text{Hypergeometric2F1}[-3/4, 1, 1/4, -\text{Tan}[c + d*x]^2]))/(15*d)$

**fricas** [A] time = 0.90, size = 377, normalized size = 3.25

$$\left[ \frac{15 \sqrt{2} (ae^2 \cos(2dx + 2c) - ae^2) \sqrt{e} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}} (\cos(2dx + 2c) - \sin(2dx + 2c) - 1) + 2e \sin(2dx + 2c)\right)}{30(d \cos(2dx + 2c))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] [1/30\*(15\*sqrt(2)\*(a\*e^2\*cos(2\*d\*x + 2\*c) - a\*e^2)\*sqrt(e)\*log(sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1) + 2\*e\*sin(2\*d\*x + 2\*c) + e) + 4\*(18\*a\*e^2\*cos(2\*d\*x + 2\*c) + 5\*a\*e^2\*sin(2\*d\*x + 2\*c) - 12\*a\*e^2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*cos(2\*d\*x + 2\*c) - d), 1/15\*(15\*sqrt(2)\*(a\*e^2\*cos(2\*d\*x + 2\*c) - a\*e^2)\*sqrt(-e)\*arctan(1/2\*sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e)) + 2\*(18\*a\*e^2\*cos(2\*d\*x + 2\*c) + 5\*a\*e^2\*sin(2\*d\*x + 2\*c) - 12\*a\*e^2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*cos(2\*d\*x + 2\*c) - d)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a) (e \cot(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(5/2), x)

**maple** [B] time = 0.50, size = 388, normalized size = 3.34

$$\frac{2a(e \cot(dx + c))^{\frac{5}{2}}}{5d} - \frac{2ae(e \cot(dx + c))^{\frac{3}{2}}}{3d} + \frac{2ae^2 \sqrt{e \cot(dx + c)}}{d} - \frac{ae^2 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2} + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2} + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2}} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(5/2)\*(a+cot(d\*x+c)\*a),x)

[Out] -2/5\*a\*(e\*cot(d\*x+c))^(5/2)/d-2/3\*a\*e\*(e\*cot(d\*x+c))^(3/2)/d+2\*a\*e^2\*(e\*cot(d\*x+c))^(1/2)/d-1/4\*a/d\*e^2\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)))-1/2\*a/d\*e^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/2\*a/d\*e^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/4\*a/d\*e^3\*2^(1/2)/(e^2)^(1/4)\*ln((e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)))+1/2\*a/d\*e^3\*2^(1/2)/(e^2)^(1/4)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/2\*a/d\*e^3\*2^(1/2)/(e^2)^(1/4)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)

**maxima** [A] time = 0.76, size = 149, normalized size = 1.28

$$\frac{\left( 15 a e^2 \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right) - 4 \left( 15 a e^2 \sqrt{\frac{e}{\tan(dx+c)}} - 5 a e \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}} - 3 \right)}{e} \right)}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c)),x, algorithm="maxima")

[Out] -1/30\*(15\*a\*e^2\*(sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e)) - 4\*(15\*a\*e^2\*sqrt(e/tan(d\*x + c)) - 5\*a\*e\*(e/tan(d\*x + c))^(3/2) - 3\*a\*(e/tan(d\*x + c))^(5/2))/e)\*e/d

**mupad** [B] time = 1.98, size = 144, normalized size = 1.24

$$\frac{2 a e^2 \sqrt{e \cot(c+d x)}}{d} - \frac{2 a e (e \cot(c+d x))^{3/2}}{3 d} - \frac{2 a (e \cot(c+d x))^{5/2}}{5 d} + \frac{(-1)^{1/4} a e^{5/2} \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+d x)} i}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(5/2)\*(a + a\*cot(c + d\*x)),x)

[Out] (2\*a\*e^2\*(e\*cot(c + d\*x))^(1/2))/d - (2\*a\*e\*(e\*cot(c + d\*x))^(3/2))/(3\*d) - (2\*a\*(e\*cot(c + d\*x))^(5/2))/(5\*d) + ((-1)^(1/4)\*a\*e^(5/2)\*atan(((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*(1 + 1i))/d + ((-1)^(1/4)\*a\*e^(5/2)\*atan(((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2)\*1i)/e^(1/2)))/d - ((-1)^(1/4)\*a\*e^(5/2)\*atanh(((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/d

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int (e \cot(c + dx))^{\frac{5}{2}} dx + \int (e \cot(c + dx))^{\frac{5}{2}} \cot(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)\*(a+a\*cot(d\*x+c)),x)

[Out] a\*(Integral((e\*cot(c + d\*x))\*\*(5/2), x) + Integral((e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x), x))

### 3.3 $\int (e \cot(c + dx))^{3/2} (a + a \cot(c + dx)) dx$

Optimal. Leaf size=94

$$-\frac{\sqrt{2} a e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e} \cot(c+dx)}\right)}{d} - \frac{2 a e \sqrt{e} \cot(c+dx)}{d} - \frac{2 a (e \cot(c+dx))^{3/2}}{3 d}$$

[Out]  $-2/3*a*(e*\cot(d*x+c))^(3/2)/d-a*e^(3/2)*\arctan(1/2*(e^(1/2)-\cot(d*x+c)*e^(1/2))*2^(1/2)/(e*\cot(d*x+c))^(1/2))*2^(1/2)/d-2*a*e*(e*\cot(d*x+c))^(1/2)/d$

**Rubi [A]** time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3528, 3532, 205}

$$-\frac{\sqrt{2} a e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e} \cot(c+dx)}\right)}{d} - \frac{2 a e \sqrt{e} \cot(c+dx)}{d} - \frac{2 a (e \cot(c+dx))^{3/2}}{3 d}$$

Antiderivative was successfully verified.

[In] `Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]`

[Out]  $-\left(\frac{\text{Sqrt}[2]*a*e^{3/2}*\text{ArcTan}[\left(\frac{\text{Sqrt}[e]-\text{Sqrt}[e]*\text{Cot}[c+d*x]}{\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]]}\right)]}{d}\right) - \left(\frac{2*a*e*\text{Sqrt}[e*\text{Cot}[c+d*x]]}{d}\right) - \left(\frac{2*a*(e*\text{Cot}[c+d*x])^{3/2}}{3*d}\right)$

#### Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

#### Rule 3528

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]`

#### Rule 3532

`Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]`



Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{3/2} (a + a \cot(c + dx)) dx &= -\frac{2a(e \cot(c + dx))^{3/2}}{3d} + \int \sqrt{e \cot(c + dx)} (-ae + ae \cot(c + dx)) dx \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2a(e \cot(c + dx))^{3/2}}{3d} + \int \frac{-ae^2 - ae^2 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2a(e \cot(c + dx))^{3/2}}{3d} - \frac{(2a^2e^4) \text{Subst}\left(\int \frac{1}{-2a^2e}\right)}{3d} \\
&= -\frac{\sqrt{2} ae^{3/2} \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} - \frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2a(e \cot(c + dx))^{3/2}}{3d}
\end{aligned}$$

**Mathematica [C]** time = 0.11, size = 67, normalized size = 0.71

$$\frac{2ae\sqrt{e \cot(c + dx)} \left( 3 {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\tan^2(c + dx)\right) + \cot(c + dx) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)\right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x]),x]

[Out] (-2\*a\*e\*Sqrt[e\*Cot[c + d\*x]]\*(Cot[c + d\*x]\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d\*x]^2] + 3\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2]))/(3\*d)

**fricas [B]** time = 0.75, size = 334, normalized size = 3.55

$$\left[ \frac{3\sqrt{2}a\sqrt{-e}e \log\left(\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c) + \sin(2dx+2c) - 1) - 2e\sin(2dx+2c) + e\right) \sin(2dx+2c)}{6d\sin(2dx+2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] [1/6\*(3\*sqrt(2)\*a\*sqrt(-e)\*e\*log(sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*e\*sin(2\*d\*x + 2\*c) + e)\*sin(2\*d\*x + 2\*c) - 4\*(a\*e\*cos(2\*d\*x + 2\*c) + 3\*a\*e\*sin(2\*

$d*x + 2*c) + a*e)*\sqrt{((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))}/(d*\sin(2*d*x + 2*c))$ ,  $-1/3*(3*\sqrt{2}*a*e^{3/2}*\arctan(-1/2*\sqrt{2}*\sqrt{e}*\sqrt{((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))*(\cos(2*d*x + 2*c) - \sin(2*d*x + 2*c) + 1)/((e*\cos(2*d*x + 2*c) + e))*\sin(2*d*x + 2*c) + 2*(a*e*\cos(2*d*x + 2*c) + 3*a*e*\sin(2*d*x + 2*c) + a*e)*\sqrt{((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))})/(d*\sin(2*d*x + 2*c))]$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a) (e \cot(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(3/2), x)

**maple [B]** time = 0.43, size = 363, normalized size = 3.86

$$\frac{2a(e \cot(dx + c))^{\frac{3}{2}}}{3d} - \frac{2ae\sqrt{e \cot(dx + c)}}{d} + \frac{ae(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{4d} + \frac{ae(e^2)^{\frac{1}{4}}\sqrt{2} \arctan\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)\*(a+cot(d\*x+c)\*a),x)

[Out]  $-2/3*a*(e*\cot(d*x+c))^{3/2}/d - 2*a*e*(e*\cot(d*x+c))^{1/2}/d + 1/4*a/d*e*(e^{-2})^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c) + (e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2} + (e^{-2})^{1/2})/(e*\cot(d*x+c) - (e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2} + (e^{-2})^{1/2})) + 1/2*a/d*e*(e^{-2})^{1/4}*2^{1/2}*\arctan(2^{1/2}/(e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2} + 1) - 1/2*a/d*e*(e^{-2})^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2} + 1) + 1/4*a/d*e*2*2^{1/2}/(e^{-2})^{1/4}*\ln((e*\cot(d*x+c) - (e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2} + (e^{-2})^{1/2})/(e*\cot(d*x+c) + (e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2} + (e^{-2})^{1/2})) + 1/2*a/d*e*2*2^{1/2}/(e^{-2})^{1/4}*\arctan(2^{1/2}/(e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2} + 1) - 1/2*a/d*e*2*2^{1/2}/(e^{-2})^{1/4}*\arctan(-2^{1/2}/(e^{-2})^{1/4}*(e*\cot(d*x+c))^{1/2} + 1)$

**maxima [A]** time = 0.62, size = 124, normalized size = 1.32

$$\left( 3ae \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e} - 2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right) - \frac{2\left(3ae\sqrt{\frac{e}{\tan(dx+c)}} + a\left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}\right)}{e} \right) e$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{3} \cdot (3 \cdot a \cdot e \cdot (\sqrt{2} \cdot \arctan(\frac{1}{2} \sqrt{2}) \cdot (\sqrt{2} \cdot \sqrt{e}) + 2 \cdot \sqrt{e/\tan(dx+c)})) / \sqrt{e}) / \sqrt{e} + \sqrt{2} \cdot \arctan(-\frac{1}{2} \sqrt{2}) \cdot (\sqrt{2} \cdot \sqrt{e}) - 2 \cdot \sqrt{e/\tan(dx+c)}) / \sqrt{e}) / \sqrt{e} - 2 \cdot (3 \cdot a \cdot e \cdot \sqrt{e/\tan(dx+c)} + a \cdot (e/\tan(dx+c))^{3/2}) / e) \cdot e/d$

**mupad [B]** time = 1.16, size = 98, normalized size = 1.04

$$\frac{2ae \cot(c+dx)^{3/2}}{3d} - \frac{2ae \sqrt{e \cot(c+dx)}}{d} + \frac{(-1)^{1/4} a e^{3/2} \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) (1-i)}{d} + \frac{(-1)^{1/4} a e^{3/2} \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) (1+i)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c+d*x))^(3/2)*(a+a*cot(c+d*x)),x)`

[Out]  $((-1)^{1/4} \cdot a \cdot e^{3/2} \cdot \operatorname{atan}((( -1)^{1/4} \cdot (e \cdot \cot(c + d \cdot x))^{1/2}) / e^{1/2})) \cdot (1 - 1i) / d - (2 \cdot a \cdot e \cdot (e \cdot \cot(c + d \cdot x))^{1/2}) / d - (2 \cdot a \cdot (e \cdot \cot(c + d \cdot x))^{3/2}) / (3 \cdot d) - ((-1)^{1/4} \cdot a \cdot e^{3/2} \cdot \operatorname{atanh}((( -1)^{1/4} \cdot (e \cdot \cot(c + d \cdot x))^{1/2}) / e^{1/2})) \cdot (1 + 1i) / d$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int (e \cot(c + dx))^{\frac{3}{2}} dx + \int (e \cot(c + dx))^{\frac{3}{2}} \cot(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c)),x)`

[Out] `a*(Integral((e*cot(c+d*x))**(3/2),x)+Integral((e*cot(c+d*x))**(3/2)*cot(c+d*x),x))`

### 3.4 $\int \sqrt{e \cot(c + dx)} (a + a \cot(c + dx)) dx$

Optimal. Leaf size=71

$$\frac{\sqrt{2} a \sqrt{e} \tanh^{-1} \left( \frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}} \right)}{d} - \frac{2a \sqrt{e \cot(c + dx)}}{d}$$

[Out]  $a \cdot \operatorname{arctanh} \left( \frac{1}{2} \cdot (e^{1/2} + \cot(dx+c)) \cdot e^{1/2} \right) \cdot 2^{1/2} / (e \cdot \cot(dx+c))^{1/2} \cdot 2^{1/2} \cdot e^{1/2} / d - 2 \cdot a \cdot (e \cdot \cot(dx+c))^{1/2} / d$

**Rubi [A]** time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3528, 3532, 208}

$$\frac{\sqrt{2} a \sqrt{e} \tanh^{-1} \left( \frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}} \right)}{d} - \frac{2a \sqrt{e \cot(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]`

[Out]  $(\sqrt{2} * a * \sqrt{e} * \operatorname{ArcTanh}[(\sqrt{e} + \sqrt{e} * \cot[c + d * x]) / (\sqrt{2} * \sqrt{e * \cot[c + d * x]})]) / d - (2 * a * \sqrt{e * \cot[c + d * x]}) / d$

#### Rule 208

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

#### Rule 3528

`Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]`

#### Rule 3532

`Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]`

Rubi steps

$$\begin{aligned}
\int \sqrt{e \cot(c+dx)} (a + a \cot(c+dx)) dx &= -\frac{2a\sqrt{e \cot(c+dx)}}{d} + \int \frac{-ae + ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx \\
&= -\frac{2a\sqrt{e \cot(c+dx)}}{d} - \frac{(2a^2e^2) \text{Subst}\left(\int \frac{1}{2a^2e^2 - ex^2} dx, x, \frac{-ae - ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}}\right)}{d} \\
&= \frac{\sqrt{2} a \sqrt{e} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} - \frac{2a\sqrt{e \cot(c+dx)}}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.29, size = 154, normalized size = 2.17

$$\frac{a\sqrt{e \cot(c+dx)} \left(8 {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\tan^2(c+dx)\right) + \sqrt{2} \sqrt{\tan(c+dx)} \left(2 \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c+dx)}\right) - 2 \tan^{-1}\left(1 + \sqrt{2} \sqrt{\tan(c+dx)}\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x]),x]

[Out] -1/4\*(a\*Sqrt[e\*Cot[c + d\*x]]\*(8\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2] + Sqrt[2]\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]]))/d

**fricas [A]** time = 0.65, size = 236, normalized size = 3.32

$$\left[ \frac{\sqrt{2} a \sqrt{e} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} (\cos(2dx+2c) - \sin(2dx+2c) - 1) + 2e \sin(2dx+2c) + e\right) - 4a\sqrt{e \cot(c+dx)}}{2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] [1/2\*(sqrt(2)\*a\*sqrt(e)\*log(-sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1) + 2\*e\*sin(2\*d\*x + 2\*c) + e) - 4\*a\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/d, -(sq

```
rt(2)*a*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x +
2*c) + e)) + 2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/d]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a) \sqrt{e \cot(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

**maple** [B] time = 0.43, size = 337, normalized size = 4.75

$$\frac{-\frac{2a\sqrt{e \cot(dx+c)}}{d} + \frac{a(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{4d} + \frac{a(e^2)^{\frac{1}{4}}\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{2d}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)*(a+cot(d*x+c)*a),x)
```

```
[Out] -2*a*(e*cot(d*x+c))^(1/2)/d+1/4*a/d*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+1/2*a/d*(e^2)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/2*a/d*(e^2)^(1/4)*2^(1/
2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/4*a/d*e*2^(1/2)/(e
^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))-
1/2*a/d*e*2^(1/2)/(e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)+1/2*a/d*e*2^(1/2)/(e^2)^(1/4)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)
```

**maxima** [A] time = 0.90, size = 108, normalized size = 1.52

$$\frac{\left( a \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right) - \frac{4a\sqrt{\frac{e}{\tan(dx+c)}}}{e} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

[Out]  $\frac{1}{2} * (a * (\sqrt{2} * \log(\sqrt{2}) * \sqrt{e}) * \sqrt{e / \tan(dx + c)} + e + e / \tan(dx + c)) / \sqrt{e} - \sqrt{2} * \log(-\sqrt{2}) * \sqrt{e}) * \sqrt{e / \tan(dx + c)} + e + e / \tan(dx + c)) / \sqrt{e} - 4 * a * \sqrt{e / \tan(dx + c)} / e * e / d$

**mupad [B]** time = 0.78, size = 128, normalized size = 1.80

$$\frac{2 a \sqrt{e \cot(c + dx)}}{d} - \frac{(-1)^{1/4} a \sqrt{e} \left( \operatorname{atan} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right) - \operatorname{atanh} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right) \right)}{d} - \frac{(-1)^{1/4} a \sqrt{e} \operatorname{atan} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x)),x)`

[Out]  $-(2 * a * (e * \cot(c + dx))^{1/2}) / d - ((-1)^{1/4} * a * e^{1/2} * \operatorname{atan}((( -1)^{1/4} * (e * \cot(c + dx))^{1/2}) / e^{1/2}) * 1i) / d - ((-1)^{1/4} * a * e^{1/2} * \operatorname{atanh}((( -1)^{1/4} * (e * \cot(c + dx))^{1/2}) / e^{1/2}) * 1i) / d - ((-1)^{1/4} * a * e^{1/2} * (\operatorname{atan}((( -1)^{1/4} * (e * \cot(c + dx))^{1/2}) / e^{1/2}) - \operatorname{atanh}((( -1)^{1/4} * (e * \cot(c + dx))^{1/2}) / e^{1/2}))) / d$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int \sqrt{e \cot(c + dx)} dx + \int \sqrt{e \cot(c + dx)} \cot(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c)),x)`

[Out] `a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*cot(c + d*x), x))`

$$3.5 \quad \int \frac{a+a \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx$$

**Optimal.** Leaf size=49

$$\frac{\sqrt{2} a \tan^{-1} \left( \frac{\sqrt{e}(1-\cot(c+dx))}{\sqrt{2} \sqrt{e \cot(c+dx)}} \right)}{d\sqrt{e}}$$

[Out] a\*arctan(1/2\*(1-cot(d\*x+c))\*e^(1/2)\*2^(1/2)/(e\*cot(d\*x+c))^(1/2))\*2^(1/2)/d/e^(1/2)

**Rubi [A]** time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {3532, 205}

$$\frac{\sqrt{2} a \tan^{-1} \left( \frac{\sqrt{e}(1-\cot(c+dx))}{\sqrt{2} \sqrt{e \cot(c+dx)}} \right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])/Sqrt[e\*Cot[c + d\*x]],x]

[Out] (Sqrt[2]\*a\*ArcTan[(Sqrt[e]\*(1 - Cot[c + d\*x]))/(Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])])/(d\*Sqrt[e])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 3532

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

#### Rubi steps



$$\int \frac{a + a \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx = -\frac{(2a^2) \text{Subst}\left(\int \frac{1}{-2a^2 - ex^2} dx, x, \frac{a - a \cot(c + dx)}{\sqrt{e \cot(c + dx)}}\right)}{d}$$

$$= \frac{\sqrt{2} a \tan^{-1}\left(\frac{\sqrt{e}(1 - \cot(c + dx))}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{d\sqrt{e}}$$

**Mathematica [C]** time = 0.22, size = 165, normalized size = 3.37

$$\frac{a \left( 8 \tan^{\frac{3}{2}}(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(c + dx)\right) + 3\sqrt{2} \left(-2 \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) + 2 \tan^{-1}\left(\sqrt{2} \sqrt{\tan(c + dx)}\right)\right) \right)}{12d\sqrt{\tan(c + dx)}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])/Sqrt[e\*Cot[c + d\*x]], x]

[Out] (a\*(3\*Sqrt[2]\*(-2\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + 8\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(3/2))/(12\*d\*Sqrt[e\*Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])

**fricas [B]** time = 0.60, size = 172, normalized size = 3.51

$$\left[ \frac{\sqrt{2} a \sqrt{-\frac{1}{e}} \log\left(-\sqrt{2} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} \sqrt{-\frac{1}{e}} (\cos(2dx+2c) + \sin(2dx+2c) - 1) - 2 \sin(2dx+2c) + 1\right)}{2d}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2), x, algorithm="fricas")

[Out] [1/2\*sqrt(2)\*a\*sqrt(-1/e)\*log(-sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sqrt(-1/e)\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*sin(2\*d\*x + 2\*c) + 1)/d, sqrt(2)\*a\*arctan(-1/2\*sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) + 1)/(sqrt(e)\*(cos(2\*d\*x + 2\*c) + 1)))/(d\*sqrt(e))]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \cot(dx + c) + a}{\sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)/sqrt(e\*cot(d\*x + c)), x)

**maple** [B] time = 0.43, size = 327, normalized size = 6.67

$$\frac{a \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + \left( e^2 \right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - \left( e^2 \right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{4de} - \frac{a \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{\left( e^2 \right)^{\frac{1}{4}}} + 1 \right)}{2de} + \frac{a \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{\left( e^2 \right)^{\frac{1}{4}}} - 1 \right)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)/(e\*cot(d\*x+c))^(1/2),x)

[Out] 
$$-1/4*a/d/e*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/2*a/d/e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2*a/d/e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/4*a/d*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/2*a/d*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2*a/d*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)$$

**maxima** [B] time = 0.69, size = 83, normalized size = 1.69

$$\frac{a \left( \frac{\sqrt{2} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e+2} \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{e-2} \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out]  $-a(\sqrt{2})\arctan(1/2\sqrt{2})(\sqrt{2})\sqrt{e} + 2\sqrt{e}/\tan(dx + c))/\sqrt{e})/\sqrt{e} + \sqrt{2})\arctan(-1/2\sqrt{2})(\sqrt{2})\sqrt{e} - 2\sqrt{e}/\tan(dx + c))/\sqrt{e})/\sqrt{e})/d$

**mupad [B]** time = 0.73, size = 65, normalized size = 1.33

$$\frac{(-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) (-1 + i)}{d \sqrt{e}} + \frac{(-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) (1 + i)}{d \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(1/2), x)`

[Out]  $((-1)^{1/4} a \operatorname{atanh}((( -1)^{1/4} (e \cot(c + d*x))^{1/2})/e^{1/2})) * (1 + i)) / (d * e^{1/2}) - ((-1)^{1/4} a \operatorname{atan}((( -1)^{1/4} (e \cot(c + d*x))^{1/2})/e^{1/2})) * (1 - i)) / (d * e^{1/2})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int \frac{1}{\sqrt{e \cot(c + dx)}} dx + \int \frac{\cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(1/2), x)`

[Out] `a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(e*cot(c + d*x)), x))`



Rubi steps

$$\begin{aligned}
\int \frac{a + a \cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx &= \frac{2a}{de\sqrt{e \cot(c + dx)}} + \frac{\int \frac{ae - ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{e^2} \\
&= \frac{2a}{de\sqrt{e \cot(c + dx)}} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{2a^2 e^2 - ex^2} dx, x, \frac{ae + ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2} a \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{3/2}} + \frac{2a}{de\sqrt{e \cot(c + dx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.26, size = 191, normalized size = 2.55

$$\frac{a \left( 8 \tan^{\frac{3}{2}}(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(c + dx)\right) + 6\sqrt{2} \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) - 6\sqrt{2} \tan^{-1}\left(\sqrt{2} \sqrt{\tan(c + dx)}\right) \right)}{12d \tan^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])/(e\*Cot[c + d\*x])^(3/2), x]

[Out] (a\*(6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - 3\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + 24\*Sqrt[Tan[c + d\*x]] + 8\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(3/2)))/(12\*d\*(e\*Cot[c + d\*x])^(3/2)\*Tan[c + d\*x]^(3/2))

**fricas [B]** time = 0.53, size = 321, normalized size = 4.28

$$\left[ \frac{4a \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} \sin(2dx+2c) + \frac{\sqrt{2}(ae \cos(2dx+2c)+ae) \log\left(\frac{\sqrt{2} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} (\cos(2dx+2c)-\sin(2dx+2c)-1)}{\sqrt{e}} + 2 \sin(2dx+2c)+1\right)}{\sqrt{e}}}{2(de^2 \cos(2dx+2c) + de^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2), x, algorithm="fricas")

```
[Out] [1/2*(4*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
+ sqrt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*log(sqrt(2)*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) +
2*sin(2*d*x + 2*c) + 1)/sqrt(e))/(d*e^2*cos(2*d*x + 2*c) + d*e^2), (sqrt(2)
)*(a*e*cos(2*d*x + 2*c) + a*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x
+ 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + 2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))*sin(2*d*x + 2*c))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \cot(dx + c) + a}{(e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

**maple [B]** time = 0.36, size = 355, normalized size = 4.73

$$\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)}{4de^2} + \frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)}{2de^2} + \frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}-1\right)}{2de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cot(d*x+c)*a)/(e*cot(d*x+c))^(3/2),x)
```

```
[Out] -1/4*a/d/e^2*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))-1/2*a/d/e^2*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1)+1/2*a/d/e^2*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)+1/4*a/d/e^2^(1/2)/(e^2)^(1/4)*ln((
e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*
x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+1/2*a/d/e^2^(1/
2)/(e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/2*a/d/e
*2^(1/2)/(e^2)^(1/4)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)+2*
a/d/e/(e*cot(d*x+c))^(1/2)
```

**maxima** [A] time = 0.72, size = 111, normalized size = 1.48

$$e \left( \frac{a \left( \frac{\sqrt{2} \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}} \right)}{\sqrt{e}} - \frac{\sqrt{2} \log \left( -\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}} \right)}{\sqrt{e}} \right)}{e^2} - \frac{4a}{e^2 \sqrt{\frac{e}{\tan(dx+c)}}} \right) \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2),x, algorithm="maxima")

[Out]  $-1/2*e*(a*(\sqrt{2}*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c)))/\sqrt{e}-\sqrt{2}*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e})/e^2-4*a/(e^2*\sqrt{e/\tan(d*x+c)})/d$

**mupad** [B] time = 0.96, size = 84, normalized size = 1.12

$$\frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) (1+1i)}{de^{3/2}} + \frac{(-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) (-1+1i)}{de^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cot(c + d\*x))/(e\*cot(c + d\*x))^(3/2),x)

[Out]  $(2*a)/(d*e*(e*\cot(c+d*x))^{1/2}) + ((-1)^{1/4}*a*\operatorname{atan}((( -1)^{1/4}*(e*\cot(c+d*x))^{1/2}))/e^{1/2})*(1+1i))/(d*e^{3/2}) - ((-1)^{1/4}*a*\operatorname{atanh}((( -1)^{1/4}*(e*\cot(c+d*x))^{1/2}))/e^{1/2})*(1-1i))/(d*e^{3/2})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int \frac{1}{(e \cot(c + dx))^{3/2}} dx + \int \frac{\cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))\*\*(3/2),x)

[Out]  $a*(\operatorname{Integral}((e*\cot(c+d*x))^{3/2}),x) + \operatorname{Integral}(\cot(c+d*x)/(e*\cot(c+d*x))^{3/2},x)$

$$3.7 \quad \int \frac{a+a \cot(c+dx)}{(e \cot(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=99

$$-\frac{\sqrt{2} a \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e} \cot(c+dx)}\right)}{de^{5/2}} + \frac{2a}{de^2 \sqrt{e} \cot(c+dx)} + \frac{2a}{3de(e \cot(c+dx))^{3/2}}$$

[Out] 2/3\*a/d/e/(e\*cot(d\*x+c))^(3/2)-a\*arctan(1/2\*(e^(1/2)-cot(d\*x+c)\*e^(1/2))\*2^(1/2)/(e\*cot(d\*x+c))^(1/2))\*2^(1/2)/d/e^(5/2)+2\*a/d/e^2/(e\*cot(d\*x+c))^(1/2)

**Rubi [A]** time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3529, 3532, 205}

$$\frac{2a}{de^2 \sqrt{e} \cot(c+dx)} - \frac{\sqrt{2} a \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e} \cot(c+dx)}\right)}{de^{5/2}} + \frac{2a}{3de(e \cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])/(e\*Cot[c + d\*x])^(5/2), x]

[Out] -((Sqrt[2]\*a\*ArcTan[(Sqrt[e] - Sqrt[e]\*Cot[c + d\*x])/(Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])])/(d\*e^(5/2))) + (2\*a)/(3\*d\*e\*(e\*Cot[c + d\*x])^(3/2)) + (2\*a)/(d\*e^2\*Sqrt[e\*Cot[c + d\*x]])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3529

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3532

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] :> Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c -



$d \cdot \tan[e + f \cdot x] / \sqrt{b \cdot \tan[e + f \cdot x]}$ , x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{a + a \cot(c + dx)}{(e \cot(c + dx))^{5/2}} dx &= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{\int \frac{ae - ae \cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx}{e^2} \\ &= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2a}{de^2 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{-ae^2 - ae^2 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{e^4} \\ &= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2a}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{-2a^2 e^4 - ex^2} dx, x, \frac{-ae^2 + ae^2 \cot(c + dx)}{\sqrt{e \cot(c + dx)}}\right)}{d} \\ &= -\frac{\sqrt{2} a \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c + dx)}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{de^{5/2}} + \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2a}{de^2 \sqrt{e \cot(c + dx)}} \end{aligned}$$

**Mathematica [C]** time = 0.42, size = 203, normalized size = 2.05

$$a \left( -8 \tan^{\frac{3}{2}}(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(c + dx)\right) + 6\sqrt{2} \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) - 6\sqrt{2} \tan^{-1}\left(\sqrt{2} \sqrt{\tan(c + dx)}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])/(e\*Cot[c + d\*x])^(5/2), x]

[Out] (a\*(6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - 3\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + 24\*Sqrt[Tan[c + d\*x]] + 8\*Tan[c + d\*x]^(3/2) - 8\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2\*Tan[c + d\*x]^(3/2)]/(12\*d\*(e\*Cot[c + d\*x])^(5/2)\*Tan[c + d\*x]^(5/2)))

**fricas** [B] time = 0.82, size = 358, normalized size = 3.62

$$\frac{3\sqrt{2}(ae\cos(2dx+2c)+ae)\sqrt{-\frac{1}{e}}\log\left(\sqrt{2}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\sqrt{-\frac{1}{e}}(\cos(2dx+2c)+\sin(2dx+2c)-1)-2\sin(2dx+2c)\right)}{6(d e^3 \cos(2dx+2c)+d e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/6\*(3\*sqrt(2)\*(a\*e\*cos(2\*d\*x + 2\*c) + a\*e)\*sqrt(-1/e)\*log(sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sqrt(-1/e)\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*sin(2\*d\*x + 2\*c) + 1) - 4\*(a\*cos(2\*d\*x + 2\*c) - 3\*a\*sin(2\*d\*x + 2\*c) - a)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e^3\*cos(2\*d\*x + 2\*c) + d\*e^3), -1/3\*(3\*sqrt(2)\*(a\*e\*cos(2\*d\*x + 2\*c) + a\*e)\*arctan(-1/2\*sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) + 1)/(sqrt(e)\*(cos(2\*d\*x + 2\*c) + 1)))/sqrt(e) + 2\*(a\*cos(2\*d\*x + 2\*c) - 3\*a\*sin(2\*d\*x + 2\*c) - a)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e^3\*cos(2\*d\*x + 2\*c) + d\*e^3)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \cot(dx+c) + a}{(e \cot(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)/(e\*cot(d\*x + c))^(5/2), x)

**maple** [B] time = 0.36, size = 374, normalized size = 3.78

$$\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{4de^3} + \frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)}{2de^3} - \frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)/(e\*cot(d\*x+c))^(5/2),x)

[Out]  $\frac{1}{4} a/d/e^3 (e^2)^{1/4} 2^{1/2} \ln\left(\frac{(e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}{(e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}\right) + \frac{1}{2} a/d/e^3 (e^2)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) - \frac{1}{2} a/d/e^3 (e^2)^{1/4} 2^{1/2} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) + \frac{1}{4} a/d/e^2 2^{1/2} / (e^2)^{1/4} \ln\left(\frac{e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2}}{(e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}\right) + \frac{1}{2} a/d/e^2 2^{1/2} / (e^2)^{1/4} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) - \frac{1}{2} a/d/e^2 2^{1/2} / (e^2)^{1/4} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) + 2 a/d/e^2 / (e \cot(dx+c))^{1/2} + 2/3 a/d/e / (e \cot(dx+c))^{3/2}$

**maxima [A]** time = 0.60, size = 123, normalized size = 1.24

$$e \left( \frac{3a \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{e^3} + \frac{2\left(ae + \frac{3ae}{\tan(dx+c)}\right)}{e^3 \left(\frac{e}{\tan(dx+c)}\right)^2} \right) / 3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")`

[Out]  $\frac{1}{3} e * (3 a * (\sqrt{2} * \arctan(1/2 * \sqrt{2}) * (\sqrt{2} * \sqrt{e}) + 2 * \sqrt{e / \tan(dx+c)})) / \sqrt{e} / \sqrt{e} + \sqrt{2} * \arctan(-1/2 * \sqrt{2}) * (\sqrt{2} * \sqrt{e}) - 2 * \sqrt{e / \tan(dx+c)}) / \sqrt{e} / \sqrt{e} / e^3 + 2 * (a * e + 3 a * e / \tan(dx+c)) / (e^3 * (e / \tan(dx+c))^{3/2}) / d$

**mupad [B]** time = 1.48, size = 103, normalized size = 1.04

$$\frac{2a}{d e^2 \sqrt{e \cot(c+dx)}} + \frac{2a}{3 d e (e \cot(c+dx))^{3/2}} + \frac{(-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) (1-i)}{d e^{5/2}} + \frac{(-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) (1+i)}{d e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)`

[Out]  $(2*a)/(d*e^2*(e*\cot(c + d*x))^{1/2}) + (2*a)/(3*d*e*(e*\cot(c + d*x))^{3/2}) + ((-1)^{1/4}*a*\operatorname{atan}((-1)^{1/4}*(e*\cot(c + d*x))^{1/2})/e^{1/2})*(1 - 1i)$

$\left. \right) / (d \cdot e^{5/2}) - ((-1)^{1/4} \cdot a \cdot \operatorname{atanh}((( -1)^{1/4} \cdot (e \cdot \cot(c + d \cdot x))^{1/2})) / e^{1/2}) \cdot (1 + 1i) / (d \cdot e^{5/2})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int \frac{1}{(e \cot(c + dx))^{\frac{5}{2}}} dx + \int \frac{\cot(c + dx)}{(e \cot(c + dx))^{\frac{5}{2}}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))/(e\*cot(d\*x+c))\*\*(5/2),x)

[Out] a\*(Integral((e\*cot(c + d\*x))\*\*(-5/2), x) + Integral(cot(c + d\*x)/(e\*cot(c + d\*x))\*\*(5/2), x))

### 3.8 $\int (e \cot(c + dx))^{5/2} (a + a \cot(c + dx))^2 dx$

**Optimal.** Leaf size=269

$$\frac{a^2 e^{5/2} \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d} - \frac{a^2 e^{5/2} \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d}$$

[Out]  $-4/5*a^2*(e*\cot(d*x+c))^{(5/2)}/d-2/7*a^2*(e*\cot(d*x+c))^{(7/2)}/d/e+1/2*a^2*e^{(5/2)}*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d*2^{(1/2)}-1/2*a^2*e^{(5/2)}*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d*2^{(1/2)}+a^2*e^{(5/2)}*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d-a^2*e^{(5/2)}*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d+4*a^2*e^2*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.29, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3543, 12, 16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} + \frac{a^2 e^{5/2} \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d} - \frac{a^2 e^{5/2} \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*\text{Cot}[c + d*x])^{(5/2)}*(a + a*\text{Cot}[c + d*x])^2, x]$

[Out]  $(\text{Sqrt}[2]*a^2*e^{(5/2)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/d - (\text{Sqrt}[2]*a^2*e^{(5/2)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/d + (4*a^2*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (4*a^2*(e*\text{Cot}[c + d*x])^{(5/2)})/(5*d) - (2*a^2*(e*\text{Cot}[c + d*x])^{(7/2)})/(7*d*e) + (a^2*e^{(5/2)}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d) - (a^2*e^{(5/2)}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d)$

#### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 16

$\text{Int}[(u_)*(v_)^{(m_)}*((b_)*(v_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

$x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 3473

$\text{Int}[(b_.)*\tan[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*(b*\tan[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\tan[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1]$

### Rule 3476

$\text{Int}[(b_.)*\tan[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\tan[c + d*x]], x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[n]$

### Rule 3543

$\text{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]^2, x\_Symbol] \rightarrow \text{Simp}[(d^2*(a + b*\tan[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Int}[(a + b*\tan[e + f*x])^m*\text{Simp}[c^2 - d^2 + 2*c*d*\tan[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{LeQ}[m, -1] \ \&\& \ !(\text{EqQ}[m, 2] \ \&\& \ \text{EqQ}[a, 0])$

### Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{5/2} (a + a \cot(c + dx))^2 dx &= -\frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} + \int 2a^2 \cot(c + dx) (e \cot(c + dx))^{5/2} dx \\
&= -\frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} + (2a^2) \int \cot(c + dx) (e \cot(c + dx))^{5/2} dx \\
&= -\frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} + \frac{(2a^2) \int (e \cot(c + dx))^{7/2} dx}{e} \\
&= -\frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} - (2a^2 e) \int (e \cot(c + dx))^{5/2} dx \\
&= \frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} \\
&= \frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} \\
&= \frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} \\
&= \frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} \\
&= \frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} \\
&= \frac{4a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4a^2 (e \cot(c + dx))^{5/2}}{5d} - \frac{2a^2 (e \cot(c + dx))^{7/2}}{7de} \\
&= \frac{\sqrt{2} a^2 e^{5/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{d} - \frac{\sqrt{2} a^2 e^{5/2} \tan^{-1} \left( 1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 1.19, size = 187, normalized size = 0.70

$$\frac{a^2 (e \cot(c + dx))^{5/2} \left( 20 \cot^{\frac{7}{2}}(c + dx) + 56 \cot^{\frac{5}{2}}(c + dx) - 280 \sqrt{\cot(c + dx)} - 35 \sqrt{2} \log \left( \cot(c + dx) - \sqrt{2} \sqrt{\cot(c + dx)} \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^2,x]



```
[Out] -1/70*(a^2*(e*Cot[c + d*x])^(5/2)*(-70*sqrt[2]*ArcTan[1 - Sqrt[2]*sqrt[Cot[c + d*x]]) + 70*sqrt[2]*ArcTan[1 + Sqrt[2]*sqrt[Cot[c + d*x]]) - 280*sqrt[Cot[c + d*x]] + 56*Cot[c + d*x]^(5/2) + 20*Cot[c + d*x]^(7/2) - 35*sqrt[2]*Log[1 - Sqrt[2]*sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 35*sqrt[2]*Log[1 + Sqrt[2]*sqrt[Cot[c + d*x]] + Cot[c + d*x]])/(d*Cot[c + d*x]^(5/2))
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a)^2 (e \cot(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2), x)
```

**maple** [A] time = 0.61, size = 234, normalized size = 0.87

$$\frac{2a^2 (e \cot(dx + c))^{\frac{7}{2}}}{7de} - \frac{4a^2 (e \cot(dx + c))^{\frac{5}{2}}}{5d} + \frac{4a^2 e^2 \sqrt{e \cot(dx + c)}}{d} + \frac{a^2 e^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx + c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)*(a+cot(d*x+c)*a)^2,x)
```

```
[Out] -2/7*a^2*(e*cot(d*x+c))^(7/2)/d/e-4/5*a^2*(e*cot(d*x+c))^(5/2)/d+4*a^2*e^2*(e*cot(d*x+c))^(1/2)/d+1/d*a^2*e^2*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/2/d*a^2*e^2*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))-1/d*a^2*e^2*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
```

**maxima** [A] time = 0.59, size = 232, normalized size = 0.86

$$\left( 35 \left( 2 \sqrt{2} e^{\frac{3}{2}} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right) + 2 \sqrt{2} e^{\frac{3}{2}} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right) + \sqrt{2} e^{\frac{3}{2}} \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $-1/70*(35*(2*\sqrt{2}*e^{3/2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e}) + 2*\sqrt{2}*e^{3/2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e}) + \sqrt{2}*e^{3/2}*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)}) + e + e/\tan(d*x + c)) - \sqrt{2}*e^{3/2}*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)}) + e + e/\tan(d*x + c))*a^2 - 4*(70*a^2*e^{3/2}*\sqrt{e/\tan(d*x + c)} - 14*a^2*e*(e/\tan(d*x + c))^{5/2} - 5*a^2*(e/\tan(d*x + c))^{7/2})/e^2)*e/d$

**mupad** [B] time = 1.73, size = 125, normalized size = 0.46

$$\frac{4 a^2 e^2 \sqrt{e \cot(c + dx)}}{d} - \frac{4 a^2 (e \cot(c + dx))^{5/2}}{5 d} - \frac{2 a^2 (e \cot(c + dx))^{7/2}}{7 d e} + \frac{(-1)^{1/4} a^2 e^{5/2} \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{d} + 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(5/2)\*(a + a\*cot(c + d\*x))^2,x)

[Out]  $(4*a^2*e^2*(e*\cot(c + d*x))^{1/2})/d - (4*a^2*(e*\cot(c + d*x))^{5/2})/(5*d) - (2*a^2*(e*\cot(c + d*x))^{7/2})/(7*d*e) + ((-1)^{1/4}*a^2*e^{5/2}*\operatorname{atan}((( -1)^{1/4}*(e*\cot(c + d*x))^{1/2})/e^{1/2})*2i)/d + (2*(-1)^{1/4}*a^2*e^{5/2})*\operatorname{atan}((( -1)^{1/4}*(e*\cot(c + d*x))^{1/2})*1i)/e^{1/2})/d$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int (e \cot(c + dx))^{\frac{5}{2}} dx + \int 2 (e \cot(c + dx))^{\frac{5}{2}} \cot(c + dx) dx + \int (e \cot(c + dx))^{\frac{5}{2}} \cot^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)\*(a+a\*cot(d\*x+c))\*\*2,x)

[Out]  $a**2*(\operatorname{Integral}((e*\cot(c + d*x))**(5/2), x) + \operatorname{Integral}(2*(e*\cot(c + d*x))**(5/2)*\cot(c + d*x), x) + \operatorname{Integral}((e*\cot(c + d*x))**(5/2)*\cot(c + d*x)**2, x))$

### 3.9 $\int (e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^2 dx$

**Optimal.** Leaf size=246

$$\frac{a^2 e^{3/2} \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d} - \frac{a^2 e^{3/2} \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d}$$

[Out]  $-4/3 a^2 (e \cot(d x + c))^{3/2} / d - 2/5 a^2 (e \cot(d x + c))^{5/2} / d e^{1/2} a^2 e^{3/2} \ln(e^{1/2} + \cot(d x + c) e^{1/2} - 2^{1/2} (e \cot(d x + c))^{1/2}) / d 2^{1/2} - 1/2 a^2 e^{3/2} \ln(e^{1/2} + \cot(d x + c) e^{1/2} + 2^{1/2} (e \cot(d x + c))^{1/2}) / d 2^{1/2} - a^2 e^{3/2} \arctan(1 - 2^{1/2} (e \cot(d x + c))^{1/2} / e^{1/2}) 2^{1/2} / d + a^2 e^{3/2} \arctan(1 + 2^{1/2} (e \cot(d x + c))^{1/2} / e^{1/2}) 2^{1/2} / d$

**Rubi [A]** time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3543, 12, 16, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{a^2 e^{3/2} \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d} - \frac{a^2 e^{3/2} \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2} d}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])^2,x]

[Out]  $-((\text{Sqrt}[2] a^2 e^{3/2} \text{ArcTan}[1 - (\text{Sqrt}[2] \text{Sqrt}[e \cot[c + d x]])] / \text{Sqrt}[e]]) / d + (\text{Sqrt}[2] a^2 e^{3/2} \text{ArcTan}[1 + (\text{Sqrt}[2] \text{Sqrt}[e \cot[c + d x]])] / \text{Sqrt}[e]) / d - (4 a^2 (e \cot[c + d x])^{3/2}) / (3 d) - (2 a^2 (e \cot[c + d x])^{5/2}) / (5 d e) + (a^2 e^{3/2} \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot[c + d x] - \text{Sqrt}[2] \text{Sqrt}[e \cot[c + d x]])] / (\text{Sqrt}[2] d) - (a^2 e^{3/2} \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot[c + d x] + \text{Sqrt}[2] \text{Sqrt}[e \cot[c + d x]])] / (\text{Sqrt}[2] d)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*c}, simplify[(a*c)/b^2], Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$eQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 3473

$Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x\_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[\{b, c, d\}, x] \&\& GtQ[n, 1]$

### Rule 3476

$Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x\_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[\{b, c, d, n\}, x] \&\& ! IntegerQ[n]$

### Rule 3543

$Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2, x\_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] /; FreeQ[\{a, b, c, d, e, f, m\}, x] \&\& NeQ[b*c - a*d, 0] \&\& !LeQ[m, -1] \&\& !(EqQ[m, 2] \&\& EqQ[a, 0])$

### Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^2 dx &= -\frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + \int 2a^2 \cot(c + dx) (e \cot(c + dx))^{3/2} dx \\
&= -\frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + (2a^2) \int \cot(c + dx) (e \cot(c + dx))^{3/2} dx \\
&= -\frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + \frac{(2a^2) \int (e \cot(c + dx))^{5/2} dx}{e} \\
&= -\frac{4a^2 (e \cot(c + dx))^{3/2}}{3d} - \frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} - (2a^2 e) \int \sqrt{e \cot(c + dx)} dx \\
&= -\frac{4a^2 (e \cot(c + dx))^{3/2}}{3d} - \frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + \frac{(2a^2 e^2) \text{Subst}\left(\int \frac{y}{e^2} dy\right)}{e^2} \\
&= -\frac{4a^2 (e \cot(c + dx))^{3/2}}{3d} - \frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + \frac{(4a^2 e^2) \text{Subst}\left(\int \frac{x}{e^2} dx\right)}{e^2} \\
&= -\frac{4a^2 (e \cot(c + dx))^{3/2}}{3d} - \frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} - \frac{(2a^2 e^2) \text{Subst}\left(\int \frac{e-x}{e^2} dx\right)}{e^2} \\
&= -\frac{4a^2 (e \cot(c + dx))^{3/2}}{3d} - \frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + \frac{(a^2 e^{3/2}) \text{Subst}\left(\int \frac{x}{e} dx\right)}{e} \\
&= -\frac{4a^2 (e \cot(c + dx))^{3/2}}{3d} - \frac{2a^2 (e \cot(c + dx))^{5/2}}{5de} + \frac{a^2 e^{3/2} \log(\sqrt{e} + \sqrt{e \cot(c + dx)})}{e} \\
&= -\frac{\sqrt{2} a^2 e^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{d} + \frac{\sqrt{2} a^2 e^{3/2} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{d}
\end{aligned}$$

**Mathematica** [C] time = 0.39, size = 52, normalized size = 0.21

$$-\frac{2a^2 (e \cot(c + dx))^{3/2} \left(-10 {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right) + 3 \cot(c + dx) + 10\right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])^2,x]

[Out] (-2\*a^2\*(e\*Cot[c + d\*x])^(3/2)\*(10 + 3\*Cot[c + d\*x] - 10\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(15\*d)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: catde  
f: division by zero

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a)^2 (e \cot(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")`

[Out] `integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)`

**maple** [A] time = 0.61, size = 213, normalized size = 0.87

$$\frac{2a^2 (e \cot(dx + c))^{\frac{5}{2}}}{5de} - \frac{4a^2 (e \cot(dx + c))^{\frac{3}{2}}}{3d} + \frac{a^2 e^2 \sqrt{2} \ln\left(\frac{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2+\sqrt{e^2}}}\right)}{2d (e^2)^{\frac{1}{4}}} + \frac{a^2 e^2 \sqrt{2} \arctan\left(\frac{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2+\sqrt{e^2}}}\right)}{d (e^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(d*x+c))^(3/2)*(a+cot(d*x+c)*a)^2,x)`

[Out]  $-2/5*a^2*(e*\cot(d*x+c))^{5/2}/d/e-4/3*a^2*(e*\cot(d*x+c))^{3/2}/d+1/2/d*a^2*e^2/(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))+1/d*a^2*e^2/(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/d*a^2*e^2/(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)$

**maxima** [A] time = 0.60, size = 213, normalized size = 0.87

$$\left( 15 a^2 e \left( \frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} + \dots \right) \right) / 30d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $\frac{1}{30} * (15 * a^2 * e * (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e} / \tan(d * x + c))) / \sqrt{e}) / \sqrt{e} + 2 * \sqrt{2} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e} / \tan(d * x + c))) / \sqrt{e}) / \sqrt{e} - \sqrt{2} * \log(\sqrt{2} * \sqrt{e} * \sqrt{e} / \tan(d * x + c)) + e + e / \tan(d * x + c)) / \sqrt{e} + \sqrt{2} * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e} / \tan(d * x + c)) + e + e / \tan(d * x + c)) / \sqrt{e} - 4 * (10 * a^2 * e * (e / \tan(d * x + c))^{3/2} + 3 * a^2 * (e / \tan(d * x + c))^{5/2}) / e^2) * e / d$

**mupad [B]** time = 0.95, size = 104, normalized size = 0.42

$$\frac{2(-1)^{1/4} a^2 e^{3/2} \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{2 a^2 (e \cot(c+dx))^{5/2}}{5 d e} - \frac{4 a^2 (e \cot(c+dx))^{3/2}}{3 d} + \frac{(-1)^{1/4} a^2 e^{3/2} \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(3/2)\*(a + a\*cot(c + d\*x))^2,x)

[Out]  $(2 * (-1)^{1/4} * a^2 * e^{3/2} * \operatorname{atan}((-1)^{1/4} * (e * \cot(c + d * x))^{1/2}) / e^{1/2}) / d - (2 * a^2 * (e * \cot(c + d * x))^{5/2}) / (5 * d * e) - (4 * a^2 * (e * \cot(c + d * x))^{3/2}) / (3 * d) + ((-1)^{1/4} * a^2 * e^{3/2} * \operatorname{atan}((-1)^{1/4} * (e * \cot(c + d * x))^{1/2}) * i) / e^{1/2}) * 2i) / d$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int (e \cot(c + dx))^{\frac{3}{2}} dx + \int 2 (e \cot(c + dx))^{\frac{3}{2}} \cot(c + dx) dx + \int (e \cot(c + dx))^{\frac{3}{2}} \cot^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(3/2)\*(a+a\*cot(d\*x+c))\*\*2,x)

[Out]  $a^{**2} * (\operatorname{Integral}((e * \cot(c + d * x))^{**3/2}), x) + \operatorname{Integral}(2 * (e * \cot(c + d * x))^{**3/2} * \cot(c + d * x), x) + \operatorname{Integral}((e * \cot(c + d * x))^{**3/2} * \cot(c + d * x)^{**2}, x))$



### 3.10 $\int \sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2 dx$

Optimal. Leaf size=244

$$\frac{2a^2(e \cot(c + dx))^{3/2}}{3de} - \frac{4a^2\sqrt{e \cot(c + dx)}}{d} - \frac{a^2\sqrt{e} \log(\sqrt{e} \cot(c + dx) - \sqrt{2}\sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2}d} + \frac{a^2\sqrt{e} \log(\sqrt{e} \cot(c + dx) + \sqrt{2}\sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2}d}$$

[Out]  $-2/3*a^2*(e*\cot(d*x+c))^(3/2)/d/e-1/2*a^2*\ln(e^(1/2)+\cot(d*x+c))*e^(1/2)-2^(1/2)*(e*\cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/2*a^2*\ln(e^(1/2)+\cot(d*x+c))*e^(1/2)+2^(1/2)*(e*\cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-a^2*\arctan(1-2^(1/2)*(e*\cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d+a^2*\arctan(1+2^(1/2)*(e*\cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d-4*a^2*(e*\cot(d*x+c))^(1/2)/d$

**Rubi [A]** time = 0.23, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3543, 12, 16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2a^2(e \cot(c + dx))^{3/2}}{3de} - \frac{4a^2\sqrt{e \cot(c + dx)}}{d} - \frac{a^2\sqrt{e} \log(\sqrt{e} \cot(c + dx) - \sqrt{2}\sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2}d} + \frac{a^2\sqrt{e} \log(\sqrt{e} \cot(c + dx) + \sqrt{2}\sqrt{e \cot(c + dx)} + \sqrt{e})}{\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^2,x]

[Out]  $-((\text{Sqrt}[2]*a^2*\text{Sqrt}[e]*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/\text{Sqrt}[e]])/d) + (\text{Sqrt}[2]*a^2*\text{Sqrt}[e]*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/\text{Sqrt}[e]])/d - (4*a^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (2*a^2*(e*\text{Cot}[c + d*x])^(3/2))/(3*d*e) - (a^2*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(\text{Sqrt}[2]*d) + (a^2*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(\text{Sqrt}[2]*d)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*c*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
```

$eQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 3473

$Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x\_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[\{b, c, d\}, x] \&\& GtQ[n, 1]$

### Rule 3476

$Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x\_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[\{b, c, d, n\}, x] \&\& ! IntegerQ[n]$

### Rule 3543

$Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2, x\_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] /; FreeQ[\{a, b, c, d, e, f, m\}, x] \&\& NeQ[b*c - a*d, 0] \&\& !LeQ[m, -1] \&\& !(EqQ[m, 2] \&\& EqQ[a, 0])$

### Rubi steps

$$\begin{aligned}
\int \sqrt{e \cot(c+dx)} (a + a \cot(c+dx))^2 dx &= -\frac{2a^2(e \cot(c+dx))^{3/2}}{3de} + \int 2a^2 \cot(c+dx) \sqrt{e \cot(c+dx)} dx \\
&= -\frac{2a^2(e \cot(c+dx))^{3/2}}{3de} + (2a^2) \int \cot(c+dx) \sqrt{e \cot(c+dx)} dx \\
&= -\frac{2a^2(e \cot(c+dx))^{3/2}}{3de} + \frac{(2a^2) \int (e \cot(c+dx))^{3/2} dx}{e} \\
&= -\frac{4a^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2a^2(e \cot(c+dx))^{3/2}}{3de} - (2a^2e) \int \frac{1}{\sqrt{e \cot(c+dx)}} dx \\
&= -\frac{4a^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2a^2(e \cot(c+dx))^{3/2}}{3de} + \frac{(2a^2e^2) \text{Subst}\left(\int \frac{1}{\sqrt{x(e^2+x^4)}} dx\right)}{e^2} \\
&= -\frac{4a^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2a^2(e \cot(c+dx))^{3/2}}{3de} + \frac{(4a^2e^2) \text{Subst}\left(\int \frac{1}{e^2+x^4} dx\right)}{e^2} \\
&= -\frac{4a^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2a^2(e \cot(c+dx))^{3/2}}{3de} + \frac{(2a^2e) \text{Subst}\left(\int \frac{e-x^2}{e^2+x^4} dx\right)}{e^2} \\
&= -\frac{4a^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2a^2(e \cot(c+dx))^{3/2}}{3de} - \frac{(a^2\sqrt{e}) \text{Subst}\left(\int \frac{\sqrt{2}y}{-e-\sqrt{2}y^2} dy\right)}{e} \\
&= -\frac{4a^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2a^2(e \cot(c+dx))^{3/2}}{3de} - \frac{a^2\sqrt{e} \log(\sqrt{e} + \sqrt{e} \cot(c+dx))}{e} \\
&= -\frac{\sqrt{2} a^2 \sqrt{e} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{\sqrt{2} a^2 \sqrt{e} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.43, size = 175, normalized size = 0.72

$$\frac{a^2 \sqrt{e \cot(c+dx)} \left(4 \cot^{\frac{3}{2}}(c+dx) + 24 \sqrt{\cot(c+dx)} + 3\sqrt{2} \log(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1) - 3\sqrt{2} \log(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1)\right)}{6d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^2,x]

[Out] -1/6\*(a^2\*Sqrt[e\*Cot[c + d\*x]]\*(6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 24\*Sqrt[Cot[c + d\*x]] + 4\*Cot[c + d\*x]^(3/2) + 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]])

+ Cot[c + d\*x]] - 3\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])))/(d\*Sqrt[Cot[c + d\*x]])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catde  
f: division by zero

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a)^2 \sqrt{e \cot(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^2\*sqrt(e\*cot(d\*x + c)), x)

**maple** [A] time = 0.59, size = 204, normalized size = 0.84

$$\frac{2a^2 (e \cot(dx + c))^{\frac{3}{2}}}{3de} - \frac{4a^2 \sqrt{e \cot(dx + c)}}{d} + \frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx + c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{d} - \frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx + c)}}{(e^2)^{\frac{1}{4}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(1/2)\*(a+cot(d\*x+c)\*a)^2,x)

[Out]  $-2/3*a^2*(e*cot(d*x+c))^{3/2}/d/e-4*a^2*(e*cot(d*x+c))^{1/2}/d+1/d*a^2*(e^2)^{1/4}*2^{1/2}*arctan(2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)-1/d*a^2*(e^2)^{1/4}*2^{1/2}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)+1/2/d*a^2*(e^2)^{1/4}*2^{1/2}*ln((e*cot(d*x+c)+(e^2)^{1/4}*(e*cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))/((e*cot(d*x+c)-(e^2)^{1/4}*(e*cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))$

**maxima** [A] time = 0.70, size = 211, normalized size = 0.86

$$3a^2 \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\sqrt{2}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/6\*(3\*a^2\*(2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c))))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c))))/sqrt(e))/sqrt(e) + sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e)) - 4\*(6\*a^2\*e\*sqrt(e/tan(d\*x + c)) + a^2\*(e/tan(d\*x + c))^(3/2))/e^2)\*e/d

mupad [B] time = 0.70, size = 104, normalized size = 0.43

$$\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{(-1)^{1/4}a^2\sqrt{e}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2(-1)^{1/4}a^2\sqrt{e}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(1/2)\*(a + a\*cot(c + d\*x))^2,x)

[Out] - (4\*a^2\*(e\*cot(c + d\*x))^(1/2))/d - (2\*a^2\*(e\*cot(c + d\*x))^(3/2))/(3\*d\*e) - ((-1)^(1/4)\*a^2\*e^(1/2)\*atan((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*2i)/d - (2\*(-1)^(1/4)\*a^2\*e^(1/2)\*atan((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2)\*1i)/e^(1/2))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \sqrt{e\cot(c+dx)} dx + \int 2\sqrt{e\cot(c+dx)} \cot(c+dx) dx + \int \sqrt{e\cot(c+dx)} \cot^2(c+dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)\*(a+a\*cot(d\*x+c))\*\*2,x)

[Out] a\*\*2\*(Integral(sqrt(e\*cot(c + d\*x)), x) + Integral(2\*sqrt(e\*cot(c + d\*x))\*cot(c + d\*x), x) + Integral(sqrt(e\*cot(c + d\*x))\*cot(c + d\*x)\*\*2, x))

$$3.11 \quad \int \frac{(a+a \cot(c+dx))^2}{\sqrt{e \cot(c+dx)}} dx$$

Optimal. Leaf size=222

$$\frac{2a^2 \sqrt{e \cot(c+dx)}}{de} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} d \sqrt{e}} + \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{\sqrt{2} d \sqrt{e}}$$

[Out]  $-1/2*a^2*\ln(e^{(1/2)+\cot(d*x+c)}*e^{(1/2)-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}}/d*2^{(1/2)}/e^{(1/2)}+1/2*a^2*\ln(e^{(1/2)+\cot(d*x+c)}*e^{(1/2)+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}}/d*2^{(1/2)}/e^{(1/2)}+a^2*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d/e^{(1/2)}-a^2*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d/e^{(1/2)}-2*a^2*(e*\cot(d*x+c))^{(1/2)}/d/e$

Rubi [A] time = 0.20, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$ , Rules used = {3543, 12, 16, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2a^2 \sqrt{e \cot(c+dx)}}{de} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} d \sqrt{e}} + \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{\sqrt{2} d \sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^2/Sqrt[e\*Cot[c + d\*x]],x]

[Out]  $(\text{Sqrt}[2]*a^2*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{d}*\text{Sqrt}[e]) - (\text{Sqrt}[2]*a^2*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{d}*\text{Sqrt}[e]) - (2*a^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{d}*e) - (a^2*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*\text{d}*\text{Sqrt}[e]) + (a^2*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*\text{d}*\text{Sqrt}[e])$

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 329

```
Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*c}, simplify[(a*c)/b^2], Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```



$eQ[\{a, c, d, e\}, x] \ \&\& \ EqQ[c*d^2 - a*e^2, 0] \ \&\& \ NegQ[d*e]$

### Rule 3476

$Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_.), x\_Symbol] \ :> \ Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] \ /; \ FreeQ[\{b, c, d, n\}, x] \ \&\& \ !IntegerQ[n]$

### Rule 3543

$Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)])^2, x\_Symbol] \ :> \ Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] \ /; \ FreeQ[\{a, b, c, d, e, f, m\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ !LeQ[m, -1] \ \&\& \ !(EqQ[m, 2] \ \&\& \ EqQ[a, 0])$

### Rubi steps

$$\begin{aligned}
\int \frac{(a + a \cot(c + dx))^2}{\sqrt{e \cot(c + dx)}} dx &= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} + \int \frac{2a^2 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} + (2a^2) \int \frac{\cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} + \frac{(2a^2) \int \sqrt{e \cot(c + dx)} dx}{e} \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} - \frac{(2a^2) \text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2} dx, x, e \cot(c + dx)\right)}{d} \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} - \frac{(4a^2) \text{Subst}\left(\int \frac{x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} + \frac{(2a^2) \text{Subst}\left(\int \frac{e^{-x^2}}{e^2+x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{e^{-x^2}} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} - \frac{a^2 \text{Subst}\left(\int \frac{1}{e^{-\sqrt{2}} \sqrt{e} x+x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} - \frac{a^2 \text{Subst}\left(\int \frac{1}{e^{-x^2}} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{2a^2 \sqrt{e \cot(c + dx)}}{de} - \frac{a^2 \log\left(\sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{\sqrt{2} d \sqrt{e}} + \frac{a^2 \log\left(\sqrt{e} + \sqrt{e \cot(c + dx)} + \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{\sqrt{2} d \sqrt{e}} \\
&= \frac{\sqrt{2} a^2 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} - \frac{\sqrt{2} a^2 \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} - \frac{2a^2 \sqrt{e \cot(c + dx)}}{de}
\end{aligned}$$

**Mathematica** [C] time = 0.26, size = 53, normalized size = 0.24

$$\frac{2a^2 \sqrt{e \cot(c + dx)} \left(2 \cot(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right) + 3\right)}{3de}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^2/Sqrt[e\*Cot[c + d\*x]],x]

[Out] (-2\*a^2\*Sqrt[e\*Cot[c + d\*x]]\*(3 + 2\*Cot[c + d\*x]\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(3\*d\*e)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catde  
f: division by zero

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^2}{\sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^2/sqrt(e\*cot(d\*x + c)), x)

maple [A] time = 0.54, size = 186, normalized size = 0.84

$$\frac{2a^2 \sqrt{e \cot(dx + c)}}{de} - \frac{a^2 \sqrt{2} \ln \left( \frac{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2d (e^2)^{\frac{1}{4}}} - \frac{a^2 \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d (e^2)^{\frac{1}{4}}} + \frac{a^2 \sqrt{2} a}{d (e^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)^2/(e\*cot(d\*x+c))^(1/2),x)

[Out]  $-2*a^2*(e*cot(d*x+c))^{(1/2)}/d/e-1/2/d*a^2/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^2/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/d*a^2/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)$

**maxima [A]** time = 0.79, size = 193, normalized size = 0.87

$$\frac{a^2 \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} + \frac{\sqrt{2} \log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right)}{e}$$


---


$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/2\*(a^2\*(2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) + sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/e + 4\*a^2\*sqrt(e/tan(d\*x + c))/e^2)\*e/d

**mupad [B]** time = 0.44, size = 86, normalized size = 0.39

$$\frac{2(-1)^{1/4} a^2 \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} - \frac{2(-1)^{1/4} a^2 \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} - \frac{2 a^2 \sqrt{e \cot(c+dx)}}{d e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cot(c + d\*x))^2/(e\*cot(c + d\*x))^(1/2),x)

[Out] (2\*(-1)^(1/4)\*a^2\*atanh(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/(d\*e^(1/2)) - (2\*(-1)^(1/4)\*a^2\*atan(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/(d\*e^(1/2)) - (2\*a^2\*(e\*cot(c + d\*x))^(1/2))/(d\*e)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{1}{\sqrt{e \cot(c+dx)}} dx + \int \frac{2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx + \int \frac{\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*cot(c + d*x)/sqrt(e*  
cot(c + d*x)), x) + Integral(cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

$$3.12 \quad \int \frac{(a+a \cot(c+dx))^2}{(e \cot(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=222

$$\frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{3/2}} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{3/2}} + \frac{\sqrt{2} a^2 t}{\sqrt{2} de^{3/2}}$$

[Out]  $\frac{1}{2} a^2 \ln(e^{1/2} + \cot(dx+c)) e^{1/2} - 2^{1/2} (e \cot(dx+c))^{1/2} / d e^{3/2} - \frac{1}{2} a^2 \ln(e^{1/2} + \cot(dx+c)) e^{1/2} + 2^{1/2} (e \cot(dx+c))^{1/2} / d e^{3/2} + a^2 \arctan(1 - 2^{1/2} (e \cot(dx+c))^{1/2} / e^{1/2}) * 2^{1/2} / d e^{3/2} - a^2 \arctan(1 + 2^{1/2} (e \cot(dx+c))^{1/2} / e^{1/2}) * 2^{1/2} / d e^{3/2} + 2 a^2 / d e / (e \cot(dx+c))^{1/2}$

**Rubi [A]** time = 0.21, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3542, 12, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{3/2}} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{3/2}} + \frac{\sqrt{2} a^2 t}{\sqrt{2} de^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(3/2), x]

[Out]  $(\text{Sqrt}[2] a^2 \text{ArcTan}[1 - (\text{Sqrt}[2] \text{Sqrt}[e \cot[c + d*x]]) / \text{Sqrt}[e]]) / (d e^{3/2}) - (\text{Sqrt}[2] a^2 \text{ArcTan}[1 + (\text{Sqrt}[2] \text{Sqrt}[e \cot[c + d*x]]) / \text{Sqrt}[e]]) / (d e^{3/2}) + (2 a^2) / (d e \text{Sqrt}[e \cot[c + d*x]]) + (a^2 \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot[c + d*x] - \text{Sqrt}[2] \text{Sqrt}[e \cot[c + d*x]]) / (\text{Sqrt}[2] d e^{3/2}) - (a^2 \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot[c + d*x] + \text{Sqrt}[2] \text{Sqrt}[e \cot[c + d*x]]) / (\text{Sqrt}[2] d e^{3/2})$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 211

```
Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 329

```
Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 3476

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```

IntegerQ[n]

Rule 3542

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m +
1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \cot(c + dx))^2}{(e \cot(c + dx))^{3/2}} dx &= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{\int \frac{2a^2 e}{\sqrt{e \cot(c + dx)}} dx}{e^2} \\
&= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{(2a^2) \int \frac{1}{\sqrt{e \cot(c + dx)}} dx}{e} \\
&= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{\sqrt{x(e^2 + x^2)}} dx, x, e \cot(c + dx)\right)}{d} \\
&= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} - \frac{(4a^2) \text{Subst}\left(\int \frac{1}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} - \frac{(2a^2) \text{Subst}\left(\int \frac{e - x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de} - \frac{(2a^2) \text{Subst}\left(\int \frac{e + x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de} \\
&= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{a^2 \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e+2x}}{-e-\sqrt{2}\sqrt{e}x-x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{\sqrt{2}de^{3/2}} + \frac{a^2 \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e-2x}}{-e+\sqrt{2}\sqrt{e}x-x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{\sqrt{2}de^{3/2}} \\
&= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{a^2 \log\left(\sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2}\sqrt{e \cot(c + dx)}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2 \log\left(\sqrt{e} - \sqrt{e \cot(c + dx)} + \sqrt{2}\sqrt{e \cot(c + dx)}\right)}{\sqrt{2}de^{3/2}} \\
&= \frac{\sqrt{2}a^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{\sqrt{2}a^2 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2}{de\sqrt{e \cot(c + dx)}}
\end{aligned}$$



**Mathematica [C]** time = 1.81, size = 236, normalized size = 1.06

$$a^2(\cot(c + dx) + 1)^2 \left( 3 \sin(c + dx) \left( 4 \cos(c + dx) {}_2F_1 \left( -\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx) \right) + \sqrt{2} \sin(c + dx) \cot^{\frac{3}{2}}(c + dx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(3/2),x]

[Out] (a^2\*(1 + Cot[c + d\*x])^2\*(-4\*Cos[c + d\*x]^2\*Cot[c + d\*x]\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] + 3\*Sin[c + d\*x]\*(4\*Cos[c + d\*x]\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + Sqrt[2]\*Cot[c + d\*x]^(3/2)\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]) + Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])\*Sin[c + d\*x]))/(6\*d\*(e\*Cot[c + d\*x])^(3/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^2)

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^2}{(e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^2/(e\*cot(d\*x + c))^(3/2), x)

**maple [A]** time = 0.48, size = 195, normalized size = 0.88

$$\frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2d e^2} - \frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d e^2} + \frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} - 1 \right)}{d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+cot(d*x+c)*a)^2/(e*cot(d*x+c))^(3/2),x)`

[Out] 
$$-1/2/d*a^2/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^2/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/d*a^2/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+2*a^2/d/e/(e*cot(d*x+c))^{(1/2)}$$

**maxima [A]** time = 0.46, size = 202, normalized size = 0.91

$$e^{\frac{2\sqrt{2}a^2 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\frac{3}{e^{\frac{3}{2}}}} + \frac{2\sqrt{2}a^2 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\frac{3}{e^{\frac{3}{2}}}} + \frac{\sqrt{2}a^2 \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\frac{3}{e^{\frac{3}{2}}}} - \frac{\sqrt{2}a^2 \log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\frac{3}{e^{\frac{3}{2}}}}}$$


---

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] 
$$-1/2*e*((2*\sqrt{2})a^2*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(dx+c)}))/\sqrt{e})/e^{(3/2)} + 2*\sqrt{2}a^2*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(dx+c)}))/\sqrt{e})/e^{(3/2)} + \sqrt{2}a^2*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))/e^{(3/2)} - \sqrt{2}a^2*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))/e^{(3/2)})/e - 4*a^2/(e^2*\sqrt{e/\tan(dx+c)})/d$$

**mupad [B]** time = 0.59, size = 86, normalized size = 0.39

$$\frac{2a^2}{de\sqrt{e}\cot(c+dx)} + \frac{(-1)^{1/4}a^2 \operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e}\cot(c+dx)}{\sqrt{e}}\right) 2i}{de^{3/2}} + \frac{(-1)^{1/4}a^2 \operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e}\cot(c+dx)}{\sqrt{e}}\right) 2i}{de^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)`

[Out] 
$$(2*a^2)/(d*e*(e*cot(c + d*x))^{(1/2)}) + ((-1)^{(1/4)}*a^2*\operatorname{atan}((( -1)^{(1/4)}*(e*cot(c + d*x))^{(1/2)})/e^{(1/2)})*2i)/(d*e^{(3/2)}) + ((-1)^{(1/4)}*a^2*\operatorname{atanh}((( -1)^{(1/4)}*(e*cot(c + d*x))^{(1/2)})/e^{(1/2)})*2i)/(d*e^{(3/2)})$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{1}{(e \cot(c + dx))^{\frac{3}{2}}} dx + \int \frac{2 \cot(c + dx)}{(e \cot(c + dx))^{\frac{3}{2}}} dx + \int \frac{\cot^2(c + dx)}{(e \cot(c + dx))^{\frac{3}{2}}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))\*\*2/(e\*cot(d\*x+c))\*\*(3/2),x)

[Out] a\*\*2\*(Integral((e\*cot(c + d\*x))\*\*(-3/2), x) + Integral(2\*cot(c + d\*x)/(e\*cot(c + d\*x))\*\*(3/2), x) + Integral(cot(c + d\*x)\*\*2/(e\*cot(c + d\*x))\*\*(3/2), x))

$$3.13 \quad \int \frac{(a+a \cot(c+dx))^2}{(e \cot(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=247

$$\frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{5/2}} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{5/2}} - \frac{\sqrt{2} a^2 \tan^{-1}\left(\frac{\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{5/2}}$$

[Out]  $2/3*a^2/d/e/(e*\cot(d*x+c))^{(3/2)}+1/2*a^2*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}-1/2*a^2*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}-a^2*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d/e^{(5/2)}+a^2*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d/e^{(5/2)}+4*a^2/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.24, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$ , Rules used = {3542, 12, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{4a^2}{de^2 \sqrt{e \cot(c+dx)}} + \frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{5/2}} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(5/2), x]

[Out]  $-((\text{Sqrt}[2]*a^2*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{d}*e^{(5/2)})) + (\text{Sqrt}[2]*a^2*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{d}*e^{(5/2)}) + (2*a^2)/(3*d*e*(e*\text{Cot}[c + d*x])^{(3/2)}) + (4*a^2)/(d*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]]) + (a^2*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d*e^{(5/2)}) - (a^2*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d*e^{(5/2)})$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x]
```

```
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

### Rule 3542

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m +
1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + a \cot(c + dx))^2}{(e \cot(c + dx))^{5/2}} dx &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{\int \frac{2a^2 e}{(e \cot(c + dx))^{3/2}} dx}{e^2} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{(2a^2) \int \frac{1}{(e \cot(c + dx))^{3/2}} dx}{e} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4a^2}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(2a^2) \int \sqrt{e \cot(c + dx)} dx}{e^3} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4a^2}{de^2 \sqrt{e \cot(c + dx)}} + \frac{(2a^2) \text{Subst} \left( \int \frac{\sqrt{x}}{e^2 + x^2} dx, x, e \cot(c + dx) \right)}{de^2} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4a^2}{de^2 \sqrt{e \cot(c + dx)}} + \frac{(4a^2) \text{Subst} \left( \int \frac{x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{de^2} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4a^2}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(2a^2) \text{Subst} \left( \int \frac{e - x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{de^2} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4a^2}{de^2 \sqrt{e \cot(c + dx)}} + \frac{a^2 \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{e} + 2x}{-e - \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)} \right)}{\sqrt{2} de^{5/2}} \\
&= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4a^2}{de^2 \sqrt{e \cot(c + dx)}} + \frac{a^2 \log \left( \sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2} \sqrt{e} \cot(c + dx) \right)}{\sqrt{2} de^{5/2}} \\
&= -\frac{\sqrt{2} a^2 \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{de^{5/2}} + \frac{\sqrt{2} a^2 \tan^{-1} \left( 1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{de^{5/2}} + \frac{2a^2}{3de(e \cot(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 1.26, size = 233, normalized size = 0.94

$$a^2(\tan(c + dx) + 1)^2 \left( 48 \cos^2(c + dx) {}_2F_1 \left( -\frac{1}{4}, 1, \frac{3}{4}; -\cot^2(c + dx) \right) + \sin(c + dx) \left( 8 \cos(c + dx) {}_2F_1 \left( -\frac{3}{4}, 1, \frac{1}{4}; -\cot^2(c + dx) \right) + \dots \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(5/2), x]

[Out] (a^2\*(48\*Cos[c + d\*x]^2\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + Sin[c + d\*x]\*(8\*Cos[c + d\*x]\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2] + 3\*Sqrt[2]\*Cot[c + d\*x]^(5/2)\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]))

- 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] \* Sin[c + d\*x])\*(1 + Tan[c + d\*x])^2)/(12\*d\*e^2\*Sqrt[e\*Cot[c + d\*x]]\*(Cos[c + d\*x] + Sin[c + d\*x])^2)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^2}{(e \cot(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^2/(e\*cot(d\*x + c))^(5/2), x)

**maple** [A] time = 0.47, size = 216, normalized size = 0.87

$$\frac{a^2 \sqrt{2} \ln \left( \frac{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2d e^2 (e^2)^{\frac{1}{4}}} + \frac{a^2 \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d e^2 (e^2)^{\frac{1}{4}}} - \frac{a^2 \sqrt{2} \arctan \left( -\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} \right)}{d e^2 (e^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)^2/(e\*cot(d\*x+c))^(5/2),x)

[Out] 1/2/d\*a^2/e^2/(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/d\*a^2/e^2/(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/d\*a^2/e^2/(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+2/3\*a^2/d/e/(e\*cot(d\*x+c))^(3/2)+4\*a^2/d/e^2/(e\*cot(d\*x+c))^(1/2)



**maxima** [A] time = 0.54, size = 211, normalized size = 0.85

$$e \left( \frac{3a^2 \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right) + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} + \frac{\sqrt{2} \log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}}}{e^3} \right) \frac{6d}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/6\*e\*(3\*a^2\*(2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) + sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/e^3 + 4\*(a^2\*e + 6\*a^2\*e/tan(d\*x + c))/(e^3\*(e/tan(d\*x + c))^(3/2))/d

**mupad** [B] time = 0.70, size = 99, normalized size = 0.40

$$\frac{4a^2 \cot(c + dx) + \frac{2a^2}{3}}{de(e \cot(c + dx))^{3/2}} + \frac{2(-1)^{1/4} a^2 \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2(-1)^{1/4} a^2 \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cot(c + d\*x))^2/(e\*cot(c + d\*x))^(5/2),x)

[Out] (4\*a^2\*cot(c + d\*x) + (2\*a^2)/3)/(d\*e\*(e\*cot(c + d\*x))^(3/2)) + (2\*(-1)^(1/4)\*a^2\*atan(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/(d\*e^(5/2)) - (2\*(-1)^(1/4)\*a^2\*atanh(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/(d\*e^(5/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{1}{(e \cot(c + dx))^{5/2}} dx + \int \frac{2 \cot(c + dx)}{(e \cot(c + dx))^{5/2}} dx + \int \frac{\cot^2(c + dx)}{(e \cot(c + dx))^{5/2}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(2*cot(c + d*x)/(e*cot(c + d*x))**(5/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(5/2), x))
```

$$3.14 \quad \int \frac{(a+a \cot(c+dx))^2}{(e \cot(c+dx))^{7/2}} dx$$

Optimal. Leaf size=249

$$\frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{7/2}} + \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{7/2}} - \frac{\sqrt{2} a^2}{de^{7/2}}$$

[Out]  $2/5*a^2/d/e/(e*\cot(d*x+c))^{(5/2)}+4/3*a^2/d/e^2/(e*\cot(d*x+c))^{(3/2)}-1/2*a^2*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(7/2)}*2^{(1/2)}+1/2*a^2*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(7/2)}*2^{(1/2)}-a^2*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d/e^{(7/2)}+a^2*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*2^{(1/2)}/d/e^{(7/2)}$

Rubi [A] time = 0.24, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$ , Rules used = {3542, 12, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{4a^2}{3de^2(e \cot(c+dx))^{3/2}} - \frac{a^2 \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{7/2}} + \frac{a^2 \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{\sqrt{2} de^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + a*\text{Cot}[c + d*x])^2/(e*\text{Cot}[c + d*x])^{(7/2)}, x]$

[Out]  $-((\text{Sqrt}[2]*a^2*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{d}*e^{(7/2)})) + (\text{Sqrt}[2]*a^2*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{d}*e^{(7/2)}) + (2*a^2)/(5*d*e*(e*\text{Cot}[c + d*x])^{(5/2)}) + (4*a^2)/(3*d*e^2*(e*\text{Cot}[c + d*x])^{(3/2)}) - (a^2*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d*e^{(7/2)}) + (a^2*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[2]*d*e^{(7/2)})$

### Rule 12

$\text{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 204

$\text{Int}[((a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x]
```

```
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

### Rule 3542

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m +
1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e +
f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + a \cot(c + dx))^2}{(e \cot(c + dx))^{7/2}} dx &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{\int \frac{2a^2 e}{(e \cot(c + dx))^{5/2}} dx}{e^2} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{(2a^2) \int \frac{1}{(e \cot(c + dx))^{5/2}} dx}{e} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4a^2}{3de^2(e \cot(c + dx))^{3/2}} - \frac{(2a^2) \int \frac{1}{\sqrt{e \cot(c + dx)}} dx}{e^3} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4a^2}{3de^2(e \cot(c + dx))^{3/2}} + \frac{(2a^2) \text{Subst} \left( \int \frac{1}{\sqrt{x}(e^2 + x^2)} dx, x, e \cot(c + dx) \right)}{de^2} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4a^2}{3de^2(e \cot(c + dx))^{3/2}} + \frac{(4a^2) \text{Subst} \left( \int \frac{1}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{de^2} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4a^2}{3de^2(e \cot(c + dx))^{3/2}} + \frac{(2a^2) \text{Subst} \left( \int \frac{e^{-x^2}}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{de^3} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4a^2}{3de^2(e \cot(c + dx))^{3/2}} - \frac{a^2 \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{e} + 2x}{-e - \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)} \right)}{\sqrt{2} de^{7/2}} \\
&= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4a^2}{3de^2(e \cot(c + dx))^{3/2}} - \frac{a^2 \log \left( \sqrt{e} + \sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} \right)}{\sqrt{2} de^{7/2}} \\
&= -\frac{\sqrt{2} a^2 \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{de^{7/2}} + \frac{\sqrt{2} a^2 \tan^{-1} \left( 1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{de^{7/2}} + \frac{2a^2}{5de(e \cot(c + dx))^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.41, size = 141, normalized size = 0.57

$$\frac{2a^2 \sin(c + dx)(\tan(c + dx) + 1)^2 \left( 10 \cos(c + dx) {}_2F_1 \left( -\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx) \right) + 15 \cos(c + dx) \cot(c + dx) {}_2F_1 \left( -\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx) \right) + 3 \text{Hypergeometric2F1}[-5/4, 1, -1/4, -\cot^2(c + dx)] \sin(c + dx) \right)}{15de^3 \sqrt{e \cot(c + dx)} (\sin(c + dx) + \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(7/2),x]

[Out] (2\*a^2\*Sin[c + d\*x]\*(10\*Cos[c + d\*x]\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2] + 15\*Cos[c + d\*x]\*Cot[c + d\*x]\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + 3\*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d\*x]^2]\*Sin[c + d\*x])/(15\*d\*e^3\*sqrt(e\*Cot[c + d\*x])\*(Sin[c + d\*x] + Cos[c + d\*x]))

+ d\*x]))\*(1 + Tan[c + d\*x])^2)/(15\*d\*e^3\*Sqrt[e\*Cot[c + d\*x]]\*(Cos[c + d\*x] + Sin[c + d\*x])^2)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catde f: division by zero

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^2}{(e \cot(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^2/(e\*cot(d\*x + c))^(7/2), x)

**maple** [A] time = 0.46, size = 216, normalized size = 0.87

$$\frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2d e^4} + \frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d e^4} - \frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} - 1 \right)}{d e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)^2/(e\*cot(d\*x+c))^(7/2),x)

[Out] 1/2/d\*a^2/e^4\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c)))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/d\*a^2/e^4\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/d\*a^2/e^4\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+2/5\*a^2/d/e/(e\*cot(d\*x+c))^(5/2)+4/3\*a^2/d/e^2/(e\*cot(d\*x+c))^(3/2)

**maxima [A]** time = 0.69, size = 221, normalized size = 0.89

$$e \left[ \frac{15 \left( \frac{2 \sqrt{2} a^2 \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\frac{3}{e^{\frac{3}{2}}}} \right) + \frac{2 \sqrt{2} a^2 \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\frac{3}{e^{\frac{3}{2}}}} \right) + \frac{\sqrt{2} a^2 \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)} \right)}{\frac{3}{e^{\frac{3}{2}}}} - \frac{\sqrt{2} a^2 \log \left( -\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)} \right)}{\frac{3}{e^{\frac{3}{2}}}} \right]}{e^3} \right]$$

$30d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x, algorithm="maxima")

[Out] 1/30\*e\*(15\*(2\*sqrt(2)\*a^2\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/e^(3/2) + 2\*sqrt(2)\*a^2\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/e^(3/2) + sqrt(2)\*a^2\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/e^(3/2) - sqrt(2)\*a^2\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/e^(3/2))/e^3 + 4\*(3\*a^2\*e + 10\*a^2\*e/tan(d\*x + c))/(e^3\*(e/tan(d\*x + c))^(5/2))/d

**mupad [B]** time = 1.29, size = 99, normalized size = 0.40

$$\frac{\frac{4a^2 \cot(c+dx)}{3} + \frac{2a^2}{5}}{de(e \cot(c+dx))^{5/2}} - \frac{(-1)^{1/4} a^2 \operatorname{atan} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right) 2i}{d e^{7/2}} - \frac{(-1)^{1/4} a^2 \operatorname{atanh} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right) 2i}{d e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cot(c + d\*x))^2/(e\*cot(c + d\*x))^(7/2),x)

[Out] ((4\*a^2\*cot(c + d\*x))/3 + (2\*a^2)/5)/(d\*e\*(e\*cot(c + d\*x))^(5/2)) - ((-1)^(1/4)\*a^2\*atan((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*2i/(d\*e^(7/2)) - ((-1)^(1/4)\*a^2\*atanh((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*2i/(d\*e^(7/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{1}{(e \cot(c+dx))^{\frac{7}{2}}} dx + \int \frac{2 \cot(c+dx)}{(e \cot(c+dx))^{\frac{7}{2}}} dx + \int \frac{\cot^2(c+dx)}{(e \cot(c+dx))^{\frac{7}{2}}} dx \right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(2*cot(c + d*x)/(e*cot(c + d*x))**(7/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(7/2), x))
```

### 3.15 $\int (e \cot(c + dx))^{5/2} (a + a \cot(c + dx))^3 dx$

**Optimal.** Leaf size=186

$$\frac{2\sqrt{2} a^3 e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} + \frac{4a^3 e^2 \sqrt{e \cot(c+dx)}}{d} - \frac{2(a^3 \cot(c+dx) + a^3)(e \cot(c+dx))^{7/2}}{9de} - \frac{40a^3(e \cot(c+dx))^{7/2}}{63de}$$

[Out]  $4/3*a^3*e*(e*\cot(d*x+c))^{(3/2)}/d-4/5*a^3*(e*\cot(d*x+c))^{(5/2)}/d-40/63*a^3*(e*\cot(d*x+c))^{(7/2)}/d/e-2/9*(e*\cot(d*x+c))^{(7/2)}*(a^3+a^3*\cot(d*x+c))/d/e+2*a^3*e^{(5/2)}*\arctan(1/2*(e^{(1/2)}-\cot(d*x+c))*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}/d+4*a^3*e^2*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.30, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3566, 3630, 3528, 3532, 205}

$$\frac{4a^3 e^2 \sqrt{e \cot(c+dx)}}{d} + \frac{2\sqrt{2} a^3 e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} - \frac{2(a^3 \cot(c+dx) + a^3)(e \cot(c+dx))^{7/2}}{9de} - \frac{40a^3(e \cot(c+dx))^{7/2}}{63de}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3,x]

[Out]  $(2*\text{Sqrt}[2]*a^3*e^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])])/d + (4*a^3*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d + (4*a^3*e*(e*\text{Cot}[c + d*x])^{(3/2)})/(3*d) - (4*a^3*(e*\text{Cot}[c + d*x])^{(5/2)})/(5*d) - (40*a^3*(e*\text{Cot}[c + d*x])^{(7/2)})/(63*d*e) - (2*(e*\text{Cot}[c + d*x])^{(7/2)}*(a^3 + a^3*\text{Cot}[c + d*x]))/(9*d*e)$

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 3528

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(d\*(a + b\*Tan[e + f\*x])^m)/(f\*m), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3532

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]
```

### Rule 3566

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n - 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3630

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{5/2} (a + a \cot(c + dx))^3 dx &= -\frac{2(e \cot(c + dx))^{7/2} (a^3 + a^3 \cot(c + dx))}{9de} - \frac{2 \int (e \cot(c + dx))^{5/2} (-a}{9de} \\
&= -\frac{40a^3 (e \cot(c + dx))^{7/2}}{63de} - \frac{2(e \cot(c + dx))^{7/2} (a^3 + a^3 \cot(c + dx))}{9de} - \frac{2 \int (e \cot(c + dx))^{5/2} (-a}{9de} \\
&= -\frac{4a^3 (e \cot(c + dx))^{5/2}}{5d} - \frac{40a^3 (e \cot(c + dx))^{7/2}}{63de} - \frac{2(e \cot(c + dx))^{7/2} (a^3 + a^3 \cot(c + dx))}{9de} \\
&= \frac{4a^3 e (e \cot(c + dx))^{3/2}}{3d} - \frac{4a^3 (e \cot(c + dx))^{5/2}}{5d} - \frac{40a^3 (e \cot(c + dx))^{7/2}}{63de} \\
&= \frac{4a^3 e^2 \sqrt{e \cot(c + dx)}}{d} + \frac{4a^3 e (e \cot(c + dx))^{3/2}}{3d} - \frac{4a^3 (e \cot(c + dx))^{5/2}}{5d} \\
&= \frac{4a^3 e^2 \sqrt{e \cot(c + dx)}}{d} + \frac{4a^3 e (e \cot(c + dx))^{3/2}}{3d} - \frac{4a^3 (e \cot(c + dx))^{5/2}}{5d} \\
&= \frac{2\sqrt{2} a^3 e^{5/2} \tan^{-1} \left( \frac{\sqrt{e} - \sqrt{e \cot(c + dx)}}{\sqrt{2} \sqrt{e \cot(c + dx)}} \right)}{d} + \frac{4a^3 e^2 \sqrt{e \cot(c + dx)}}{d} + \frac{4a^3 e (e \cot(c + dx))^{3/2}}{3d}
\end{aligned}$$

**Mathematica [C]** time = 6.10, size = 729, normalized size = 3.92

$$\frac{4 \sin^3(c + dx) \tan(c + dx) (a \cot(c + dx) + a)^3 (e \cot(c + dx))^{5/2} {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right)}{3d(\sin(c + dx) + \cos(c + dx))^3} - \frac{2 \sin(c + dx) \cos^2(c + dx)}{9d(\sin(c + dx) + \cos(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3,x]

[Out] (-2\*Cos[c + d\*x]^2\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Sin[c + d\*x])/(9\*d\*(Cos[c + d\*x] + Sin[c + d\*x])^3) - (6\*Cos[c + d\*x]\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Sin[c + d\*x]^2)/(7\*d\*(Cos[c + d\*x] + Sin[c + d\*x])^3) - (4\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Sin[c + d\*x]^3)/(5\*d\*(Cos[c + d\*x] + Sin[c + d\*x])^3) + (Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Sin[c + d\*x]^3)/(d\*Cot[c + d\*x]^(5/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^3) - (Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Sin[c + d\*x]^3)/(d\*Cot[c + d\*x]^(5/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^3) + ((e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]\*Sin[c + d\*x]^3)/(Sqrt[2]\*d\*Cot[c + d\*x]^(5/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^3) - ((e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]\*Sin[c + d\*x]^3)/(Sqrt[2]\*d\*Cot[c + d\*x]^(5/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^3)

$$2] * d * \cot[c + d*x]^{(5/2)} * (\cos[c + d*x] + \sin[c + d*x])^3 + (4 * (e * \cot[c + d*x])^{(5/2)} * (a + a * \cot[c + d*x])^3 * \sin[c + d*x]^3 * \tan[c + d*x]) / (3 * d * (\cos[c + d*x] + \sin[c + d*x])^3) - (4 * (e * \cot[c + d*x])^{(5/2)} * (a + a * \cot[c + d*x])^3 * \text{Hypergeometric2F1}[3/4, 1, 7/4, -\cot[c + d*x]^2] * \sin[c + d*x]^3 * \tan[c + d*x]) / (3 * d * (\cos[c + d*x] + \sin[c + d*x])^3) + (4 * (e * \cot[c + d*x])^{(5/2)} * (a + a * \cot[c + d*x])^3 * \sin[c + d*x]^3 * \tan[c + d*x]^2) / (d * (\cos[c + d*x] + \sin[c + d*x])^3)$$

**fricas [A]** time = 0.95, size = 535, normalized size = 2.88

$$\left[ \frac{315 \sqrt{2} (a^3 e^2 \cos(2dx + 2c)^2 - 2a^3 e^2 \cos(2dx + 2c) + a^3 e^2) \sqrt{-e} \log\left(-\sqrt{2} \sqrt{-e} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}} (\cos(2dx + 2c) + \sin(2dx + 2c) - 1) - 2e \sin(2dx + 2c) + e\right) + 2 * (721 a^3 e^2 \cos(2dx + 2c)^2 - 1330 a^3 e^2 \cos(2dx + 2c) + 469 a^3 e^2 - 15 * (23 a^3 e^2 \cos(2dx + 2c) - 5 a^3 e^2) \sin(2dx + 2c)) \sqrt{(e \cos(2dx + 2c) + e) / \sin(2dx + 2c)}}{(d \cos(2dx + 2c))^2 - 2 * d * \cos(2dx + 2c) + d}, \frac{2 / 315 * (315 \sqrt{2} * (a^3 e^2 \cos(2dx + 2c)^2 - 2a^3 e^2 \cos(2dx + 2c) + a^3 e^2) \sqrt{e} * \arctan(-1/2 \sqrt{2} * \sqrt{e} * \sqrt{(e \cos(2dx + 2c) + e) / \sin(2dx + 2c)}) * (\cos(2dx + 2c) - \sin(2dx + 2c) + 1) / (e \cos(2dx + 2c) + e)) + (721 a^3 e^2 \cos(2dx + 2c)^2 - 1330 a^3 e^2 \cos(2dx + 2c) + 469 a^3 e^2 - 15 * (23 a^3 e^2 \cos(2dx + 2c) - 5 a^3 e^2) \sin(2dx + 2c)) \sqrt{(e \cos(2dx + 2c) + e) / \sin(2dx + 2c)}}{(d \cos(2dx + 2c))^2 - 2 * d * \cos(2dx + 2c) + d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [1/315\*(315\*sqrt(2)\*(a^3\*e^2\*cos(2\*d\*x + 2\*c)^2 - 2\*a^3\*e^2\*cos(2\*d\*x + 2\*c) + a^3\*e^2)\*sqrt(-e)\*log(-sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*e\*sin(2\*d\*x + 2\*c) + e) + 2\*(721\*a^3\*e^2\*cos(2\*d\*x + 2\*c)^2 - 1330\*a^3\*e^2\*cos(2\*d\*x + 2\*c) + 469\*a^3\*e^2 - 15\*(23\*a^3\*e^2\*cos(2\*d\*x + 2\*c) - 5\*a^3\*e^2)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*cos(2\*d\*x + 2\*c)^2 - 2\*d\*cos(2\*d\*x + 2\*c) + d), 2/315\*(315\*sqrt(2)\*(a^3\*e^2\*cos(2\*d\*x + 2\*c)^2 - 2\*a^3\*e^2\*cos(2\*d\*x + 2\*c) + a^3\*e^2)\*sqrt(e)\*arctan(-1/2\*sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e)) + (721\*a^3\*e^2\*cos(2\*d\*x + 2\*c)^2 - 1330\*a^3\*e^2\*cos(2\*d\*x + 2\*c) + 469\*a^3\*e^2 - 15\*(23\*a^3\*e^2\*cos(2\*d\*x + 2\*c) - 5\*a^3\*e^2)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*cos(2\*d\*x + 2\*c)^2 - 2\*d\*cos(2\*d\*x + 2\*c) + d)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a)^3 (e \cot(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^3\*(e\*cot(d\*x + c))^(5/2), x)

**maple [B]** time = 0.79, size = 446, normalized size = 2.40

$$\frac{2a^3 (e \cot(dx+c))^{\frac{9}{2}}}{9de^2} - \frac{6a^3 (e \cot(dx+c))^{\frac{7}{2}}}{7de} - \frac{4a^3 (e \cot(dx+c))^{\frac{5}{2}}}{5d} + \frac{4a^3 e (e \cot(dx+c))^{\frac{3}{2}}}{3d} + \frac{4a^3 e^2 \sqrt{e \cot(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(5/2)\*(a+cot(d\*x+c)\*a)^3,x)

[Out] 
$$-2/9/d*a^3/e^2*(e*\cot(d*x+c))^{(9/2)}-6/7*a^3*(e*\cot(d*x+c))^{(7/2)}/d/e-4/5*a^3*(e*\cot(d*x+c))^{(5/2)}/d+4/3*a^3*e*(e*\cot(d*x+c))^{(3/2)}/d+4*a^3*e^2*(e*\cot(d*x+c))^{(1/2)}/d-1/2/d*a^3*e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^3*e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/d*a^3*e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/2/d*a^3*e^3*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^3*e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/d*a^3*e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)$$

**maxima [A]** time = 0.50, size = 193, normalized size = 1.04

$$\frac{2 \left( 315 a^3 e^2 \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right) - \frac{630 a^3 e^4 \sqrt{\frac{e}{\tan(dx+c)}} + 210 a^3 e^3 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{315 d} \right)}{315 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-2/315*(315*a^3*e^2*(\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e}/\tan(d*x+c)))/\sqrt{e}))/\sqrt{e} + \sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e}/\tan(d*x+c)))/\sqrt{e}))/\sqrt{e} - (630*a^3*e^4*\sqrt{e/\tan(d*x+c)} + 210*a^3*e^3*(e/\tan(d*x+c))^{(3/2)} - 126*a^3*e^2*(e/\tan(d*x+c))^{(5/2)} - 135*a^3*e*(e/\tan(d*x+c))^{(7/2)} - 35*a^3*(e/\tan(d*x+c))^{(9/2)})/e^3)*e/d$$

**mupad [B]** time = 2.44, size = 177, normalized size = 0.95

$$\frac{4a^3 e^2 \sqrt{e \cot(c+dx)}}{d} - \frac{4a^3 (e \cot(c+dx))^{5/2}}{5d} - \frac{6a^3 (e \cot(c+dx))^{7/2}}{7de} - \frac{2a^3 (e \cot(c+dx))^{9/2}}{9de^2} + \frac{4a^3 e (e \cot(c+dx))^{11/2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(5/2)\*(a + a\*cot(c + d\*x))^3,x)

[Out] (4\*a^3\*e^2\*(e\*cot(c + d\*x))^(1/2))/d - (4\*a^3\*(e\*cot(c + d\*x))^(5/2))/(5\*d) - (6\*a^3\*(e\*cot(c + d\*x))^(7/2))/(7\*d\*e) - (2\*a^3\*(e\*cot(c + d\*x))^(9/2))/(9\*d\*e^2) + (4\*a^3\*e\*(e\*cot(c + d\*x))^(3/2))/(3\*d) - (2^(1/2)\*a^3\*e^(5/2)\*(2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2))) + 2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2))) + (2^(1/2)\*(e\*cot(c + d\*x))^(3/2))/(2\*e^(3/2)))))/d

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left( \int (e \cot(c+dx))^{\frac{5}{2}} dx + \int 3 (e \cot(c+dx))^{\frac{5}{2}} \cot(c+dx) dx + \int 3 (e \cot(c+dx))^{\frac{5}{2}} \cot^2(c+dx) dx + \int (e \cot(c+dx))^{\frac{5}{2}} \cot^3(c+dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)\*(a+a\*cot(d\*x+c))\*\*3,x)

[Out] a\*\*3\*(Integral((e\*cot(c + d\*x))\*\*(5/2), x) + Integral(3\*(e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x), x) + Integral(3\*(e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x)\*\*2, x) + Integral((e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x)\*\*3, x))

### 3.16 $\int (e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^3 dx$

**Optimal.** Leaf size=160

$$\frac{2\sqrt{2} a^3 e^{3/2} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} - \frac{32a^3 (e \cot(c + dx))^{5/2}}{35de} - \frac{4a^3 (e \cot(c + dx))^{3/2}}{3d} + \frac{4a^3 e \sqrt{e \cot(c + dx)}}{d} - \frac{2(a^3 \cot(c + dx))^{3/2}}{d}$$

[Out]  $-4/3*a^3*(e*\cot(d*x+c))^{(3/2)}/d-32/35*a^3*(e*\cot(d*x+c))^{(5/2)}/d/e-2/7*(e*\cot(d*x+c))^{(5/2)}*(a^3+a^3*\cot(d*x+c))/d/e-2*a^3*e^{(3/2)}*\operatorname{arctanh}(1/2*(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}/d+4*a^3*e*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.26, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3566, 3630, 3528, 3532, 208}

$$\frac{2\sqrt{2} a^3 e^{3/2} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{d} - \frac{32a^3 (e \cot(c + dx))^{5/2}}{35de} - \frac{4a^3 (e \cot(c + dx))^{3/2}}{3d} + \frac{4a^3 e \sqrt{e \cot(c + dx)}}{d} - \frac{2(a^3 \cot(c + dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*\operatorname{Cot}[c + d*x])^{(3/2)}*(a + a*\operatorname{Cot}[c + d*x])^3, x]$

[Out]  $(-2*\operatorname{Sqrt}[2]*a^3*e^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[e]*\operatorname{Cot}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]])])/d + (4*a^3*e*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]])/d - (4*a^3*(e*\operatorname{Cot}[c + d*x])^{(3/2)})/(3*d) - (32*a^3*(e*\operatorname{Cot}[c + d*x])^{(5/2)})/(35*d*e) - (2*(e*\operatorname{Cot}[c + d*x])^{(5/2)}*(a^3 + a^3*\operatorname{Cot}[c + d*x]))/(7*d*e)$

#### Rule 208

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

#### Rule 3528

$\operatorname{Int}[(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Simp}[(d*(a + b*\tan[e + f*x])^m)/(f*m), x] + \operatorname{Int}[(a + b*\tan[e + f*x])^{(m-1)}*\operatorname{Simp}[a*c - b*d + (b*c + a*d)*\tan[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3532

$\operatorname{Int}[(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/(\operatorname{Sqrt}[(b_.)*\tan[(e_.) + (f_.)*(x_.)]]), x\_Symbol] \rightarrow \operatorname{Dist}[(-2*d^2)/f, \operatorname{Subst}[\operatorname{Int}[1/(2*c*d + b*x^2), x], x, (c -$



$d \cdot \tan[e + f \cdot x] / \sqrt{b \cdot \tan[e + f \cdot x]}$ , x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3566

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1))/(d\*f\*(m + n - 1)), x] + Dist[1/(d\*(m + n - 1)), Int[(a + b\*Tan[e + f\*x])^(m - 3)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^3\*d\*(m + n - 1) - b^2\*(b\*c\*(m - 2) + a\*d\*(1 + n)) + b\*d\*(m + n - 1)\*(3\*a^2 - b^2)\*Tan[e + f\*x] - b^2\*(b\*c\*(m - 2) - a\*d\*(3\*m + 2\*n - 4))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3630

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] := Simp[(C\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int (e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^3 dx &= -\frac{2(e \cot(c + dx))^{5/2} (a^3 + a^3 \cot(c + dx))}{7de} - \frac{2 \int (e \cot(c + dx))^{3/2} (-}{ \\
 &= -\frac{32a^3(e \cot(c + dx))^{5/2}}{35de} - \frac{2(e \cot(c + dx))^{5/2} (a^3 + a^3 \cot(c + dx))}{7de} \\
 &= -\frac{4a^3(e \cot(c + dx))^{3/2}}{3d} - \frac{32a^3(e \cot(c + dx))^{5/2}}{35de} - \frac{2(e \cot(c + dx))^{5/2}}{35de} \\
 &= \frac{4a^3 e \sqrt{e \cot(c + dx)}}{d} - \frac{4a^3 (e \cot(c + dx))^{3/2}}{3d} - \frac{32a^3 (e \cot(c + dx))^{5/2}}{35de} \\
 &= \frac{4a^3 e \sqrt{e \cot(c + dx)}}{d} - \frac{4a^3 (e \cot(c + dx))^{3/2}}{3d} - \frac{32a^3 (e \cot(c + dx))^{5/2}}{35de} \\
 &= -\frac{2\sqrt{2} a^3 e^{3/2} \tanh^{-1} \left( \frac{\sqrt{e} + \sqrt{e \cot(c + dx)}}{\sqrt{2} \sqrt{e \cot(c + dx)}} \right)}{d} + \frac{4a^3 e \sqrt{e \cot(c + dx)}}{d} - \frac{4a^3 (e \cot(c + dx))^{3/2}}{3d}
 \end{aligned}$$

**Mathematica [C]** time = 2.81, size = 332, normalized size = 2.08

$$a^3 \sin(c + dx)(\cot(c + dx) + 1)^3 (e \cot(c + dx))^{3/2} \left( 280 \sin^2(c + dx) \cot^2(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right) - 60 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])^3,x]

[Out] (a^3\*(e\*Cot[c + d\*x])^(3/2)\*(1 + Cot[c + d\*x])^3\*Sin[c + d\*x]\*(-60\*Cos[c + d\*x]^2\*Cot[c + d\*x]^(3/2) + 210\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]])\*Sin[c + d\*x]^2 - 210\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])\*Sin[c + d\*x]^2 + 840\*Sqrt[Cot[c + d\*x]]\*Sin[c + d\*x]^2 - 280\*Cot[c + d\*x]^(3/2)\*Sin[c + d\*x]^2 + 280\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]\*Sin[c + d\*x]^2 + 105\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]\*Sin[c + d\*x]^2 - 105\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]\*Sin[c + d\*x]^2 - 126\*Cot[c + d\*x]^(3/2)\*Sin[2\*(c + d\*x)]))/(210\*d\*Cot[c + d\*x]^(3/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^3)

**fricas [A]** time = 0.72, size = 487, normalized size = 3.04

$$\left[ \frac{105 \sqrt{2} (a^3 e \cos(2 dx + 2 c) - a^3 e) \sqrt{e} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e \cos(2 dx + 2 c) + e}{\sin(2 dx + 2 c)}} (\cos(2 dx + 2 c) - \sin(2 dx + 2 c) - 1) + 2 e \sin(2 dx + 2 c) + e\right)}{(d \cos(2 dx + 2 c) - d) \sin(2 dx + 2 c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [1/105\*(105\*sqrt(2)\*(a^3\*e\*cos(2\*d\*x + 2\*c) - a^3\*e)\*sqrt(e)\*log(sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1) + 2\*e\*sin(2\*d\*x + 2\*c) + e)\*sin(2\*d\*x + 2\*c) - 2\*(55\*a^3\*e\*cos(2\*d\*x + 2\*c)^2 - 30\*a^3\*e\*cos(2\*d\*x + 2\*c) - 85\*a^3\*e - 21\*(13\*a^3\*e\*cos(2\*d\*x + 2\*c) - 7\*a^3\*e)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/((d\*cos(2\*d\*x + 2\*c) - d)\*sin(2\*d\*x + 2\*c)), 2/105\*(105\*sqrt(2)\*(a^3\*e\*cos(2\*d\*x + 2\*c) - a^3\*e)\*sqrt(-e)\*arctan(1/2\*sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e))\*sin(2\*d\*x + 2\*c) - (55\*a^3\*e\*cos(2\*d\*x + 2\*c)^2 - 30\*a^3\*e\*cos(2\*d\*x + 2\*c) - 85\*a^3\*e - 21\*(13\*a^3\*e\*cos(2\*d\*x + 2\*c) - 7\*a^3\*e)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/((d\*cos(2\*d\*x + 2\*c) - d)\*sin(2\*d\*x + 2\*c))]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a)^3 (e \cot(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^3\*(e\*cot(d\*x + c))^(3/2), x)

**maple** [B] time = 0.87, size = 419, normalized size = 2.62

$$\frac{2a^3 (e \cot(dx + c))^{\frac{7}{2}}}{7d e^2} - \frac{6a^3 (e \cot(dx + c))^{\frac{5}{2}}}{5de} - \frac{4a^3 (e \cot(dx + c))^{\frac{3}{2}}}{3d} + \frac{4a^3 e \sqrt{e \cot(dx + c)}}{d} - \frac{a^3 e (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}}}\right)}{e \cot(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)\*(a+cot(d\*x+c)\*a)^3,x)

[Out] 
$$\begin{aligned} & -2/7/d*a^3/e^2*(e*cot(d*x+c))^(7/2)-6/5*a^3*(e*cot(d*x+c))^(5/2)/d/e-4/3*a^3 \\ & * (e*cot(d*x+c))^(3/2)/d+4*a^3*e*(e*cot(d*x+c))^(1/2)/d-1/2/d*a^3*e*(e^2)^(1/4) \\ & *2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)) \\ & )^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)) \\ & )-1/d*a^3*e*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1) \\ & +1/d*a^3*e*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1) \\ & +1/2/d*a^3*e^2*2^(1/2)/(e^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2) \\ & *2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)) \\ & )+1/d*a^3*e^2*2^(1/2)/(e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1) \\ & -1/d*a^3*e^2*2^(1/2)/(e^2)^(1/4)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1) \end{aligned}$$

**maxima** [A] time = 0.47, size = 175, normalized size = 1.09

$$\frac{\left( 105 a^3 e \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right) - 2 \left( 210 a^3 e^3 \sqrt{\frac{e}{\tan(dx+c)}} - 70 a^3 e^2 \left( \frac{e}{\tan(dx+c)} \right) \right)}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="maxima")

```
[Out] -1/105*(105*a^3*e*(sqrt(2)*log(sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e
/tan(d*x + c))/sqrt(e) - sqrt(2)*log(-sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c))
+ e + e/tan(d*x + c))/sqrt(e) - 2*(210*a^3*e^3*sqrt(e/tan(d*x + c)) - 70*a
^3*e^2*(e/tan(d*x + c))^(3/2) - 63*a^3*e*(e/tan(d*x + c))^(5/2) - 15*a^3*(e
/tan(d*x + c))^(7/2))/e^3)*e/d
```

**mupad [B]** time = 1.62, size = 143, normalized size = 0.89

$$\frac{4a^3 e \sqrt{e \cot(c+dx)}}{d} - \frac{6a^3 (e \cot(c+dx))^{5/2}}{5de} - \frac{2a^3 (e \cot(c+dx))^{7/2}}{7de^2} - \frac{4a^3 (e \cot(c+dx))^{3/2}}{3d} + \frac{\sqrt{2} a^3 e^{3/2} \operatorname{atan}\left(\frac{1}{\sqrt{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3,x)
```

```
[Out] (4*a^3*e*(e*cot(c + d*x))^(1/2))/d - (6*a^3*(e*cot(c + d*x))^(5/2))/(5*d*e)
- (2*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e^2) - (4*a^3*(e*cot(c + d*x))^(3/2)
)/(3*d) + (2^(1/2)*a^3*e^(3/2)*atan((2^(1/2)*a^6*e^(9/2)*(e*cot(c + d*x))^(
1/2)*32i)/(32*a^6*e^5 + 32*a^6*e^5*cot(c + d*x)))*2i)/d
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left( \int (e \cot(c+dx))^{\frac{3}{2}} dx + \int 3(e \cot(c+dx))^{\frac{3}{2}} \cot(c+dx) dx + \int 3(e \cot(c+dx))^{\frac{3}{2}} \cot^2(c+dx) dx + \int (e \cot(c+dx))^{\frac{3}{2}} \cot^3(c+dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(3*(e*cot(c + d*x))**(
3/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(3/2)*cot(c + d*x)**2,
x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3, x))
```

### 3.17 $\int \sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^3 dx$

**Optimal.** Leaf size=138

$$\frac{8a^3(e \cot(c + dx))^{3/2}}{5de} - \frac{4a^3\sqrt{e \cot(c + dx)}}{d} - \frac{2(a^3 \cot(c + dx) + a^3)(e \cot(c + dx))^{3/2}}{5de} - \frac{2\sqrt{2}a^3\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{d}$$

[Out]  $-8/5*a^3*(e*\cot(d*x+c))^{(3/2)}/d/e-2/5*(e*\cot(d*x+c))^{(3/2)}*(a^3+a^3*\cot(d*x+c))/d/e-2*a^3*\arctan(1/2*(e^{(1/2)}-\cot(d*x+c)*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)})*2^{(1/2)}*e^{(1/2)}/d-4*a^3*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3566, 3630, 3528, 3532, 205}

$$\frac{8a^3(e \cot(c + dx))^{3/2}}{5de} - \frac{4a^3\sqrt{e \cot(c + dx)}}{d} - \frac{2(a^3 \cot(c + dx) + a^3)(e \cot(c + dx))^{3/2}}{5de} - \frac{2\sqrt{2}a^3\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^3,x]

[Out]  $(-2*\text{Sqrt}[2]*a^3*\text{Sqrt}[e]*\text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])])/d - (4*a^3*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (8*a^3*(e*\text{Cot}[c + d*x])^{(3/2)})/(5*d*e) - (2*(e*\text{Cot}[c + d*x])^{(3/2)}*(a^3 + a^3*\text{Cot}[c + d*x]))/(5*d*e)$

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 3528

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(d\*(a + b\*Tan[e + f\*x])^m)/(f\*m), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

#### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&

EqQ[c^2 - d^2, 0]

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
+ f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x, x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a +
b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^3 dx &= -\frac{2(e \cot(c + dx))^{3/2} (a^3 + a^3 \cot(c + dx))}{5de} - \frac{2 \int \sqrt{e \cot(c + dx)} (-a^3 e}{5de} \\
&= -\frac{8a^3(e \cot(c + dx))^{3/2}}{5de} - \frac{2(e \cot(c + dx))^{3/2} (a^3 + a^3 \cot(c + dx))}{5de} - \frac{2 \int \sqrt{e \cot(c + dx)} (-a^3 e}{5de} \\
&= -\frac{4a^3 \sqrt{e \cot(c + dx)}}{d} - \frac{8a^3(e \cot(c + dx))^{3/2}}{5de} - \frac{2(e \cot(c + dx))^{3/2} (a^3 + a^3 \cot(c + dx))}{5de} \\
&= -\frac{4a^3 \sqrt{e \cot(c + dx)}}{d} - \frac{8a^3(e \cot(c + dx))^{3/2}}{5de} - \frac{2(e \cot(c + dx))^{3/2} (a^3 + a^3 \cot(c + dx))}{5de} \\
&= -\frac{2\sqrt{2} a^3 \sqrt{e} \tan^{-1} \left( \frac{\sqrt{e} - \sqrt{e} \cot(c + dx)}{\sqrt{2} \sqrt{e \cot(c + dx)}} \right)}{d} - \frac{4a^3 \sqrt{e \cot(c + dx)}}{d} - \frac{8a^3(e \cot(c + dx))^{3/2}}{5de}
\end{aligned}$$

**Mathematica** [C] time = 1.55, size = 315, normalized size = 2.28

$$\frac{a^3 \sin(c + dx)(\cot(c + dx) + 1)^3 \sqrt{e \cot(c + dx)} \left( 3 \left( 4 \cos^2(c + dx) \sqrt{\cot(c + dx)} + 40 \sin^2(c + dx) \sqrt{\cot(c + dx)} \right) + \dots \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^3,x]

[Out] 
$$-1/30*(a^3*\text{Sqrt}[e*\text{Cot}[c + d*x]]*(1 + \text{Cot}[c + d*x])^3*\text{Sin}[c + d*x]*(-20*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Hypergeometric2F1}[3/4, 1, 7/4, -\text{Cot}[c + d*x]^2]*\text{Sin}[2*(c + d*x)] + 3*(4*\text{Cos}[c + d*x]^2*\text{Sqrt}[\text{Cot}[c + d*x]] + 10*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]*\text{Sin}[c + d*x]^2 - 10*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]*\text{Sin}[c + d*x]^2 + 40*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sin}[c + d*x]^2 + 5*\text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]*\text{Sin}[c + d*x]^2 - 5*\text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]*\text{Sin}[c + d*x]^2 + 10*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sin}[2*(c + d*x)])))/(d*\text{Sqrt}[\text{Cot}[c + d*x]]*(\text{Cos}[c + d*x] + \text{Sin}[c + d*x])^3)$$

**fricas** [A] time = 0.72, size = 366, normalized size = 2.65

$$\frac{5\sqrt{2}(a^3 \cos(2dx + 2c) - a^3)\sqrt{-e} \log\left(\sqrt{2}\sqrt{-e} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} (\cos(2dx + 2c) + \sin(2dx + 2c) - 1) - 2e \sin(2dx + 2c) + e\right)}{5(d \cos(2dx + 2c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$\frac{1}{5}*(5*\text{sqrt}(2)*(a^3*\text{cos}(2*d*x + 2*c) - a^3)*\text{sqrt}(-e)*\text{log}(\text{sqrt}(2)*\text{sqrt}(-e)*\text{sqrt}((e*\text{cos}(2*d*x + 2*c) + e)/\text{sin}(2*d*x + 2*c))*(\text{cos}(2*d*x + 2*c) + \text{sin}(2*d*x + 2*c) - 1) - 2*e*\text{sin}(2*d*x + 2*c) + e) - 2*(9*a^3*\text{cos}(2*d*x + 2*c) - 5*a^3*\text{sin}(2*d*x + 2*c) - 11*a^3)*\text{sqrt}((e*\text{cos}(2*d*x + 2*c) + e)/\text{sin}(2*d*x + 2*c)))/(d*\text{cos}(2*d*x + 2*c) - d), -2/5*(5*\text{sqrt}(2)*(a^3*\text{cos}(2*d*x + 2*c) - a^3)*\text{sqrt}(e)*\text{arctan}(-1/2*\text{sqrt}(2)*\text{sqrt}(e)*\text{sqrt}((e*\text{cos}(2*d*x + 2*c) + e)/\text{sin}(2*d*x + 2*c)))*(\text{cos}(2*d*x + 2*c) - \text{sin}(2*d*x + 2*c) + 1)/(e*\text{cos}(2*d*x + 2*c) + e)) + (9*a^3*\text{cos}(2*d*x + 2*c) - 5*a^3*\text{sin}(2*d*x + 2*c) - 11*a^3)*\text{sqrt}((e*\text{cos}(2*d*x + 2*c) + e)/\text{sin}(2*d*x + 2*c)))/(d*\text{cos}(2*d*x + 2*c) - d]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cot(dx + c) + a)^3 \sqrt{e \cot(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^3\*sqrt(e\*cot(d\*x + c)), x)

**maple [B]** time = 0.96, size = 391, normalized size = 2.83

$$\frac{2a^3 (e \cot(dx+c))^{\frac{5}{2}}}{5d e^2} - \frac{2a^3 (e \cot(dx+c))^{\frac{3}{2}}}{de} - \frac{4a^3 \sqrt{e \cot(dx+c)}}{d} + \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{2}} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(1/2)\*(a+cot(d\*x+c)\*a)^3,x)

[Out]  $-2/5/d*a^3/e^2*(e*\cot(d*x+c))^{5/2}-2*a^3*(e*\cot(d*x+c))^{3/2}/d/e-4*a^3*(e*\cot(d*x+c))^{1/2}/d+1/2/d*a^3*(e^2)^{1/4}*2^{1/2}*ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}))/((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}))+1/d*a^3*(e^2)^{1/4}*2^{1/2}*arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/d*a^3*(e^2)^{1/4}*2^{1/2}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+1/2/d*a^3*e*2^{1/2}/(e^2)^{1/4}*ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}))/((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}))+1/d*a^3*e*2^{1/2}/(e^2)^{1/4}*arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/d*a^3*e*2^{1/2}/(e^2)^{1/4}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)$

**maxima [A]** time = 0.76, size = 149, normalized size = 1.08

$$\frac{2 \left( 5a^3 \left( \frac{\sqrt{2} \arctan \left( \frac{\sqrt{2} (\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan \left( -\frac{\sqrt{2} (\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}} \right)}{\sqrt{e}} \right) \right)}{5d} - \frac{10a^3 e^2 \sqrt{\frac{e}{\tan(dx+c)}} + 5a^3 e \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}} + a^3 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{5}{2}}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+a\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out]  $2/5*(5*a^3*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x+c)))/sqrt(e))/sqrt(e) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e/tan(d*x+c)))/sqrt(e))/sqrt(e)) - (10*a^3*e^2*sqrt(e/tan(d*x+c)) + 5*a^3*e*(e/tan(d*x+c))^{3/2} + a^3*(e/tan(d*x+c))^{5/2})/e^3)*e/d$

**mapad [B]** time = 0.99, size = 136, normalized size = 0.99

$$\frac{\sqrt{2} a^3 \sqrt{e} \left( 2 \operatorname{atan} \left( \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2 \sqrt{e}} \right) + 2 \operatorname{atan} \left( \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2 \sqrt{e}} + \frac{\sqrt{2} (e \cot(c+dx))^{3/2}}{2e^{3/2}} \right) \right)}{d} - \frac{2a^3 (e \cot(c+dx))^{3/2}}{de} - \frac{2a^3 (e \cot(c+dx))^{5/2}}{de}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3,x)
```

```
[Out] (2^(1/2)*a^3*e^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)))
+ 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c +
d*x))^(3/2))/(2*e^(3/2)))))/d - (2*a^3*(e*cot(c + d*x))^(3/2))/(d*e) - (2*
a^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2) - (4*a^3*(e*cot(c + d*x))^(1/2))/d
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$a^3 \left( \int \sqrt{e \cot(c + dx)} dx + \int 3\sqrt{e \cot(c + dx)} \cot(c + dx) dx + \int 3\sqrt{e \cot(c + dx)} \cot^2(c + dx) dx + \int \sqrt{e \cot(c + dx)} \cot^3(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(3*sqrt(e*cot(c + d*x))*c
ot(c + d*x), x) + Integral(3*sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x) + Int
egral(sqrt(e*cot(c + d*x))*cot(c + d*x)**3, x))
```

$$3.18 \quad \int \frac{(a+a \cot(c+dx))^3}{\sqrt{e \cot(c+dx)}} dx$$

Optimal. Leaf size=117

$$\frac{16a^3\sqrt{e \cot(c+dx)}}{3de} - \frac{2(a^3 \cot(c+dx) + a^3)\sqrt{e \cot(c+dx)}}{3de} + \frac{2\sqrt{2}a^3 \tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{d\sqrt{e}}$$

[Out]  $2*a^3*\operatorname{arctanh}(1/2*(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)})*2^{(1/2)}/d/e^{(1/2)}-16/3*a^3*(e*\cot(d*x+c))^{(1/2)}/d/e-2/3*(a^3+a^3*\cot(d*x+c))*(e*\cot(d*x+c))^{(1/2)}/d/e$

**Rubi [A]** time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {3566, 3630, 3532, 208}

$$\frac{16a^3\sqrt{e \cot(c+dx)}}{3de} - \frac{2(a^3 \cot(c+dx) + a^3)\sqrt{e \cot(c+dx)}}{3de} + \frac{2\sqrt{2}a^3 \tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + a*\operatorname{Cot}[c + d*x])^3/\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]], x]$

[Out]  $(2*\operatorname{Sqrt}[2]*a^3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[e]*\operatorname{Cot}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]])]/(d*\operatorname{Sqrt}[e]) - (16*a^3*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]])/(3*d*e) - (2*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]]*(a^3 + a^3*\operatorname{Cot}[c + d*x]))/(3*d*e)$

### Rule 208

$\operatorname{Int}(((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

### Rule 3532

$\operatorname{Int}(((c_) + (d_)*\tan[(e_) + (f_)*(x_)])/(\operatorname{Sqrt}[(b_)*\tan[(e_) + (f_)*(x_)]]), x\_Symbol] := \operatorname{Dist}[(-2*d^2)/f, \operatorname{Subst}[\operatorname{Int}[1/(2*c*d + b*x^2), x], x, (c - d*\tan[e + f*x])/(\operatorname{Sqrt}[b*\tan[e + f*x]])], x] /; \operatorname{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \ \operatorname{EqQ}[c^2 - d^2, 0]$

### Rule 3566

$\operatorname{Int}(((a_) + (b_)*\tan[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*\tan[(e_) + (f_)*(x_)])^{(n_)}), x\_Symbol] := \operatorname{Simp}[(b^2*(a + b*\tan[e + f*x])^{(m-2)}*(c + d*\tan[e + f*x])^{(n+1)})/(d*f*(m+n-1)), x] + \operatorname{Dist}[1/(d*(m+n-1)),$

```
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n - 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + a \cot(c + dx))^3}{\sqrt{e \cot(c + dx)}} dx &= -\frac{2\sqrt{e \cot(c + dx)} (a^3 + a^3 \cot(c + dx))}{3de} - \frac{2 \int \frac{-a^3 e - 3a^3 e \cot(c + dx) - 4a^3 e \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{3e} \\ &= -\frac{16a^3 \sqrt{e \cot(c + dx)}}{3de} - \frac{2\sqrt{e \cot(c + dx)} (a^3 + a^3 \cot(c + dx))}{3de} - \frac{2 \int \frac{3a^3 e - 3a^3 e \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{3e} \\ &= -\frac{16a^3 \sqrt{e \cot(c + dx)}}{3de} - \frac{2\sqrt{e \cot(c + dx)} (a^3 + a^3 \cot(c + dx))}{3de} + \frac{(12a^6 e) \text{Subst}\left(\int \frac{3a^3 e - 3a^3 e \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx\right)}{3e} \\ &= \frac{2\sqrt{2} a^3 \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e \cot(c + dx)}}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{d\sqrt{e}} - \frac{16a^3 \sqrt{e \cot(c + dx)}}{3de} - \frac{2\sqrt{e \cot(c + dx)} (a^3 + a^3 \cot(c + dx))}{3de} \end{aligned}$$

**Mathematica [C]** time = 5.17, size = 292, normalized size = 2.50

$$a^3 \sin(c + dx) (\cot(c + dx) + 1)^3 \left( 8 \cos^2(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right) + 18 \sin(2(c + dx)) + 4 \cos^2(c + dx) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]], x]
```

```
[Out] -1/6*(a^3*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(4*Cos[c + d*x]^2 + 8*Cos[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 6*Sqrt[2]*ArcTan[1
```

- Sqrt[2]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Cot[c + d\*x]]\*Sin[c + d\*x]^2 - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]\*Sqrt[Cot[c + d\*x]]\*Sin[c + d\*x]^2 + 3\*Sqrt[2]\*Sqrt[Cot[c + d\*x]]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]\*Sin[c + d\*x]^2 - 3\*Sqrt[2]\*Sqrt[Cot[c + d\*x]]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]\*Sin[c + d\*x]^2 + 18\*Sin[2\*(c + d\*x)]))/(d\*Sqrt[e\*Cot[c + d\*x]]\*(Cos[c + d\*x] + Sin[c + d\*x])^3)

**fricas** [A] time = 0.84, size = 349, normalized size = 2.98

$$\left[ \frac{3\sqrt{2}a^3\sqrt{e} \log\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)-\sin(2dx+2c)-1)}{\sqrt{e}} + 2\sin(2dx+2c) + 1\right)}{3de\sin(2dx+2c)} \right] \sin(2dx+2c) - 2(a^3\cos(2dx+2c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/3\*(3\*sqrt(2)\*a^3\*sqrt(e)\*log(-sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1)/sqrt(e) + 2\*sin(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) - 2\*(a^3\*cos(2\*d\*x + 2\*c) + 9\*a^3\*sin(2\*d\*x + 2\*c) + a^3)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e\*sin(2\*d\*x + 2\*c)), -2/3\*(3\*sqrt(2)\*a^3\*e\*sqrt(-1/e)\*arctan(1/2\*sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sqrt(-1/e)\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)/(cos(2\*d\*x + 2\*c) + 1))\*sin(2\*d\*x + 2\*c) + (a^3\*cos(2\*d\*x + 2\*c) + 9\*a^3\*sin(2\*d\*x + 2\*c) + a^3)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e\*sin(2\*d\*x + 2\*c))]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^3}{\sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^3/sqrt(e\*cot(d\*x + c)), x)

**maple** [B] time = 0.57, size = 379, normalized size = 3.24

$$\frac{2a^3(e \cot(dx + c))^{\frac{3}{2}}}{3de^2} - \frac{6a^3\sqrt{e \cot(dx + c)}}{de} + \frac{a^3(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{2de} + a^3(e^2)^{\frac{1}{4}}\sqrt{2} \arctan\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+\cot(dx+c))a^3/(e\cot(dx+c))^{1/2}, x)$

[Out] 
$$-2/3/d*a^3/e^2*(e\cot(dx+c))^{3/2}-6*a^3*(e\cot(dx+c))^{1/2}/d/e+1/2/d*a^3/e*(e^2)^{1/4}*2^{1/2}*\ln((e\cot(dx+c)+(e^2)^{1/4}*(e\cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e\cot(dx+c)-(e^2)^{1/4}*(e\cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))+1/d*a^3/e*(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(e^2)^{1/4}*(e\cot(dx+c))^{1/2}+1)-1/d*a^3/e*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e\cot(dx+c))^{1/2}+1)-1/2/d*a^3*2^{1/2}/(e^2)^{1/4}*\ln((e\cot(dx+c)-(e^2)^{1/4}*(e\cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e\cot(dx+c)+(e^2)^{1/4}*(e\cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))-1/d*a^3*2^{1/2}/(e^2)^{1/4}*\arctan(2^{1/2}/(e^2)^{1/4}*(e\cot(dx+c))^{1/2}+1)+1/d*a^3*2^{1/2}/(e^2)^{1/4}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e\cot(dx+c))^{1/2}+1)$$

**maxima [A]** time = 0.55, size = 136, normalized size = 1.16

$$\left( \frac{3a^3 \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right) - \sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{e} - \frac{2 \left( 9a^3 e \sqrt{\frac{e}{\tan(dx+c)}} + a^3 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}} \right)}{e^3} \right) e$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+a\cot(dx+c))^3/(e\cot(dx+c))^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] 
$$1/3*(3*a^3*(\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(e)*\text{sqrt}(e/\tan(dx+c)) + e + e/\tan(dx+c))/\text{sqrt}(e) - \text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(e)*\text{sqrt}(e/\tan(dx+c)) + e + e/\tan(dx+c))/\text{sqrt}(e)))/e - 2*(9*a^3*e*\text{sqrt}(e/\tan(dx+c)) + a^3*(e/\tan(dx+c))^{3/2})/e^3)*e/d$$

**mupad [B]** time = 0.62, size = 100, normalized size = 0.85

$$\frac{2\sqrt{2}a^3 \operatorname{atanh}\left(\frac{32\sqrt{2}a^6\sqrt{e}\sqrt{e\cot(c+dx)}}{32a^6e+32a^6e\cot(c+dx)}\right)}{d\sqrt{e}} - \frac{2a^3(e\cot(c+dx))^{3/2}}{3de^2} - \frac{6a^3\sqrt{e\cot(c+dx)}}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a + a\cot(c + dx))^3/(e\cot(c + dx))^{1/2}, x)$

[Out] 
$$(2*2^{1/2}*a^3*\operatorname{atanh}((32*2^{1/2}*a^6*e^{1/2}*(e\cot(c + dx))^{1/2})/(32*a^6*e + 32*a^6*e*\cot(c + dx))))/(d*e^{1/2}) - (2*a^3*(e\cot(c + dx))^{3/2})/(3*d*e^2) - (6*a^3*(e\cot(c + dx))^{1/2})/(d*e)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left( \int \frac{1}{\sqrt{e \cot(c + dx)}} dx + \int \frac{3 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx + \int \frac{3 \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}} dx + \int \frac{\cot^3(c + dx)}{\sqrt{e \cot(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))\*\*3/(e\*cot(d\*x+c))\*\*(1/2),x)

[Out] a\*\*3\*(Integral(1/sqrt(e\*cot(c + d\*x)), x) + Integral(3\*cot(c + d\*x)/sqrt(e\*cot(c + d\*x)), x) + Integral(3\*cot(c + d\*x)\*\*2/sqrt(e\*cot(c + d\*x)), x) + Integral(cot(c + d\*x)\*\*3/sqrt(e\*cot(c + d\*x)), x))

$$3.19 \quad \int \frac{(a+a \cot(c+dx))^3}{(e \cot(c+dx))^{3/2}} dx$$

Optimal. Leaf size=114

$$\frac{2\sqrt{2} a^3 \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3 \sqrt{e \cot(c+dx)}}{de^2} + \frac{2(a^3 \cot(c+dx) + a^3)}{de \sqrt{e \cot(c+dx)}}$$

[Out]  $2*a^3*\arctan(1/2*(e^{(1/2)}-\cot(d*x+c))*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)} *2^{(1/2)}/d/e^{(3/2)}+2*(a^3+a^3*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^{(1/2)}-4*a^3*(e*\cot(d*x+c))^{(1/2)}/d/e^2$

**Rubi [A]** time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {3565, 3630, 3532, 205}

$$-\frac{4a^3 \sqrt{e \cot(c+dx)}}{de^2} + \frac{2\sqrt{2} a^3 \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{3/2}} + \frac{2(a^3 \cot(c+dx) + a^3)}{de \sqrt{e \cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(3/2),x]

[Out]  $(2*\text{Sqrt}[2]*a^3*\text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(d*e^{(3/2)}) - (4*a^3*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(d*e^2) + (2*(a^3 + a^3*\text{Cot}[c + d*x]))/(d*e*\text{Sqrt}[e*\text{Cot}[c + d*x]])$

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 3532

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

#### Rule 3565

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[((b\*c - a\*d)^2\*(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1))/(d\*f\*(n + 1)\*(c^2 + d^2)), x] - Dist[1

```
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && IntegerQ[2*m]
```

### Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)] + (C_.)*tan[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + a \cot(c + dx))^3}{(e \cot(c + dx))^{3/2}} dx &= \frac{2(a^3 + a^3 \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} - \frac{2 \int \frac{-2a^3e^2 - a^3e^2 \cot(c+dx) - a^3e^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{e^3} \\ &= -\frac{4a^3\sqrt{e \cot(c + dx)}}{de^2} + \frac{2(a^3 + a^3 \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} - \frac{2 \int \frac{-a^3e^2 - a^3e^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{e^3} \\ &= -\frac{4a^3\sqrt{e \cot(c + dx)}}{de^2} + \frac{2(a^3 + a^3 \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} + \frac{(4a^6e) \text{Subst}\left(\int \frac{1}{-2a^6e^4 - ex^2} dx, x, \frac{-a^3}{\sqrt{e \cot(c + dx)}}\right)}{d} \\ &= \frac{2\sqrt{2}a^3 \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3\sqrt{e \cot(c + dx)}}{de^2} + \frac{2(a^3 + a^3 \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} \end{aligned}$$

**Mathematica [C]** time = 2.89, size = 311, normalized size = 2.73

$$\frac{a^3(\cot(c + dx) + 1)^3 \left( \sin(c + dx) \left( 2 \sin(2(c + dx)) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right) - 4 \cos^2(c + dx) + \sqrt{2} \sin^2(c + dx) \right) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2), x]
```



```
[Out] (a^3*(1 + Cot[c + d*x])^3*(-4*Cos[c + d*x]^3*Hypergeometric2F1[3/4, 1, 7/4,
-Cot[c + d*x]^2] + Sin[c + d*x]*(-4*Cos[c + d*x]^2 + 2*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]])*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 - 2*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]])*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 + S
qrt[2]*Cot[c + d*x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]
]*Sin[c + d*x]^2 - Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x])*Sin[c + d*x]^2 + 2*Hypergeometric2F1[-1/4, 1, 3/4, -C
ot[c + d*x]^2]*Sin[2*(c + d*x)])))/(2*d*(e*Cot[c + d*x])^(3/2)*(Cos[c + d*x
] + Sin[c + d*x])^3)
```

**fricas** [A] time = 0.67, size = 372, normalized size = 3.26

$$\frac{\sqrt{2} \left( a^3 e \cos(2dx + 2c) + a^3 e \right) \sqrt{-\frac{1}{e}} \log \left( -\sqrt{2} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}} \sqrt{-\frac{1}{e}} (\cos(2dx + 2c) + \sin(2dx + 2c) - 1) - 2 \sin(2dx + 2c) + 1 \right) - 2 \left( a^3 \cos(2dx + 2c) - a^3 \sin(2dx + 2c) + a^3 \right) \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}}}{d e^2 \cos(2dx + 2c) + d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*sqrt(-1/e)*log(-sqrt(2)*sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(
2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c) + 1) - 2*(a^3*cos(2*d*x + 2*c) - a^3
*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(
d*e^2*cos(2*d*x + 2*c) + d*e^2), 2*(sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e
)*arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(
2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)))/sqrt
(e) - (a^3*cos(2*d*x + 2*c) - a^3*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

[Out] integrate((a\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(3/2), x)

**maple [B]** time = 0.48, size = 388, normalized size = 3.40

$$\frac{2a^3\sqrt{e\cot(dx+c)}}{de^2} - \frac{a^3(e^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{2de^2} - \frac{a^3(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{de^2} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)^3/(e\*cot(d\*x+c))^(3/2), x)

[Out]  $-2*a^3*(e*\cot(d*x+c))^{(1/2)}/d/e^2-1/2/d*a^3/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^3/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/d*a^3/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/2/d*a^3/e^2*(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^3/e^2*(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/d*a^3/e^2*(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+2/d*a^3/e/(e*\cot(d*x+c))^{(1/2)}$

**maxima [A]** time = 0.68, size = 130, normalized size = 1.14

$$\frac{2 \left( \frac{a^3 \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\tan(dx+c)})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\tan(dx+c)})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{e^2} - \frac{a^3}{e^2\sqrt{\frac{e}{\tan(dx+c)}}} + \frac{a^3\sqrt{\frac{e}{\tan(dx+c)}}}{e^3} \right)}{d} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(3/2), x, algorithm="maxima")

[Out]  $-2*(a^3*(\sqrt{2})*\arctan(1/2*\sqrt{2}*(\sqrt{2})*\sqrt{e} + 2*\sqrt{e/\tan(dx+c)}))/\sqrt{e}))/\sqrt{e} + \sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2})*\sqrt{e} - 2*\sqrt{e/\tan(dx+c)})/\sqrt{e} + \sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2})*\sqrt{e} + 2*\sqrt{e/\tan(dx+c)})/\sqrt{e} - 2*\sqrt{e/\tan(dx+c)})/\sqrt{e} + \sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2})*\sqrt{e} - 2*\sqrt{e/\tan(dx+c)})/\sqrt{e}$

$t(e/\tan(dx + c))/\sqrt{e}/\sqrt{e})/e^2 - a^3/(e^2\sqrt{e/\tan(dx + c)}) + a^3\sqrt{e/\tan(dx + c)}/e^3 * e/d$

**mupad [B]** time = 0.58, size = 119, normalized size = 1.04

$$\frac{2a^3}{de\sqrt{e\cot(c+dx)}} - \frac{2a^3\sqrt{e\cot(c+dx)}}{de^2} - \frac{\sqrt{2}a^3\left(2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right) + 2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}} + \frac{\sqrt{2}(e\cot(c+dx))^{3/2}}{2e^{3/2}}\right)\right)}{de^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2), x)`

[Out]  $(2a^3)/(de*(e*\cot(c + dx))^{(1/2)}) - (2a^3*(e*\cot(c + dx))^{(1/2)})/(de^2) - (2^{(1/2)}*a^3*(2*\operatorname{atan}((2^{(1/2)}*(e*\cot(c + dx))^{(1/2)})/(2*e^{(1/2)}))) + 2*\operatorname{atan}((2^{(1/2)}*(e*\cot(c + dx))^{(1/2)})/(2*e^{(1/2)}) + (2^{(1/2)}*(e*\cot(c + dx))^{(3/2)})/(2*e^{(3/2)}))))/(de^{(3/2)})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{(e\cot(c+dx))^{3/2}} dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{3/2}} dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}} dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{3/2}} dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2), x)`

[Out] `a**3*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(3*cot(c + d*x)/(e*cot(c + d*x))**(3/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(3/2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(3/2), x))`

$$3.20 \quad \int \frac{(a+a \cot(c+dx))^3}{(e \cot(c+dx))^{5/2}} dx$$

Optimal. Leaf size=117

$$-\frac{2\sqrt{2} a^3 \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx)+\sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2 \sqrt{e \cot(c+dx)}} + \frac{2(a^3 \cot(c+dx) + a^3)}{3de(e \cot(c+dx))^{3/2}}$$

[Out]  $2/3*(a^3+a^3*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^(3/2)-2*a^3*\operatorname{arctanh}(1/2*(e^(1/2)+\cot(d*x+c)*e^(1/2))*2^(1/2)/(e*\cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+16/3*a^3/d/e^2/(e*\cot(d*x+c))^(1/2)$

**Rubi [A]** time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {3565, 3628, 3532, 208}

$$\frac{16a^3}{3de^2 \sqrt{e \cot(c+dx)}} - \frac{2\sqrt{2} a^3 \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx)+\sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{5/2}} + \frac{2(a^3 \cot(c+dx) + a^3)}{3de(e \cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(5/2), x]

[Out]  $(-2*\operatorname{Sqrt}[2]*a^3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[e]*\operatorname{Cot}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]])]/(d*e^(5/2)) + (16*a^3)/(3*d*e^2*\operatorname{Sqrt}[e*\operatorname{Cot}[c + d*x]]) + (2*(a^3 + a^3*\operatorname{Cot}[c + d*x]))/(3*d*e*(e*\operatorname{Cot}[c + d*x])^(3/2))$

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3565

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[((b\*c - a\*d)^2\*(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1))/(d\*f\*(n + 1)\*(c^2 + d^2)), x] - Dist[1

/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 3)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a^2\*d\*(b\*d\*(m - 2) - a\*c\*(n + 1)) + b\*(b\*c - 2\*a\*d)\*(b\*c\*(m - 2) + a\*d\*(n + 1)) - d\*(n + 1)\*(3\*a^2\*b\*c - b^3\*c - a^3\*d + 3\*a\*b^2\*d)\*Tan[e + f\*x] - b\*(a\*d\*(2\*b\*c - a\*d)\*(m + n - 1) - b^2\*(c^2\*(m - 2) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && IntegerQ[2\*m]

### Rule 3628

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)^2], x\_Symbol] :> Simp[(A\*b^2 - a\*b\*B + a^2\*C)\*(a + b\*Tan[e + f\*x])^(m + 1)/(b\*f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{(a + a \cot(c + dx))^3}{(e \cot(c + dx))^{5/2}} dx &= \frac{2(a^3 + a^3 \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{2 \int \frac{-4a^3e^2 - 3a^3e^2 \cot(c+dx) - a^3e^2 \cot^2(c+dx)}{(e \cot(c+dx))^{3/2}} dx}{3e^3} \\ &= \frac{16a^3}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{2 \int \frac{-3a^3e^3 + 3a^3e^3 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{3e^5} \\ &= \frac{16a^3}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} + \frac{(12a^6e) \text{Subst}\left(\int \frac{1}{18a^6e^6 - ex^2} dx, x, \frac{1}{\sqrt{e \cot(c + dx)}}\right)}{d} \\ &= -\frac{2\sqrt{2}a^3 \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} \end{aligned}$$

**Mathematica [C]** time = 6.11, size = 417, normalized size = 3.56

$$-\frac{2 \cos^3(c + dx) \cot(c + dx)(a \cot(c + dx) + a)^3 {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right)}{3d(e \cot(c + dx))^{5/2}(\sin(c + dx) + \cos(c + dx))^3} + \frac{6 \sin(c + dx) \cos^2(c + dx)(a \cot(c + dx) + a)^3}{d(e \cot(c + dx))^{5/2}(\sin(c + dx) + \cos(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(5/2), x]

```
[Out] (-2*Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Cot[c + d*x])^3*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (6*Cos[c + d*x]^2*(a + a*Cot[c + d*x])^3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2]*Sin[c + d*x])/(d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*Cos[c + d*x]*(a + a*Cot[c + d*x])^3*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]*Sin[c + d*x]^2)/(3*d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (3*Cot[c + d*x]^(5/2)*(a + a*Cot[c + d*x])^3*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]) - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]) + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sin[c + d*x]^3)/(4*d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3)
```

**fricas** [A] time = 1.54, size = 378, normalized size = 3.23

$$\frac{3\sqrt{2}(a^3 e \cos(2dx+2c) + a^3 e) \log\left(\frac{\sqrt{2} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} (\cos(2dx+2c) - \sin(2dx+2c) - 1)}{\sqrt{e}} + 2 \sin(2dx+2c) + 1\right)}{\sqrt{e}} - 2(a^3 \cos(2dx+2c) - 9a^3 \sin(2dx+2c))}{3(de^3 \cos(2dx+2c) + de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*log(sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) + 2*sin(2*d*x + 2*c) + 1)/sqrt(e) - 2*(a^3*cos(2*d*x + 2*c) - 9*a^3*sin(2*d*x + 2*c) - a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3), 2/3*(3*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) - (a^3*cos(2*d*x + 2*c) - 9*a^3*sin(2*d*x + 2*c) - a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

[Out] integrate((a\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(5/2), x)

**maple [B]** time = 0.50, size = 388, normalized size = 3.32

$$\frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2d e^3} - \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d e^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)^3/(e\*cot(d\*x+c))^(5/2), x)

[Out] 
$$-1/2/d*a^3/e^3*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/d*a^3/e^3*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/d*a^3/e^3*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2/d*a^3/e^2*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))+1/d*a^3/e^2*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/d*a^3/e^2*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+2/3/d*a^3/e/(e*cot(d*x+c))^{(3/2)}+6*a^3/d/e^2/(e*cot(d*x+c))^{(1/2)}$$

**maxima [A]** time = 0.79, size = 133, normalized size = 1.14

$$\frac{e \left( \frac{3a^3 \left( \frac{\sqrt{2} \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)} \right)}{\sqrt{e}} - \frac{\sqrt{2} \log \left( -\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)} \right)}{\sqrt{e}} \right)}{e^3} - \frac{2 \left( a^3 e + \frac{9a^3 e}{\tan(dx+c)} \right)}{e^3 \left( \frac{e}{\tan(dx+c)} \right)^2} \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] 
$$-1/3*e*(3*a^3*(\sqrt{2})*\log(\sqrt{2})*\sqrt{e}*\sqrt{e/\tan(d*x+c)} + e + e/\tan(d*x+c))/\sqrt{e} - \sqrt{2}*\log(-\sqrt{2})*\sqrt{e}*\sqrt{e/\tan(d*x+c)} + e + e/\tan(d*x+c))/\sqrt{e})/e^3 - 2*(a^3*e + 9*a^3*e/\tan(d*x+c))/(e^3*(e/\tan(d*x+c))^{(3/2)})/d$$

**mupad [B]** time = 0.71, size = 101, normalized size = 0.86

$$\frac{\frac{2a^3 e}{3} + 6a^3 e \cot(c + dx)}{d e^2 (e \cot(c + dx))^{3/2}} - \frac{2\sqrt{2} a^3 \operatorname{atanh} \left( \frac{32\sqrt{2} a^6 d e^{5/2} \sqrt{e \cot(c+dx)}}{32a^6 d e^3 + 32a^6 d e^3 \cot(c+dx)} \right)}{d e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2), x)`

[Out]  $((2*a^3*e)/3 + 6*a^3*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - (2*2^(1/2)*a^3*atanh((32*2^(1/2)*a^6*d*e^(5/2)*(e*cot(c + d*x))^(1/2))/(32*a^6*d*e^3 + 32*a^6*d*e^3*cot(c + d*x))))/(d*e^(5/2))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left( \int \frac{1}{(e \cot(c + dx))^{\frac{5}{2}}} dx + \int \frac{3 \cot(c + dx)}{(e \cot(c + dx))^{\frac{5}{2}}} dx + \int \frac{3 \cot^2(c + dx)}{(e \cot(c + dx))^{\frac{5}{2}}} dx + \int \frac{\cot^3(c + dx)}{(e \cot(c + dx))^{\frac{5}{2}}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2), x)`

[Out] `a**3*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(3*cot(c + d*x)/(e*cot(c + d*x))**(5/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(5/2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(5/2), x))`



$$3.21 \quad \int \frac{(a+a \cot(c+dx))^3}{(e \cot(c+dx))^{7/2}} dx$$

**Optimal.** Leaf size=141

$$-\frac{2\sqrt{2} a^3 \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{7/2}} + \frac{4a^3}{de^3 \sqrt{e \cot(c+dx)}} + \frac{8a^3}{5de^2 (e \cot(c+dx))^{3/2}} + \frac{2(a^3 \cot(c+dx) + a^3)}{5de (e \cot(c+dx))^{5/2}}$$

[Out]  $8/5*a^3/d/e^2/(e*\cot(d*x+c))^{(3/2)}+2/5*(a^3+a^3*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^{(5/2)}-2*a^3*\arctan(1/2*(e^{(1/2)}-\cot(d*x+c)*e^{(1/2)})*2^{(1/2)/(e*\cot(d*x+c))^{(1/2)})}*2^{(1/2)/d/e^{(7/2)}}+4*a^3/d/e^3/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3565, 3628, 3529, 3532, 205}

$$\frac{4a^3}{de^3 \sqrt{e \cot(c+dx)}} + \frac{8a^3}{5de^2 (e \cot(c+dx))^{3/2}} - \frac{2\sqrt{2} a^3 \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{7/2}} + \frac{2(a^3 \cot(c+dx) + a^3)}{5de (e \cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(7/2), x]

[Out]  $(-2*\text{Sqrt}[2]*a^3*\text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(d*e^{(7/2)}) + (8*a^3)/(5*d*e^2*(e*\text{Cot}[c + d*x])^{(3/2)}) + (4*a^3)/(d*e^3*\text{Sqrt}[e*\text{Cot}[c + d*x]]) + (2*(a^3 + a^3*\text{Cot}[c + d*x]))/(5*d*e*(e*\text{Cot}[c + d*x])^{(5/2)})$

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 3529

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c -

$d*\text{Tan}[e + f*x])/ \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{EqQ}[c^2 - d^2, 0]$

### Rule 3565

$\text{Int}[(a + b*\text{tan}[(e + f*x)])^m * ((c + d*\text{tan}[(e + f*x)] + (f*x))^{n-2}), x\_Symbol] :> \text{Simp}[(b*c - a*d)^2 * (a + b*\text{Tan}[e + f*x])^{m-2} * (c + d*\text{Tan}[e + f*x])^{n+1} / (d*f*(n+1)*(c^2 + d^2)), x] - \text{Dist}[1 / (d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m-3} * (c + d*\text{Tan}[e + f*x])^{n+1} * \text{Simp}[a^2*d*(b*d*(m-2) - a*c*(n+1)) + b*(b*c - 2*a*d)*(b*c*(m-2) + a*d*(n+1)) - d*(n+1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*\text{Tan}[e + f*x] - b*(a*d*(2*b*c - a*d)*(m+n-1) - b^2*(c^2*(m-2) - d^2*(n+1)))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*m]$

### Rule 3628

$\text{Int}[(a + b*\text{tan}[(e + f*x)])^m * ((A + B*\text{tan}[(e + f*x)] + (f*x)) + (C + d*\text{tan}[(e + f*x)] + (f*x))^2), x\_Symbol] :> \text{Simp}[(A*b^2 - a*b*B + a^2*C) * (a + b*\text{Tan}[e + f*x])^{m+1} / (b*f*(m+1)*(a^2 + b^2)), x] + \text{Dist}[1 / (a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m+1} * \text{Simp}[b*B + a*(A - C) - (A*b - a*B - b*C)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \&\& \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(a + a \cot(c + dx))^3}{(e \cot(c + dx))^{7/2}} dx &= \frac{2(a^3 + a^3 \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{2 \int \frac{-6a^3e^2 - 5a^3e^2 \cot(c+dx) - a^3e^2 \cot^2(c+dx)}{(e \cot(c+dx))^{5/2}} dx}{5e^3} \\ &= \frac{8a^3}{5de^2(e \cot(c + dx))^{3/2}} + \frac{2(a^3 + a^3 \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{2 \int \frac{-5a^3e^3 + 5a^3e^3 \cot(c+dx)}{(e \cot(c+dx))^{3/2}} dx}{5e^5} \\ &= \frac{8a^3}{5de^2(e \cot(c + dx))^{3/2}} + \frac{4a^3}{de^3 \sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{2 \int \frac{5a^3e^4 + 5a^3e^4 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{5e^7} \\ &= \frac{8a^3}{5de^2(e \cot(c + dx))^{3/2}} + \frac{4a^3}{de^3 \sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} + \frac{(20a^6e) \text{Subst}(\int \frac{1}{\sqrt{u}} du, u = e \cot(c + dx))}{5e^7} \\ &= -\frac{2\sqrt{2} a^3 \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e \cot(c+dx)}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{7/2}} + \frac{8a^3}{5de^2(e \cot(c + dx))^{3/2}} + \frac{4a^3}{de^3 \sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} \end{aligned}$$

**Mathematica [C]** time = 3.34, size = 269, normalized size = 1.91

$$a^3(\tan(c + dx) + 1)^3 \left( 120 \cos^3(c + dx) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right) + \sin(c + dx) \left( 40 \cos^2(c + dx) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; \frac{1}{\cot^2(c + dx)}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(7/2), x]

[Out] (a^3\*(120\*Cos[c + d\*x]^3\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + Sin[c + d\*x]\*(40\*Cos[c + d\*x]^2\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2] + Sin[c + d\*x]\*(8\*Cos[c + d\*x]\*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d\*x]^2] + 5\*Sqrt[2]\*Cot[c + d\*x]^(7/2)\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])\*Sin[c + d\*x]))\*(1 + Tan[c + d\*x])^3)/(20\*d\*e^3\*Sqrt[e\*Cot[c + d\*x]]\*(Cos[c + d\*x] + Sin[c + d\*x])^3)

**fricas [A]** time = 0.45, size = 485, normalized size = 3.44

$$\frac{5\sqrt{2}\left(a^3e\cos(2dx+2c)^2+2a^3e\cos(2dx+2c)+a^3e\right)\sqrt{-\frac{1}{e}}\log\left(\sqrt{2}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\sqrt{-\frac{1}{e}}(\cos(2dx+2c)+\sin(2dx+2c))\right)}{5\left(de^4\cos(2dx+2c)+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2), x, algorithm="fricas")

[Out] [1/5\*(5\*sqrt(2)\*(a^3\*e\*cos(2\*d\*x + 2\*c)^2 + 2\*a^3\*e\*cos(2\*d\*x + 2\*c) + a^3\*e)\*sqrt(-1/e)\*log(sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))\*sqrt(-1/e)\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*sin(2\*d\*x + 2\*c) + 1) - 2\*(5\*a^3\*cos(2\*d\*x + 2\*c)^2 - 5\*a^3 - (9\*a^3\*cos(2\*d\*x + 2\*c) + 11\*a^3)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e^4\*cos(2\*d\*x + 2\*c)^2 + 2\*d\*e^4\*cos(2\*d\*x + 2\*c) + d\*e^4), -2/5\*(5\*sqrt(2)\*(a^3\*e\*cos(2\*d\*x + 2\*c)^2 + 2\*a^3\*e\*cos(2\*d\*x + 2\*c) + a^3\*e)\*arctan(-1/2\*sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) + 1)/(sqrt(e)\*(cos(2\*d\*x + 2\*c) + 1)))/sqrt(e) + (5\*a^3\*cos(

$(2dx + 2c)^2 - 5a^3 - (9a^3 \cos(2dx + 2c) + 11a^3) \sin(2dx + 2c) \sqrt{(e \cos(2dx + 2c) + e) / \sin(2dx + 2c)} / (d e^4 \cos(2dx + 2c)^2 + 2 d e^4 \cos(2dx + 2c) + d e^4]$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(7/2), x)

**maple [B]** time = 0.53, size = 409, normalized size = 2.90

$$\frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2d e^4} + \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d e^4} - \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} - 1 \right)}{d e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cot(d\*x+c)\*a)^3/(e\*cot(d\*x+c))^(7/2),x)

[Out]  $\frac{1}{2} d a^3 / e^4 * (e^2)^{(1/4)} * 2^{(1/2)} * \ln((e \cot(d*x+c) + (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e \cot(d*x+c) - (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) + 1/d a^3 / e^4 * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} + 1) - 1/d a^3 / e^4 * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} + 1) + 1/2 d a^3 / e^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e \cot(d*x+c) - (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e \cot(d*x+c) + (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) + 1/d a^3 / e^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} + 1) - 1/d a^3 / e^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e \cot(d*x+c))^{(1/2)} + 1) + 2/5 d a^3 / e / (e \cot(d*x+c))^{(5/2)} + 2 a^3 / d / e^2 / (e \cot(d*x+c))^{(3/2)} + 4 a^3 / d / e^3 / (e \cot(d*x+c))^{(1/2)}$

**maxima** [A] time = 0.77, size = 148, normalized size = 1.05

$$2e \left( \frac{5a^3 \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}}\right) + \sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{e^4} + \frac{a^3e^2 + \frac{5a^3e^2}{\tan(dx+c)} + \frac{10a^3e^2}{\tan(dx+c)^2}}{e^4 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{5}{2}}} \right)$$

$5d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2),x, algorithm="maxima")

[Out] 2/5\*e\*(5\*a^3\*(sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e))/e^4 + (a^3\*e^2 + 5\*a^3\*e^2/tan(d\*x + c) + 10\*a^3\*e^2/tan(d\*x + c)^2)/(e^4\*(e/tan(d\*x + c))^(5/2))/d

**mupad** [B] time = 1.26, size = 126, normalized size = 0.89

$$\frac{4ea^3 \cot(c+dx)^2 + 2ea^3 \cot(c+dx) + \frac{2ea^3}{5} + \frac{\sqrt{2}a^3 \left( 2 \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}\cot(c+dx)}{2\sqrt{e}}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}\cot(c+dx)}{2\sqrt{e}}\right) + \frac{\sqrt{2}}{2\sqrt{e}} \right)}{de^2(e\cot(c+dx))^{5/2}} + \frac{\sqrt{2}a^3 \left( 2 \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}\cot(c+dx)}{2\sqrt{e}}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e}\cot(c+dx)}{2\sqrt{e}}\right) + \frac{\sqrt{2}}{2\sqrt{e}} \right)}{de^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cot(c + d\*x))^3/(e\*cot(c + d\*x))^(7/2),x)

[Out] ((2\*a^3\*e)/5 + 4\*a^3\*e\*cot(c + d\*x)^2 + 2\*a^3\*e\*cot(c + d\*x))/(d\*e^2\*(e\*cot(c + d\*x))^(5/2)) + (2^(1/2)\*a^3\*(2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2))) + 2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2))) + (2^(1/2)\*(e\*cot(c + d\*x))^(3/2))/(2\*e^(3/2))))/(d\*e^(7/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left( \int \frac{1}{(e \cot(c + dx))^{\frac{7}{2}}} dx + \int \frac{3 \cot(c + dx)}{(e \cot(c + dx))^{\frac{7}{2}}} dx + \int \frac{3 \cot^2(c + dx)}{(e \cot(c + dx))^{\frac{7}{2}}} dx + \int \frac{\cot^3(c + dx)}{(e \cot(c + dx))^{\frac{7}{2}}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
```

```
[Out] a**3*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(3*cot(c + d*x)/(e*cot(c + d*x))**(7/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(7/2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(7/2), x))
```

$$3.22 \quad \int \frac{(a+a \cot(c+dx))^3}{(e \cot(c+dx))^{9/2}} dx$$

Optimal. Leaf size=165

$$\frac{2\sqrt{2} a^3 \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx)+\sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{9/2}} - \frac{4a^3}{de^4 \sqrt{e \cot(c+dx)}} + \frac{4a^3}{3de^3 (e \cot(c+dx))^{3/2}} + \frac{32a^3}{35de^2 (e \cot(c+dx))^{5/2}} + \frac{2(a^3 \cot(c+dx))^{3/2}}{7de(e \cot(c+dx))^{7/2}}$$

[Out]  $32/35*a^3/d/e^2/(e*\cot(d*x+c))^(5/2)+4/3*a^3/d/e^3/(e*\cot(d*x+c))^(3/2)+2/7*(a^3+a^3*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^(7/2)+2*a^3*arctanh(1/2*(e^(1/2)+\cot(d*x+c)*e^(1/2))*2^(1/2)/(e*\cot(d*x+c))^(1/2))*2^(1/2)/d/e^(9/2)-4*a^3/d/e^4/(e*\cot(d*x+c))^(1/2)$

Rubi [A] time = 0.30, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3565, 3628, 3529, 3532, 208}

$$-\frac{4a^3}{de^4 \sqrt{e \cot(c+dx)}} + \frac{4a^3}{3de^3 (e \cot(c+dx))^{3/2}} + \frac{32a^3}{35de^2 (e \cot(c+dx))^{5/2}} + \frac{2\sqrt{2} a^3 \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx)+\sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{9/2}} + \frac{2(a^3 \cot(c+dx))^{3/2}}{7de(e \cot(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(9/2), x]

[Out]  $(2*\text{Sqrt}[2]*a^3*\text{ArcTanh}[(\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])])/(d*e^(9/2)) + (32*a^3)/(35*d*e^2*(e*\text{Cot}[c + d*x])^(5/2)) + (4*a^3)/(3*d*e^3*(e*\text{Cot}[c + d*x])^(3/2)) - (4*a^3)/(d*e^4*\text{Sqrt}[e*\text{Cot}[c + d*x]]) + (2*(a^3 + a^3*\text{Cot}[c + d*x]))/(7*d*e*(e*\text{Cot}[c + d*x])^(7/2))$

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3529

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

### Rule 3532

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]
```

### Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && IntegerQ[2*m]
```

### Rule 3628

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A - C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + a \cot(c + dx))^3}{(e \cot(c + dx))^{9/2}} dx &= \frac{2(a^3 + a^3 \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} - \frac{2 \int \frac{-8a^3 e^2 - 7a^3 e^2 \cot(c+dx) - a^3 e^2 \cot^2(c+dx)}{(e \cot(c+dx))^{7/2}} dx}{7e^3} \\
&= \frac{32a^3}{35de^2(e \cot(c + dx))^{5/2}} + \frac{2(a^3 + a^3 \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} - \frac{2 \int \frac{-7a^3 e^3 + 7a^3 e^3 \cot(c+dx)}{(e \cot(c+dx))^{5/2}} dx}{7e^5} \\
&= \frac{32a^3}{35de^2(e \cot(c + dx))^{5/2}} + \frac{4a^3}{3de^3(e \cot(c + dx))^{3/2}} + \frac{2(a^3 + a^3 \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} - \frac{2 \int \frac{7a^3}{(e \cot(c+dx))^{3/2}} dx}{7e^5} \\
&= \frac{32a^3}{35de^2(e \cot(c + dx))^{5/2}} + \frac{4a^3}{3de^3(e \cot(c + dx))^{3/2}} - \frac{4a^3}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{32a^3}{35de^2(e \cot(c + dx))^{5/2}} + \frac{4a^3}{3de^3(e \cot(c + dx))^{3/2}} - \frac{4a^3}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2(a^3 + a^3 \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{2\sqrt{2} a^3 \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{de^{9/2}} + \frac{32a^3}{35de^2(e \cot(c + dx))^{5/2}} + \frac{4a^3}{3de^3(e \cot(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 2.00, size = 174, normalized size = 1.05

$$\frac{2a^3 \cos(c + dx)(\cot(c + dx) + 1)^3 \left( 35 \cos^2(c + dx) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx)\right) + 35 \cos^2(c + dx) \cot(c + dx) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right) + 5 \operatorname{Hypergeometric2F1}\left[-\frac{7}{4}, 1, -\frac{3}{4}, -\cot(c + dx)^2\right] \operatorname{Sin}[c + dx]^2 + (21 \operatorname{Hypergeometric2F1}[-\frac{5}{4}, 1, -\frac{1}{4}, -\cot(c + dx)^2] \operatorname{Sin}[2(c + dx)]) / 2 \right)}{35d(e \cot(c + dx))^{9/2} (\cos[c + dx] + \sin[c + dx])^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(9/2), x]

[Out] (2\*a^3\*Cos[c + d\*x]\*(1 + Cot[c + d\*x])^3\*(35\*Cos[c + d\*x]^2\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2] + 35\*Cos[c + d\*x]^2\*Cot[c + d\*x]\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + 5\*Hypergeometric2F1[-7/4, 1, -3/4, -Cot[c + d\*x]^2]\*Sin[c + d\*x]^2 + (21\*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d\*x]^2]\*Sin[2\*(c + d\*x)]/2))/(35\*d\*(e\*Cot[c + d\*x])^(9/2)\*(Cos[c + d\*x] + Sin[c + d\*x])^3)

**fricas** [A] time = 0.87, size = 514, normalized size = 3.12

$$\frac{105 \sqrt{2} \left( a^3 e \cos(2 dx + 2 c)^2 + 2 a^3 e \cos(2 dx + 2 c) + a^3 e \right) \log \left( -\frac{\sqrt{2} \sqrt{\frac{e \cos(2 dx + 2 c) + e}{\sin(2 dx + 2 c)}} (\cos(2 dx + 2 c) - \sin(2 dx + 2 c) - 1)}{\sqrt{e}} + 2 \sin(2 dx + 2 c) + 1 \right)}{\sqrt{e}} - 2 \left( 55 a^3 \cos(2 dx + 2 c)^2 + 30 a^3 \cos(2 dx + 2 c) - 85 a^3 + 21 (13 a^3 \cos(2 dx + 2 c) + 7 a^3) \sin(2 dx + 2 c) \right) \sqrt{\frac{e \cos(2 dx + 2 c) + e}{\sin(2 dx + 2 c)}} / \left( d e^5 \cos(2 dx + 2 c)^2 + 2 d e^5 \cos(2 dx + 2 c) + d e^5 \right) + \frac{2}{105} \sqrt{\frac{e \cos(2 dx + 2 c) + e}{\sin(2 dx + 2 c)}} \arctan \left( \frac{1}{\sqrt{2}} \sqrt{\frac{e \cos(2 dx + 2 c) + e}{\sin(2 dx + 2 c)}} \sqrt{\frac{-1}{e}} (\cos(2 dx + 2 c) + \sin(2 dx + 2 c) + 1) / (\cos(2 dx + 2 c) + 1) + (55 a^3 \cos(2 dx + 2 c)^2 + 30 a^3 \cos(2 dx + 2 c) - 85 a^3 + 21 (13 a^3 \cos(2 dx + 2 c) + 7 a^3) \sin(2 dx + 2 c)) \sqrt{\frac{e \cos(2 dx + 2 c) + e}{\sin(2 dx + 2 c)}} / (d e^5 \cos(2 dx + 2 c)^2 + 2 d e^5 \cos(2 dx + 2 c) + d e^5) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(9/2),x, algorithm="fricas")

[Out] [1/105\*(105\*sqrt(2)\*(a^3\*e\*cos(2\*d\*x + 2\*c)^2 + 2\*a^3\*e\*cos(2\*d\*x + 2\*c) + a^3\*e)\*log(-sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1)/sqrt(e) + 2\*sin(2\*d\*x + 2\*c) + 1)/sqrt(e) - 2\*(55\*a^3\*cos(2\*d\*x + 2\*c)^2 + 30\*a^3\*cos(2\*d\*x + 2\*c) - 85\*a^3 + 21\*(13\*a^3\*cos(2\*d\*x + 2\*c) + 7\*a^3)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e^5\*cos(2\*d\*x + 2\*c)^2 + 2\*d\*e^5\*cos(2\*d\*x + 2\*c) + d\*e^5), -2/105\*(105\*sqrt(2)\*(a^3\*e\*cos(2\*d\*x + 2\*c)^2 + 2\*a^3\*e\*cos(2\*d\*x + 2\*c) + a^3\*e)\*sqrt(-1/e)\*arctan(1/2\*sqrt(2)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sqrt(-1/e)\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)/(cos(2\*d\*x + 2\*c) + 1)) + (55\*a^3\*cos(2\*d\*x + 2\*c)^2 + 30\*a^3\*cos(2\*d\*x + 2\*c) - 85\*a^3 + 21\*(13\*a^3\*cos(2\*d\*x + 2\*c) + 7\*a^3)\*sin(2\*d\*x + 2\*c))\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(d\*e^5\*cos(2\*d\*x + 2\*c)^2 + 2\*d\*e^5\*cos(2\*d\*x + 2\*c) + d\*e^5)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(9/2),x, algorithm="giac")

[Out] integrate((a\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(9/2), x)

**maple** [B] time = 0.54, size = 430, normalized size = 2.61

$$\frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2 d e^5} + \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1 \right)}{d e^5} - \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} - 1 \right)}{d e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+\cot(dx+c))^3/(e*\cot(dx+c))^{9/2}, x)$

[Out]  $\frac{1}{2}d*a^3/e^5*(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(dx+c)+(e^2)^{1/4}*(e*\cot(dx+c))^{1/2})^{1/2}*2^{1/2}+(e^2)^{1/2})/(e*\cot(dx+c)-(e^2)^{1/4}*(e*\cot(dx+c))^{1/2})^{1/2}+(e^2)^{1/2})) + 1/d*a^3/e^5*(e^2)^{1/4}*2^{1/2}*arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(dx+c))^{1/2}+1) - 1/d*a^3/e^5*(e^2)^{1/4}*2^{1/2}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(dx+c))^{1/2}+1) - 1/2/d*a^3/e^4*2^{1/2}/(e^2)^{1/4}*ln((e*\cot(dx+c)-(e^2)^{1/4}*(e*\cot(dx+c))^{1/2})^{1/2}*2^{1/2}+(e^2)^{1/2})/(e*\cot(dx+c)+(e^2)^{1/4}*(e*\cot(dx+c))^{1/2})^{1/2}+(e^2)^{1/2})) - 1/d*a^3/e^4*2^{1/2}/(e^2)^{1/4}*arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(dx+c))^{1/2}+1) + 1/d*a^3/e^4*2^{1/2}/(e^2)^{1/4}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(dx+c))^{1/2}+1) + 2/7/d*a^3/e/(e*\cot(dx+c))^{7/2} + 6/5*a^3/d/e^2/(e*\cot(dx+c))^{5/2} - 4*a^3/d/e^4/(e*\cot(dx+c))^{1/2} + 4/3*a^3/d/e^3/(e*\cot(dx+c))^{3/2}$

**maxima [A]** time = 0.85, size = 170, normalized size = 1.03

$$e \left( \frac{105 a^3 \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{e^5} + \frac{2 \left( 15 a^3 e^3 + \frac{63 a^3 e^3}{\tan(dx+c)} + \frac{70 a^3 e^3}{\tan(dx+c)^2} - \frac{210 a^3 e^3}{\tan(dx+c)^3} \right)}{e^5 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{7}{2}}} \right) \frac{1}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+a*\cot(dx+c))^3/(e*\cot(dx+c))^{9/2}, x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{105} * e * (105 * a^3 * (\sqrt{2} * \log(\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx+c)}) + e + e / \tan(dx+c)) / \sqrt{e} - \sqrt{2} * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx+c)}) + e + e / \tan(dx+c)) / \sqrt{e}) / e^5 + 2 * (15 * a^3 * e^3 + 63 * a^3 * e^3 / \tan(dx+c) + 70 * a^3 * e^3 / \tan(dx+c)^2 - 210 * a^3 * e^3 / \tan(dx+c)^3) / (e^5 * (e / \tan(dx+c))^{7/2}) / d$

**mupad [B]** time = 1.92, size = 129, normalized size = 0.78

$$\frac{-4 e a^3 \cot(c+dx)^3 + \frac{4 e a^3 \cot(c+dx)^2}{3} + \frac{6 e a^3 \cot(c+dx)}{5} + \frac{2 e a^3}{7}}{d e^2 (e \cot(c+dx))^{7/2}} + \frac{2 \sqrt{2} a^3 \operatorname{atanh}\left(\frac{32 \sqrt{2} a^6 d e^{9/2} \sqrt{e \cot(c+dx)}}{32 a^6 d e^5 + 32 a^6 d e^5 \cot(c+dx)}\right)}{d e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a + a*\cot(c + dx))^3/(e*\cot(c + dx))^{9/2}, x)$

[Out]  $((2*a^3*e)/7 + (4*a^3*e*\cot(c + dx)^2)/3 - 4*a^3*e*\cot(c + dx)^3 + (6*a^3*e*\cot(c + dx))/5)/(d*e^2*(e*\cot(c + dx))^{7/2}) + (2*2^{1/2})*a^3*\operatorname{atanh}(($

$32*2^{(1/2)}*a^6*d*e^{(9/2)}*(e*\cot(c + d*x))^{(1/2)}/(32*a^6*d*e^5 + 32*a^6*d*e^{5*\cot(c + d*x)})/(d*e^{(9/2)})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left( \int \frac{1}{(e \cot(c + dx))^{\frac{9}{2}}} dx + \int \frac{3 \cot(c + dx)}{(e \cot(c + dx))^{\frac{9}{2}}} dx + \int \frac{3 \cot^2(c + dx)}{(e \cot(c + dx))^{\frac{9}{2}}} dx + \int \frac{\cot^3(c + dx)}{(e \cot(c + dx))^{\frac{9}{2}}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cot(d\*x+c))\*\*3/(e\*cot(d\*x+c))\*\*(9/2),x)

[Out] a\*\*3\*(Integral((e\*cot(c + d\*x))\*\*(-9/2), x) + Integral(3\*cot(c + d\*x)/(e\*cot(c + d\*x))\*\*(9/2), x) + Integral(3\*cot(c + d\*x)\*\*2/(e\*cot(c + d\*x))\*\*(9/2), x) + Integral(cot(c + d\*x)\*\*3/(e\*cot(c + d\*x))\*\*(9/2), x))

$$3.23 \quad \int \frac{(e \cot(c+dx))^{5/2}}{a+a \cot(c+dx)} dx$$

**Optimal.** Leaf size=111

$$\frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} - \frac{2e^2 \sqrt{e \cot(c+dx)}}{ad}$$

[Out]  $e^{(5/2)*\arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a/d-1/2*e^{(5/2)*\arctan(1/2*(e^{(1/2)}-\cot(d*x+c)*e^{(1/2)})*2^{(1/2)/(e*\cot(d*x+c))^{(1/2)})/a/d*2^{(1/2)}-2*e^{2*(e*\cot(d*x+c))^{(1/2)}/a/d}$

**Rubi [A]** time = 0.45, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {3566, 3653, 3532, 205, 3634, 63}

$$-\frac{2e^2 \sqrt{e \cot(c+dx)}}{ad} + \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)/(a + a\*Cot[c + d\*x]),x]

[Out]  $(e^{(5/2)*\text{ArcTan}[\text{Sqrt}[e*\text{Cot}[c + d*x]]/\text{Sqrt}[e]]}/(a*d) - (e^{(5/2)*\text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])}]/(\text{Sqrt}[2]*a*d) - (2*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(a*d)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 3532

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c -

```
d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
EqQ[c^2 - d^2, 0]
```

### Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
+ f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{5/2}}{a + a \cot(c + dx)} dx &= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{ad} - \frac{2 \int \frac{\frac{ae^3}{2} + \frac{1}{2}ae^3 \cot(c+dx) + \frac{1}{2}ae^3 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{a} \\
&= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{ad} - \frac{\int \frac{\frac{a^2e^3}{2} + \frac{1}{2}a^2e^3 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a^3} - \frac{1}{2}e^3 \int \frac{1 + \cot^2(c + dx)}{\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))} dx \\
&= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{ad} - \frac{e^3 \text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c + dx)\right)}{2d} + \frac{(ae^6) \text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c + dx)\right)}{2d} \\
&= -\frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} - \frac{2e^2 \sqrt{e \cot(c + dx)}}{ad} + \frac{e^2 \text{Subst}\left(\int \frac{1}{a + \frac{ax^2}{e}} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} - \frac{2e^2 \sqrt{e \cot(c + dx)}}{ad}
\end{aligned}$$

**Mathematica [A]** time = 0.88, size = 110, normalized size = 0.99

$$\frac{(e \cot(c + dx))^{5/2} \left(4\sqrt{\cot(c + dx)} + \sqrt{2} \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c + dx)}\right) - \sqrt{2} \tan^{-1}\left(\sqrt{2} \sqrt{\cot(c + dx)} + 1\right) - 2\sqrt{\cot(c + dx)}\right)}{2ad \cot^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)/(a + a\*Cot[c + d\*x]),x]

[Out] -1/2\*((Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*ArcTan[Sqrt[Cot[c + d\*x]]] + 4\*Sqrt[Cot[c + d\*x]])\*(e\*Cot[c + d\*x])^(5/2)/(a\*d\*Cot[c + d\*x]^(5/2))

**fricas [A]** time = 0.90, size = 400, normalized size = 3.60

$$\left[ \frac{\sqrt{2} \sqrt{-e} e^2 \log\left(\left(\sqrt{2} \cos(2dx + 2c) + \sqrt{2} \sin(2dx + 2c) - \sqrt{2}\right) \sqrt{-e} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}} - 2e \sin(2dx + 2c) + e\right)}{4ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] [1/4\*(sqrt(2)\*sqrt(-e)\*e^2\*log((sqrt(2)\*cos(2\*d\*x + 2\*c) + sqrt(2)\*sin(2\*d\*x + 2\*c) - sqrt(2))\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)) - 2\*e\*sin(2\*d\*x + 2\*c) + e) + 2\*sqrt(-e)\*e^2\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) + 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) - 8\*e^2\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a\*d), -1/2\*(sqrt(2)\*e^(5/2)\*arctan(-1/2\*(sqrt(2)\*cos(2\*d\*x + 2\*c) - sqrt(2)\*sin(2\*d\*x + 2\*c) + sqrt(2))\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(e\*cos(2\*d\*x + 2\*c) + e) - 2\*e^(5/2)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)) + 4\*e^2\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a\*d)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{5}{2}}}{a \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(5/2)/(a\*cot(d\*x + c) + a), x)

**maple** [B] time = 0.73, size = 394, normalized size = 3.55

$$\frac{2e^2 \sqrt{e \cot(dx + c)}}{da} + \frac{e^{\frac{5}{2}} \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{da} + \frac{e^2 (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2} + \sqrt{e^2}}\right)}{8da} + \frac{e^2 (e^2)^{\frac{1}{4}} \sqrt{2} a}{8da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(5/2)/(a+cot(d\*x+c)\*a),x)

[Out] -2\*e^2\*(e\*cot(d\*x+c))^(1/2)/d/a+e^(5/2)\*arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))/d/a+1/8/d/a\*e^2\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/4/d/a\*e^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/4/d/a\*e^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/8/d/a\*e^3\*2^(1/2)/(e^2)^(1/4)\*ln((e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/4/d/a\*e^3\*2^(1/2)/(e^2)^(1/4)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-



$$1/4/d/a*e^3*2^{(1/2)/(e^2)^{(1/4)}*\arctan(-2^{(1/2)/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1})}$$

**maxima [A]** time = 1.57, size = 134, normalized size = 1.21

$$\left( \frac{e^2 \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{a} + \frac{2e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{a} - \frac{4e\sqrt{\frac{e}{\tan(dx+c)}}}{a} \right) e}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{2} * (e^{5/2} * (\sqrt{2} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e/\tan(dx+c)})) / \sqrt{e})) / \sqrt{e} + \sqrt{2} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e/\tan(dx+c)})) / \sqrt{e})) / \sqrt{e} / a + 2 * e^{3/2} * \arctan(\sqrt{e/\tan(dx+c)}) / \sqrt{e} / a - 4 * e * \sqrt{e/\tan(dx+c)} / a) * e/d$

**mupad [B]** time = 0.68, size = 123, normalized size = 1.11

$$\frac{e^{5/2} \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} - \frac{2e^2 \sqrt{e \cot(c+dx)}}{ad} + \frac{\sqrt{2} e^{5/2} \left( 2 \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2\sqrt{e}}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2\sqrt{e}}\right) + \frac{\sqrt{2} (e \cot(c+dx))^{1/2}}{2\sqrt{e}} \right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c+d\*x))^(5/2)/(a+a\*cot(c+d\*x)),x)

[Out]  $(e^{5/2} * \operatorname{atan}((e * \cot(c + dx))^{1/2} / e^{1/2})) / (a * d) - (2 * e^{5/2} * (e * \cot(c + dx))^{1/2}) / (a * d) + (2^{1/2} * e^{5/2} * (2 * \operatorname{atan}((2^{1/2} * (e * \cot(c + dx))^{1/2}) / (2 * e^{1/2}))) + 2 * \operatorname{atan}((2^{1/2} * (e * \cot(c + dx))^{1/2}) / (2 * e^{1/2})) + (2^{1/2} * (e * \cot(c + dx))^{3/2}) / (2 * e^{3/2}))) / (4 * a * d)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e \cot(c+dx))^{5/2}}{\cot(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x) + 1), x)/a
```

$$3.24 \quad \int \frac{(e \cot(c+dx))^{3/2}}{a+a \cot(c+dx)} dx$$

Optimal. Leaf size=87

$$\frac{e^{3/2} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad}$$

[Out]  $-e^{(3/2)} \arctan((e \cot(d*x+c))^{(1/2)}/e^{(1/2)})/a/d + 1/2 * e^{(3/2)} \operatorname{arctanh}(1/2 * (e^{(1/2)} + \cot(d*x+c) * e^{(1/2)}) * 2^{(1/2)} / (e \cot(d*x+c))^{(1/2)})/a/d * 2^{(1/2)}$

**Rubi [A]** time = 0.24, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {3573, 3532, 208, 3634, 63, 205}

$$\frac{e^{3/2} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} - \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e \cot(c + d*x))^{(3/2)}/(a + a \cot(c + d*x)), x]$

[Out]  $-((e^{(3/2)} \operatorname{ArcTan}[\operatorname{Sqrt}[e \cot(c + d*x)]/\operatorname{Sqrt}[e]])/(a*d)) + (e^{(3/2)} \operatorname{ArcTanh}[(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[e] \cot(c + d*x))/(\operatorname{Sqrt}[2] \operatorname{Sqrt}[e \cot(c + d*x)])]) / (\operatorname{Sqrt}[2] * a*d)$

### Rule 63

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_. + (d_.)(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 205

$\text{Int}[(a_. + (b_.)(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] * \operatorname{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

### Rule 208

$\text{Int}[(a_. + (b_.)(x_.)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \operatorname{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$

Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] :> Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

Rule 3573

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(3/2)/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Dist[1/(c^2 + d^2), Int[Simp[a^2\*c - b^2\*c + 2\*a\*b\*d + (2\*a\*b\*c - a^2\*d + b^2\*d)\*Tan[e + f\*x], x]/Sqrt[a + b\*Tan[e + f\*x]], x], x] + Dist[(b\*c - a\*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f\*x]^2)/(Sqrt[a + b\*Tan[e + f\*x]]\*(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] :> Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

Rubi steps

$$\begin{aligned} \int \frac{(e \cot(c + dx))^{3/2}}{a + a \cot(c + dx)} dx &= \frac{\int \frac{-ae^2 + ae^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{2a^2} + \frac{1}{2} e^2 \int \frac{1 + \cot^2(c + dx)}{\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))} dx \\ &= \frac{e^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c + dx)\right)}{2d} - \frac{e^4 \operatorname{Subst}\left(\int \frac{1}{2a^2 e^4 - ex^2} dx, x, \frac{-ae^2 - ae^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}}\right)}{d} \\ &= \frac{e^{3/2} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} - \frac{e \operatorname{Subst}\left(\int \frac{1}{a + \frac{ax^2}{e}} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\ &= -\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{e^{3/2} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} \end{aligned}$$

**Mathematica** [A] time = 4.05, size = 107, normalized size = 1.23

$$\frac{(e \cot(c + dx))^{3/2} \left( \sqrt{2} \left( \log(-\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)}) - 1 \right) - \log(\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)} + 1) \right)}{4ad \cot^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)/(a + a\*Cot[c + d\*x]),x]

[Out] 
$$-1/4*((e*\text{Cot}[c + d*x])^{3/2}*(4*\text{ArcTan}[\text{Sqrt}[\text{Cot}[c + d*x]]] + \text{Sqrt}[2]*(\text{Log}[-1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] - \text{Cot}[c + d*x]] - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])))/(a*d*\text{Cot}[c + d*x]^{3/2})$$

**fricas** [A] time = 0.79, size = 333, normalized size = 3.83

$$\frac{\sqrt{2} \sqrt{-e} e \arctan\left(\frac{(\sqrt{2} \cos(2dx+2c) + \sqrt{2} \sin(2dx+2c) + \sqrt{2}) \sqrt{-e} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}}}{2(e \cos(2dx+2c)+e)}\right) - \sqrt{-e} e \log\left(\frac{e \cos(2dx+2c)-e \sin(2dx+2c)-2}{\cos(2dx+2c)}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] 
$$[-1/2*(\text{sqrt}(2)*\text{sqrt}(-e)*e*\arctan(1/2*(\text{sqrt}(2)*\cos(2*d*x + 2*c) + \text{sqrt}(2)*\sin(2*d*x + 2*c) + \text{sqrt}(2))*\text{sqrt}(-e)*\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)))/(e*\cos(2*d*x + 2*c) + e) - \text{sqrt}(-e)*e*\log((e*\cos(2*d*x + 2*c) - e*\sin(2*d*x + 2*c) - 2*\text{sqrt}(-e)*\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)))*\sin(2*d*x + 2*c) + e)/(\cos(2*d*x + 2*c) + \sin(2*d*x + 2*c) + 1)))/(a*d), 1/4*(\text{sqrt}(2)*e^{3/2}*\log(-(\text{sqrt}(2)*\cos(2*d*x + 2*c) - \text{sqrt}(2)*\sin(2*d*x + 2*c) - \text{sqrt}(2))*\text{sqrt}(e)*\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)) + 2*e*\sin(2*d*x + 2*c) + e) - 4*e^{3/2}*\arctan(\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))/\text{sqrt}(e)))/(a*d)]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{a \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(3/2)/(a\*cot(d\*x + c) + a), x)

**maple [B]** time = 0.73, size = 368, normalized size = 4.23

$$\frac{e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{da} + \frac{e \left(e^2\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + \left(e^2\right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - \left(e^2\right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8da} + \frac{e \left(e^2\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right)}{4da} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)/(a+cot(d\*x+c)\*a), x)

[Out]  $-e^{(3/2)} * \arctan((e * \cot(d * x + c))^{(1/2)} / e^{(1/2)}) / d / a + 1/8 / d / a * e * (e^2)^{(1/4)} * 2^{(1/2)} * \ln((e * \cot(d * x + c) + (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(d * x + c) - (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) + 1/4 / d / a * e * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} + 1) - 1/4 / d / a * e * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} + 1) - 1/8 / d / a * e^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(d * x + c) - (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(d * x + c) + (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) - 1/4 / d / a * e^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} + 1) + 1/4 / d / a * e^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(d * x + c))^{(1/2)} + 1)$

**maxima [A]** time = 0.70, size = 118, normalized size = 1.36

$$\frac{e \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{a} - \frac{4 \sqrt{e} \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{a} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c)), x, algorithm="maxima")

[Out]  $1/4 * e * (e * (\sqrt{2} * \log(\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(d * x + c)})) + e + e / \tan(d * x + c)) / \sqrt{e} - \sqrt{2} * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(d * x + c)}) + e + e / \tan(d * x + c) / \sqrt{e} / a - 4 * \sqrt{e} * \arctan(\sqrt{e / \tan(d * x + c)} / \sqrt{e}) / a / d$

**mupad [B]** time = 0.49, size = 79, normalized size = 0.91

$$\frac{\sqrt{2} e^{3/2} \operatorname{atanh}\left(\frac{12 \sqrt{2} e^{25/2} \sqrt{e \cot(c+dx)}}{12 e^{13} \cot(c+dx) + 12 e^{13}}\right)}{2 a d} - \frac{e^{3/2} \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x)),x)`

[Out]  $(2^{1/2} * e^{3/2} * \operatorname{atanh}((12 * 2^{1/2} * e^{25/2} * (e * \cot(c + d * x))^{1/2}) / (12 * e^{13} * \cot(c + d * x) + 12 * e^{13}))) / (2 * a * d) - (e^{3/2} * \operatorname{atan}((e * \cot(c + d * x))^{1/2} / e^{1/2})) / (a * d)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e \cot(c+dx))^{\frac{3}{2}}}{\cot(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)`

[Out] `Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x) + 1), x)/a`

$$3.25 \quad \int \frac{\sqrt{e \cot(c+dx)}}{a+a \cot(c+dx)} dx$$

Optimal. Leaf size=87

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad}$$

[Out] arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))\*e^(1/2)/a/d+1/2\*arctan(1/2\*(e^(1/2)-cot(d\*x+c)\*e^(1/2))\*2^(1/2)/(e\*cot(d\*x+c))^(1/2))\*e^(1/2)/a/d\*2^(1/2)

**Rubi [A]** time = 0.22, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3572, 3532, 205, 3634, 63}

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]/(a + a\*Cot[c + d\*x]),x]

[Out] (Sqrt[e]\*ArcTan[Sqrt[e\*Cot[c + d\*x]]/Sqrt[e]]/(a\*d) + (Sqrt[e]\*ArcTan[(Sqrt[e] - Sqrt[e]\*Cot[c + d\*x])/(Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])])/(Sqrt[2]\*a\*d)

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 3532

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] :> Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]



Rule 3572

```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c -
a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[(d*(b*c - a*d
))/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{e \cot(c+dx)}}{a + a \cot(c+dx)} dx &= \frac{\int \frac{ae+ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{2a^2} - \frac{1}{2} e \int \frac{1 + \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a + a \cot(c+dx))} dx \\ &= -\frac{e \operatorname{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c+dx)\right)}{2d} - \frac{e^2 \operatorname{Subst}\left(\int \frac{1}{-2a^2e^2-ex^2} dx, x, \frac{ae-ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}} dx, x, \sqrt{e \cot(c+dx)}\right)}{d} \\ &= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad} + \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ad} \end{aligned}$$

**Mathematica [A]** time = 0.25, size = 98, normalized size = 1.13

$$\frac{\sqrt{e \cot(c+dx)} \left( \sqrt{2} \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c+dx)} \right) - \sqrt{2} \tan^{-1} \left( \sqrt{2} \sqrt{\cot(c+dx)} + 1 \right) + 2 \tan^{-1} \left( \sqrt{\cot(c+dx)} \right) \right)}{2ad\sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]/(a + a\*Cot[c + d\*x]),x]

[Out] ((Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 2\*ArcTan[Sqrt[Cot[c + d\*x]]])\*Sqrt[e\*Cot[c + d\*x]])/(2\*a\*d\*Sqrt[Cot[c + d\*x]])

**fricas** [A] time = 0.84, size = 331, normalized size = 3.80

$$\frac{\sqrt{2} \sqrt{-e} \log\left(-(\sqrt{2} \cos(2dx + 2c) + \sqrt{2} \sin(2dx + 2c) - \sqrt{2})\sqrt{-e} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}} - 2e \sin(2dx + 2c) + e\right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] [1/4\*(sqrt(2)\*sqrt(-e)\*log(-(sqrt(2)\*cos(2\*d\*x + 2\*c) + sqrt(2)\*sin(2\*d\*x + 2\*c) - sqrt(2))\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)) - 2\*e\*sin(2\*d\*x + 2\*c) + e) + 2\*sqrt(-e)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) + 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)))/(a\*d), 1/2\*(sqrt(2)\*sqrt(e)\*arctan(-1/2\*(sqrt(2)\*cos(2\*d\*x + 2\*c) - sqrt(2)\*sin(2\*d\*x + 2\*c) + sqrt(2))\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(e\*cos(2\*d\*x + 2\*c) + e) + 2\*sqrt(e)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)))/(a\*d)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(dx + c)}}{a \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(e\*cot(d\*x + c))/(a\*cot(d\*x + c) + a), x)

**maple** [B] time = 0.81, size = 358, normalized size = 4.11

$$\frac{\arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right) \sqrt{e}}{da} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8da} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(d*x+c))^(1/2)/(a+cot(d*x+c)*a),x)`

[Out]  $\arctan\left(\frac{(e*\cot(d*x+c))^{1/2}/e^{1/2}}{d/a-1/8/d/a*(e^2)^{1/4}*2^{1/2}}\right)*\ln\left(\frac{(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}{(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}\right)-1/4/d/a*(e^2)^{1/4}*2^{1/2}*\arctan\left(\frac{2^{1/2}}{(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1}\right)+1/4/d/a*(e^2)^{1/4}*2^{1/2}*\arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1}\right)-1/8/d/a*e*2^{1/2}/(e^2)^{1/4}*\ln\left(\frac{(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}{(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}\right)-1/4/d/a*e*2^{1/2}/(e^2)^{1/4}*\arctan\left(\frac{2^{1/2}}{(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1}\right)+1/4/d/a*e*2^{1/2}/(e^2)^{1/4}*\arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1}\right)$

**maxima [A]** time = 0.81, size = 113, normalized size = 1.30

$$\frac{e \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} - 2 \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right) \right)}{a \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/2*e*((\sqrt{2}*\arctan(1/2*\sqrt{2})*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(dx+c)}))/\sqrt{e}))/\sqrt{e} + \sqrt{2}*\arctan(-1/2*\sqrt{2})*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(dx+c)})/\sqrt{e}))/\sqrt{e}))/a - 2*\arctan(\sqrt{e/\tan(dx+c)})/\sqrt{e}))/a*\sqrt{e}))/d$

**mupad [B]** time = 0.37, size = 102, normalized size = 1.17

$$\frac{\sqrt{e} \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{a d} - \frac{\sqrt{2} \sqrt{e} \left( 2 \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2 \sqrt{e}}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2 \sqrt{e}} + \frac{\sqrt{2} (e \cot(c+dx))^{3/2}}{2 e^{3/2}}\right) \right)}{4 a d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x)),x)`

[Out]  $(e^{1/2}*\operatorname{atan}((e*\cot(c + d*x))^{1/2}/e^{1/2}))/a*d - (2^{1/2}*e^{1/2}*(2*\operatorname{atan}((2^{1/2}*(e*\cot(c + d*x))^{1/2})/(2*e^{1/2}))) + 2*\operatorname{atan}((2^{1/2}*(e*\cot(c + d*x))^{1/2})/(2*e^{1/2})) + (2^{1/2}*(e*\cot(c + d*x))^{3/2})/(2*e^{3/2}))))/(4*a*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e \cot(c+dx)}}{\cot(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)/(a+a\*cot(d\*x+c)),x)

[Out] Integral(sqrt(e\*cot(c + d\*x))/(cot(c + d\*x) + 1), x)/a

$$3.26 \quad \int \frac{1}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}(\cot(c+dx)+1)}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

[Out]  $-\arctan((e \cot(dx+c))^{1/2}/e^{1/2})/a/d/e^{1/2}-1/2*\operatorname{arctanh}(1/2*(1+\cot(dx+c))*e^{1/2}*2^{1/2}/(e \cot(dx+c))^{1/2})/a/d*2^{1/2}/e^{1/2}$

**Rubi [A]** time = 0.22, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {3574, 3532, 208, 3634, 63, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}(\cot(c+dx)+1)}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])),x]

[Out]  $-(\operatorname{ArcTan}[\operatorname{Sqrt}[e \cot[c + dx]]/\operatorname{Sqrt}[e]]/(a*d*\operatorname{Sqrt}[e])) - \operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*(1 + \cot[c + dx]))/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e \cot[c + dx]])]/(\operatorname{Sqrt}[2]*a*d*\operatorname{Sqrt}[e])$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3574

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/(c^2 + d^2), Int[(a + b\*Tan[e + f\*x])^m\*(c - d\*Tan[e + f\*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[((a + b\*Tan[e + f\*x])^m\*(1 + Tan[e + f\*x]^2))/(c + d\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx &= \frac{1}{2} \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx + \frac{\int \frac{a-a \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{2a^2} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{2a^2-ex^2} dx, x, \frac{a+a \cot(c+dx)}{\sqrt{e \cot(c+dx)}}\right)}{d} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}(1+\cot(c+dx))}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}} dx, x, \sqrt{e \cot(c+dx)}\right)}{de} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}(1+\cot(c+dx))}{\sqrt{2}\sqrt{e \cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}} \end{aligned}$$

**Mathematica** [A] time = 0.52, size = 107, normalized size = 1.29

$$\frac{\sqrt{\cot(c+dx)}\left(\sqrt{2}\left(\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)-\log\left(-\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}-1\right)\right)+4\right)}{4ad\sqrt{e \cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])),x]

[Out] 
$$-1/4*(\text{Sqrt}[\text{Cot}[c + d*x]]*(4*\text{ArcTan}[\text{Sqrt}[\text{Cot}[c + d*x]]] + \text{Sqrt}[2]*(-\text{Log}[-1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] - \text{Cot}[c + d*x]] + \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])))/(a*d*\text{Sqrt}[e*\text{Cot}[c + d*x]])$$

**fricas** [A] time = 0.62, size = 321, normalized size = 3.87

$$\frac{\sqrt{2} \sqrt{-e} \arctan\left(\frac{\sqrt{2} \sqrt{-e} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} (\cos(2dx+2c)+\sin(2dx+2c)+1)}{2(e \cos(2dx+2c)+e)}\right) - \sqrt{-e} \log\left(\frac{e \cos(2dx+2c)-e \sin(2dx+2c)+2 \sqrt{-e} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}}}{\cos(2dx+2c)+\sin(2dx+2c)}\right)}{2ade}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{2}*(\text{sqrt}(2)*\text{sqrt}(-e)*\text{arctan}(1/2*\text{sqrt}(2)*\text{sqrt}(-e)*\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))*(\cos(2*d*x + 2*c) + \sin(2*d*x + 2*c) + 1)/(e*\cos(2*d*x + 2*c) + e)) - \text{sqrt}(-e)*\log((e*\cos(2*d*x + 2*c) - e*\sin(2*d*x + 2*c) + 2*\text{sqrt}(-e)*\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))*\sin(2*d*x + 2*c) + e)/(\cos(2*d*x + 2*c) + \sin(2*d*x + 2*c) + 1)))/(a*d*e), 1/4*(\text{sqrt}(2)*\text{sqrt}(e)*\log(\text{sqrt}(2)*\text{sqrt}(e)*\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))*(\cos(2*d*x + 2*c) - \sin(2*d*x + 2*c) - 1) + 2*e*\sin(2*d*x + 2*c) + e) - 4*\text{sqrt}(e)*\text{arctan}(\text{sqrt}((e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c))/\text{sqrt}(e)))/(a*d*e)]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx+c) + a)\sqrt{e \cot(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)\*sqrt(e\*cot(d\*x + c))), x)

**maple** [B] time = 0.88, size = 365, normalized size = 4.40

$$\frac{\arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8dae} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{4dae} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*cot(d*x+c))^(1/2)/(a+cot(d*x+c)*a),x)`

[Out] 
$$-\arctan\left(\frac{(e \cot(dx+c))^{1/2}/e^{1/2}}{a/d/e^{1/2}-1/8/d/a/e*(e^2)^{1/4}*2^{1/2}+(e^2)^{1/2}}\right) + \frac{1}{2} \ln\left(\frac{(e \cot(dx+c)+(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}{(e \cot(dx+c)-(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}\right) - \frac{1}{4} \frac{d}{a/e*(e^2)^{1/4}*2^{1/2}} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1}\right) + \frac{1}{4} \frac{d}{a/e*(e^2)^{1/4}*2^{1/2}} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1}\right) + \frac{1}{8} \frac{d}{a*2^{1/2}} \frac{1}{(e^2)^{1/4}} \ln\left(\frac{(e \cot(dx+c)-(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}{(e \cot(dx+c)+(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2})}\right) + \frac{1}{4} \frac{d}{a*2^{1/2}} \frac{1}{(e^2)^{1/4}} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1}\right) - \frac{1}{4} \frac{d}{a*2^{1/2}} \frac{1}{(e^2)^{1/4}} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1}\right)$$

**maxima** [A] time = 0.63, size = 120, normalized size = 1.45

$$\frac{e \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right) + 4 \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{ae} + \frac{4 \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{ae^{\frac{3}{2}}}$$


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$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-\frac{1}{4} e \left( \frac{\sqrt{2} \log(\sqrt{2} \sqrt{e} \sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))}{\sqrt{e}} - \frac{\sqrt{2} \log(-\sqrt{2} \sqrt{e} \sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))}{\sqrt{e}} \right) / (a e) + 4 \arctan\left(\frac{\sqrt{e/\tan(dx+c)}}{\sqrt{e}}\right) / (a e^{3/2}) / d$$

**mupad** [B] time = 0.52, size = 79, normalized size = 0.95

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ad \sqrt{e}} - \frac{\sqrt{2} \operatorname{atanh}\left(\frac{12 \sqrt{2} e^{9/2} \sqrt{e \cot(c+dx)}}{12 e^5 \cot(c+dx) + 12 e^5}\right)}{2ad \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((e*cot(c+d*x))^(1/2)*(a+a*cot(c+d*x))),x)`

[Out] 
$$-\operatorname{atan}\left(\frac{(e \cot(c+dx))^{1/2}/e^{1/2}}{a*d*e^{1/2}}\right) - \frac{2^{1/2} \operatorname{atanh}\left(\frac{12*2^{1/2}*e^{9/2}*(e \cot(c+dx))^{1/2}}{12*e^5*\cot(c+dx)+12*e^5}\right)}{2*a*d*e^{1/2}}$$



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \cot(c+dx)} \cot(c+dx) + \sqrt{e \cot(c+dx)}} dx$$


---


$$a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(1/2)/(a+a\*cot(d\*x+c)), x)

[Out] Integral(1/(sqrt(e\*cot(c + d\*x))\*cot(c + d\*x) + sqrt(e\*cot(c + d\*x))), x)/a

$$3.27 \quad \int \frac{1}{(e \cot(c+dx))^{3/2}(a+a \cot(c+dx))} dx$$

**Optimal.** Leaf size=111

$$\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{3/2}} + \frac{2}{ade \sqrt{e \cot(c+dx)}}$$

[Out] arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)-1/2\*arctan(1/2\*(e^(1/2)-cot(d\*x+c)\*e^(1/2))\*2^(1/2)/(e\*cot(d\*x+c))^(1/2))/a/d/e^(3/2)\*2^(1/2)+2/a/d/e/(e\*cot(d\*x+c))^(1/2)

**Rubi [A]** time = 0.45, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$ , Rules used = {3569, 3653, 3532, 205, 3634, 63}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{3/2}} + \frac{2}{ade \sqrt{e \cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])),x]

[Out] ArcTan[Sqrt[e\*Cot[c + d\*x]]/Sqrt[e]]/(a\*d\*e^(3/2)) - ArcTan[(Sqrt[e] - Sqrt[e]\*Cot[c + d\*x])/(Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])]/(Sqrt[2]\*a\*d\*e^(3/2)) + 2/(a\*d\*e\*Sqrt[e\*Cot[c + d\*x]])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 3532

Int[((c\_) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]]], x\_Symbol] :> Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c -

$d \cdot \tan[e + f \cdot x] / \sqrt{b \cdot \tan[e + f \cdot x]}$ , x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3569

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), x] + Dist[1/((m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3634

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3653

Int[(((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2)/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[((c + d\*Tan[e + f\*x])^n\*(1 + Tan[e + f\*x]^2))/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c + dx))^{3/2}(a + a \cot(c + dx))} dx &= \frac{2}{ade\sqrt{e \cot(c + dx)}} + \frac{2 \int \frac{-\frac{ae^2}{2} - \frac{1}{2}ae^2 \cot(c+dx) - \frac{1}{2}ae^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{ae^3} \\
&= \frac{2}{ade\sqrt{e \cot(c + dx)}} + \frac{\int \frac{-\frac{1}{2}a^2e^2 - \frac{1}{2}a^2e^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a^3e^3} - \frac{\int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{2e} \\
&= \frac{2}{ade\sqrt{e \cot(c + dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c + dx)\right)}{2de} - \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}} dx, x, \cot(c + dx)\right)}{2e} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{3/2}} + \frac{2}{ade\sqrt{e \cot(c + dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}} dx, x, \cot(c + dx)\right)}{2e} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{3/2}} + \frac{2}{ade\sqrt{e \cot(c + dx)}}
\end{aligned}$$

**Mathematica [A]** time = 2.03, size = 176, normalized size = 1.59

$$\frac{2 \sin^4(c + dx) \left( \cot^4(c + dx) + 2 \cot^2(c + dx) - \sqrt{2} \cot^{\frac{5}{2}}(c + dx) \csc^2(2(c + dx)) \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right) \right) + \sqrt{2} \sqrt{e} \cot(c + dx)}{ade\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])),x]

[Out] (2\*(1 + 2\*Cot[c + d\*x]^2 + Cot[c + d\*x]^4 - Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*Cot[c + d\*x]^(5/2)\*Csc[2\*(c + d\*x)]^2 + Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*Cot[c + d\*x]^(5/2)\*Csc[2\*(c + d\*x)]^2 + 2\*ArcTan[Sqrt[Cot[c + d\*x]]]\*Cot[c + d\*x]^(5/2)\*Csc[2\*(c + d\*x)]^2)\*Sin[c + d\*x]^4)/(a\*d\*e\*Sqrt[e\*Cot[c + d\*x]])

**fricas [B]** time = 1.13, size = 472, normalized size = 4.25

$$\left[ \frac{\sqrt{2} \sqrt{-e} (\cos(2dx + 2c) + 1) \log\left(-\sqrt{2} \sqrt{-e} \sqrt{\frac{e \cos(2dx+2c)+e}{\sin(2dx+2c)}} (\cos(2dx + 2c) + \sin(2dx + 2c) - 1) - 2e \sin(2c)\right)}{ade\sqrt{e}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*(\sqrt{2}*\sqrt{-e}*(\cos(2*d*x + 2*c) + 1)*\log(-\sqrt{2}*\sqrt{-e}*\sqrt{(e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)}*(\cos(2*d*x + 2*c) + \sin(2*d*x + 2*c) - 1) - 2*e*\sin(2*d*x + 2*c) + e) + 2*\sqrt{-e}*(\cos(2*d*x + 2*c) + 1)*\log((e*\cos(2*d*x + 2*c) - e*\sin(2*d*x + 2*c) - 2*\sqrt{-e}*\sqrt{(e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)}*\sin(2*d*x + 2*c) + e)/(\cos(2*d*x + 2*c) + \sin(2*d*x + 2*c) + 1)) - 8*\sqrt{(e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)}*\sin(2*d*x + 2*c))/(a*d*e^2*\cos(2*d*x + 2*c) + a*d*e^2), -1/2*(\sqrt{2}*\sqrt{e}*(\cos(2*d*x + 2*c) + 1)*\arctan(-1/2*\sqrt{2}*\sqrt{e}*\sqrt{(e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)}*(\cos(2*d*x + 2*c) - \sin(2*d*x + 2*c) + 1)/(e*\cos(2*d*x + 2*c) + e)) - 2*\sqrt{e}*(\cos(2*d*x + 2*c) + 1)*\arctan(\sqrt{(e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)})/\sqrt{e}) - 4*\sqrt{(e*\cos(2*d*x + 2*c) + e)/\sin(2*d*x + 2*c)}*\sin(2*d*x + 2*c))/(a*d*e^2*\cos(2*d*x + 2*c) + a*d*e^2)] \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx + c) + a) (e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(3/2)), x)

**maple** [B] time = 0.81, size = 394, normalized size = 3.55

$$\frac{\arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{a d e^{\frac{3}{2}}} + \frac{\left(e^2\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + \left(e^2\right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - \left(e^2\right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8 d a e^2} + \frac{\left(e^2\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}} + 1\right)}{4 d a e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(3/2)/(a+cot(d\*x+c)\*a),x)

[Out] 
$$\begin{aligned} & \arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a/d/e^{(3/2)}+1/8/d/a/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))+1/4/d/a/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/4/d/a/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/8/d/a/e^2*(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*\sqrt{e*\cot(dx+c)}*\sqrt{2}+\sqrt{e^2})) \end{aligned}$$

$\cot(dx+c)^{1/2} \cdot 2^{1/2} + (e^2)^{1/2} / (e \cot(dx+c) + (e^2)^{1/4}) \cdot (e \cot(dx+c))^{1/2} \cdot 2^{1/2} + (e^2)^{1/2} + 1/4/d/a/e \cdot 2^{1/2} / (e^2)^{1/4} \cdot \arctan(2^{1/2} / (e^2)^{1/4} \cdot (e \cot(dx+c))^{1/2} + 1) - 1/4/d/a/e \cdot 2^{1/2} / (e^2)^{1/4} \cdot \arctan(-2^{1/2} / (e^2)^{1/4} \cdot (e \cot(dx+c))^{1/2} + 1) + 2/a/d/e / (e \cot(dx+c))^{1/2}$

**maxima [A]** time = 1.64, size = 136, normalized size = 1.23

$$\frac{e \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + 2 \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right) + \frac{4}{ae^2 \sqrt{\frac{e}{\tan(dx+c)}}} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*e\*((sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e))/(a\*e^2) + 2\*arctan(sqrt(e/tan(d\*x + c))/sqrt(e))/(a\*e^(5/2)) + 4/(a\*e^2\*sqrt(e/tan(d\*x + c))))/d

**mupad [B]** time = 0.64, size = 123, normalized size = 1.11

$$\frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{\operatorname{atan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}} + \frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right) + 2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}} + \frac{\sqrt{2}(e\cot(c+dx))^{3/2}}{2e^{3/2}}\right)\right)}{4ade^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c + d\*x))^(3/2)\*(a + a\*cot(c + d\*x))),x)

[Out] 2/(a\*d\*e\*(e\*cot(c + d\*x))^(1/2)) + atan((e\*cot(c + d\*x))^(1/2)/e^(1/2))/(a\*d\*e^(3/2)) + (2^(1/2)\*(2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2))) + 2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2)) + (2^(1/2)\*(e\*cot(c + d\*x))^(3/2))/(2\*e^(3/2)))))/(4\*a\*d\*e^(3/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e \cot(c+dx))^{\frac{3}{2}} \cot(c+dx) + (e \cot(c+dx))^{\frac{3}{2}}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

```
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2))  
, x)/a
```

$$3.28 \quad \int \frac{1}{(e \cot(c+dx))^{5/2}(a+a \cot(c+dx))} dx$$

**Optimal.** Leaf size=135

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{5/2}} - \frac{2}{ade^2 \sqrt{e \cot(c+dx)}} + \frac{2}{3ade(e \cot(c+dx))^{3/2}}$$

[Out]  $-\arctan((e \cot(d*x+c))^{(1/2)}/e^{(1/2)})/a/d/e^{(5/2)}+2/3/a/d/e/(e \cot(d*x+c))^{(3/2)}+1/2*\operatorname{arctanh}(1/2*(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)})*2^{(1/2)}/(e \cot(d*x+c))^{(1/2)})/a/d/e^{(5/2)}*2^{(1/2)}-2/a/d/e^2/(e \cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.54, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3569, 3649, 12, 16, 3573, 3532, 208, 3634, 63, 205}

$$-\frac{2}{ade^2 \sqrt{e \cot(c+dx)}} - \frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{5/2}} + \frac{2}{3ade(e \cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]`

[Out]  $-(\operatorname{ArcTan}[\operatorname{Sqrt}[e \cot[c + d*x]]/\operatorname{Sqrt}[e]]/(a*d*e^{(5/2)})) + \operatorname{ArcTanh}[(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[e]*\cot[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e \cot[c + d*x]])]/(\operatorname{Sqrt}[2]*a*d*e^{(5/2)}) + 2/(3*a*d*e*(e \cot[c + d*x])^{(3/2)}) - 2/(a*d*e^2*\operatorname{Sqrt}[e \cot[c + d*x]])$

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

### Rule 16

`Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`



Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

Rule 3569

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), x] + Dist[1/((m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m] || (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3573

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(3/2)/((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/(c^2 + d^2), Int[Simp[a^2\*c - b^2\*c + 2\*a\*b\*d + (2\*a\*b\*c - a^2\*d + b^2\*d)\*Tan[e + f\*x], x]/Sqrt[a + b\*Tan[e + f\*x]], x], x] + Dist[(b\*c - a\*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f\*x]^2)/(Sqrt[a + b\*Tan[e + f\*x]]\*(c + d\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

## Rule 3649

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_.)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)] + (C_.)*tan[(e_.)
+ (f_.)*(x_.)]^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

## Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c + dx))^{5/2} (a + a \cot(c + dx))} dx &= \frac{2}{3ade(e \cot(c + dx))^{3/2}} + \frac{2 \int \frac{-\frac{3ae^2}{2} - \frac{3}{2}ae^2 \cot(c+dx) - \frac{3}{2}ae^2 \cot^2(c+dx)}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))} dx}{3ae^3} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2}{ade^2 \sqrt{e \cot(c + dx)}} + \frac{4 \int \frac{3a^2 e^4 \cot^2(c+dx)}{4 \sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{3a^2 e^6} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2}{ade^2 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{\cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{e^2} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2}{ade^2 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{(e \cot(c+dx))^{3/2}}{a+a \cot(c+dx)} dx}{e^4} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2}{ade^2 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{-ae^2 + ae^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{2a^2 e^4} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2}{ade^2 \sqrt{e \cot(c + dx)}} - \frac{\text{Subst}\left(\int \frac{1}{2a^2 e^4 - ex^2} dx, d\right)}{d} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e \cot(c+dx)}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{5/2}} + \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2}{ade^2 \sqrt{e \cot(c + dx)}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e \cot(c+dx)}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} ade^{5/2}} + \frac{2}{3ade(e \cot(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 1.33, size = 131, normalized size = 0.97

$$\frac{8(\tan(c + dx) - 3) - 3\sqrt{2}\sqrt{\cot(c + dx)} \left( \log(-\cot(c + dx) + \sqrt{2}\sqrt{\cot(c + dx)} - 1) - \log(\cot(c + dx) + \sqrt{2}\sqrt{\cot(c + dx)}) \right)}{12ade^2\sqrt{e\cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])),x]

[Out] (-12\*ArcTan[Sqrt[Cot[c + d\*x]]]\*Sqrt[Cot[c + d\*x]] - 3\*Sqrt[2]\*Sqrt[Cot[c + d\*x]]\*(Log[-1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] - Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]) + 8\*(-3 + Tan[c + d\*x]))/(12\*a\*d\*e^2\*Sqrt[e\*Cot[c + d\*x]])

**fricas [A]** time = 0.94, size = 500, normalized size = 3.70

$$\left[ \frac{3\sqrt{2}\sqrt{-e}(\cos(2dx + 2c) + 1) \arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)+\sin(2dx+2c)+1)}{2(e\cos(2dx+2c)+e)}\right) + 3\sqrt{-e}(\cos(2dx + 2c) + 1) \arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)-\sin(2dx+2c)-1)}{2(e\cos(2dx+2c)+e)}\right)}{6(ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c)),x, algorithm="fricas")

[Out] [-1/6\*(3\*sqrt(2)\*sqrt(-e)\*(cos(2\*d\*x + 2\*c) + 1)\*arctan(1/2\*sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e)) + 3\*sqrt(-e)\*(cos(2\*d\*x + 2\*c) + 1)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) + 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) + 4\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + 3\*sin(2\*d\*x + 2\*c) - 1))/(a\*d\*e^3\*cos(2\*d\*x + 2\*c) + a\*d\*e^3), 1/12\*(3\*sqrt(2)\*sqrt(e)\*(cos(2\*d\*x + 2\*c) + 1)\*log(-sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1) + 2\*e\*sin(2\*d\*x + 2\*c) + e) - 12\*sqrt(e)\*(cos(2\*d\*x + 2\*c) + 1)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)) - 8\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + 3\*sin(2\*d\*x + 2\*c) - 1))/(a\*d\*e^3\*cos(2\*d\*x + 2\*c) + a\*d\*e^3)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx + c) + a)(e \cot(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(5/2)), x)

**maple [B]** time = 0.76, size = 416, normalized size = 3.08

$$\frac{\arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{ad e^{\frac{5}{2}}} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8da e^3} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{4da e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(5/2)/(a+cot(d\*x+c)\*a),x)

[Out]  $-\arctan\left(\frac{e \cot(dx+c)}{e^{1/2}}\right) / a / d / e^{5/2} + 1/8 / d / a / e^3 * (e^2)^{1/4} * 2^{1/2} * \ln\left(\frac{e \cot(dx+c) + (e^2)^{1/4} * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}}{e \cot(dx+c) - (e^2)^{1/4} * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}}\right) + 1/4 / d / a / e^3 * (e^2)^{1/4} * 2^{1/2} * \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4}} * (e \cot(dx+c))^{1/2} + 1\right) - 1/4 / d / a / e^3 * (e^2)^{1/4} * 2^{1/2} * \arctan\left(-\frac{2^{1/2}}{(e^2)^{1/4}} * (e \cot(dx+c))^{1/2} + 1\right) - 1/8 / d / a / e^2 * 2^{1/2} / (e^2)^{1/4} * \ln\left(\frac{e \cot(dx+c) - (e^2)^{1/4} * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}}{e \cot(dx+c) + (e^2)^{1/4} * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}}\right) - 1/4 / d / a / e^2 * 2^{1/2} / (e^2)^{1/4} * \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4}} * (e \cot(dx+c))^{1/2} + 1\right) + 1/4 / d / a / e^2 * 2^{1/2} / (e^2)^{1/4} * \arctan\left(-\frac{2^{1/2}}{(e^2)^{1/4}} * (e \cot(dx+c))^{1/2} + 1\right) + 2/3 / a / d / e / (e \cot(dx+c))^{3/2} - 2/a / d / e^2 / (e \cot(dx+c))^{1/2}$

**maxima [A]** time = 0.44, size = 154, normalized size = 1.14

$$e \left[ \frac{3 \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{ae^3} - \frac{12 \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{ae^{\frac{7}{2}}} + \frac{8 \left( e - \frac{3e}{\tan(dx+c)} \right)}{ae^3 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}}} \right] / 12d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c)),x, algorithm="maxima")

[Out]  $1/12 * e * (3 * (\sqrt{2} * \log(\sqrt{2} * \sqrt{e} * \sqrt{e/\tan(dx+c)}) + e + e/\tan(dx+c))/\sqrt{e} - \sqrt{2} * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e/\tan(dx+c)}) + e + e/\tan(dx+c))/\sqrt{e}) / (a * e^3) - 12 * \arctan(\sqrt{e/\tan(dx+c)}) / \sqrt{e} / (a * e^{7/2}) + 8 * (e - 3 * e/\tan(dx+c)) / (a * e^3 * (e/\tan(dx+c))^{3/2}) / d$

**mupad [B]** time = 0.93, size = 132, normalized size = 0.98

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{12\sqrt{2}a^3d^3e^{21/2}\sqrt{e\cot(c+dx)}}{12a^3d^3e^{11}+12a^3d^3e^{11}\cot(c+dx)}\right)}{2ad e^{5/2}} - \frac{\operatorname{atan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad e^{5/2}} - \frac{\frac{2\cot(c+dx)}{e} - \frac{2}{3e}}{ad(e\cot(c+dx))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))),x)`

[Out]  $(2^{1/2} \operatorname{atanh}((12 \cdot 2^{1/2} \cdot a^3 \cdot d^3 \cdot e^{21/2} \cdot (e \cdot \cot(c + d \cdot x))^{1/2})) / (12 \cdot a^3 \cdot d^3 \cdot e^{11} + 12 \cdot a^3 \cdot d^3 \cdot e^{11} \cdot \cot(c + d \cdot x))) / (2 \cdot a \cdot d \cdot e^{5/2}) - \operatorname{atan}((e \cdot \cot(c + d \cdot x))^{1/2} / e^{1/2}) / (a \cdot d \cdot e^{5/2}) - ((2 \cdot \cot(c + d \cdot x)) / e - 2 / (3 \cdot e)) / (a \cdot d \cdot (e \cdot \cot(c + d \cdot x))^{3/2})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e \cot(c+dx))^{5/2} \cot(c+dx) + (e \cot(c+dx))^{5/2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)`

[Out] `Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)), x)/a`

$$3.29 \quad \int \frac{(e \cot(c+dx))^{5/2}}{(a+a \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=281

$$\frac{e^{5/2} \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} + \frac{e^{5/2} \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} - 3e^5$$

[Out]  $-3/2 * e^{(5/2)} * \arctan((e * \cot(d * x + c))^{(1/2)} / e^{(1/2)}) / a^2 / d - 1/4 * e^{(5/2)} * \arctan(1 - 2^{(1/2)} * (e * \cot(d * x + c))^{(1/2)} / e^{(1/2)}) / a^2 / d * 2^{(1/2)} + 1/4 * e^{(5/2)} * \arctan(1 + 2^{(1/2)} * (e * \cot(d * x + c))^{(1/2)} / e^{(1/2)}) / a^2 / d * 2^{(1/2)} - 1/8 * e^{(5/2)} * \ln(e^{(1/2)} + \cot(d * x + c) * e^{(1/2)} - 2^{(1/2)} * (e * \cot(d * x + c))^{(1/2)}) / a^2 / d * 2^{(1/2)} + 1/8 * e^{(5/2)} * \ln(e^{(1/2)} + \cot(d * x + c) * e^{(1/2)} + 2^{(1/2)} * (e * \cot(d * x + c))^{(1/2)}) / a^2 / d * 2^{(1/2)} + 1/2 * e^2 * (e * \cot(d * x + c))^{(1/2)} / d / (a^2 + a^2 * \cot(d * x + c))$

**Rubi [A]** time = 0.54, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$ , Rules used = {3565, 3653, 12, 3476, 329, 211, 1165, 628, 1162, 617, 204, 3634, 63, 205}

$$\frac{e^2 \sqrt{e \cot(c+dx)}}{2d(a^2 \cot(c+dx) + a^2)} - \frac{e^{5/2} \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} + \frac{e^{5/2} \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)/(a + a\*Cot[c + d\*x])^2,x]

[Out]  $(-3 * e^{(5/2)} * \text{ArcTan}[\text{Sqrt}[e * \text{Cot}[c + d * x]] / \text{Sqrt}[e]]) / (2 * a^2 * d) - (e^{(5/2)} * \text{ArcTan}[1 - (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d * x]]) / \text{Sqrt}[e]]) / (2 * \text{Sqrt}[2] * a^2 * d) + (e^{(5/2)} * \text{ArcTan}[1 + (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d * x]]) / \text{Sqrt}[e]]) / (2 * \text{Sqrt}[2] * a^2 * d) + (e^2 * \text{Sqrt}[e * \text{Cot}[c + d * x]]) / (2 * d * (a^2 + a^2 * \text{Cot}[c + d * x])) - (e^{(5/2)} * \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] * \text{Cot}[c + d * x] - \text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d * x]])] / (4 * \text{Sqrt}[2] * a^2 * d) + (e^{(5/2)} * \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] * \text{Cot}[c + d * x] + \text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d * x]])] / (4 * \text{Sqrt}[2] * a^2 * d)$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{LtQ}\{-1, m, 0\} \&\& \text{LeQ}\{-1, n, 0\} \&\& \text{LeQ}\{\text{Denominator}[n], \text{Denominator}[m]\} \&\& \text{IntLinearQ}\{a, b, c, d, m, n, x\}$

### Rule 204

$\text{Int}\{((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol\} \rightarrow -\text{Simp}\{\text{ArcTan}\{(\text{Rt}\{-b, 2\}*x)/\text{Rt}\{-a, 2\}\}/(\text{Rt}\{-a, 2\}*\text{Rt}\{-b, 2\}), x\} /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}\{a/b\} \&\& (\text{LtQ}\{a, 0\} \parallel \text{LtQ}\{b, 0\})$

### Rule 205

$\text{Int}\{((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol\} \rightarrow \text{Simp}\{(\text{Rt}\{a/b, 2\}*\text{ArcTan}\{x/\text{Rt}\{a/b, 2\}\})/a, x\} /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}\{a/b\}$

### Rule 211

$\text{Int}\{((a_) + (b_)*(x_)^4)^{-1}, x\_Symbol\} \rightarrow \text{With}\{r = \text{Numerator}\{\text{Rt}\{a/b, 2\}\}, s = \text{Denominator}\{\text{Rt}\{a/b, 2\}\}\}, \text{Dist}\{1/(2*r), \text{Int}\{(r - s*x^2)/(a + b*x^4), x\}, x\} + \text{Dist}\{1/(2*r), \text{Int}\{(r + s*x^2)/(a + b*x^4), x\}, x\} /; \text{FreeQ}\{a, b\}, x\} \&\& (\text{GtQ}\{a/b, 0\} \parallel (\text{PosQ}\{a/b\} \&\& \text{AtomQ}\{\text{SplitProduct}\{\text{SumBaseQ}, a\}\} \&\& \text{AtomQ}\{\text{SplitProduct}\{\text{SumBaseQ}, b\}\}))$

### Rule 329

$\text{Int}\{((c_)*(x_))^{(m_)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol\} \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}\{k/c, \text{Subst}\{(\text{Int}\{x^{(k*(m+1)-1}*(a + (b*x^{(k*n)}/c^n)^p, x\}, x, (c*x)^{(1/k)\}, x\} /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}\{n, 0\} \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}\{a, b, c, n, m, p, x\}$

### Rule 617

$\text{Int}\{((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol\} \rightarrow \text{With}\{q = 1 - 4*S\text{implify}\{(a*c)/b^2\}\}, \text{Dist}\{-2/b, \text{Subst}\{(\text{Int}\{1/(q - x^2), x\}, x, 1 + (2*c*x)/b\}, x\} /; \text{RationalQ}[q] \&\& (\text{EqQ}\{q^2, 1\} \parallel \text{!RationalQ}\{b^2 - 4*a*c\}) /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}\{b^2 - 4*a*c, 0\}$

### Rule 628

$\text{Int}\{((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x\_Symbol\} \rightarrow \text{Simp}\{(\text{d*Log}\{\text{RemoveContent}\{a + b*x + c*x^2, x\}\})/b, x\} /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}\{2*c*d - b*e, 0\}$

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 3476

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

### Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)^(2*(a + b*Tan[e + f*x])^(
m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3653

```
Int((((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
```



+ f\*x]^2))/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e \cot(c + dx))^{5/2}}{(a + a \cot(c + dx))^2} dx &= \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} - \frac{\int \frac{-\frac{1}{2}a^2 e^3 + a^2 e^3 \cot(c+dx) - \frac{3}{2}a^2 e^3 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{2a^3} \\
 &= \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} - \frac{\int \frac{2a^3 e^3}{\sqrt{e \cot(c+dx)}} dx}{4a^5} + \frac{(3e^3) \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{4a} \\
 &= \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} - \frac{e^3 \int \frac{1}{\sqrt{e \cot(c+dx)}} dx}{2a^2} + \frac{(3e^3) \text{Subst} \left( \int \frac{1}{\sqrt{-ex} (a-ax)} dx, x, -\frac{1}{\sqrt{e \cot(c+dx)}} \right)}{4ad} \\
 &= \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} - \frac{(3e^2) \text{Subst} \left( \int \frac{1}{a+\frac{ax^2}{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{2ad} + \frac{e^4 \text{Subst} \left( \int \frac{1}{e^2+x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{a^2d} \\
 &= -\frac{3e^{5/2} \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d} + \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} + \frac{e^4 \text{Subst} \left( \int \frac{1}{e^2+x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{a^2d} \\
 &= -\frac{3e^{5/2} \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d} + \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} + \frac{e^3 \text{Subst} \left( \int \frac{e-x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{2a^2d} \\
 &= -\frac{3e^{5/2} \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d} + \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} - \frac{e^{5/2} \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{e+2x}}{-e-\sqrt{2} \sqrt{e} x-x^2} dx, x, \sqrt{e \cot(c + dx)} \right)}{4\sqrt{2} a^2d} \\
 &= -\frac{3e^{5/2} \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d} + \frac{e^2 \sqrt{e \cot(c + dx)}}{2d (a^2 + a^2 \cot(c + dx))} - \frac{e^{5/2} \log \left( \sqrt{e} + \sqrt{e \cot(c + dx)} \right)}{4\sqrt{2} a^2d} \\
 &= -\frac{3e^{5/2} \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d} - \frac{e^{5/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2\sqrt{2} a^2d} + \frac{e^{5/2} \tan^{-1} \left( 1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2\sqrt{2} a^2d}
 \end{aligned}$$

**Mathematica [A]** time = 2.02, size = 224, normalized size = 0.80

$$(e \cot(c + dx))^{5/2} (\sin(c + dx) + \cos(c + dx)) \left( 2 \cot^2(c + dx) \sec(c + dx) - \frac{1}{2} (\cot(c + dx) + 1) \csc(c + dx) \right) (\sqrt{2} \log$$


---

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)/(a + a\*Cot[c + d\*x])^2,x]

[Out] ((e\*Cot[c + d\*x])^(5/2)\*(-1/2\*((1 + Cot[c + d\*x])\*Csc[c + d\*x]\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]) + 12\*ArcTan[Sqrt[Cot[c + d\*x]]) + Sqrt[2]\*Log[-1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] - Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])) + 2\*Cot[c + d\*x]^(3/2)\*Sec[c + d\*x]\*(Cos[c + d\*x] + Sin[c + d\*x]))/(4\*a^2\*d\*Cot[c + d\*x]^(5/2)\*(1 + Cot[c + d\*x])^2)

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{5/2}}{(a \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(5/2)/(a\*cot(d\*x + c) + a)^2, x)

**maple [A]** time = 0.83, size = 234, normalized size = 0.83

$$\frac{e^3 \sqrt{e \cot(dx + c)}}{2d a^2 (e \cot(dx + c) + e)} - \frac{3e^{5/2} \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{2a^2 d} + \frac{e^2 (e^2)^{1/4} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{1/4} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}{e \cot(dx + c) - (e^2)^{1/4} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}\right)}{8d a^2} + \frac{e^2 (e^2)^{1/4} \sqrt{2}}{8d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int ((e \cdot \cot(dx+c))^{5/2} / (a + \cot(dx+c) \cdot a)^2, x)$

[Out]  $\frac{1}{2} \frac{d}{a^2} e^3 (e \cdot \cot(dx+c))^{1/2} / (e \cdot \cot(dx+c) + e) - \frac{3}{2} e^{5/2} \arctan\left(\frac{(e \cdot \cot(dx+c))^{1/2} / e^{1/2}}{a^2/d + 1/8/d/a^2 e^2 (e^2)^{1/4} 2^{1/2} \ln\left(\frac{(e \cdot \cot(dx+c) + (e^2)^{1/4} (e \cdot \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}{(e \cdot \cot(dx+c) - (e^2)^{1/4} (e \cdot \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}\right)} + \frac{1}{4} \frac{d}{a^2} e^2 (e^2)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cdot \cot(dx+c))^{1/2} + 1}\right) - \frac{1}{4} \frac{d}{a^2} e^2 (e^2)^{1/4} 2^{1/2} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cdot \cot(dx+c))^{1/2} + 1}\right)$

**maxima [A]** time = 0.44, size = 233, normalized size = 0.83

$$\left( \frac{4 e^2 \sqrt{\frac{e}{\tan(dx+c)}}}{a^2 e + \frac{a^2 e}{\tan(dx+c)}} - \frac{12 e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{a^2} + \frac{2 \sqrt{2} e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{a^2} + 2 \sqrt{2} e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right) + \sqrt{2} e^{\frac{3}{2}} \right)$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e \cdot \cot(dx+c))^{5/2} / (a + a \cdot \cot(dx+c))^2, x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{8} (4 e^2 \sqrt{e/\tan(dx+c)} / (a^2 e + a^2 e/\tan(dx+c)) - 12 e^{3/2} a \arctan(\sqrt{e/\tan(dx+c)} / \sqrt{e}) / a^2 + (2 \sqrt{2} e^{3/2} \arctan(1/2 \sqrt{2} (\sqrt{2} \sqrt{e} + 2 \sqrt{e/\tan(dx+c)}) / \sqrt{e}) + 2 \sqrt{2} e^{3/2} \arctan(-1/2 \sqrt{2} (\sqrt{2} \sqrt{e} - 2 \sqrt{e/\tan(dx+c)}) / \sqrt{e}) + \sqrt{2} e^{3/2} \log(\sqrt{2} \sqrt{e} \sqrt{e/\tan(dx+c)}) + e + e/\tan(dx+c)) - \sqrt{2} e^{3/2} \log(-\sqrt{2} \sqrt{e} \sqrt{e/\tan(dx+c)}) + e + e/\tan(dx+c)) / a^2) e/d$

**mupad [B]** time = 0.90, size = 375, normalized size = 1.33

$$\frac{e^3 \sqrt{e \cot(c+dx)}}{2 (a^2 d e + a^2 d e \cot(c+dx))} - \text{atan} \left( \frac{e^{20} \sqrt{e \cot(c+dx)} \left(-\frac{e^{10}}{256 a^8 d^4}\right)^{1/4} 16i}{\frac{36 e^{23}}{a^2 d} + 64 a^2 d e^{18} \sqrt{-\frac{e^{10}}{256 a^8 d^4}}} - \frac{e^{15} \sqrt{e \cot(c+dx)} \left(-\frac{e^{10}}{256 a^8 d^4}\right)^{3/4} 2}{\frac{36 e^{23}}{a^6 d^3} + \frac{64 e^{18} \sqrt{-\frac{e^{10}}{256 a^8 d^4}}}{a^2 d}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int ((e \cdot \cot(c + dx))^{5/2} / (a + a \cdot \cot(c + dx))^2, x)$

[Out]  $(e^3 (e \cdot \cot(c + dx))^{1/2}) / (2 (a^2 d e + a^2 d e \cdot \cot(c + dx))) - \text{atan}\left(\frac{e^{20} (e \cdot \cot(c + dx))^{1/2} (-e^{10} / (256 a^8 d^4))^{1/4} 16i}{(36 e^{23}) / (a^2 d)}\right)$

```
*d) + 64*a^2*d*e^18*(-e^10/(256*a^8*d^4))^(1/2)) - (e^15*(e*cot(c + d*x))^(1/2)*(-e^10/(256*a^8*d^4))^(3/4)*2304i)/((36*e^23)/(a^6*d^3) + (64*e^18*(-e^10/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^10/(256*a^8*d^4))^(1/4)*2i - (atan((4*e^20*(e*cot(c + d*x))^(1/2)*(-e^10/(a^8*d^4))^(1/4))/((36*e^23)/(a^2*d) - 4*a^2*d*e^18*(-e^10/(a^8*d^4))^(1/2)) + (36*e^15*(e*cot(c + d*x))^(1/2)*(-e^10/(a^8*d^4))^(3/4))/((36*e^23)/(a^6*d^3) - (4*e^18*(-e^10/(a^8*d^4))^(1/2))/(a^2*d)))*(-e^10/(a^8*d^4))^(1/4))/2 - (atan(((e*cot(c + d*x))^(1/2)*(-e^5)^(1/2)*1i)/e^3)*(-e^5)^(1/2)*3i)/(2*a^2*d)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c+dx))^{\frac{5}{2}}}{\cot^2(c+dx)+2 \cot(c+dx)+1} dx$$

$a^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)/(a+a\*cot(d\*x+c))\*\*2,x)

[Out] Integral((e\*cot(c + d\*x))\*\*(5/2)/(cot(c + d\*x)\*\*2 + 2\*cot(c + d\*x) + 1), x)  
/a\*\*2

$$3.30 \quad \int \frac{(e \cot(c+dx))^{3/2}}{(a+a \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=279

$$\frac{e^{3/2} \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} + \frac{e^{3/2} \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} + \dots$$

[Out]  $\frac{1}{2} e^{3/2} \arctan\left(\frac{e \cot(dx+c)^{1/2}}{e^{1/2}}\right) / a^2/d + \frac{1}{4} e^{3/2} \arctan\left(1 - 2^{1/2} \frac{e \cot(dx+c)^{1/2}}{e^{1/2}}\right) / a^2/d - \frac{1}{8} e^{3/2} \arctan\left(1 + 2^{1/2} \frac{e \cot(dx+c)^{1/2}}{e^{1/2}}\right) / a^2/d - \frac{1}{8} e^{3/2} \ln\left(\frac{e^{1/2} + \cot(dx+c) e^{1/2} - 2^{1/2} \frac{e \cot(dx+c)^{1/2}}{e^{1/2}}}{e^{1/2} + \cot(dx+c) e^{1/2} + 2^{1/2} \frac{e \cot(dx+c)^{1/2}}{e^{1/2}}}\right) / a^2/d - \frac{1}{2} e \frac{e \cot(dx+c)^{1/2}}{d(a^2 + a^2 \cot(dx+c))}$

**Rubi [A]** time = 0.56, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {3567, 3653, 12, 16, 3476, 329, 297, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{e^{3/2} \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} + \frac{e^{3/2} \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e \cot[c + dx])^{3/2} / (a + a \cot[c + dx])^2, x]$

[Out]  $(e^{3/2} \text{ArcTan}[\text{Sqrt}[e \cot[c + dx]] / \text{Sqrt}[e]]) / (2 a^2 d) + (e^{3/2} \text{ArcTan}[1 - (\text{Sqrt}[2] \text{Sqrt}[e \cot[c + dx]]) / \text{Sqrt}[e]]) / (2 \text{Sqrt}[2] a^2 d) - (e^{3/2} \text{ArcTan}[1 + (\text{Sqrt}[2] \text{Sqrt}[e \cot[c + dx]]) / \text{Sqrt}[e]]) / (2 \text{Sqrt}[2] a^2 d) - (e \text{Sqrt}[e \cot[c + dx]]) / (2 d (a^2 + a^2 \cot[c + dx])) - (e^{3/2} \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot[c + dx] - \text{Sqrt}[2] \text{Sqrt}[e \cot[c + dx]])] / (4 \text{Sqrt}[2] a^2 d) + (e^{3/2} \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot[c + dx] + \text{Sqrt}[2] \text{Sqrt}[e \cot[c + dx]])] / (4 \text{Sqrt}[2] a^2 d)$

**Rule 12**

$\text{Int}[(a\_)(u\_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[u, (b\_)(v\_)] /; \text{FreeQ}[b, x]$

**Rule 16**

$\text{Int}[(u\_)(v\_)^{(m\_)}((b\_)(v\_))^{(n\_)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u(b^m v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3476

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

### Rule 3567

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 2)\*Simp[a\*c^2\*(m + 1) + a\*d^2\*(n - 1) + b\*c\*d\*(m - n + 2) - (b\*c^2 - 2\*a\*c\*d - b\*d^2)\*(m + 1)\*Tan[e + f\*x] - d\*(b\*c - a\*d)\*(m + n)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2\*m]

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3653

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])

```

+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

Rubi steps



$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{3/2}}{(a + a \cot(c + dx))^2} dx &= -\frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} - \frac{\int \frac{\frac{ae^2}{2} - ae^2 \cot(c+dx) - \frac{1}{2}ae^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{2a^2} \\
&= -\frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} - \frac{\int -\frac{2a^2e^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{4a^4} - \frac{e^2 \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{4a} \\
&= -\frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} + \frac{e^2 \int \frac{\cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{2a^2} - \frac{e^2 \text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c+dx)\right)}{4ad} \\
&= -\frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} + \frac{e \int \sqrt{e \cot(c + dx)} dx}{2a^2} + \frac{e \text{Subst}\left(\int \frac{1}{a+\frac{ax^2}{e}} dx, x, \sqrt{e \cot(c+dx)}\right)}{2ad} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} - \frac{e^2 \text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2} dx, x, e \cot(c+dx)\right)}{2a^2d} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} - \frac{e^2 \text{Subst}\left(\int \frac{x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{a^2d} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} + \frac{e^2 \text{Subst}\left(\int \frac{e-x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{2a^2d} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} - \frac{e^{3/2} \text{Subst}\left(\int \frac{\sqrt{2} \sqrt{e+2x}}{-e-\sqrt{2} \sqrt{e}x-x^2} dx, x, \sqrt{e \cot(c+dx)}\right)}{4\sqrt{2} a^2d} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} - \frac{e\sqrt{e \cot(c + dx)}}{2d(a^2 + a^2 \cot(c + dx))} - \frac{e^{3/2} \log(\sqrt{e} + \sqrt{e \cot(c + dx)})}{4\sqrt{2} a^2d} \\
&= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{e^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2} a^2d} - \frac{e^{3/2} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2} a^2d}
\end{aligned}$$

**Mathematica [A]** time = 2.83, size = 312, normalized size = 1.12

$$\sin^2(c + dx)(e \cot(c + dx))^{3/2} \left( 4 \cot^7(c + dx) - 4 \cot^5(c + dx) + 4 \cot^3(c + dx) - 4\sqrt{\cot(c + dx)} + \sqrt{2} \cos(2(c + dx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)/(a + a\*Cot[c + d\*x])^2,x]

[Out] -1/8\*((e\*Cot[c + d\*x])^(3/2)\*(-4\*Sqrt[Cot[c + d\*x]] + 4\*Cot[c + d\*x])^(3/2) - 4\*Cot[c + d\*x]^(5/2) + 4\*Cot[c + d\*x]^(7/2) - 2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*Cos[2\*(c + d\*x)]\*Csc[c + d\*x]^4 + 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*Cos[2\*(c + d\*x)]\*Csc[c + d\*x]^4 - 4\*ArcTan[Sqrt[Cot[c + d\*x]]]\*Cos[2\*(c + d\*x)]\*Csc[c + d\*x]^4 + Sqrt[2]\*Cos[2\*(c + d\*x)]\*Csc[c + d\*x]^4\*Log[-1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] - Cot[c + d\*x]] - Sqrt[2]\*Cos[2\*(c + d\*x)]\*Csc[c + d\*x]^4\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])\*Sin[c + d\*x]^2/(a^2\*d\*Cot[c + d\*x]^(3/2)\*(-1 + Cot[c + d\*x])^2))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{(a \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(3/2)/(a\*cot(d\*x + c) + a)^2, x)

**maple** [A] time = 0.74, size = 234, normalized size = 0.84

$$\frac{e^2 \sqrt{e \cot(dx + c)}}{2d a^2 (e \cot(dx + c) + e)} + \frac{e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{2a^2 d} - \frac{e^2 \sqrt{2} \ln\left(\frac{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}\right)}{8d a^2 (e^2)^{\frac{1}{4}}} - \frac{e^2 \sqrt{2} \arctan\left(\frac{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}\right)}{4d a^2 (e^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)/(a+cot(d\*x+c)\*a)^2,x)

[Out]  $-1/2/d/a^2*e^2*(e*\cot(d*x+c))^{(1/2)}/(e*\cot(d*x+c)+e)+1/2*e^{(3/2)}*\arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^2/d-1/8/d/a^2*e^2/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/4/d/a^2*e^2/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/4/d/a^2*e^2/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)$

**maxima [A]** time = 0.49, size = 232, normalized size = 0.83

$$e \frac{4e \sqrt{\frac{e}{\tan(dx+c)}}}{a^2 e + \frac{a^2 e}{\tan(dx+c)}} + \frac{e \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right)}{a^2}$$


---

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")`

[Out]  $-1/8*e*(4*e*\sqrt{e/\tan(d*x+c)})/(a^2*e+a^2*e/\tan(d*x+c))+e*(2*\sqrt{2})*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(d*x+c)})/\sqrt{e})/\sqrt{e}+2*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(d*x+c)})/\sqrt{e})/\sqrt{e}-\sqrt{2}*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e}+\sqrt{2}*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e})/a^2-4*\sqrt{e}*\arctan(\sqrt{e/\tan(d*x+c)})/\sqrt{e}/a^2)/d$

**mupad [B]** time = 0.82, size = 376, normalized size = 1.35

$$\frac{\operatorname{atan}\left(\frac{4e^{16}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{a^8d^4}\right)^{1/4}}{\frac{4e^{18}}{a^2d}+4a^2de^{15}\sqrt{-\frac{e^6}{a^8d^4}}}+\frac{4e^{13}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{a^8d^4}\right)^{3/4}}{\frac{4e^{18}}{a^6d^3}+\frac{4e^{15}\sqrt{-\frac{e^6}{a^8d^4}}}{a^2d}}\right)\left(-\frac{e^6}{a^8d^4}\right)^{1/4}}{2}-\operatorname{atan}\left(\frac{e^{16}\sqrt{e\cot(c+dx)}\left(-\frac{e^6}{256a^8d^4}\right)^{1/4}}{\frac{4e^{18}}{a^2d}-64a^2de^{15}\sqrt{-\frac{e^6}{256a^8d^4}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^2,x)`

[Out]  $(\operatorname{atan}(((e \cot(c + dx))^{1/2} * (-e^3)^{1/2} * 1i) / e^2) * (-e^3)^{1/2} * 1i) / (2 * a^2 * d) - \operatorname{atan}((e^{16} * (e \cot(c + dx))^{1/2} * (-e^6 / (256 * a^8 * d^4))^{1/4} * 16i) / ((4 * e^{18}) / (a^2 * d) - 64 * a^2 * d * e^{15} * (-e^6 / (256 * a^8 * d^4))^{1/2})) - (e^{13} * (e \cot(c + dx))^{1/2} * (-e^6 / (256 * a^8 * d^4))^{3/4} * 256i) / ((4 * e^{18}) / (a^6 * d^3) - (64 * e^{15} * (-e^6 / (256 * a^8 * d^4))^{1/2}) / (a^2 * d))) * (-e^6 / (256 * a^8 * d^4))^{1/4} * 2i - (e^2 * (e \cot(c + dx))^{1/2}) / (2 * (a^2 * d * e + a^2 * d * e * \cot(c + dx))) - (\operatorname{atan}((4 * e^{16} * (e \cot(c + dx))^{1/2} * (-e^6 / (a^8 * d^4))^{1/4}) / ((4 * e^{18}) / (a^2 * d) + 4 * a^2 * d * e^{15} * (-e^6 / (a^8 * d^4))^{1/2})) + (4 * e^{13} * (e \cot(c + dx))^{1/2} * (-e^6 / (a^8 * d^4))^{3/4}) / ((4 * e^{18}) / (a^6 * d^3) + (4 * e^{15} * (-e^6 / (a^8 * d^4))^{1/2}) / (a^2 * d))) * (-e^6 / (a^8 * d^4))^{1/4}) / 2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c+dx))^{\frac{3}{2}}}{\frac{\cot^2(c+dx)+2 \cot(c+dx)+1}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)`

[Out] `Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x) / a**2`

$$3.31 \quad \int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^2} dx$$

Optimal. Leaf size=278

$$\frac{\sqrt{e \cot(c+dx)}}{2d(a^2 \cot(c+dx) + a^2)} + \frac{\sqrt{e} \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} - \frac{\sqrt{e} \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{4\sqrt{2} a^2 d}$$

[Out]  $1/2 \cdot \arctan((e \cdot \cot(d \cdot x + c))^{1/2} / e^{1/2}) \cdot e^{1/2} / a^2 / d + 1/4 \cdot \arctan(1 - 2^{1/2}) \cdot (e \cdot \cot(d \cdot x + c))^{1/2} / e^{1/2} \cdot e^{1/2} / a^2 / d \cdot 2^{1/2} - 1/4 \cdot \arctan(1 + 2^{1/2}) \cdot (e \cdot \cot(d \cdot x + c))^{1/2} / e^{1/2} \cdot e^{1/2} / a^2 / d \cdot 2^{1/2} + 1/8 \cdot \ln(e^{1/2} + \cot(d \cdot x + c)) \cdot e^{1/2} - 2^{1/2} \cdot (e \cdot \cot(d \cdot x + c))^{1/2} \cdot e^{1/2} / a^2 / d \cdot 2^{1/2} - 1/8 \cdot \ln(e^{1/2} + \cot(d \cdot x + c)) \cdot e^{1/2} + 2^{1/2} \cdot (e \cdot \cot(d \cdot x + c))^{1/2} \cdot e^{1/2} / a^2 / d \cdot 2^{1/2} + 1/2 \cdot (e \cdot \cot(d \cdot x + c))^{1/2} / d / (a^2 + a^2 \cdot \cot(d \cdot x + c))$

**Rubi [A]** time = 0.53, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$ , Rules used = {3568, 3653, 12, 3476, 329, 211, 1165, 628, 1162, 617, 204, 3634, 63, 205}

$$\frac{\sqrt{e \cot(c+dx)}}{2d(a^2 \cot(c+dx) + a^2)} + \frac{\sqrt{e} \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d} - \frac{\sqrt{e} \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{4\sqrt{2} a^2 d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]/(a + a\*Cot[c + d\*x])^2,x]

[Out]  $(\text{Sqrt}[e] \cdot \text{ArcTan}[\text{Sqrt}[e \cdot \text{Cot}[c + d \cdot x]] / \text{Sqrt}[e]]) / (2 \cdot a^2 \cdot d) + (\text{Sqrt}[e] \cdot \text{ArcTan}[1 - (\text{Sqrt}[2] \cdot \text{Sqrt}[e \cdot \text{Cot}[c + d \cdot x]]) / \text{Sqrt}[e]]) / (2 \cdot \text{Sqrt}[2] \cdot a^2 \cdot d) - (\text{Sqrt}[e] \cdot \text{ArcTan}[1 + (\text{Sqrt}[2] \cdot \text{Sqrt}[e \cdot \text{Cot}[c + d \cdot x]]) / \text{Sqrt}[e]]) / (2 \cdot \text{Sqrt}[2] \cdot a^2 \cdot d) + \text{Sqrt}[e \cdot \text{Cot}[c + d \cdot x]] / (2 \cdot d \cdot (a^2 + a^2 \cdot \text{Cot}[c + d \cdot x])) + (\text{Sqrt}[e] \cdot \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cdot \text{Cot}[c + d \cdot x] - \text{Sqrt}[2] \cdot \text{Sqrt}[e \cdot \text{Cot}[c + d \cdot x]])] / (4 \cdot \text{Sqrt}[2] \cdot a^2 \cdot d) - (\text{Sqrt}[e] \cdot \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cdot \text{Cot}[c + d \cdot x] + \text{Sqrt}[2] \cdot \text{Sqrt}[e \cdot \text{Cot}[c + d \cdot x]])] / (4 \cdot \text{Sqrt}[2] \cdot a^2 \cdot d)$

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]\}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

### Rule 211

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^4\}^{-1}, x\_Symbol] \ :> \ \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 329

$\text{Int}[\{(c\_)*(x\_)\}^m * \{(a\_)+ (b\_)*(x\_)^n\}^p, x\_Symbol] \ :> \ \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k*(m+1)-1} * (a + (b*x^{k*n})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 617

$\text{Int}[\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\{(d\_)+ (e\_)*(x\_)\}/\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}, x\_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3476

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

### Rule 3568

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n)/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*c\*(m + 1) - b\*d\*n - (b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(m + n + 1)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[2\*m]

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3653

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n \* Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[((c + d\*Tan[e + f\*x])^n\*(1 + Tan[e + f\*x]^2))/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&

!GtQ[n, 0] &amp;&amp; !LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^2} dx &= \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} - \frac{\int \frac{-\frac{ae}{2}-ae \cot(c+dx)+\frac{1}{2}ae \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{2a^2} \\
&= \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} - \frac{\int -\frac{2a^2e}{\sqrt{e \cot(c+dx)}} dx}{4a^4} - \frac{e \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{4a} \\
&= \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} + \frac{e \int \frac{1}{\sqrt{e \cot(c+dx)}} dx}{2a^2} - \frac{e \operatorname{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -\cot(c+dx)\right)}{4ad} \\
&= \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{ax^2}{a+e}} dx, x, \sqrt{e \cot(c+dx)}\right)}{2ad} - \frac{e^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(e-x^2)} dx, x, \sqrt{e \cot(c+dx)}\right)}{2ad} \\
&= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} - \frac{e^2 \operatorname{Subst}\left(\int \frac{1}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{a^2d} \\
&= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} - \frac{e \operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{2a^2d} \\
&= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} + \frac{\sqrt{e} \operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e+2x}}{-e-\sqrt{2}\sqrt{e}x-x^2} dx, x, \sqrt{e \cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\
&= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e \cot(c+dx)}}{2d(a^2+a^2 \cot(c+dx))} + \frac{\sqrt{e} \log(\sqrt{e} + \sqrt{e} \cot(c+dx))}{4\sqrt{2}a^2d} \\
&= \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{\sqrt{e} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d}
\end{aligned}$$

**Mathematica** [A] time = 1.30, size = 207, normalized size = 0.74

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$$\frac{\csc(c+dx)\sqrt{e \cot(c+dx)}(\sin(c+dx) + \cos(c+dx))\left(\frac{1}{2}(\tan(c+dx) + 1)\sqrt{\cot(c+dx)}\right)\left(\sqrt{2} \log(-\cot(c+dx)) + \sqrt{2} \log(\cot(c+dx))\right)}{2a^2d}$$



Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cos[c + d*x] + Sin[c + d*x])*(2 + (Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]) - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]) + 4*ArcTan[Sqrt[Cot[c + d*x]]) + Sqrt[2]*Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*(1 + Tan[c + d*x]))/2))/(4*a^2*d*(1 + Cot[c + d*x])^2)
```

```
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{e \cot(dx + c)}}{(a \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^2, x)
```

```
maple [A] time = 0.79, size = 223, normalized size = 0.80
```

$$\frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8d a^2} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{4d a^2} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{4d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+cot(d*x+c)*a)^2,x)
```

```
[Out] -1/8/d/a^2*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))-1/4/d/a^2*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4))
```

4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/4/d/a^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/2/d/a^2\*e\*(e\*cot(d\*x+c))^(1/2)/(e\*cot(d\*x+c)+e)+1/2\*arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))\*e^(1/2)/a^2/d

**maxima [A]** time = 0.58, size = 231, normalized size = 0.83

$$e \left( \frac{4 \sqrt{\frac{e}{\tan(dx+c)}}}{a^2 e + \frac{a^2 e}{\tan(dx+c)}} - \frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} - e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right)}{a^2} \cdot 8d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/8\*e\*(4\*sqrt(e/tan(d\*x + c))/(a^2\*e + a^2\*e/tan(d\*x + c)) - (2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/a^2 + 4\*arctan(sqrt(e/tan(d\*x + c))/sqrt(e))/(a^2\*sqrt(e))/d

**mupad [B]** time = 0.72, size = 366, normalized size = 1.32

$$\text{atan} \left( \frac{4e^{12} \sqrt{e \cot(c+dx)} \left(-\frac{e^2}{a^8 d^4}\right)^{1/4} + 4e^{11} \sqrt{e \cot(c+dx)} \left(-\frac{e^2}{a^8 d^4}\right)^{3/4}}{\frac{4e^{13}}{a^2 d} - 4a^2 d e^{12} \sqrt{-\frac{e^2}{a^8 d^4}}} + \frac{4e^{13}}{a^6 d^3} - \frac{4e^{12} \sqrt{-\frac{e^2}{a^8 d^4}}}{a^2 d}}{2} \right) + \text{atan} \left( \frac{e^{12} \sqrt{e \cot(c+dx)} \left(-\frac{e^2}{256 a^8 d^4}\right)^{1/4}}{\frac{4e^{13}}{a^2 d} + 64 a^2 d e^{12} \sqrt{-\frac{e^2}{256 a^8 d^4}}} \right) 16$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(1/2)/(a + a\*cot(c + d\*x))^2,x)

[Out] (atan((4\*e^12\*(e\*cot(c + d\*x))^(1/2)\*(-e^2/(a^8\*d^4))^(1/4))/((4\*e^13)/(a^2\*d) - 4\*a^2\*d\*e^12\*(-e^2/(a^8\*d^4))^(1/2)) + (4\*e^11\*(e\*cot(c + d\*x))^(1/2)\*(-e^2/(a^8\*d^4))^(3/4))/((4\*e^13)/(a^6\*d^3) - (4\*e^12\*(-e^2/(a^8\*d^4))^(1/2))/(a^2\*d)))\*(-e^2/(a^8\*d^4))^(1/4))/2 + atan((e^12\*(e\*cot(c + d\*x))^(1/2)\*(-e^2/(256\*a^8\*d^4))^(1/4)\*16i)/((4\*e^13)/(a^2\*d) + 64\*a^2\*d\*e^12\*(-e^2/(2

$$56*a^8*d^4)^{(1/2)} - (e^{11}*(e*\cot(c + d*x))^{(1/2)}*(-e^2/(256*a^8*d^4))^{(3/4)}*256i)/((4*e^{13})/(a^6*d^3) + (64*e^{12}*(-e^2/(256*a^8*d^4))^{(1/2)})/(a^2*d)))*(-e^2/(256*a^8*d^4))^{(1/4)}*2i + (e*(e*\cot(c + d*x))^{(1/2)})/(2*(a^2*d*e + a^2*d*e*\cot(c + d*x))) - ((-e)^{(1/2)}*atan(((e*\cot(c + d*x))^{(1/2)}*1i)/(-e)^{(1/2)})*1i)/(2*a^2*d)$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(c+dx)}}{\cot^2(c+dx)+2 \cot(c+dx)+1} dx$$

$a^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)/(a+a\*cot(d\*x+c))\*\*2,x)

[Out] Integral(sqrt(e\*cot(c + d\*x))/(cot(c + d\*x)\*\*2 + 2\*cot(c + d\*x) + 1), x)/a\*\*2

$$3.32 \quad \int \frac{1}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=281

$$-\frac{\sqrt{e \cot(c+dx)}}{2de(a^2 \cot(c+dx) + a^2)} + \frac{\log(\sqrt{e \cot(c+dx)} - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d \sqrt{e}} - \frac{\log(\sqrt{e \cot(c+dx)} + \sqrt{2} \sqrt{e \cot(c+dx)})}{4\sqrt{2} a^2 d \sqrt{e}}$$

[Out]  $-3/2*\arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^2/d/e^{(1/2)}-1/4*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^2/d*2^{(1/2)}/e^{(1/2)}+1/4*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^2/d*2^{(1/2)}/e^{(1/2)}+1/8*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^2/d*2^{(1/2)}/e^{(1/2)}-1/8*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^2/d*2^{(1/2)}/e^{(1/2)}-1/2*(e*\cot(d*x+c))^{(1/2)}/d/e/(a^2+a^2*\cot(d*x+c))$

**Rubi [A]** time = 0.57, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {3569, 3653, 12, 16, 3476, 329, 297, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$-\frac{\sqrt{e \cot(c+dx)}}{2de(a^2 \cot(c+dx) + a^2)} + \frac{\log(\sqrt{e \cot(c+dx)} - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d \sqrt{e}} - \frac{\log(\sqrt{e \cot(c+dx)} + \sqrt{2} \sqrt{e \cot(c+dx)})}{4\sqrt{2} a^2 d \sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^2), x]

[Out]  $(-3*\text{ArcTan}[\text{Sqrt}[e*\text{Cot}[c + d*x]]/\text{Sqrt}[e]])/(2*a^2*d*\text{Sqrt}[e]) - \text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]]/(2*\text{Sqrt}[2]*a^2*d*\text{Sqrt}[e]) + \text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]]/(2*\text{Sqrt}[2]*a^2*d*\text{Sqrt}[e]) - \text{Sqrt}[e*\text{Cot}[c + d*x]]/(2*d*e*(a^2 + a^2*\text{Cot}[c + d*x])) + \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]]]/(4*\text{Sqrt}[2]*a^2*d*\text{Sqrt}[e]) - \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]]]/(4*\text{Sqrt}[2]*a^2*d*\text{Sqrt}[e])$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 3476

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

### Rule 3569

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), x] + Dist[1/((m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3653

Int((((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])

```

+ (f_.)(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))^2} dx &= -\frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} - \frac{\int \frac{-\frac{3a^2e}{2}+a^2e \cot(c+dx)-\frac{1}{2}a^2e \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{2a^3e} \\
&= -\frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} + \frac{3 \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{4a} - \frac{\int \frac{2a^3e \cot(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{4a^3} \\
&= -\frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} - \frac{\int \frac{\cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{2a^2} + \frac{3 \text{Subst} \left( \int \frac{1}{\sqrt{-ex} (a-ax)} dx \right)}{4a^3} \\
&= -\frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} - \frac{\int \sqrt{e \cot(c+dx)} dx}{2a^2e} - \frac{3 \text{Subst} \left( \int \frac{1}{a+\frac{ax^2}{e}} dx \right)}{4a^3} \\
&= -\frac{3 \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} + \frac{\text{Subst} \left( \int \frac{\sqrt{x}}{e^2+x^2} dx \right)}{2a^3} \\
&= -\frac{3 \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} + \frac{\text{Subst} \left( \int \frac{x^2}{e^2+x^4} dx \right)}{2a^3} \\
&= -\frac{3 \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} - \frac{\text{Subst} \left( \int \frac{e-x^2}{e^2+x^4} dx \right)}{2a^3} \\
&= -\frac{3 \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} + \frac{\text{Subst} \left( \int \frac{1}{e-\sqrt{2}\sqrt{x}} dx \right)}{2a^3} \\
&= -\frac{3 \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{2de (a^2+a^2 \cot(c+dx))} + \frac{\log(\sqrt{e} + \sqrt{e \cot(c+dx)})}{2a^3} \\
&= -\frac{3 \tan^{-1} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2a^2d\sqrt{e}} - \frac{\tan^{-1} \left( 1 - \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2\sqrt{2}a^2d\sqrt{e}} + \frac{\tan^{-1} \left( 1 + \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{2\sqrt{2}a^2d\sqrt{e}}
\end{aligned}$$

**Mathematica [A]** time = 0.84, size = 337, normalized size = 1.20

$$\frac{\sqrt{\cot(c+dx)} \left( 4 \sin(c+dx) \sqrt{\cot(c+dx)} - \sqrt{2} \cos(c+dx) \log(-\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} - 1) + \sqrt{2} \cos(c+dx) \log(-\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1) \right)}{2\sqrt{2}a^2d\sqrt{e}}$$

Antiderivative was successfully verified.



[In] Integrate[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^2),x]

[Out] 
$$\frac{-1/8*(\text{Sqrt}[\text{Cot}[c + d*x]]*(12*\text{ArcTan}[\text{Sqrt}[\text{Cot}[c + d*x]]]*\text{Cos}[c + d*x] - \text{Sqrt}[2]*\text{Cos}[c + d*x]*\text{Log}[-1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] - \text{Cot}[c + d*x]] + \text{Sqrt}[2]*\text{Cos}[c + d*x]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]] + 12*\text{ArcTan}[\text{Sqrt}[\text{Cot}[c + d*x]]]*\text{Sin}[c + d*x] + 4*\text{Sqrt}[\text{Cot}[c + d*x]]*\text{Sin}[c + d*x] - \text{Sqrt}[2]*\text{Log}[-1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] - \text{Cot}[c + d*x]]*\text{Sin}[c + d*x] + \text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]]*\text{Sin}[c + d*x] + 2*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]]*(\text{Cos}[c + d*x] + \text{Sin}[c + d*x]) - 2*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]*(\text{Cos}[c + d*x] + \text{Sin}[c + d*x])))/(\text{a}^2*d*\text{Sqrt}[e*\text{Cot}[c + d*x]]*(\text{Cos}[c + d*x] + \text{Sin}[c + d*x]))}{(a^2*d*\text{Sqrt}[e*\text{Cot}[c + d*x]]*(\text{Cos}[c + d*x] + \text{Sin}[c + d*x]))}$$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx + c) + a)^2 \sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)^2\*sqrt(e\*cot(d\*x + c))), x)

**maple** [A] time = 0.71, size = 222, normalized size = 0.79

$$\frac{\frac{\sqrt{e \cot(dx + c)}}{2d a^2 (e \cot(dx + c) + e)} - \frac{3 \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{2a^2 d \sqrt{e}} + \frac{\sqrt{2} \ln\left(\frac{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}\right)}{8d a^2 (e^2)^{\frac{1}{4}}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}}{4d a^2}\right)}{4d a^2}}{2d a^2 (e \cot(dx + c) + e) - \frac{3 \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{2a^2 d \sqrt{e}} + \frac{\sqrt{2} \ln\left(\frac{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}\right)}{8d a^2 (e^2)^{\frac{1}{4}}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}}{4d a^2}\right)}{4d a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(1/2)/(a+cot(d\*x+c)\*a)^2,x)

[Out] 
$$-1/2/d/a^2*(e*\text{cot}(d*x+c))^{1/2}/(e*\text{cot}(d*x+c)+e)-3/2*\text{arctan}((e*\text{cot}(d*x+c))^{1/2}/e^{1/2})/a^2/d/e^{1/2}+1/8/d/a^2/(e^2)^{1/4}*2^{1/2}*\ln((e*\text{cot}(d*x+c))^{1/2}/e^{1/2})/a^2/d/e^{1/2}+1/8/d/a^2/(e^2)^{1/4}*2^{1/2}*\ln((e*\text{cot}(d*x+c))^{1/2}/e^{1/2})/a^2/d/e^{1/2}$$

$$-(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)} / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) + 1/4/d/a^2/(e^2)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) - 1/4/d/a^2/(e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1)$$

**maxima [A]** time = 0.55, size = 238, normalized size = 0.85

$$e \left( \frac{4 \sqrt{\frac{e}{\tan(dx+c)}}}{a^2 e^2 + \frac{a^2 e^2}{\tan(dx+c)}} - \frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e} - 2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} + \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}} - e - \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right) / a^2 e$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $-1/8 * e * (4 * \sqrt{e/\tan(dx+c)}) / (a^2 * e^2 + a^2 * e^2 / \tan(dx+c)) - (2 * \sqrt{2} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e/\tan(dx+c)})) / \sqrt{e}) / \sqrt{e} + 2 * \sqrt{2} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e/\tan(dx+c)})) / \sqrt{e}) / \sqrt{e} - \sqrt{2} * \log(\sqrt{2} * \sqrt{e} * \sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c)) / \sqrt{e} + \sqrt{2} * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e/\tan(dx+c)} - e - e/\tan(dx+c)) / \sqrt{e}) / (a^2 * e) + 12 * \arctan(\sqrt{e/\tan(dx+c)}) / \sqrt{e} / (a^2 * e^{3/2}) / d$

**mupad [B]** time = 0.81, size = 366, normalized size = 1.30

$$\frac{\operatorname{atan}\left(\frac{4e^8 \sqrt{e \cot(c+dx)} \left(-\frac{1}{a^8 d^4 e^2}\right)^{1/4} + \frac{36e^9 \sqrt{e \cot(c+dx)} \left(-\frac{1}{a^8 d^4 e^2}\right)^{3/4}}{\frac{4e^8}{a^2 d} + 36a^2 d e^9 \sqrt{-\frac{1}{a^8 d^4 e^2}}}}{\frac{4e^8}{a^6 d^3} + \frac{36e^9 \sqrt{-\frac{1}{a^8 d^4 e^2}}}{a^2 d}}\right) \left(-\frac{1}{a^8 d^4 e^2}\right)^{1/4}}{2} + \operatorname{atan}\left(\frac{e^8 \sqrt{e \cot(c+dx)} \left(-\frac{1}{256 a^8 d^4 e^2}\right)^{1/4}}{\frac{4e^8}{a^2 d} - 576 a^2 d e^9 \sqrt{-\frac{1}{256 a^8 d^4 e^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c+d\*x))^(1/2)\*(a+a\*cot(c+d\*x))^2),x)

[Out]  $(\operatorname{atan}((4 * e^8 * (e * \cot(c + dx))^{(1/2)} * (-1 / (a^8 * d^4 * e^2))^{(1/4)}) / ((4 * e^8) / (a^2 * d) + 36 * a^2 * d * e^9 * (-1 / (a^8 * d^4 * e^2))^{(1/2)})) + (36 * e^9 * (e * \cot(c + dx))^{(1/2)} * (-1 / (a^8 * d^4 * e^2))^{(3/4)}) / ((4 * e^8) / (a^6 * d^3) + (36 * e^9 * (-1 / (a^8 * d^4 * e^2))^{(1/2)})) / (a^2 * d)) * (-1 / (a^8 * d^4 * e^2))^{(1/4)}) / 2 + \operatorname{atan}((e^8 * (e * \cot(c + dx))^{(1/2)} * (-1 / (256 * a^8 * d^4 * e^2))^{(1/4)}) * 16i) / ((4 * e^8) / (a^2 * d) - 576 * a^2 * d * e^9 * (-1 / (256 * a^8 * d^4 * e^2))^{(1/2)}))$

$$-1/(256*a^8*d^4*e^2))^{(1/2)} - (e^9*(e*\cot(c + d*x))^{(1/2)}*(-1/(256*a^8*d^4*e^2))^{(3/4)}*2304i)/((4*e^8)/(a^6*d^3) - (576*e^9*(-1/(256*a^8*d^4*e^2))^{(1/2)})/(a^2*d)))*(-1/(256*a^8*d^4*e^2))^{(1/4)}*2i - (e*\cot(c + d*x))^{(1/2)}/(2*(a^2*d*e + a^2*d*e*\cot(c + d*x))) - (\operatorname{atan}(((e*\cot(c + d*x))^{(1/2)}*1i)/(-e)^{(1/2)})*3i)/(2*a^2*d*(-e)^{(1/2)})$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{\sqrt{e \cot(c+dx)} \cot^2(c+dx) + 2\sqrt{e \cot(c+dx)} \cot(c+dx) + \sqrt{e \cot(c+dx)}}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(1/2)/(a+a\*cot(d\*x+c))\*\*2,x)

[Out] Integral(1/(sqrt(e\*cot(c + d\*x))\*cot(c + d\*x)\*\*2 + 2\*sqrt(e\*cot(c + d\*x))\*cot(c + d\*x) + sqrt(e\*cot(c + d\*x))), x)/a\*\*2

$$3.33 \quad \int \frac{1}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=306

$$\frac{\log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{3/2}} + \frac{\log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{3/2}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}}{2a^2 d}\right)}{2a^2 d}$$

[Out]  $\frac{5}{2} \arctan\left(\frac{e \cot(dx+c)^{1/2}/e^{1/2}}{a^2/d/e^{3/2}}\right) - \frac{1}{4} \arctan\left(1 - \frac{2}{e \cot(dx+c)^{1/2}/e^{1/2}}\right) / a^2/d/e^{3/2} + \frac{1}{4} \arctan\left(1 + \frac{2}{e \cot(dx+c)^{1/2}/e^{1/2}}\right) / a^2/d/e^{3/2} + \frac{1}{8} \ln\left(\frac{e^{1/2} + \cot(dx+c)}{e^{1/2} - 2 \cot(dx+c)}\right) / a^2/d/e^{3/2} + \frac{1}{8} \ln\left(\frac{e^{1/2} + \cot(dx+c)}{e^{1/2} + 2 \cot(dx+c)}\right) / a^2/d/e^{3/2} + \frac{5}{2} \arctan\left(\frac{e \cot(dx+c)^{1/2}/e^{1/2}}{a^2/d/e^{3/2}}\right) - \frac{1}{2} \arctan\left(\frac{e \cot(dx+c)^{1/2}/e^{1/2}}{a^2+a^2 \cot(dx+c)}\right) / (e \cot(dx+c))^{1/2}$

**Rubi [A]** time = 0.80, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {3569, 3649, 3653, 12, 3476, 329, 211, 1165, 628, 1162, 617, 204, 3634, 63, 205}

$$\frac{\log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{3/2}} + \frac{\log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{3/2}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}}{2a^2 d}\right)}{2a^2 d}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])^2), x]

[Out]  $\frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right]}{(2 a^2 d e^{3/2})} - \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right] / (2 \sqrt{2} a^2 d e^{3/2}) + \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right] / (2 \sqrt{2} a^2 d e^{3/2}) + \frac{5}{(2 a^2 d e \sqrt{e \cot(c+dx)})} - \frac{1}{(2 d e \sqrt{e \cot(c+dx)})} \left(\frac{a^2 + a^2 \cot(c+dx)}{e \cot(c+dx)}\right) - \frac{\log\left[\frac{\sqrt{e} + \sqrt{e \cot(c+dx)}}{\sqrt{e} - \sqrt{2} \sqrt{e \cot(c+dx)}}\right]}{(4 \sqrt{2} a^2 d e^{3/2})} + \frac{\log\left[\frac{\sqrt{e} + \sqrt{e \cot(c+dx)}}{\sqrt{e} + \sqrt{2} \sqrt{e \cot(c+dx)}}\right]}{(4 \sqrt{2} a^2 d e^{3/2})}$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-(a\*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x / \text{Rt}[a/b, 2]]) / a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

### Rule 211

$\text{Int}[(a + (b \cdot x)^4)^{-1}, x\_Symbol] \rightarrow \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2 \cdot r), \text{Int}[(r - s \cdot x^2)/(a + b \cdot x^4), x], x] + \text{Dist}[1/(2 \cdot r), \text{Int}[(r + s \cdot x^2)/(a + b \cdot x^4), x], x]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 329

$\text{Int}[(c \cdot x)^m \cdot (a + (b \cdot x)^n)^p, x\_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k \cdot (m + 1) - 1)} \cdot (a + (b \cdot x^{k \cdot n})) / c^n]^p, x], x, (c \cdot x)^{(1/k)}], x]] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 617

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c) / b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x) / b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

### Rule 628

$\text{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x\_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 3476

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

### Rule 3569

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)^2]), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3649

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) +
(f_)*(x_)^2]), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
```

```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))^2} dx &= -\frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} - \int \frac{-\frac{5a^2e}{2}+a^2e \cot(c+dx)-\frac{3}{2}a^2e}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))} dx \\
&= \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} - \int \frac{-\frac{5a^2e}{2}+a^2e \cot(c+dx)-\frac{3}{2}a^2e}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))} dx \\
&= \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} - \int \frac{-\frac{5a^2e}{2}+a^2e \cot(c+dx)-\frac{3}{2}a^2e}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))} dx \\
&= \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} - \int \frac{-\frac{5a^2e}{2}+a^2e \cot(c+dx)-\frac{3}{2}a^2e}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))} dx \\
&= \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} + \int \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} dx \\
&= \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} + \int \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} dx \\
&= \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} + \int \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} dx \\
&= \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} + \frac{5}{2a^2de\sqrt{e \cot(c+dx)}} - \frac{1}{2de\sqrt{e \cot(c+dx)} (a^2+a^2 \cot(c+dx))} + \int \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} dx \\
&= \frac{5 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2} a^2de^{3/2}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2} a^2de^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 1.33, size = 203, normalized size = 0.66

$$\frac{\cot^{\frac{3}{2}}(c+dx) \left( \frac{\log(\cot(c+dx)+\sqrt{2} \sqrt{\cot(c+dx)}+1)-\log(-\cot(c+dx)+\sqrt{2} \sqrt{\cot(c+dx)}-1)}{\sqrt{2}} - \sqrt{2} \tan^{-1}\left(1 - \sqrt{2} \sqrt{\cot(c+dx)}\right) + \sqrt{2} \tan^{-1}\left(1 + \sqrt{2} \sqrt{\cot(c+dx)}\right) \right)}{4a^2d(e \cot(c+dx))^{3/2}}$$



Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

```
[Out] (Cot[c + d*x]^(3/2)*(-(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]) + Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 10*ArcTan[Sqrt[Cot[c + d*x]]] + (-Log[-1 + Sqrt[2]*Sqrt[Cot[c + d*x]] - Cot[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])/Sqrt[2] + (2*(5*Cos[c + d*x] + 4*Sin[c + d*x]))/(Sqrt[Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x]))) / (4*a^2*d*(e*Cot[c + d*x])^(3/2))
```

```
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{(a \cot(dx + c) + a)^2 (e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2)), x)
```

```
maple [A] time = 0.65, size = 255, normalized size = 0.83
```

$$\frac{\sqrt{e \cot(dx + c)}}{2d a^2 e (e \cot(dx + c) + e)} + \frac{5 \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{2a^2 d e^{\frac{3}{2}}} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)} \sqrt{2 + \sqrt{e^2}}}\right)}{8d a^2 e^2} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} a}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+cot(d*x+c)*a)^2,x)
```

```
[Out] 1/2/d/a^2/e*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+5/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)+1/8/d/a^2/e^2*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))/a^2/d/e^(3/2)
```

$$e^{-2} \wedge (1/4) * (e * \cot(dx+c)) \wedge (1/2) * 2 \wedge (1/2) + (e^{-2}) \wedge (1/2) \Big) + 1/4/d/a^2/e^{-2} * (e^{-2}) \wedge (1/4) * 2 \wedge (1/2) * \arctan(2 \wedge (1/2) / (e^{-2}) \wedge (1/4) * (e * \cot(dx+c)) \wedge (1/2) + 1) - 1/4/d/a^2/e^{-2} * (e^{-2}) \wedge (1/4) * 2 \wedge (1/2) * \arctan(-2 \wedge (1/2) / (e^{-2}) \wedge (1/4) * (e * \cot(dx+c)) \wedge (1/2) + 1) + 2/a^2/d/e / (e * \cot(dx+c)) \wedge (1/2)$$

**maxima [A]** time = 0.87, size = 256, normalized size = 0.84

$$e \left( \frac{4 \left( 4e + \frac{5e}{\tan(dx+c)} \right)}{a^2 e^3 \sqrt{\frac{e}{\tan(dx+c)}} + a^2 e^2 \left( \frac{e}{\tan(dx+c)} \right)^2} + \frac{2 \sqrt{2} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{e^{3/2}} + \frac{2 \sqrt{2} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{e^{3/2}} + \frac{\sqrt{2} \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + \frac{2e}{\sqrt{\frac{e}{\tan(dx+c)}}} \right)}{a^2 e} \right) \frac{1}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/8\*e\*(4\*(4\*e + 5\*e/tan(d\*x + c))/(a^2\*e^3\*sqrt(e/tan(d\*x + c)) + a^2\*e^2\*(e/tan(d\*x + c))^(3/2)) + (2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/e^(3/2) + 2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/e^(3/2) + sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/e^(3/2) - sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/e^(3/2))/(a^2\*e) + 20\*arctan(sqrt(e/tan(d\*x + c))/sqrt(e))/(a^2\*e^(5/2))/d

**mupad [B]** time = 0.92, size = 414, normalized size = 1.35

$$\frac{\frac{5 \cot(c+dx)}{2} + 2}{a^2 d (e \cot(c + dx))^{3/2} + a^2 d e \sqrt{e \cot(c + dx)}} - \frac{\operatorname{atan} \left( \frac{2048 a^{10} d^5 e^{13} \sqrt{e \cot(c+dx)} \left( -\frac{1}{a^8 d^4 e^6} \right)^{1/4}}{51200 a^8 d^4 e^{12} - 2048 a^{12} d^6 e^{15} \sqrt{-\frac{1}{a^8 d^4 e^6}}} \right) + \frac{51200 a^{14} d^7 e^{16} \sqrt{e \cot(c+dx)}}{51200 a^8 d^4 e^{12} - 2048 a^{12} d^6 e^{15}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c + d\*x))^(3/2)\*(a + a\*cot(c + d\*x))^2),x)

[Out] ((5\*cot(c + d\*x))/2 + 2)/(a^2\*d\*(e\*cot(c + d\*x))^(3/2) + a^2\*d\*e\*(e\*cot(c + d\*x))^(1/2)) - (atan((2048\*a^10\*d^5\*e^13\*(e\*cot(c + d\*x))^(1/2)\*(-1/(a^8\*d^4\*e^6))^(1/4))/(51200\*a^8\*d^4\*e^12 - 2048\*a^12\*d^6\*e^15\*(-1/(a^8\*d^4\*e^6))^(1/2)) + (51200\*a^14\*d^7\*e^16\*(e\*cot(c + d\*x))^(1/2)\*(-1/(a^8\*d^4\*e^6))^(3/4))/(51200\*a^8\*d^4\*e^12 - 2048\*a^12\*d^6\*e^15\*(-1/(a^8\*d^4\*e^6))^(1/2)))\*(-

$$\frac{1}{(a^8 d^4 e^6)^{1/4}} / 2 - \operatorname{atan}\left(\frac{a^{10} d^5 e^{13} (e \cot(c + dx))^{1/2} (-1/(256 a^8 d^4 e^6))^{1/4} * 8192i}{(51200 a^8 d^4 e^{12} + 32768 a^{12} d^6 e^{15} (-1/(256 a^8 d^4 e^6))^{1/2})} - \frac{a^{14} d^7 e^{16} (e \cot(c + dx))^{1/2} (-1/(256 a^8 d^4 e^6))^{3/4} * 3276800i}{(51200 a^8 d^4 e^{12} + 32768 a^{12} d^6 e^{15} (-1/(256 a^8 d^4 e^6))^{1/2})} * (-1/(256 a^8 d^4 e^6))^{1/4} * 2i + \operatorname{atan}\left(\frac{(e \cot(c + dx))^{1/2} (-e^3)^{1/2} * 1i}{e^2} * (-e^3)^{1/2} * 5i\right) / (2 a^2 d e^3)\right)$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{(e \cot(c+dx))^{\frac{3}{2}} \cot^2(c+dx) + 2(e \cot(c+dx))^{\frac{3}{2}} \cot(c+dx) + (e \cot(c+dx))^{\frac{3}{2}}}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(3/2)/(a+a\*cot(d\*x+c))\*\*2,x)

[Out] Integral(1/((e\*cot(c + d\*x))\*\*(3/2)\*cot(c + d\*x)\*\*2 + 2\*(e\*cot(c + d\*x))\*\*(3/2)\*cot(c + d\*x) + (e\*cot(c + d\*x))\*\*(3/2)), x)/a\*\*2

$$3.34 \quad \int \frac{1}{(e \cot(c+dx))^{5/2} (a+a \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=331

$$\frac{\log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{5/2}} + \frac{\log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{5/2}} - \frac{7 \tan^{-1}\left(\frac{\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}}{2a^2 d e^{5/2}}\right)}{2a^2 d e^{5/2}}$$

[Out]  $-7/2 \arctan((e \cot(d*x+c))^{1/2}/e^{1/2})/a^2/d/e^{5/2} + 7/6/a^2/d/e/(e \cot(d*x+c))^{3/2} - 1/2/d/e/(e \cot(d*x+c))^{3/2}/(a^2+a^2 \cot(d*x+c)) + 1/4 \arctan(1-2^{1/2}*(e \cot(d*x+c))^{1/2}/e^{1/2})/a^2/d/e^{5/2} * 2^{1/2} - 1/4 \arctan(1+2^{1/2}*(e \cot(d*x+c))^{1/2}/e^{1/2})/a^2/d/e^{5/2} * 2^{1/2} - 1/8 \ln(e^{1/2} + \cot(d*x+c)*e^{1/2} - 2^{1/2}*(e \cot(d*x+c))^{1/2})/a^2/d/e^{5/2} * 2^{1/2} + 1/8 \ln(e^{1/2} + \cot(d*x+c)*e^{1/2} + 2^{1/2}*(e \cot(d*x+c))^{1/2})/a^2/d/e^{5/2} * 2^{1/2} - 9/2/a^2/d/e^2/(e \cot(d*x+c))^{1/2}$

**Rubi [A]** time = 1.08, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$ , Rules used = {3569, 3649, 3653, 12, 16, 3476, 329, 297, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{9}{2a^2 d e^2 \sqrt{e \cot(c+dx)}} - \frac{\log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{5/2}} + \frac{\log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{4\sqrt{2} a^2 d e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^2), x]

[Out]  $(-7 \text{ArcTan}[\text{Sqrt}[e \text{Cot}[c + d*x]]/\text{Sqrt}[e]])/(2*a^2*d*e^{5/2}) + \text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e \text{Cot}[c + d*x]])/\text{Sqrt}[e]]/(2*\text{Sqrt}[2]*a^2*d*e^{5/2}) - \text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e \text{Cot}[c + d*x]])/\text{Sqrt}[e]]/(2*\text{Sqrt}[2]*a^2*d*e^{5/2}) + 7/(6*a^2*d*e*(e \text{Cot}[c + d*x])^{3/2}) - 9/(2*a^2*d*e^2*\text{Sqrt}[e \text{Cot}[c + d*x]]) - 1/(2*d*e*(e \text{Cot}[c + d*x])^{3/2}*(a^2 + a^2*\text{Cot}[c + d*x])) - \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e \text{Cot}[c + d*x]]]/(4*\text{Sqrt}[2]*a^2*d*e^{5/2}) + \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e \text{Cot}[c + d*x]]]/(4*\text{Sqrt}[2]*a^2*d*e^{5/2})$

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 16

$\text{Int}[(u_.)*(v_)^{(m_.)}*((b_)*(v_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x\} \&\& \text{IntegerQ}[m]$

### Rule 63

$\text{Int}[((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[((a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[((a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

### Rule 297

$\text{Int}[x^2/((a_.) + (b_.)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /; \text{FreeQ}\{a, b\}, x\} \&\& (\text{GtQ}[a/b, 0] \parallel (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 329

$\text{Int}[((c_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_}))^{(p_)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 617

$\text{Int}[((a_.) + (b_.)*(x_.) + (c_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3476

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3569

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)^2]), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3649

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c+dx))^{5/2} (a+a \cot(c+dx))^2} dx &= -\frac{1}{2de(e \cot(c+dx))^{3/2} (a^2+a^2 \cot(c+dx))} - \frac{\int \frac{-\frac{7a^2e}{2}+a^2e \cot(c+dx)-\frac{5}{2}a}{(e \cot(c+dx))^{5/2} (a+a \cot(c+dx))} dx}{2a^3e} \\
&= \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{1}{2de(e \cot(c+dx))^{3/2} (a^2+a^2 \cot(c+dx))} \\
&= \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} - \frac{1}{2de(e \cot(c+dx))} \\
&= \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} - \frac{1}{2de(e \cot(c+dx))} \\
&= \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} - \frac{1}{2de(e \cot(c+dx))} \\
&= \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} - \frac{1}{2de(e \cot(c+dx))} \\
&= \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} - \frac{1}{2de(e \cot(c+dx))} \\
&= -\frac{7 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} \\
&= -\frac{7 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} \\
&= -\frac{7 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} \\
&= -\frac{7 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} \\
&= -\frac{7 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{7}{6a^2de(e \cot(c+dx))^{3/2}} - \frac{9}{2a^2de^2\sqrt{e \cot(c+dx)}} \\
&= -\frac{7 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}} + \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}} - \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}
\end{aligned}$$



**Mathematica [A]** time = 6.34, size = 467, normalized size = 1.41

$$\frac{\cot^3(c+dx) \csc^2(c+dx) (\sin(c+dx) + \cos(c+dx))^2 \left( -4 \tan(c+dx) + \frac{2}{3} \sec^2(c+dx) - \frac{\sin(c+dx)}{2(\sin(c+dx) + \cos(c+dx))} - \frac{2}{3} \right)}{d(a \cot(c+dx) + a)^2 (e \cot(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^2),x]

[Out] (Cot[c + d\*x]^3\*Csc[c + d\*x]^2\*(Cos[c + d\*x] + Sin[c + d\*x])^2\*(-2/3 + (2\*Sec[c + d\*x]^2)/3 - Sin[c + d\*x]/(2\*(Cos[c + d\*x] + Sin[c + d\*x]))) - 4\*Tan[c + d\*x]))/(d\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^2) + (Cot[c + d\*x]^(5/2)\*Csc[c + d\*x]^2\*(Cos[c + d\*x] + Sin[c + d\*x])^2\*((-16\*ArcTan[Sqrt[Cot[c + d\*x]]]\*(1 + Cot[c + d\*x])\*Csc[c + d\*x]^3\*Sec[c + d\*x]))/((1 + Cot[c + d\*x])^2\*(1 + Tan[c + d\*x])) + (Cos[2\*(c + d\*x)]\*Csc[c + d\*x]^3\*(-Log[-1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] - Cot[c + d\*x]] + Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])\*Sec[c + d\*x])/(Sqrt[2]\*(-1 + Cot[c + d\*x])\*(1 + Cot[c + d\*x])^2\*(1 + Tan[c + d\*x])) + ((-(Sqrt[2]\*(-ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]) + ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]])) + 2\*ArcTan[Sqrt[Cot[c + d\*x]]]\*(1 + Cot[c + d\*x])\*Csc[c + d\*x]^2\*Sec[c + d\*x]^2\*Sin[2\*(c + d\*x)])/(2\*(1 + Cot[c + d\*x])^2\*(1 + Tan[c + d\*x])))))/(4\*d\*(e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^2)

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: catdef: division by zero

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx+c) + a)^2 (e \cot(dx+c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)^2\*(e\*cot(d\*x + c))^(5/2)), x)

**maple [A]** time = 0.67, size = 276, normalized size = 0.83

$$\frac{\frac{\sqrt{e \cot(dx+c)}}{2d a^2 e^2 (e \cot(dx+c) + e)} \frac{7 \arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{2a^2 d e^{\frac{5}{2}}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{8d a^2 e^2 (e^2)^{\frac{1}{4}}} \sqrt{2} \arctan\left(\frac{\sqrt{2}}{\sqrt{e}}\right)}{4d a^2 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(5/2)/(a+cot(d\*x+c)\*a)^2,x)

[Out]  $-\frac{1}{2} \frac{d}{a^2} \frac{e^{-2} (e \cot(dx+c))^{1/2}}{(e \cot(dx+c) + e)} - \frac{7}{2} \arctan\left(\frac{(e \cot(dx+c))^{1/2}}{e^{1/2}}\right) \frac{1}{a^2 d e^{5/2}} - \frac{1}{8} \frac{d}{a^2} \frac{e^{-2/2}}{e^{1/2}} \frac{2^{1/2} \ln\left(\frac{(e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}{(e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/2})}\right) - \frac{1}{4} \frac{d}{a^2} \frac{e^{-2/2}}{e^{1/2}} \frac{2^{1/2} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) + \frac{1}{4} \frac{d}{a^2} \frac{2}{e^2} \frac{e^{-2/2}}{(e^2)^{1/4}} \frac{2^{1/2} \arctan\left(-\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) + \frac{2}{3} \frac{d}{a^2} \frac{e}{d} \frac{1}{(e \cot(dx+c))^{3/2}} - \frac{4}{a^2} \frac{d}{e^2} \frac{1}{(e \cot(dx+c))^{1/2}}$

**maxima [A]** time = 1.01, size = 274, normalized size = 0.83

$$\frac{e \left( \frac{4 \left( 4e^2 - \frac{20e^2}{\tan(dx+c)} - \frac{27e^2}{\tan(dx+c)^2} \right)}{a^2 e^4 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}} + a^2 e^3 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{5}{2}}} - \frac{3 \left( \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{a^2 e^3} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $\frac{1}{24} e \left( \frac{4 \left( 4e^2 - \frac{20e^2}{\tan(dx+c)} - \frac{27e^2}{\tan(dx+c)^2} \right)}{a^2 e^4 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}} + a^2 e^3 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{5}{2}}} - \frac{3 \left( 2\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right) + 2\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right) \right)}{a^2 e^3} - \frac{\sqrt{2} \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} + \frac{e + e/\tan(dx+c)}{\sqrt{e}} + \frac{\sqrt{2} \log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right) + e + e/\tan(dx+c)}{\sqrt{e}} \right) / (a^2 e^7)^{1/2} / d$

**mupad [B]** time = 1.23, size = 425, normalized size = 1.28

$$\frac{\operatorname{atan}\left(\frac{2048 a^{10} d^5 e^{18} \sqrt{e \cot(c+dx)} \left(-\frac{1}{a^8 d^4 e^{10}}\right)^{1/4}}{2048 a^8 d^4 e^{16} + 100352 a^{12} d^6 e^{21} \sqrt{-\frac{1}{a^8 d^4 e^{10}}}} + \frac{100352 a^{14} d^7 e^{23} \sqrt{e \cot(c+dx)} \left(-\frac{1}{a^8 d^4 e^{10}}\right)^{3/4}}{2048 a^8 d^4 e^{16} + 100352 a^{12} d^6 e^{21} \sqrt{-\frac{1}{a^8 d^4 e^{10}}}}\right) \left(-\frac{1}{a^8 d^4 e^{10}}\right)^{1/4}}{2} - \operatorname{atan}\left(\frac{a^{10} d^5 e^{18}}{2048 a^8 d^4 e^{16} + 100352 a^{12} d^6 e^{21} \sqrt{-\frac{1}{a^8 d^4 e^{10}}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2), x)`

[Out]  $-\left(\operatorname{atan}\left(\frac{2048 a^{10} d^5 e^{18} (e \cot(c + dx))^{1/2} (-1/(a^8 d^4 e^{10}))^{1/4}}{(2048 a^8 d^4 e^{16} + 100352 a^{12} d^6 e^{21} (-1/(a^8 d^4 e^{10}))^{1/2}) + (100352 a^{14} d^7 e^{23} (e \cot(c + dx))^{1/2} (-1/(a^8 d^4 e^{10}))^{3/4})/(2048 a^8 d^4 e^{16} + 100352 a^{12} d^6 e^{21} (-1/(a^8 d^4 e^{10}))^{1/2})}\right) (-1/(a^8 d^4 e^{10}))^{1/4}\right)/2 - \operatorname{atan}\left(\frac{a^{10} d^5 e^{18} (e \cot(c + dx))^{1/2} (-1/(256 a^8 d^4 e^{10}))^{1/4} * 8192i}{(2048 a^8 d^4 e^{16} - 1605632 a^{12} d^6 e^{21} (-1/(256 a^8 d^4 e^{10}))^{1/2}) - (a^{14} d^7 e^{23} (e \cot(c + dx))^{1/2} (-1/(256 a^8 d^4 e^{10}))^{3/4} * 6422528i)/(2048 a^8 d^4 e^{16} - 1605632 a^{12} d^6 e^{21} (-1/(256 a^8 d^4 e^{10}))^{1/2})}\right) (-1/(256 a^8 d^4 e^{10}))^{1/4} * 2i - ((10 \cot(c + dx))/3 + (9 \cot(c + dx)^2)/2 - 2/3)/(a^2 d (e \cot(c + dx))^{5/2} + a^2 d e (e \cot(c + dx))^{3/2}) - \operatorname{atan}\left(\frac{(e \cot(c + dx))^{1/2} (-e^5)^{1/2} * 1i}{e^3} * (-e^5)^{1/2} * 7i\right)/(2 a^2 d e^5)$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e \cot(c+dx))^{\frac{5}{2}} \cot^2(c+dx) + 2(e \cot(c+dx))^{\frac{5}{2}} \cot(c+dx) + (e \cot(c+dx))^{\frac{5}{2}}} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2, x)`

[Out] `Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)), x)/a**2`

$$3.35 \quad \int \frac{(e \cot(c+dx))^{5/2}}{(a+a \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=164

$$-\frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2} \tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d} - \frac{5e^2 \sqrt{e \cot(c+dx)}}{8a^3d(\cot(c+dx)+1)} + \frac{e^2 \sqrt{e \cot(c+dx)}}{4ad(a \cot(c+dx)+a)^2}$$

[Out]  $-1/8*e^{(5/2)}*\arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^3/d+1/4*e^{(5/2)}*\operatorname{arctanh}(1/2*(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)})/a^3/d*2^{(1/2)}-5/8*e^2*(e*\cot(d*x+c))^{(1/2)}/a^3/d/(1+\cot(d*x+c))+1/4*e^2*(e*\cot(d*x+c))^{(1/2)}/a/d/(a+a*\cot(d*x+c))^2$

**Rubi [A]** time = 0.62, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {3565, 3649, 3654, 3532, 208, 3634, 63, 205}

$$-\frac{5e^2 \sqrt{e \cot(c+dx)}}{8a^3d(\cot(c+dx)+1)} - \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2} \tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d} + \frac{e^2 \sqrt{e \cot(c+dx)}}{4ad(a \cot(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)/(a + a\*Cot[c + d\*x])^3, x]

[Out]  $-(e^{(5/2)}*\operatorname{ArcTan}[\operatorname{Sqrt}[e*\cot[c+d*x]]/\operatorname{Sqrt}[e]])/(8*a^3*d) + (e^{(5/2)}*\operatorname{ArcTan}[\operatorname{h}[(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[e]*\cot[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[e*\cot[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^3*d) - (5*e^2*\operatorname{Sqrt}[e*\cot[c+d*x]])/(8*a^3*d*(1 + \cot[c+d*x])) + (e^2*\operatorname{Sqrt}[e*\cot[c+d*x]])/(4*a*d*(a + a*\cot[c+d*x])^2)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3565

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[((b\*c - a\*d)^2\*(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1))/(d\*f\*(n + 1)\*(c^2 + d^2)), x] - Dist[1/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 3)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a^2\*d\*(b\*d\*(m - 2) - a\*c\*(n + 1)) + b\*(b\*c - 2\*a\*d)\*(b\*c\*(m - 2) + a\*d\*(n + 1)) - d\*(n + 1)\*(3\*a^2\*b\*c - b^3\*c - a^3\*d + 3\*a\*b^2\*d)\*Tan[e + f\*x] - b\*(a\*d\*(2\*b\*c - a\*d)\*(m + n - 1) - b^2\*(c^2\*(m - 2) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && IntegerQ[2\*m]

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3649

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[((A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

## Rule 3654

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e + f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

## Rubi steps

$$\begin{aligned} \int \frac{(e \cot(c + dx))^{5/2}}{(a + a \cot(c + dx))^3} dx &= \frac{e^2 \sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} - \frac{\int \frac{-\frac{1}{2}a^2 e^3 + 2a^2 e^3 \cot(c + dx) - \frac{5}{2}a^2 e^3 \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}(a + a \cot(c + dx))^2} dx}{4a^3} \\ &= -\frac{5e^2 \sqrt{e \cot(c + dx)}}{8a^3 d(1 + \cot(c + dx))} + \frac{e^2 \sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{\int \frac{-\frac{3}{2}a^4 e^4 + \frac{5}{2}a^4 e^4 \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}(a + a \cot(c + dx))} dx}{8a^6 e} \\ &= -\frac{5e^2 \sqrt{e \cot(c + dx)}}{8a^3 d(1 + \cot(c + dx))} + \frac{e^2 \sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{\int \frac{-4a^5 e^4 + 4a^5 e^4 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{16a^8 e} + \frac{e^3 \int}{16a^8 e} \\ &= -\frac{5e^2 \sqrt{e \cot(c + dx)}}{8a^3 d(1 + \cot(c + dx))} + \frac{e^2 \sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{e^3 \text{Subst}\left(\int \frac{1}{\sqrt{-x}(a - ax)} dx, x, -\cot(c + dx)\right)}{16a^2 d} \\ &= \frac{e^{5/2} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e \cot(c + dx)}}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{2\sqrt{2} a^3 d} - \frac{5e^2 \sqrt{e \cot(c + dx)}}{8a^3 d(1 + \cot(c + dx))} + \frac{e^2 \sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} - \frac{e^2}{16a^2 d} \\ &= -\frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{8a^3 d} + \frac{e^{5/2} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e \cot(c + dx)}}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{2\sqrt{2} a^3 d} - \frac{5e^2 \sqrt{e \cot(c + dx)}}{8a^3 d(1 + \cot(c + dx))} + \frac{e^2}{16a^2 d} \end{aligned}$$

**Mathematica [A]** time = 2.13, size = 192, normalized size = 1.17

$$\frac{\csc(c + dx)(e \cot(c + dx))^{5/2}(\sin(c + dx) + \cos(c + dx))^3 \left( \frac{\sec^4(c + dx)(-5 \sin(2(c + dx)) + 3 \cos(2(c + dx)) - 3)}{(\tan(c + dx) + 1)^2} - \frac{2 \csc(c + dx) \sec(c + dx)}{16a^3 d(\cot(c + dx) + 1)^3} \right)}{16a^3 d(\cot(c + dx) + 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)/(a + a\*Cot[c + d\*x])^3,x]

```
[Out] ((e*Cot[c + d*x])^(5/2)*Csc[c + d*x]*(Cos[c + d*x] + Sin[c + d*x])^3*((-2*C
sc[c + d*x]*(ArcTan[Sqrt[Cot[c + d*x]])] + Sqrt[2]*(Log[-1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] - Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
d*x]])))*Sec[c + d*x])/Cot[c + d*x]^(3/2) + (Sec[c + d*x]^4*(-3 + 3*Cos[2*(c
+ d*x)] - 5*Sin[2*(c + d*x)]))/(1 + Tan[c + d*x]^2))/(16*a^3*d*(1 + Cot[c
+ d*x])^3)
```

**fricas** [A] time = 0.47, size = 567, normalized size = 3.46

$$\frac{4\left(\sqrt{2}e^2\sin(2dx+2c)+\sqrt{2}e^2\right)\sqrt{-e}\arctan\left(\frac{\left(\sqrt{2}\cos(2dx+2c)+\sqrt{2}\sin(2dx+2c)+\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}}{2(e\cos(2dx+2c)+e)}\right)}{e^2\sin(2dx+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(sqrt(2)*e^2*sin(2*d*x + 2*c) + sqrt(2)*e^2)*sqrt(-e)*arctan(1/2*
(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(e*cos(2*d*x + 2*c) + e) - (
e^2*sin(2*d*x + 2*c) + e^2)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x
+ 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d
*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - (3*e^2*cos(2*d*
x + 2*c) - 5*e^2*sin(2*d*x + 2*c) - 3*e^2)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d), -1/16*(2*(e^2*sin(2*d*x
+ 2*c) + e^2)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)/sqrt(e)) - 2*(sqrt(2)*e^2*sin(2*d*x + 2*c) + sqrt(2)*e^2)*sqrt(e)*log(-(s
qrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt(e)*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + 2*e*sin(2*d*x + 2*c) + e) - (3
*e^2*cos(2*d*x + 2*c) - 5*e^2*sin(2*d*x + 2*c) - 3*e^2)*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{5}{2}}}{(a \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^3, x)
```

**maple [B]** time = 0.83, size = 440, normalized size = 2.68

$$\frac{5e^3 (e \cot(dx+c))^{\frac{3}{2}}}{8da^3 (e \cot(dx+c)+e)^2} - \frac{3e^4 \sqrt{e \cot(dx+c)}}{8da^3 (e \cot(dx+c)+e)^2} - \frac{e^{\frac{5}{2}} \arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{8a^3 d} + \frac{e^2 (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}} \sqrt{e}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}} \sqrt{e}}\right)}{16da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(5/2)/(a+cot(d\*x+c)\*a)^3,x)

[Out] 
$$-5/8/d/a^3*e^3/(e*\cot(d*x+c)+e)^2*(e*\cot(d*x+c))^{(3/2)}-3/8/d/a^3*e^4/(e*\cot(d*x+c)+e)^2*(e*\cot(d*x+c))^{(1/2)}-1/8*e^{(5/2)}*\arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^3/d+1/16/d/a^3*e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))+1/8/d/a^3*e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/8/d/a^3*e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/16/d/a^3*e^3*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/8/d/a^3*e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/8/d/a^3*e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)$$

**maxima [A]** time = 0.46, size = 197, normalized size = 1.20

$$\frac{e \left( \frac{3e^3 \sqrt{\frac{e}{\tan(dx+c)}} + 5e^2 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}}}{a^3 e^2 + \frac{2a^3 e^2}{\tan(dx+c)} + \frac{a^3 e^2}{\tan(dx+c)^2}} - \frac{e^2 \left( \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right) - \sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e + \frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{a^3} \right) + \frac{e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-1/8*e*((3*e^3*\sqrt{e/\tan(d*x+c)}+5*e^2*(e/\tan(d*x+c))^{(3/2)})/(a^3*e^2+2*a^3*e^2/\tan(d*x+c)+a^3*e^2/\tan(d*x+c)^2)-e^2*(\sqrt{2}*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e}-\sqrt{2}*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e}))/a^3+e^{(3/2)}*\arctan(\sqrt{e/\tan(d*x+c)}/\sqrt{e})/a^3)/d$$



**mupad** [B] time = 1.04, size = 154, normalized size = 0.94

$$\frac{\sqrt{2} e^{5/2} \operatorname{atanh}\left(\frac{9\sqrt{2} e^{33/2} \sqrt{e \cot(c+dx)}}{32\left(\frac{9e^{17} \cot(c+dx)}{32} + \frac{9e^{17}}{32}\right)}\right)}{4a^3 d} - \frac{e^{5/2} \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d} - \frac{\frac{3e^4 \sqrt{e \cot(c+dx)}}{8} + \frac{5e^3 (e \cot(c+dx))^{3/2}}{8}}{d a^3 e^2 \cot(c+dx)^2 + 2d a^3 e^2 \cot(c+dx) + d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^3,x)`

[Out]  $(2^{(1/2)} * e^{(5/2)} * \operatorname{atanh}((9 * 2^{(1/2)} * e^{(33/2)} * (e * \cot(c + d * x))^{(1/2)}) / (32 * ((9 * e^{17} * \cot(c + d * x)) / 32 + (9 * e^{17}) / 32)))) / (4 * a^3 * d) - (e^{(5/2)} * \operatorname{atan}((e * \cot(c + d * x))^{(1/2)} / e^{(1/2)})) / (8 * a^3 * d) - ((3 * e^4 * (e * \cot(c + d * x))^{(1/2)}) / 8 + (5 * e^3 * (e * \cot(c + d * x))^{(3/2)}) / 8) / (a^3 * d * e^2 + a^3 * d * e^2 * \cot(c + d * x)^2 + 2 * a^3 * d * e^2 * \cot(c + d * x))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c+dx))^{\frac{5}{2}}}{\cot^3(c+dx) + 3 \cot^2(c+dx) + 3 \cot(c+dx) + 1} dx$$

$$a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)`

[Out] `Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*cot(c + d*x) + 1), x)/a**3`

$$3.36 \quad \int \frac{(e \cot(c+dx))^{3/2}}{(a+a \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=164

$$\frac{5e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d} + \frac{e\sqrt{e \cot(c+dx)}}{8d(a^3 \cot(c+dx) + a^3)} - \frac{e\sqrt{e \cot(c+dx)}}{4ad(a \cot(c+dx) + a)^2}$$

[Out]  $5/8 * e^{(3/2)} * \arctan((e * \cot(d * x + c))^{(1/2)} / e^{(1/2)}) / a^3 / d + 1/4 * e^{(3/2)} * \arctan(1/2 * (e^{(1/2)} - \cot(d * x + c) * e^{(1/2)}) * 2^{(1/2)} / (e * \cot(d * x + c))^{(1/2)}) / a^3 / d^2^{(1/2)} - 1/4 * e * (e * \cot(d * x + c))^{(1/2)} / a / d / (a + a * \cot(d * x + c))^{(1/2)} + 1/8 * e * (e * \cot(d * x + c))^{(1/2)} / d / (a^3 + a^3 * \cot(d * x + c))$

**Rubi [A]** time = 0.66, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {3567, 3649, 3653, 3532, 205, 3634, 63}

$$\frac{5e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d} + \frac{e\sqrt{e \cot(c+dx)}}{8d(a^3 \cot(c+dx) + a^3)} - \frac{e\sqrt{e \cot(c+dx)}}{4ad(a \cot(c+dx) + a)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e * \text{Cot}[c + d * x])^{(3/2)} / (a + a * \text{Cot}[c + d * x])^3, x]$

[Out]  $(5 * e^{(3/2)} * \text{ArcTan}[\text{Sqrt}[e * \text{Cot}[c + d * x]] / \text{Sqrt}[e]]) / (8 * a^3 * d) + (e^{(3/2)} * \text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e] * \text{Cot}[c + d * x]) / (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d * x]])]) / (2 * \text{Sqrt}[2] * a^3 * d) - (e * \text{Sqrt}[e * \text{Cot}[c + d * x]]) / (4 * a * d * (a + a * \text{Cot}[c + d * x])^2) + (e * \text{Sqrt}[e * \text{Cot}[c + d * x]]) / (8 * d * (a^3 + a^3 * \text{Cot}[c + d * x]))$

### Rule 63

$\text{Int}(((a_.) + (b_.) * (x_.))^{(m_.)} * ((c_.) + (d_.) * (x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p * (m + 1) - 1)} * (c - (a * d) / b + (d * x^p) / b)^n, x], x, (a + b * x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 205

$\text{Int}(((a_.) + (b_.) * (x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] * \text{ArcTan}[x / \text{Rt}[a/b, 2]]) / a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

### Rule 3532

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]
```

### Rule 3567

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2*m]
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3649

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3653

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
```

+ f\*x]^2))/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e \cot(c + dx))^{3/2}}{(a + a \cot(c + dx))^3} dx &= -\frac{e\sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} - \frac{\int \frac{\frac{ae^2}{2} - 2ae^2 \cot(c+dx) - \frac{3}{2}ae^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))^2} dx}{4a^2} \\
 &= -\frac{e\sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{e\sqrt{e \cot(c + dx)}}{8d(a^3 + a^3 \cot(c + dx))} + \frac{\int \frac{-\frac{1}{2}a^3e^3 + 4a^3e^3 \cot(c+dx) - \frac{1}{2}a^3e^3 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))} dx}{8a^5e} \\
 &= -\frac{e\sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{e\sqrt{e \cot(c + dx)}}{8d(a^3 + a^3 \cot(c + dx))} + \frac{\int \frac{4a^4e^3 + 4a^4e^3 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{16a^7e} \quad (5e) \\
 &= -\frac{e\sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{e\sqrt{e \cot(c + dx)}}{8d(a^3 + a^3 \cot(c + dx))} - \frac{(5e^2) \text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, \frac{e \cot(c + dx)}{a}\right)}{16a^2d} \\
 &= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e \cot(c+dx)}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d} - \frac{e\sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \frac{e\sqrt{e \cot(c + dx)}}{8d(a^3 + a^3 \cot(c + dx))} + \dots \\
 &= \frac{5e^{3/2} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e \cot(c+dx)}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d} - \frac{e\sqrt{e \cot(c + dx)}}{4ad(a + a \cot(c + dx))^2} + \dots
 \end{aligned}$$

**Mathematica [A]** time = 2.02, size = 131, normalized size = 0.80

$$\frac{e\sqrt{e \cot(c + dx)} \left( \frac{2\sqrt{2} \tan^{-1}(1 - \sqrt{2} \sqrt{\cot(c+dx)}) - 2\sqrt{2} \tan^{-1}(\sqrt{2} \sqrt{\cot(c+dx)} + 1) + 5 \tan^{-1}(\sqrt{\cot(c+dx)})}{\sqrt{\cot(c+dx)}} + \frac{\tan(c+dx) - \sec^2(c+dx) + 1}{(\tan(c+dx) + 1)^2} \right)}{8a^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)/(a + a\*Cot[c + d\*x])^3,x]

[Out] (e\*Sqrt[e\*Cot[c + d\*x]]\*((2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 5\*ArcTan[Sqrt[Cot[c + d\*x]]])/Sqrt[Cot[c + d\*x]] + (1 - Sec[c + d\*x]^2 + Tan[c + d\*x])/(1 + Tan[c + d\*x])^2))/(8\*a^3\*d)

**fricas** [A] time = 0.54, size = 533, normalized size = 3.25

$$2 \left( \sqrt{2} e \sin(2dx + 2c) + \sqrt{2} e \right) \sqrt{-e} \log \left( - \left( \sqrt{2} \cos(2dx + 2c) + \sqrt{2} \sin(2dx + 2c) - \sqrt{2} \right) \sqrt{-e} \sqrt{\frac{e \cos(2dx + 2c)}{\sin(2dx + 2c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [1/16\*(2\*(sqrt(2)\*e\*sin(2\*d\*x + 2\*c) + sqrt(2)\*e)\*sqrt(-e)\*log(-(sqrt(2)\*cos(2\*d\*x + 2\*c) + sqrt(2)\*sin(2\*d\*x + 2\*c) - sqrt(2))\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)) - 2\*e\*sin(2\*d\*x + 2\*c) + e) + 5\*(e\*sin(2\*d\*x + 2\*c) + e)\*sqrt(-e)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) + 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) + (e\*cos(2\*d\*x + 2\*c) + e\*sin(2\*d\*x + 2\*c) - e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a^3\*d\*sin(2\*d\*x + 2\*c) + a^3\*d), 1/16\*(4\*(sqrt(2)\*e\*sin(2\*d\*x + 2\*c) + sqrt(2)\*e)\*sqrt(e)\*arctan(-1/2\*(sqrt(2)\*cos(2\*d\*x + 2\*c) - sqrt(2)\*sin(2\*d\*x + 2\*c) + sqrt(2))\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(e\*cos(2\*d\*x + 2\*c) + e) + 10\*(e\*sin(2\*d\*x + 2\*c) + e)\*sqrt(e)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/sqrt(e) + (e\*cos(2\*d\*x + 2\*c) + e\*sin(2\*d\*x + 2\*c) - e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a^3\*d\*sin(2\*d\*x + 2\*c) + a^3\*d)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{(a \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(3/2)/(a\*cot(d\*x + c) + a)^3, x)

**maple** [B] time = 0.84, size = 434, normalized size = 2.65

$$\frac{e^2 (e \cot(dx + c))^{\frac{3}{2}}}{8d a^3 (e \cot(dx + c) + e)^2} - \frac{e^3 \sqrt{e \cot(dx + c)}}{8d a^3 (e \cot(dx + c) + e)^2} + \frac{5e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{8a^3 d} - \frac{e (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{2}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{2}}\right)}{16d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(d*x+c))^(3/2)/(a+cot(d*x+c)*a)^3,x)`

[Out]  $\frac{1}{8} \frac{d}{a^3} \frac{e^2}{(e \cot(dx+c)+e)^2} \frac{e^2}{(e \cot(dx+c)+e)^2} \frac{e^2}{(e \cot(dx+c)+e)^2} - \frac{1}{8} \frac{d}{a^3} \frac{e^3}{(e \cot(dx+c)+e)^2} \frac{e^3}{(e \cot(dx+c)+e)^2} \frac{e^3}{(e \cot(dx+c)+e)^2} + \frac{5}{8} \frac{e^{3/2}}{a^3} \frac{e^{3/2}}{a^3} \frac{e^{3/2}}{a^3} \arctan\left(\frac{(e \cot(dx+c))^{1/2}}{e^{1/2}}\right) \frac{e^{3/2}}{a^3} \arctan\left(\frac{(e \cot(dx+c))^{1/2}}{e^{1/2}}\right) \frac{e^{3/2}}{a^3} \arctan\left(\frac{(e \cot(dx+c))^{1/2}}{e^{1/2}}\right) - \frac{1}{16} \frac{d}{a^3} \frac{e}{a^3} \frac{e}{a^3} \frac{e}{a^3} (e^2)^{1/4} 2^{1/2} \ln\left(\frac{(e \cot(dx+c)+(e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/4})}{(e \cot(dx+c)-(e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/4})}\right) - \frac{1}{8} \frac{d}{a^3} \frac{e}{a^3} \frac{e}{a^3} \frac{e}{a^3} (e^2)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) + \frac{1}{8} \frac{d}{a^3} \frac{e}{a^3} \frac{e}{a^3} \frac{e}{a^3} (e^2)^{1/4} 2^{1/2} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) - \frac{1}{16} \frac{d}{a^3} \frac{e}{a^3} \frac{e}{a^3} \frac{e}{a^3} 2^{1/2} \frac{e}{a^3} \ln\left(\frac{(e \cot(dx+c)-(e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/4})}{(e \cot(dx+c)+(e^2)^{1/4} (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/4})}\right) - \frac{1}{8} \frac{d}{a^3} \frac{e}{a^3} \frac{e}{a^3} \frac{e}{a^3} 2^{1/2} \frac{e}{a^3} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right) + \frac{1}{8} \frac{d}{a^3} \frac{e}{a^3} \frac{e}{a^3} \frac{e}{a^3} 2^{1/2} \frac{e}{a^3} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1}\right)$

**maxima** [A] time = 0.92, size = 189, normalized size = 1.15

$$\frac{e^2 \sqrt{\frac{e}{\tan(dx+c)}} - e \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{a^3 e^2 + \frac{2a^3 e^2}{\tan(dx+c)} + \frac{a^3 e^2}{\tan(dx+c)^2}} + \frac{2e \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{a^3} - \frac{5\sqrt{e} \arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{a^3}$$


---

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")`

[Out]  $-\frac{1}{8} \frac{e \left( (e^2 \sqrt{e/\tan(dx+c)}) - e (e/\tan(dx+c))^{3/2} \right)}{a^3 e^2 + 2a^3 e^2/\tan(dx+c) + a^3 e^2/\tan(dx+c)^2} + \frac{2e \left( \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left( \sqrt{2} \sqrt{e} + 2\sqrt{e/\tan(dx+c)} \right) / \sqrt{e} \right) / \sqrt{e} + \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left( \sqrt{2} \sqrt{e} - 2\sqrt{e/\tan(dx+c)} \right) / \sqrt{e} \right) / \sqrt{e} \right)}{a^3} - \frac{5\sqrt{e} \arctan\left(\frac{\sqrt{e/\tan(dx+c)}}{\sqrt{e}}\right)}{a^3} \frac{1}{d}$

**mupad** [B] time = 0.94, size = 178, normalized size = 1.09

$$\frac{5e^{3/2} \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d} - \frac{\frac{e^3 \sqrt{e \cot(c+dx)}}{8} - \frac{e^2 (e \cot(c+dx))^{3/2}}{8}}{d a^3 e^2 \cot(c+dx)^2 + 2d a^3 e^2 \cot(c+dx) + d a^3 e^2} - \frac{\sqrt{2} e^{3/2} \left( 2 \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2\sqrt{e}}\right) \right)}{8a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^3,x)`

[Out]  $(5*e^{3/2}*atan((e*cot(c + d*x))^{1/2}/e^{1/2}))/8*a^3*d - ((e^3*(e*cot(c + d*x))^{1/2})/8 - (e^2*(e*cot(c + d*x))^{3/2})/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (2^{1/2}*e^{3/2}*(2*atan((2^{1/2}*(e*cot(c + d*x))^{1/2})/(2*e^{1/2}))) + 2*atan((2^{1/2}*(e*cot(c + d*x))^{1/2})/(2*e^{1/2})) + (2^{1/2}*(e*cot(c + d*x))^{3/2})/(2*e^{3/2}))))/(8*a^3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c+dx))^{\frac{3}{2}}}{\frac{\cot^3(c+dx)+3 \cot^2(c+dx)+3 \cot(c+dx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)`

[Out] `Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*cot(c + d*x) + 1), x)/a**3`

$$3.37 \quad \int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^3} dx$$

Optimal. Leaf size=161

$$\frac{3\sqrt{e \cot(c+dx)}}{8d(a^3 \cot(c+dx) + a^3)} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d} + \frac{\sqrt{e \cot(c+dx)}}{4ad(a \cot(c+dx) + a)^2}$$

[Out]  $-1/8*\arctan((e*\cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^3/d-1/4*\operatorname{arctanh}(1/2*(e^(1/2)+\cot(d*x+c))*e^(1/2))*2^(1/2)/(e*\cot(d*x+c))^(1/2))*e^(1/2)/a^3/d*2^(1/2)+1/4*(e*\cot(d*x+c))^(1/2)/a/d/(a+a*\cot(d*x+c))^2+3/8*(e*\cot(d*x+c))^(1/2)/d/(a^3+a^3*\cot(d*x+c))$

**Rubi [A]** time = 0.59, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {3568, 3649, 3654, 3532, 208, 3634, 63, 205}

$$\frac{3\sqrt{e \cot(c+dx)}}{8d(a^3 \cot(c+dx) + a^3)} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{e} \cot(c+dx) + \sqrt{e}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d} + \frac{\sqrt{e \cot(c+dx)}}{4ad(a \cot(c+dx) + a)^2}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]`

[Out]  $-(\text{Sqrt}[e]*\text{ArcTan}[\text{Sqrt}[e*\text{Cot}[c + d*x]]/\text{Sqrt}[e]])/(8*a^3*d) - (\text{Sqrt}[e]*\text{ArcTan}[\text{h}[(\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]])/(2*\text{Sqrt}[2]*a^3*d) + \text{Sqrt}[e*\text{Cot}[c + d*x]]/(4*a*d*(a + a*\text{Cot}[c + d*x])^2) + (3*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(8*d*(a^3 + a^3*\text{Cot}[c + d*x]))$

### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

### Rule 208



Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3568

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n)/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n - 1)\*Simp[a\*c\*(m + 1) - b\*d\*n - (b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b\*d\*(m + n + 1)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[2\*m]

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3649

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[((A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && ! (ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3654

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_)

```

+ (f_.)*(x_)^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[
e + f*x])^n*(1 + Tan[e + f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a,
b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e \cot(c+dx)}}{(a+a \cot(c+dx))^3} dx &= \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a \cot(c+dx))^2} - \frac{\int \frac{-\frac{ae}{2} - 2ae \cot(c+dx) + \frac{3}{2}ae \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))^2} dx}{4a^2} \\
&= \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a \cot(c+dx))^2} + \frac{3\sqrt{e \cot(c+dx)}}{8d(a^3+a^3 \cot(c+dx))} + \frac{\int \frac{\frac{5a^3e^2}{2} - \frac{3}{2}a^3e^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+a \cot(c+dx))} dx}{8a^5e} \\
&= \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a \cot(c+dx))^2} + \frac{3\sqrt{e \cot(c+dx)}}{8d(a^3+a^3 \cot(c+dx))} + \frac{\int \frac{4a^4e^2 - 4a^4e^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{16a^7e} + \frac{e \int \dots}{\dots} \\
&= \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a \cot(c+dx))^2} + \frac{3\sqrt{e \cot(c+dx)}}{8d(a^3+a^3 \cot(c+dx))} + \frac{e \operatorname{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-ax)} dx, x, -c\right)}{16a^2d} \\
&= -\frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d} + \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a \cot(c+dx))^2} + \frac{3\sqrt{e \cot(c+dx)}}{8d(a^3+a^3 \cot(c+dx))} \\
&= -\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d} + \frac{\sqrt{e \cot(c+dx)}}{4ad(a+a \cot(c+dx))^2} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.82, size = 181, normalized size = 1.12

$$\frac{\sqrt{e \cot(c+dx)} \left( \sqrt{\cot(c+dx)} (-3 \sin(2(c+dx)) + 5 \cos(2(c+dx)) - 5) + 2(\sin(2(c+dx)) + 1) \tan^{-1} \left( \sqrt{\cot(c+dx)} \right) \right)}{16a^3 d \sqrt{\cot(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]/(a + a\*Cot[c + d\*x])^3, x]

[Out] -1/16\*(Sqrt[e\*Cot[c + d\*x]]\*(-2\*Sqrt[2]\*(Log[-1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] - Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])\*(Co

$s[c + d*x] + \text{Sin}[c + d*x])^2 + \text{Sqrt}[\text{Cot}[c + d*x]]*(-5 + 5*\text{Cos}[2*(c + d*x)] - 3*\text{Sin}[2*(c + d*x)]) + 2*\text{ArcTan}[\text{Sqrt}[\text{Cot}[c + d*x]]*(1 + \text{Sin}[2*(c + d*x)])]) / (a^3*d*\text{Sqrt}[\text{Cot}[c + d*x]]*(\text{Cos}[c + d*x] + \text{Sin}[c + d*x])^2)$

**fricas** [A] time = 0.53, size = 518, normalized size = 3.22

$$\frac{4(\sqrt{2} \sin(2dx + 2c) + \sqrt{2})\sqrt{-e} \arctan\left(\frac{(\sqrt{2} \cos(2dx + 2c) + \sqrt{2} \sin(2dx + 2c) + \sqrt{2})\sqrt{-e} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}}}{2(e \cos(2dx + 2c) + e)}\right) + \sqrt{-e} (\sin(2dx + 2c) + 1) \log\left(\frac{(\sqrt{2} \cos(2dx + 2c) + \sqrt{2} \sin(2dx + 2c) + \sqrt{2})\sqrt{-e} \sqrt{\frac{e \cos(2dx + 2c) + e}{\sin(2dx + 2c)}}}{2(e \cos(2dx + 2c) + e)}\right)}{a^3 d \sin(2dx + 2c) + a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [1/16\*(4\*(sqrt(2)\*sin(2\*d\*x + 2\*c) + sqrt(2))\*sqrt(-e)\*arctan(1/2\*(sqrt(2)\*cos(2\*d\*x + 2\*c) + sqrt(2)\*sin(2\*d\*x + 2\*c) + sqrt(2))\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(e\*cos(2\*d\*x + 2\*c) + e) + sqrt(-e)\*(sin(2\*d\*x + 2\*c) + 1)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) - 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) - sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(5\*cos(2\*d\*x + 2\*c) - 3\*sin(2\*d\*x + 2\*c) - 5))/(a^3\*d\*sin(2\*d\*x + 2\*c) + a^3\*d), -1/16\*(2\*sqrt(e)\*(sin(2\*d\*x + 2\*c) + 1)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)) - 2\*(sqrt(2)\*sin(2\*d\*x + 2\*c) + sqrt(2))\*sqrt(e)\*log((sqrt(2)\*cos(2\*d\*x + 2\*c) - sqrt(2)\*sin(2\*d\*x + 2\*c) - sqrt(2))\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)) + 2\*e\*sin(2\*d\*x + 2\*c) + e) + sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(5\*cos(2\*d\*x + 2\*c) - 3\*sin(2\*d\*x + 2\*c) - 5))/(a^3\*d\*sin(2\*d\*x + 2\*c) + a^3\*d)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(dx + c)}}{(a \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(e\*cot(d\*x + c))/(a\*cot(d\*x + c) + a)^3, x)

**maple [B]** time = 0.86, size = 423, normalized size = 2.63

$$\frac{3e(e \cot(dx+c))^{\frac{3}{2}}}{8da^3(e \cot(dx+c)+e)^2} + \frac{5e^2\sqrt{e \cot(dx+c)}}{8da^3(e \cot(dx+c)+e)^2} - \frac{\arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)\sqrt{e}}{8a^3d} - \frac{(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}}\right)}{16da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(1/2)/(a+cot(d\*x+c)\*a)^3,x)

[Out] 3/8/d/a^3\*e/(e\*cot(d\*x+c)+e)^2\*(e\*cot(d\*x+c))^(3/2)+5/8/d/a^3\*e^2/(e\*cot(d\*x+c)+e)^2\*(e\*cot(d\*x+c))^(1/2)-1/8\*arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))\*e^(1/2)/a^3/d-1/16/d/a^3\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))-1/8/d/a^3\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/8/d/a^3\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/16/d/a^3\*e\*2^(1/2)/(e^2)^(1/4)\*ln((e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/8/d/a^3\*e\*2^(1/2)/(e^2)^(1/4)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/8/d/a^3\*e\*2^(1/2)/(e^2)^(1/4)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)

**maxima [A]** time = 0.60, size = 190, normalized size = 1.18

$$e \left( \frac{5e\sqrt{\frac{e}{\tan(dx+c)}} + 3\left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{a^3e^2 + \frac{2a^3e^2}{\tan(dx+c)} + \frac{a^3e^2}{\tan(dx+c)^2}} - \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e} + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e} + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}}\right)}{a^3\sqrt{e}} \right) / 8d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/8\*e\*((5\*e\*sqrt(e/tan(d\*x+c)) + 3\*(e/tan(d\*x+c))^(3/2))/(a^3\*e^2 + 2\*a^3\*e^2/tan(d\*x+c) + a^3\*e^2/tan(d\*x+c)^2) - (sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x+c)) + e + e/tan(d\*x+c))/sqrt(e) - sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x+c)) + e + e/tan(d\*x+c))/sqrt(e))/a^3 - arctan(sqrt(e/tan(d\*x+c))/sqrt(e))/(a^3\*sqrt(e)))/d

mupad [B] time = 0.90, size = 151, normalized size = 0.94

$$\frac{\frac{3e(e \cot(c+dx))^{3/2}}{8} + \frac{5e^2 \sqrt{e \cot(c+dx)}}{8}}{d a^3 e^2 \cot(c+dx)^2 + 2 d a^3 e^2 \cot(c+dx) + d a^3 e^2} - \frac{\sqrt{e} \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8 a^3 d} - \frac{\sqrt{2} \sqrt{e} \operatorname{atanh}\left(\frac{9 \sqrt{2} e^{17/2} \sqrt{e \cot(c+dx)}}{32 \left(\frac{9 e^9 \cot(c+dx)}{32} + \frac{9 e^9}{32}\right)}\right)}{4 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^3,x)`

[Out]  $((3e*(e \cot(c + dx))^{3/2})/8 + (5e^2*(e \cot(c + dx))^{1/2})/8)/(a^3 d e^2 + a^3 d e^2 \cot(c + dx) + 2 a^3 d e^2 \cot(c + dx)) - (e^{1/2} \operatorname{atan}((e \cot(c + dx))^{1/2}/e^{1/2}))/8 a^3 d - (2^{1/2} e^{1/2} \operatorname{atanh}((9 * 2^{1/2} * e^{17/2} * (e \cot(c + dx))^{1/2})/(32 * ((9 e^9 \cot(c + dx))/32 + (9 e^9)/32))))/(4 a^3 d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(c+dx)}}{\cot^3(c+dx) + 3 \cot^2(c+dx) + 3 \cot(c+dx) + 1} dx$$

$$a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)`

[Out] `Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*cot(c + d*x) + 1), x)/a**3`

$$3.38 \quad \int \frac{1}{\sqrt{e \cot(c+dx)} (a+a \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=165

$$\frac{7\sqrt{e \cot(c+dx)}}{8a^3de(\cot(c+dx)+1)} - \frac{11 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{4ade(a \cot(c+dx)+a)^2}$$

[Out]  $-11/8*\arctan((e*\cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(1/2)-1/4*\arctan(1/2*(e^(1/2)-\cot(d*x+c)*e^(1/2))*2^(1/2)/(e*\cot(d*x+c))^(1/2))/a^3/d*2^(1/2)/e^(1/2)-7/8*(e*\cot(d*x+c))^(1/2)/a^3/d/e/(1+\cot(d*x+c))-1/4*(e*\cot(d*x+c))^(1/2)/a/d/e/(a+a*\cot(d*x+c))^2$

**Rubi [A]** time = 0.65, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$ , Rules used = {3569, 3649, 3653, 3532, 205, 3634, 63}

$$\frac{7\sqrt{e \cot(c+dx)}}{8a^3de(\cot(c+dx)+1)} - \frac{11 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3d\sqrt{e}} - \frac{\sqrt{e \cot(c+dx)}}{4ade(a \cot(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^3), x]

[Out]  $(-11*\text{ArcTan}[\text{Sqrt}[e*\text{Cot}[c + d*x]]/\text{Sqrt}[e]])/(8*a^3*d*\text{Sqrt}[e]) - \text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c + d*x])]/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*a^3*d*\text{Sqrt}[e]) - (7*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(8*a^3*d*e*(1 + \text{Cot}[c + d*x])) - \text{Sqrt}[e*\text{Cot}[c + d*x]]/(4*a*d*e*(a + a*\text{Cot}[c + d*x])^2)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 3532

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]
```

### Rule 3569

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3649

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3653

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
```

$+ f*x]^2)/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0] \&\& !\text{LeQ}[n, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{e \cot(c+dx)} (a + a \cot(c+dx))^3} dx &= -\frac{\sqrt{e \cot(c+dx)}}{4ade(a + a \cot(c+dx))^2} - \frac{\int \frac{-\frac{7a^2e}{2} + 2a^2e \cot(c+dx) - \frac{3}{2}a^2e \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a + a \cot(c+dx))^2} dx}{4a^3e} \\ &= -\frac{7\sqrt{e \cot(c+dx)}}{8a^3de(1 + \cot(c+dx))} - \frac{\sqrt{e \cot(c+dx)}}{4ade(a + a \cot(c+dx))^2} + \frac{\int \frac{\frac{7a^4e^2}{2} - 4a^4e^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{16a^3de} \\ &= -\frac{7\sqrt{e \cot(c+dx)}}{8a^3de(1 + \cot(c+dx))} - \frac{\sqrt{e \cot(c+dx)}}{4ade(a + a \cot(c+dx))^2} + \frac{11 \int \frac{1 + \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{16a^3de} \\ &= -\frac{7\sqrt{e \cot(c+dx)}}{8a^3de(1 + \cot(c+dx))} - \frac{\sqrt{e \cot(c+dx)}}{4ade(a + a \cot(c+dx))^2} + \frac{11 \text{Subst}\left(\int \frac{1 + \cot(u)}{\sqrt{e \cot(u)}} du\right)}{16a^3de} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d \sqrt{e}} - \frac{7\sqrt{e \cot(c+dx)}}{8a^3de(1 + \cot(c+dx))} - \frac{\sqrt{e \cot(c+dx)}}{4ade(a + a \cot(c+dx))^2} \\ &= -\frac{11 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d \sqrt{e}} - \frac{\tan^{-1}\left(\frac{\sqrt{e} - \sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d \sqrt{e}} - \frac{7\sqrt{e \cot(c+dx)}}{8a^3de(1 + \cot(c+dx))} \end{aligned}$$

**Mathematica** [A] time = 1.27, size = 217, normalized size = 1.32

$$\frac{\sqrt{\cot(c+dx)} \left( -9\sqrt{\cot(c+dx)} + 9 \cos(2(c+dx))\sqrt{\cot(c+dx)} - 7 \sin(2(c+dx))\sqrt{\cot(c+dx)} - 22 \tan^{-1}\left(\sqrt{\cot(c+dx)}\right) \right)}{16a^3d\sqrt{e}(\cos(c+dx) + \sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^3),x]

[Out] (Sqrt[Cot[c + d\*x]]\*(-22\*ArcTan[Sqrt[Cot[c + d\*x]]] - 9\*Sqrt[Cot[c + d\*x]] + 9\*Cos[2\*(c + d\*x)]\*Sqrt[Cot[c + d\*x]] - 4\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*(Cos[c + d\*x] + Sin[c + d\*x])^2 + 4\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]\*(Cos[c + d\*x] + Sin[c + d\*x])^2 - 22\*ArcTan[Sqrt[Cot[c + d\*x]]]\*Sin[2\*(c + d\*x)] - 7\*Sqrt[Cot[c + d\*x]]\*Sin[2\*(c + d\*x)]))/(16\*a^3\*d\*Sqrt[e\*Cot[c + d\*x]]\*(Cos[c + d\*x] + Sin[c + d\*x])^2)



**fricas** [A] time = 0.45, size = 504, normalized size = 3.05

$$\left[ \frac{2\sqrt{2}\sqrt{-e}(\sin(2dx+2c)+1)\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)+\sin(2dx+2c)-1)-2e\sin(2dx+2c)\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [-1/16\*(2\*sqrt(2)\*sqrt(-e)\*(sin(2\*d\*x + 2\*c) + 1)\*log(-sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*e\*sin(2\*d\*x + 2\*c) + e) + 11\*sqrt(-e)\*(sin(2\*d\*x + 2\*c) + 1)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) + 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) - sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(9\*cos(2\*d\*x + 2\*c) - 7\*sin(2\*d\*x + 2\*c) - 9))/(a^3\*d\*e\*sin(2\*d\*x + 2\*c) + a^3\*d\*e), -1/16\*(4\*sqrt(2)\*sqrt(e)\*(sin(2\*d\*x + 2\*c) + 1)\*arctan(-1/2\*sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e)) + 22\*sqrt(e)\*(sin(2\*d\*x + 2\*c) + 1)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)) - sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(9\*cos(2\*d\*x + 2\*c) - 7\*sin(2\*d\*x + 2\*c) - 9))/(a^3\*d\*e\*sin(2\*d\*x + 2\*c) + a^3\*d\*e)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx + c) + a)^3 \sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)^3\*sqrt(e\*cot(d\*x + c))), x)

**maple** [B] time = 0.83, size = 426, normalized size = 2.58

$$\frac{7(e \cot(dx + c))^{\frac{3}{2}}}{8d a^3 (e \cot(dx + c) + e)^2} - \frac{9e\sqrt{e \cot(dx + c)}}{8d a^3 (e \cot(dx + c) + e)^2} - \frac{11 \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{8a^3 d \sqrt{e}} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e}}\right)}{16d a^3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*cot(d*x+c))^(1/2)/(a+cot(d*x+c)*a)^3,x)`

[Out] 
$$-7/8/d/a^3/(e*\cot(d*x+c)+e)^2*(e*\cot(d*x+c))^{3/2}-9/8/d/a^3*e/(e*\cot(d*x+c)+e)^2*(e*\cot(d*x+c))^{1/2}-11/8*\arctan((e*\cot(d*x+c))^{1/2}/e^{1/2})/a^3/d/e^{1/2}+1/16/d/a^3/e*(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))+1/8/d/a^3/e*(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/8/d/a^3/e*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+1/16/d/a^3*2^{1/2}/(e^2)^{1/4}*\ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))+1/8/d/a^3*2^{1/2}/(e^2)^{1/4}*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/8/d/a^3*2^{1/2}/(e^2)^{1/4}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)$$

**maxima** [A] time = 0.51, size = 189, normalized size = 1.15

$$\frac{e \left( \frac{9e \sqrt{\frac{e}{\tan(dx+c)}} + 7 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}}}{a^3 e^3 + \frac{2a^3 e^3}{\tan(dx+c)} + \frac{a^3 e^3}{\tan(dx+c)^2}} - \frac{2 \left( \frac{\sqrt{2} \arctan \left( \frac{\sqrt{2} (\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan \left( -\frac{\sqrt{2} (\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}} \right)}{\sqrt{e}} \right)}{a^3 e} + \frac{11 \arctan \left( \frac{\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{e}} \right)}{a^3 e^{\frac{3}{2}}} \right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")`

[Out] 
$$-1/8*e*((9*e*\sqrt{e/\tan(d*x+c)}+7*(e/\tan(d*x+c))^{3/2})/(a^3*e^3+2*a^3*e^3/\tan(d*x+c)+a^3*e^3/\tan(d*x+c)^2)-2*(\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e}))/\sqrt{e}+\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e}))/\sqrt{e}+(a^3*e+11*\arctan(\sqrt{e/\tan(d*x+c)}/\sqrt{e}))/a^3*e^{3/2})/d$$

**mupad** [B] time = 0.94, size = 173, normalized size = 1.05

$$\frac{\sqrt{2} \left( 2 \operatorname{atan} \left( \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2 \sqrt{e}} \right) + 2 \operatorname{atan} \left( \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{2 \sqrt{e}} + \frac{\sqrt{2} (e \cot(c+dx))^{3/2}}{2 e^{3/2}} \right) \right)}{8 a^3 d \sqrt{e}} - \frac{11 \operatorname{atan} \left( \frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{8 a^3 d \sqrt{e}} - \frac{9 e}{d a^3 e^2 \cot(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3),x)`

[Out]  $(2^{1/2} * (2 * \operatorname{atan}((2^{1/2} * (e * \cot(c + d * x))^{1/2}) / (2 * e^{1/2}))) + 2 * \operatorname{atan}((2^{1/2} * (e * \cot(c + d * x))^{1/2}) / (2 * e^{1/2}))) / (2 * e^{3/2})) / (8 * a^3 * d * e^{1/2}) - (11 * \operatorname{atan}((e * \cot(c + d * x))^{1/2} / e^{1/2})) / (8 * a^3 * d * e^{1/2}) - ((9 * e * (e * \cot(c + d * x))^{1/2}) / 8 + (7 * (e * \cot(c + d * x))^{3/2}) / 8) / (a^3 * d * e^2 + a^3 * d * e^2 * \cot(c + d * x)^2 + 2 * a^3 * d * e^2 * \cot(c + d * x))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \cot(c+dx)} \cot^3(c+dx) + 3\sqrt{e \cot(c+dx)} \cot^2(c+dx) + 3\sqrt{e \cot(c+dx)} \cot(c+dx) + \sqrt{e \cot(c+dx)}} \frac{dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)`

[Out] `Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**3 + 3*sqrt(e*cot(c + d*x))*cot(c + d*x)**2 + 3*sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**3`

$$3.39 \quad \int \frac{1}{(e \cot(c+dx))^{3/2} (a+a \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=189

$$\frac{31 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d e^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d e^{3/2}} + \frac{27}{8a^3 d e \sqrt{e \cot(c+dx)}} - \frac{9}{8a^3 d e (\cot(c+dx)+1) \sqrt{e \cot(c+dx)}} - \frac{1}{4}$$

[Out] 31/8\*arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))/a^3/d/e^(3/2)+1/4\*arctanh(1/2\*(e^(1/2)+cot(d\*x+c)\*e^(1/2))\*2^(1/2)/(e\*cot(d\*x+c))^(1/2))/a^3/d/e^(3/2)\*2^(1/2)+27/8/a^3/d/e/(e\*cot(d\*x+c))^(1/2)-9/8/a^3/d/e/(1+cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2)-1/4/a/d/e/(a+a\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2)

**Rubi [A]** time = 0.86, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {3569, 3649, 3654, 3532, 208, 3634, 63, 205}

$$\frac{31 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 d e^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e \cot(c+dx)+\sqrt{e}}}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 d e^{3/2}} + \frac{27}{8a^3 d e \sqrt{e \cot(c+dx)}} - \frac{9}{8a^3 d e (\cot(c+dx)+1) \sqrt{e \cot(c+dx)}} - \frac{1}{4}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])^3), x]

[Out] (31\*ArcTan[Sqrt[e\*Cot[c + d\*x]]/Sqrt[e]]/(8\*a^3\*d\*e^(3/2)) + ArcTanh[(Sqrt[e] + Sqrt[e]\*Cot[c + d\*x])/(Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])]/(2\*Sqrt[2]\*a^3\*d\*e^(3/2)) + 27/(8\*a^3\*d\*e\*Sqrt[e\*Cot[c + d\*x]]) - 9/(8\*a^3\*d\*e\*Sqrt[e\*Cot[c + d\*x]]\*(1 + Cot[c + d\*x])) - 1/(4\*a\*d\*e\*Sqrt[e\*Cot[c + d\*x]]\*(a + a\*Cot[c + d\*x])^2)

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c-(a\*d)/b+(d\*x^p)/b)^n, x], x, (a+b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3532

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*c\*d + b\*x^2), x], x, (c - d\*Tan[e + f\*x])/Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]

### Rule 3569

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), x] + Dist[1/((m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3649

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)] + (C\_)\*tan[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] := Simp[((A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3654

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[
e + f*x])^n*(1 + Tan[e + f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a,
b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^3} dx &= -\frac{1}{4ade\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2} - \int \frac{-\frac{9a^2e}{2} + 2a^2e \cot(c + dx) - \frac{5}{2}a^2e}{(e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^3} dx \\
&= -\frac{9}{8a^3de\sqrt{e \cot(c + dx)} (1 + \cot(c + dx))} - \frac{1}{4ade\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2} \\
&= \frac{27}{8a^3de\sqrt{e \cot(c + dx)}} - \frac{9}{8a^3de\sqrt{e \cot(c + dx)} (1 + \cot(c + dx))} - \frac{1}{4ade\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2} \\
&= \frac{27}{8a^3de\sqrt{e \cot(c + dx)}} - \frac{9}{8a^3de\sqrt{e \cot(c + dx)} (1 + \cot(c + dx))} - \frac{1}{4ade\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2} \\
&= \frac{27}{8a^3de\sqrt{e \cot(c + dx)}} - \frac{9}{8a^3de\sqrt{e \cot(c + dx)} (1 + \cot(c + dx))} - \frac{1}{4ade\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c + dx)}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{2\sqrt{2} a^3 de^{3/2}} + \frac{27}{8a^3de\sqrt{e \cot(c + dx)}} - \frac{1}{8a^3de\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2} \\
&= \frac{31 \tan^{-1}\left(\frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{8a^3 de^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e} + \sqrt{e} \cot(c + dx)}{\sqrt{2} \sqrt{e \cot(c + dx)}}\right)}{2\sqrt{2} a^3 de^{3/2}} + \frac{27}{8a^3de\sqrt{e \cot(c + dx)}} - \frac{1}{8a^3de\sqrt{e \cot(c + dx)} (a + a \cot(c + dx))^2}
\end{aligned}$$

**Mathematica [A]** time = 1.21, size = 156, normalized size = 0.83

$$\frac{\cot^{\frac{3}{2}}(c + dx) \left( -2\sqrt{2} \left( \log(-\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)}) - 1 \right) - \log(\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)} + 1) \right) + 6}{16a^3 d (e \cot(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(3/2)\*(a + a\*Cot[c + d\*x])^3),x]

[Out] (Cot[c + d\*x]^(3/2)\*(62\*ArcTan[Sqrt[Cot[c + d\*x]]] - 2\*Sqrt[2]\*(Log[-1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] - Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]) + (43 + 11\*Cos[2\*(c + d\*x)] + 45\*Sin[2\*(c + d\*x)])/(Sqrt[Cot[c + d\*x]]\*(Cos[c + d\*x] + Sin[c + d\*x])^2))/(16\*a^3\*d\*(e\*Cot[c + d\*x])^(3/2))

**fricas** [B] time = 0.66, size = 697, normalized size = 3.69

$$\frac{4\sqrt{2}((\cos(2dx+2c)+1)\sin(2dx+2c)+\cos(2dx+2c)+1)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)+1)}{2(e\cos(2dx+2c)+e)}\right)}{16a^3d(e\cot(dx+c))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [-1/16\*(4\*sqrt(2)\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(-e)\*arctan(1/2\*sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e)) + 31\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(-e)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) - 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) + (45\*cos(2\*d\*x + 2\*c)^2 - (11\*cos(2\*d\*x + 2\*c) + 43)\*sin(2\*d\*x + 2\*c) - 45)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a^3\*d\*e^2\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^2 + (a^3\*d\*e^2\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^2)\*sin(2\*d\*x + 2\*c)), 1/16\*(2\*sqrt(2)\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(e)\*log(-sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) - 1) + 2\*e\*sin(2\*d\*x + 2\*c) + e) + 62\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(e)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)) - (45\*cos(2\*d\*x + 2\*c)^2 - (11\*cos(2\*d\*x + 2\*c) + 43)\*sin(2\*d\*x + 2\*c) - 45)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a^3\*d\*e^2\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^2 + (a^3\*d\*e^2\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^2)\*sin(2\*d\*x + 2\*c)]]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx+c) + a)^3 (e \cot(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)^3\*(e\*cot(d\*x + c))^(3/2)), x)

**maple [B]** time = 0.80, size = 458, normalized size = 2.42

$$\frac{11 (e \cot(dx + c))^{\frac{3}{2}}}{8d a^3 e (e \cot(dx + c) + e)^2} + \frac{13 \sqrt{e \cot(dx + c)}}{8d a^3 (e \cot(dx + c) + e)^2} + \frac{31 \arctan\left(\frac{\sqrt{e \cot(dx + c)}}{\sqrt{e}}\right)}{8a^3 d e^{\frac{3}{2}}} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx + c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)}}{e \cot(dx + c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx + c)}}\right)}{16d a^3 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(3/2)/(a+cot(d\*x+c)\*a)^3,x)

[Out] 11/8/d/a^3/e/(e\*cot(d\*x+c)+e)^2\*(e\*cot(d\*x+c))^(3/2)+13/8/d/a^3/(e\*cot(d\*x+c)+e)^2\*(e\*cot(d\*x+c))^(1/2)+31/8\*arctan((e\*cot(d\*x+c))^(1/2)/e^(1/2))/a^3/d/e^(3/2)+1/16/d/a^3/e^2\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/8/d/a^3/e^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/8/d/a^3/e^2\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/16/d/a^3/e^2\*(1/2)/e^(1/4)\*ln((e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))-1/8/d/a^3/e^2\*(1/2)/(e^2)^(1/4)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/8/d/a^3/e^2\*(1/2)/(e^2)^(1/4)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+2/a^3/d/e/(e\*cot(d\*x+c))^(1/2)

**maxima [A]** time = 0.71, size = 214, normalized size = 1.13

$$e \left( \frac{16e^2 + \frac{45e^2}{\tan(dx+c)} + \frac{27e^2}{\tan(dx+c)^2}}{a^3 e^4 \sqrt{\frac{e}{\tan(dx+c)}} + 2a^3 e^3 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}} + a^3 e^2 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{5}{2}}} + \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e} + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} - \frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)} + e} + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right) + \frac{31}{a^3 e^2}$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/8\*e\*((16\*e^2 + 45\*e^2/tan(d\*x + c) + 27\*e^2/tan(d\*x + c)^2)/(a^3\*e^4\*sqrt(e/tan(d\*x + c)) + 2\*a^3\*e^3\*(e/tan(d\*x + c))^(3/2) + a^3\*e^2\*(e/tan(d\*x + c))^(5/2)) + (sqrt(2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e +



$e/\tan(dx + c)/\sqrt{e}/(a^3e^2) + 31\arctan(\sqrt{e/\tan(dx + c)/\sqrt{e}})/(a^3e^{5/2})/d$

**mupad [B]** time = 1.14, size = 175, normalized size = 0.93

$$\frac{\frac{27e\cot(c+dx)^2}{8} + \frac{45e\cot(c+dx)}{8} + 2e}{a^3d(e\cot(c+dx))^{5/2} + 2a^3de(e\cot(c+dx))^{3/2} + a^3de^2\sqrt{e\cot(c+dx)}} + \frac{31\operatorname{atan}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}} + \frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{4a^3de^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3), x)`

[Out]  $(2e + (45e\cot(c + dx))/8 + (27e\cot(c + dx)^2)/8)/(a^3d(e\cot(c + dx))^{5/2} + 2a^3de(e\cot(c + dx))^{3/2} + a^3de^2(e\cot(c + dx))^{1/2}) + (31\operatorname{atan}((e\cot(c + dx))^{1/2}/e^{1/2}))/8a^3de^{3/2} + (2^{1/2}\operatorname{atanh}((63504384*2^{1/2}*a^9d^3e^{15/2})(e\cot(c + dx))^{1/2}))/63504384a^9d^3e^8 + 63504384a^9d^3e^8\cot(c + dx))/4a^3de^{3/2}$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{(e\cot(c+dx))^{\frac{3}{2}}\cot^3(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3, x)`

[Out] `Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**(3/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**3`

$$3.40 \quad \int \frac{1}{(e \cot(c+dx))^{5/2} (a+a \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=215

$$-\frac{59 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 de^{5/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 de^{5/2}} - \frac{63}{8a^3 de^2 \sqrt{e \cot(c+dx)}} - \frac{11}{8a^3 de (\cot(c+dx)+1) (e \cot(c+dx))^{3/2}}$$

[Out]  $-59/8*\arctan((e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/a^3/d/e^{(5/2)}+55/24/a^3/d/e/(e*\cot(d*x+c))^{(3/2)}-11/8/a^3/d/e/(e*\cot(d*x+c))^{(3/2)}/(1+\cot(d*x+c))-1/4/a/d/e/(e*\cot(d*x+c))^{(3/2)}/(a+a*\cot(d*x+c))^{2+1/4}*\arctan(1/2*(e^{(1/2)}-\cot(d*x+c))*e^{(1/2)})*2^{(1/2)}/(e*\cot(d*x+c))^{(1/2)}/a^3/d/e^{(5/2)}*2^{(1/2)}-63/8/a^3/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 1.10, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {3569, 3649, 3650, 3653, 3532, 205, 3634, 63}

$$-\frac{63}{8a^3 de^2 \sqrt{e \cot(c+dx)}} - \frac{59 \tan^{-1}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8a^3 de^{5/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e} \cot(c+dx)}{\sqrt{2} \sqrt{e \cot(c+dx)}}\right)}{2\sqrt{2} a^3 de^{5/2}} - \frac{11}{8a^3 de (\cot(c+dx)+1) (e \cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((e*\text{Cot}[c+d*x])^{(5/2)}*(a+a*\text{Cot}[c+d*x])^3),x]$

[Out]  $(-59*\text{ArcTan}[\text{Sqrt}[e*\text{Cot}[c+d*x]]/\text{Sqrt}[e]])/(8*a^3*d*e^{(5/2)}) + \text{ArcTan}[(\text{Sqrt}[e] - \text{Sqrt}[e]*\text{Cot}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*a^3*d*e^{(5/2)}) + 55/(24*a^3*d*e*(e*\text{Cot}[c+d*x])^{(3/2)}) - 63/(8*a^3*d*e^2*\text{Sqrt}[e*\text{Cot}[c+d*x]]) - 11/(8*a^3*d*e*(e*\text{Cot}[c+d*x])^{(3/2)}*(1+\text{Cot}[c+d*x])) - 1/(4*a*d*e*(e*\text{Cot}[c+d*x])^{(3/2)}*(a+a*\text{Cot}[c+d*x])^2)$

### Rule 63

$\text{Int}[(a_. + (b_.)*(x_)^{(m_)})*((c_.) + (d_.)*(x_)^{(n_)})], x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 205

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}], x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$

Rule 3532

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*c*d + b*x^2), x], x, (c - d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 - d^2, 0]
```

Rule 3569

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3649

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3650

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n
```

```

+ 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
+ n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
-1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)])], x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c + dx))^{5/2} (a + a \cot(c + dx))^3} dx &= -\frac{1}{4ade(e \cot(c + dx))^{3/2} (a + a \cot(c + dx))^2} - \int \frac{-\frac{11a^2e}{2} + 2a^2e \cot(c+dx)}{(e \cot(c+dx))^{5/2} (a+a \cot(c+dx))^3} dx \\
&= -\frac{11}{8a^3de(e \cot(c + dx))^{3/2} (1 + \cot(c + dx))} - \frac{1}{4ade(e \cot(c + dx))^{3/2}} \\
&= \frac{55}{24a^3de(e \cot(c + dx))^{3/2}} - \frac{11}{8a^3de(e \cot(c + dx))^{3/2} (1 + \cot(c + dx))} \\
&= \frac{55}{24a^3de(e \cot(c + dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e \cot(c + dx)}} - \frac{1}{8a^3de(e \cot(c + dx))^{3/2}} \\
&= \frac{55}{24a^3de(e \cot(c + dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e \cot(c + dx)}} - \frac{1}{8a^3de(e \cot(c + dx))^{3/2}} \\
&= \frac{55}{24a^3de(e \cot(c + dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e \cot(c + dx)}} - \frac{1}{8a^3de(e \cot(c + dx))^{3/2}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e}\cot(c+dx)}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e \cot(c + dx))^{3/2}} - \frac{63}{8a^3de^2\sqrt{e \cot(c + dx)}} \\
&= -\frac{59 \tan^{-1}\left(\frac{\sqrt{e}\cot(c+dx)}{\sqrt{e}}\right)}{8a^3de^{5/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e}\cot(c+dx)}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e \cot(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 3.19, size = 167, normalized size = 0.78

$$\frac{\cot^{\frac{5}{2}}(c + dx) \left( 4\sqrt{2} \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\cot(c + dx)} \right) - 4\sqrt{2} \tan^{-1} \left( \sqrt{2} \sqrt{\cot(c + dx)} + 1 \right) - 118 \tan^{-1} \left( \sqrt{\cot(c + dx)} \right) \right)}{16a^3d(e \cot(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(5/2)\*(a + a\*Cot[c + d\*x])^3), x]

[Out] (Cot[c + d\*x]^(5/2)\*(4\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 4\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 118\*ArcTan[Sqrt[Cot[c + d\*x]]] - (Sqrt[Cot[c + d\*x]]\*(614 + 678\*Cos[2\*(c + d\*x)] + 679\*Cot[c + d\*x] + 77\*Cos[3\*(c + d\*x)]\*Csc[c + d\*x])\*Sec[c + d\*x]^2)/(6\*(1 + Cot[c + d\*x])^2))/(16\*a^3\*d\*(e\*Cot[c + d\*x])^(5/2))

**fricas** [A] time = 0.48, size = 718, normalized size = 3.34

$$6\sqrt{2}((\cos(2dx+2c)+1)\sin(2dx+2c)+\cos(2dx+2c)+1)\sqrt{-e}\log\left(\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)+1)\sqrt{-e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] [-1/48\*(6\*sqrt(2)\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(-e)\*log(sqrt(2)\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))\*(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) - 1) - 2\*e\*sin(2\*d\*x + 2\*c) + e) + 177\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(-e)\*log((e\*cos(2\*d\*x + 2\*c) - e\*sin(2\*d\*x + 2\*c) + 2\*sqrt(-e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))\*sin(2\*d\*x + 2\*c) + e)/(cos(2\*d\*x + 2\*c) + sin(2\*d\*x + 2\*c) + 1)) - (339\*cos(2\*d\*x + 2\*c)^2 - 7\*(11\*cos(2\*d\*x + 2\*c) + 43)\*sin(2\*d\*x + 2\*c) - 32\*cos(2\*d\*x + 2\*c) - 307)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a^3\*d\*e^3\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^3 + (a^3\*d\*e^3\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^3)\*sin(2\*d\*x + 2\*c)), 1/48\*(12\*sqrt(2)\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(e)\*arctan(-1/2\*sqrt(2)\*sqrt(e)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))\*(cos(2\*d\*x + 2\*c) - sin(2\*d\*x + 2\*c) + 1)/(e\*cos(2\*d\*x + 2\*c) + e)) - 354\*((cos(2\*d\*x + 2\*c) + 1)\*sin(2\*d\*x + 2\*c) + cos(2\*d\*x + 2\*c) + 1)\*sqrt(e)\*arctan(sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c))/sqrt(e)) + (339\*cos(2\*d\*x + 2\*c)^2 - 7\*(11\*cos(2\*d\*x + 2\*c) + 43)\*sin(2\*d\*x + 2\*c) - 32\*cos(2\*d\*x + 2\*c) - 307)\*sqrt((e\*cos(2\*d\*x + 2\*c) + e)/sin(2\*d\*x + 2\*c)))/(a^3\*d\*e^3\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^3 + (a^3\*d\*e^3\*cos(2\*d\*x + 2\*c) + a^3\*d\*e^3)\*sin(2\*d\*x + 2\*c))]]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(dx+c) + a)^3 (e \cot(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+a\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a\*cot(d\*x + c) + a)^3\*(e\*cot(d\*x + c))^(5/2)), x)

**maple [B]** time = 0.86, size = 482, normalized size = 2.24

$$\frac{15 (e \cot(dx+c))^{\frac{3}{2}}}{8d a^3 e^2 (e \cot(dx+c)+e)^2} - \frac{17 \sqrt{e \cot(dx+c)}}{8d a^3 e (e \cot(dx+c)+e)^2} - \frac{59 \arctan\left(\frac{\sqrt{e \cot(dx+c)}}{\sqrt{e}}\right)}{8a^3 d e^{\frac{5}{2}}} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}}}\right)}{16d a^3 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*cot(d*x+c))^(5/2)/(a+cot(d*x+c)*a)^3,x)`

[Out] 
$$-15/8/d/a^3/e^2/(e*\cot(d*x+c)+e)^2*(e*\cot(d*x+c))^{3/2}-17/8/d/a^3/e/(e*\cot(d*x+c)+e)^2*(e*\cot(d*x+c))^{1/2}-59/8*\arctan((e*\cot(d*x+c))^{1/2}/e^{1/2})/a^3/d/e^{5/2}-1/16/d/a^3/e^3*(e^2)^{1/4}*2^{1/2}*ln((e*\cot(d*x+c)+(e^2)^{1/4})*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}))-1/8/d/a^3/e^3*(e^2)^{1/4}*2^{1/2}*a*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+1/8/d/a^3/e^3*(e^2)^{1/4}*2^{1/2}*a*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/16/d/a^3/e^2*2^{1/2}/(e^2)^{1/4}*ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}))-1/8/d/a^3/e^2*2^{1/2}/(e^2)^{1/4}*a*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+1/8/d/a^3/e^2*2^{1/2}/(e^2)^{1/4}*a*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+2/3/a^3/d/e/(e*\cot(d*x+c))^{3/2}-6/a^3/d/e^2/(e*\cot(d*x+c))^{1/2}$$

**maxima [A]** time = 0.78, size = 224, normalized size = 1.04

$$e^{\frac{16e^3 - \frac{112e^3}{\tan(dx+c)} - \frac{323e^3}{\tan(dx+c)^2} - \frac{189e^3}{\tan(dx+c)^3}}{a^3 e^5 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}} + 2a^3 e^4 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{5}{2}} + a^3 e^3 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{7}{2}}} - \frac{6 \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{a^3 e^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")`

[Out] 
$$1/24*e*((16*e^3 - 112*e^3/\tan(d*x + c) - 323*e^3/\tan(d*x + c)^2 - 189*e^3/\tan(d*x + c)^3)/(a^3*e^5*(e/\tan(d*x + c))^{3/2} + 2*a^3*e^4*(e/\tan(d*x + c))$$

$$\begin{aligned} & \sqrt[5]{2} + a^3 e^3 \left( \frac{e}{\tan(dx + c)} \right)^{7/2} - 6 \left( \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left( \sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx + c)}} \right) / \sqrt{e} \right) / \sqrt{e} + \sqrt{2} \arctan\left(\frac{-1}{2} \sqrt{2} \left( \sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx + c)}} \right) / \sqrt{e} \right) / \sqrt{e} \right) / \left( a^3 e^3 \right) - 177 \arctan\left(\sqrt{\frac{e}{\tan(dx + c)}} / \sqrt{e}\right) / \left( a^3 e^{7/2} \right) / d \end{aligned}$$

**mupad [B]** time = 1.38, size = 193, normalized size = 0.90

$$\frac{\frac{63 e \cot(c+dx)^3}{8} + \frac{323 e \cot(c+dx)^2}{24} + \frac{14 e \cot(c+dx)}{3} - \frac{2 e}{3}}{a^3 d (e \cot(c+dx))^{7/2} + 2 a^3 d e (e \cot(c+dx))^{5/2} + a^3 d e^2 (e \cot(c+dx))^{3/2}} - \frac{59 \operatorname{atan}\left(\frac{\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{8 a^3 d e^{5/2}} - \frac{\sqrt{2}}{2 a^3 d e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c + d\*x))^(5/2)\*(a + a\*cot(c + d\*x))^3), x)

[Out] - ((14\*e\*cot(c + d\*x))/3 - (2\*e)/3 + (323\*e\*cot(c + d\*x)^2)/24 + (63\*e\*cot(c + d\*x)^3)/8)/(a^3\*d\*(e\*cot(c + d\*x))^(7/2) + 2\*a^3\*d\*e\*(e\*cot(c + d\*x))^(5/2) + a^3\*d\*e^2\*(e\*cot(c + d\*x))^(3/2)) - (59\*atan((e\*cot(c + d\*x))^(1/2)/e^(1/2)))/(8\*a^3\*d\*e^(5/2)) - (2^(1/2)\*(2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2)))) + 2\*atan((2^(1/2)\*(e\*cot(c + d\*x))^(1/2))/(2\*e^(1/2)) + (2^(1/2)\*(e\*cot(c + d\*x))^(3/2))/(2\*e^(3/2)))))/(8\*a^3\*d\*e^(5/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{(e \cot(c+dx))^{\frac{5}{2}} \cot^3(c+dx) + 3(e \cot(c+dx))^{\frac{5}{2}} \cot^2(c+dx) + 3(e \cot(c+dx))^{\frac{5}{2}} \cot(c+dx) + (e \cot(c+dx))^{\frac{5}{2}}}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(5/2)/(a+a\*cot(d\*x+c))\*\*3, x)

[Out] Integral(1/((e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x)\*\*3 + 3\*(e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x)\*\*2 + 3\*(e\*cot(c + d\*x))\*\*(5/2)\*cot(c + d\*x) + (e\*cot(c + d\*x))\*\*(5/2)), x)/a\*\*3



### 3.41 $\int \cot^2(x) \sqrt{1 + \cot(x)} dx$

Optimal. Leaf size=223

$$-\frac{2}{3}(\cot(x)+1)^{3/2} + \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{2\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}\right)}{2\sqrt{2(1+\sqrt{2})}}$$

[Out]  $-2/3*(1+\cot(x))^{3/2}-1/2*\arctan((-2*(1+\cot(x))^{1/2}+(2+2*2^{1/2}))^{1/2})/(-2+2*2^{1/2})^{1/2}*(2+2*2^{1/2})^{1/2}+1/2*\arctan((2*(1+\cot(x))^{1/2}+(2+2*2^{1/2}))^{1/2})/(-2+2*2^{1/2})^{1/2}*(2+2*2^{1/2})^{1/2}+1/2*\ln(1+\cot(x))+2^{1/2}-(1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2})/(2+2*2^{1/2})^{1/2}-1/2*\ln(1+\cot(x)+2^{1/2}+(1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2})/(2+2*2^{1/2})^{1/2}$

**Rubi [A]** time = 0.27, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$ , Rules used = {3543, 3485, 700, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{2}{3}(\cot(x)+1)^{3/2} + \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{2\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}\right)}{2\sqrt{2(1+\sqrt{2})}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^2\*Sqrt[1 + Cot[x]], x]

[Out]  $-(\text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])) - 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]]) + \text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])) + 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]]) - (2*(1 + \text{Cot}[x])^{3/2})/3 + \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] - \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*\text{Sqrt}[1 + \text{Cot}[x]]]/(2*\text{Sqrt}[2*(1 + \text{Sqrt}[2])]) - \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] + \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*\text{Sqrt}[1 + \text{Cot}[x]]]/(2*\text{Sqrt}[2*(1 + \text{Sqrt}[2])])$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 700

```
Int[Sqrt[(d_) + (e_)*(x_)]/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
- 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 3485

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3543

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
```

```
(f_.)*(x_)]^2, x_Symbol] := Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

### Rubi steps

$$\begin{aligned}
\int \cot^2(x)\sqrt{1+\cot(x)} dx &= -\frac{2}{3}(1+\cot(x))^{3/2} - \int \sqrt{1+\cot(x)} dx \\
&= -\frac{2}{3}(1+\cot(x))^{3/2} + \text{Subst}\left(\int \frac{\sqrt{1+x}}{1+x^2} dx, x, \cot(x)\right) \\
&= -\frac{2}{3}(1+\cot(x))^{3/2} + 2 \text{Subst}\left(\int \frac{x^2}{2-2x^2+x^4} dx, x, \sqrt{1+\cot(x)}\right) \\
&= -\frac{2}{3}(1+\cot(x))^{3/2} - \text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4} dx, x, \sqrt{1+\cot(x)}\right) + \text{Subst}\left(\int \frac{\sqrt{2}}{2-2x^2} dx, x, \sqrt{1+\cot(x)}\right) \\
&= -\frac{2}{3}(1+\cot(x))^{3/2} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+\cot(x)}\right) + \frac{1}{2} \log\left(\frac{1+\sqrt{2}+\cot(x)-\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot(x)}}{1+\sqrt{2}-\cot(x)+\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot(x)}}\right) \\
&= -\frac{2}{3}(1+\cot(x))^{3/2} + \frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot(x)}\right)}{2\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(1+\sqrt{2}-\cot(x)+\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot(x)}\right)}{2\sqrt{2(1+\sqrt{2})}} \\
&= \frac{\tan^{-1}\left(\frac{-\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\cot(x)}}{\sqrt{2(-1+\sqrt{2})}}\right)}{\sqrt{2(-1+\sqrt{2})}} + \frac{\tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\cot(x)}}{\sqrt{2(-1+\sqrt{2})}}\right)}{\sqrt{2(-1+\sqrt{2})}} - \frac{2}{3}(1+\cot(x))^{3/2} + \frac{1}{2} \log\left(\frac{1+\sqrt{2}+\cot(x)-\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot(x)}}{1+\sqrt{2}-\cot(x)+\sqrt{2(1+\sqrt{2})}\sqrt{1+\cot(x)}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.17, size = 69, normalized size = 0.31

$$-\frac{2}{3}(\cot(x)+1)^{3/2} - i\sqrt{1-i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}}\right) + i\sqrt{1+i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^2\*Sqrt[1 + Cot[x]], x]

[Out] (-I)\*Sqrt[1 - I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + I\*Sqrt[1 + I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - (2\*(1 + Cot[x])^(3/2))/3

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2\*(1+cot(x))^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x) + 1} \cot(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2\*(1+cot(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cot(x) + 1)\*cot(x)^2, x)

**maple** [B] time = 0.27, size = 356, normalized size = 1.60

$$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3} + \frac{\sqrt{2\sqrt{2}+2}\sqrt{2}\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2\sqrt{2}+2}\right)}{4} + \frac{\sqrt{2}(2\sqrt{2}+2)\arctan\left(\frac{2\sqrt{1+\cot(x)}}{2\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2\*(1+cot(x))^(1/2),x)

[Out] 
$$-2/3*(1+\cot(x))^{3/2}+1/4*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/2*2^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)}/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})-1/4*(2*2^{(1/2)+2})^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})-1/2*(2*2^{(1/2)+2})^{(1/2)}/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})-1/4*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/2*2^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)}/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/4*(2*2^{(1/2)+2})^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})-1/2*(2*2^{(1/2)+2})^{(1/2)}/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x) + 1} \cot(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2\*(1+cot(x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cot(x) + 1)\*cot(x)^2, x)

**mupad [B]** time = 0.63, size = 119, normalized size = 0.53

$$\operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)-\frac{2(\cot(x)+1)^{3/2}}{3}+\operatorname{atanh}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2\*(cot(x) + 1)^(1/2),x)

[Out]  $\operatorname{atanh}(4*(\cot(x) + 1)^{(1/2)}*((-2^{(1/2)}/8 - 1/8)^{(1/2)} + (2^{(1/2)}/8 - 1/8)^{(1/2)})^3)*(2*(-2^{(1/2)}/8 - 1/8)^{(1/2)} + 2*(2^{(1/2)}/8 - 1/8)^{(1/2)}) - (2*(\cot(x) + 1)^{(3/2)})/3 + \operatorname{atanh}(4*(\cot(x) + 1)^{(1/2)}*((-2^{(1/2)}/8 - 1/8)^{(1/2)} - (2^{(1/2)}/8 - 1/8)^{(1/2)})^3)*(2*(-2^{(1/2)}/8 - 1/8)^{(1/2)} - 2*(2^{(1/2)}/8 - 1/8)^{(1/2)})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x)+1} \cot^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*\*2\*(1+cot(x))\*\*(1/2),x)

[Out] Integral(sqrt(cot(x) + 1)\*cot(x)\*\*2, x)

### 3.42 $\int \cot(x)\sqrt{1 + \cot(x)} dx$

Optimal. Leaf size=135

$$-2\sqrt{\cot(x)+1} + \sqrt{\frac{1}{2}(\sqrt{2}-1)} \tan^{-1}\left(\frac{(2-\sqrt{2})\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right) + \sqrt{\frac{1}{2}(1+\sqrt{2})} \tanh^{-1}\left(\frac{(2+\sqrt{2})\cot(x)+3}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}}\right)$$

[Out]  $-2*(1+\cot(x))^{(1/2)}+1/2*\arctan(1/2*(4+\cot(x))*(2-2^{(1/2)})-3*2^{(1/2)})/(1+\cot(x))^{(1/2)}/(-7+5*2^{(1/2)})^{(1/2)}*(-2+2*2^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(1/2*(4+3*2^{(1/2)}+\cot(x)*(2+2^{(1/2)})))/(1+\cot(x))^{(1/2)}/(7+5*2^{(1/2)})^{(1/2)}*(2+2*2^{(1/2)})^{(1/2)}$

**Rubi [A]** time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$ , Rules used = {3528, 3536, 3535, 203, 207}

$$-2\sqrt{\cot(x)+1} + \sqrt{\frac{1}{2}(\sqrt{2}-1)} \tan^{-1}\left(\frac{(2-\sqrt{2})\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right) + \sqrt{\frac{1}{2}(1+\sqrt{2})} \tanh^{-1}\left(\frac{(2+\sqrt{2})\cot(x)+3}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]\*Sqrt[1 + Cot[x]], x]

[Out]  $\text{Sqrt}[(-1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(4 - 3*\text{Sqrt}[2] + (2 - \text{Sqrt}[2])* \text{Cot}[x])/(2*\text{Sqrt}[-7 + 5*\text{Sqrt}[2]]*\text{Sqrt}[1 + \text{Cot}[x]])] + \text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTanh}[(4 + 3*\text{Sqrt}[2] + (2 + \text{Sqrt}[2])* \text{Cot}[x])/(2*\text{Sqrt}[7 + 5*\text{Sqrt}[2]]*\text{Sqrt}[1 + \text{Cot}[x]])] - 2*\text{Sqrt}[1 + \text{Cot}[x]]$

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rule 3528

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(d\*(a + b\*Tan[e + f\*x])^m)/(f\*m), x] + Int

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3535

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(-2*d^2)/f, Subst[Int[1/(2*b*c*d - 4*a*d^2 +
x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

### Rule 3536

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

### Rubi steps

$$\begin{aligned}
\int \cot(x)\sqrt{1+\cot(x)} \, dx &= -2\sqrt{1+\cot(x)} - \int \frac{1-\cot(x)}{\sqrt{1+\cot(x)}} \, dx \\
&= -2\sqrt{1+\cot(x)} + \frac{\int \frac{-\sqrt{2}-(-2-\sqrt{2})\cot(x)}{\sqrt{1+\cot(x)}} \, dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}-(-2+\sqrt{2})\cot(x)}{\sqrt{1+\cot(x)}} \, dx}{2\sqrt{2}} \\
&= -2\sqrt{1+\cot(x)} + (-4+3\sqrt{2}) \operatorname{Subst}\left(\int \frac{1}{-2\sqrt{2}(-2+\sqrt{2})-4(-2+\sqrt{2})^2+x^2} \, dx\right) \\
&= \sqrt{\frac{1}{2}(-1+\sqrt{2})} \tan^{-1}\left(\frac{4-3\sqrt{2}+(2-\sqrt{2})\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \sqrt{\frac{1}{2}(1+\sqrt{2})} \tanh^{-1}\left(\frac{4+3\sqrt{2}+(2+\sqrt{2})\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.09, size = 61, normalized size = 0.45

$$-2\sqrt{\cot(x)+1} + \sqrt{1-i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}}\right) + \sqrt{1+i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]\*Sqrt[1 + Cot[x]],x]

[Out] Sqrt[1 - I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + Sqrt[1 + I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - 2\*Sqrt[1 + Cot[x]]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x)+1} \cot(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cot(x) + 1)\*cot(x), x)

**maple** [B] time = 0.13, size = 249, normalized size = 1.84

$$-2\sqrt{1+\cot(x)} - \frac{\sqrt{2\sqrt{2}+2} \ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2\sqrt{2}+2}\right)}{4} + \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)\*(1+cot(x))^(1/2),x)

[Out]  $-2*(1+\cot(x))^{1/2}-1/4*(2*2^{1/2}+2)^{1/2}*\ln(1+\cot(x)+2^{1/2}-(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2})+1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})*2^{1/2}-1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})+1/4*(2*2^{1/2}+2)^{1/2}*\ln(1+\cot(x)+2^{1/2}+(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2})-1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})+1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})*2^{1/2}$



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x) + 1} \cot(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cot(x) + 1)\*cot(x), x)

**mupad** [B] time = 0.48, size = 210, normalized size = 1.56

$$\operatorname{atanh} \left( \frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} \right) \left( 2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} + 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}} \right) - \operatorname{atanh} \left( \frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)\*(cot(x) + 1)^(1/2),x)

[Out]  $\operatorname{atanh}((\cot(x) + 1)^{1/2}/(4*(1/8 - 2^{1/2}/8)^{1/2})) + (\cot(x) + 1)^{1/2}/(4*(2^{1/2}/8 + 1/8)^{1/2}) - (2^{1/2}*(\cot(x) + 1)^{1/2})/(8*(1/8 - 2^{1/2}/8)^{1/2}) + (2^{1/2}*(\cot(x) + 1)^{1/2})/(8*(2^{1/2}/8 + 1/8)^{1/2})) * (2*(1/8 - 2^{1/2}/8)^{1/2} + 2*(2^{1/2}/8 + 1/8)^{1/2}) - \operatorname{atanh}((\cot(x) + 1)^{1/2}/(4*(2^{1/2}/8 + 1/8)^{1/2}) - (\cot(x) + 1)^{1/2}/(4*(1/8 - 2^{1/2}/8)^{1/2})) + (2^{1/2}*(\cot(x) + 1)^{1/2})/(8*(1/8 - 2^{1/2}/8)^{1/2}) + (2^{1/2}*(\cot(x) + 1)^{1/2})/(8*(2^{1/2}/8 + 1/8)^{1/2})) * (2*(1/8 - 2^{1/2}/8)^{1/2} - 2*(2^{1/2}/8 + 1/8)^{1/2}) - 2*(\cot(x) + 1)^{1/2}$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot(x) + 1} \cot(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))\*\*(1/2),x)

[Out] Integral(sqrt(cot(x) + 1)\*cot(x), x)

### 3.43 $\int \cot^2(x)(1 + \cot(x))^{3/2} dx$

Optimal. Leaf size=139

$$-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{(1-\sqrt{2})\cot(x)-2\sqrt{2}+3}{\sqrt{2}(5\sqrt{2}-7)\sqrt{\cot(x)+1}}\right)-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{(1+\sqrt{2})\cot(x)}{\sqrt{2}(7+5\sqrt{2})}\right)$$

[Out]  $-2/5*(1+\cot(x))^{(5/2)}+2*(1+\cot(x))^{(1/2)}-\arctan((3+\cot(x))*(1-2^{(1/2)}))-2*2^{(1/2)}(1/2)/(1+\cot(x))^{(1/2)/(-14+10*2^{(1/2)})^{(1/2)}}*(2^{(1/2)}-1)^{(1/2)}-\operatorname{arctanh}((3+2*2^{(1/2)}+\cot(x)*(1+2^{(1/2)}))/(1+\cot(x))^{(1/2)/(14+10*2^{(1/2)})^{(1/2)}}*(1+2^{(1/2)})^{(1/2)})$

**Rubi [A]** time = 0.23, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$ , Rules used = {3543, 3482, 12, 3536, 3535, 203, 207}

$$-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{(1-\sqrt{2})\cot(x)-2\sqrt{2}+3}{\sqrt{2}(5\sqrt{2}-7)\sqrt{\cot(x)+1}}\right)-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{(1+\sqrt{2})\cot(x)}{\sqrt{2}(7+5\sqrt{2})}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^2\*(1 + Cot[x])^(3/2), x]

[Out]  $-(\operatorname{Sqrt}[-1 + \operatorname{Sqrt}[2]]*\operatorname{ArcTan}[(3 - 2*\operatorname{Sqrt}[2] + (1 - \operatorname{Sqrt}[2])* \operatorname{Cot}[x])]/(\operatorname{Sqrt}[2*(-7 + 5*\operatorname{Sqrt}[2]])*\operatorname{Sqrt}[1 + \operatorname{Cot}[x]])) - \operatorname{Sqrt}[1 + \operatorname{Sqrt}[2]]*\operatorname{ArcTanh}[(3 + 2*\operatorname{Sqrt}[2] + (1 + \operatorname{Sqrt}[2])* \operatorname{Cot}[x])]/(\operatorname{Sqrt}[2*(7 + 5*\operatorname{Sqrt}[2]])*\operatorname{Sqrt}[1 + \operatorname{Cot}[x]])] + 2*\operatorname{Sqrt}[1 + \operatorname{Cot}[x]] - (2*(1 + \operatorname{Cot}[x])^{(5/2)})/5$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a

, 0] || GtQ[b, 0])

### Rule 3482

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(a + b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Int[(a^2 - b^2 + 2\*a\*b\*Tan[c + d\*x])\*(a + b\*Tan[c + d\*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[n, 1]

### Rule 3535

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*b\*c\*d - 4\*a\*d^2 + x^2), x], x, (b\*c - 2\*a\*d - b\*d\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[2\*a\*c\*d - b\*(c^2 - d^2), 0]

### Rule 3536

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2\*q), Int[(a\*c + b\*d + c\*q + (b\*c - a\*d + d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] - Dist[1/(2\*q), Int[(a\*c + b\*d - c\*q + (b\*c - a\*d - d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2\*a\*c\*d - b\*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])

### Rule 3543

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(d^2\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[c^2 - d^2 + 2\*c\*d\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])

### Rubi steps

$$\begin{aligned}
\int \cot^2(x)(1 + \cot(x))^{3/2} dx &= -\frac{2}{5}(1 + \cot(x))^{5/2} - \int (1 + \cot(x))^{3/2} dx \\
&= 2\sqrt{1 + \cot(x)} - \frac{2}{5}(1 + \cot(x))^{5/2} - \int \frac{2 \cot(x)}{\sqrt{1 + \cot(x)}} dx \\
&= 2\sqrt{1 + \cot(x)} - \frac{2}{5}(1 + \cot(x))^{5/2} - 2 \int \frac{\cot(x)}{\sqrt{1 + \cot(x)}} dx \\
&= 2\sqrt{1 + \cot(x)} - \frac{2}{5}(1 + \cot(x))^{5/2} - \frac{\int \frac{-1 - (-1 - \sqrt{2}) \cot(x)}{\sqrt{1 + \cot(x)}} dx}{\sqrt{2}} + \frac{\int \frac{-1 - (-1 + \sqrt{2}) \cot(x)}{\sqrt{1 + \cot(x)}} dx}{\sqrt{2}} \\
&= 2\sqrt{1 + \cot(x)} - \frac{2}{5}(1 + \cot(x))^{5/2} - (-4 + 3\sqrt{2}) \text{Subst} \left( \int \frac{1}{2(-1 + \sqrt{2}) - 4(-1 + \sqrt{2})} dx \right) \\
&= -\sqrt{-1 + \sqrt{2}} \tan^{-1} \left( \frac{3 - 2\sqrt{2} + (1 - \sqrt{2}) \cot(x)}{\sqrt{2}(-7 + 5\sqrt{2}) \sqrt{1 + \cot(x)}} \right) - \sqrt{1 + \sqrt{2}} \tanh^{-1} \left( \frac{3 + 2\sqrt{2} + (1 + \sqrt{2}) \cot(x)}{\sqrt{2}(7 + 5\sqrt{2}) \sqrt{1 + \cot(x)}} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.32, size = 96, normalized size = 0.69

$$\frac{\sin(x) \left( -\frac{2}{5} \sin(x)(\cot(x) + 1)^{5/2} (2 \cot(x) + \csc^2(x) - 5) - 2 \sin(x)(\cot(x) + 1)^2 \left( \frac{\tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}} \right)}{\sqrt{1-i}} + \frac{\tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}} \right)}{\sqrt{1+i}} \right) \right)}{(\sin(x) + \cos(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^2\*(1 + Cot[x])^(3/2), x]

[Out] (Sin[x]\*(-2\*(ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]]/Sqrt[1 + I]])\*(1 + Cot[x])^2\*Sin[x] - (2\*(1 + Cot[x])^(5/2)\*(-5 + 2\*Cot[x] + Csc[x]^2)\*Sin[x])/5)/(Cos[x] + Sin[x])^2

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2\*(1+cot(x))^(3/2), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cot(x) + 1)^{\frac{3}{2}} \cot(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2\*(1+cot(x))^(3/2),x, algorithm="giac")

[Out] integrate((cot(x) + 1)^(3/2)\*cot(x)^2, x)

**maple** [B] time = 0.18, size = 265, normalized size = 1.91

$$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5} + 2\sqrt{1+\cot(x)} + \frac{\sqrt{2\sqrt{2}+2}\sqrt{2}\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2\sqrt{2}+2}\right)}{4} - \arctan\left(\frac{2\sqrt{2\sqrt{2}+2}}{\sqrt{1+\cot(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2\*(1+cot(x))^(3/2),x)

[Out]  $-2/5*(1+\cot(x))^{5/2}+2*(1+\cot(x))^{1/2}+1/4*(2*2^{1/2}+2)^{1/2}*2^{1/2}*\ln(1+\cot(x)+2^{1/2}-(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2})-1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})*2^{1/2}+2/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})-1/4*(2*2^{1/2}+2)^{1/2}*2^{1/2}*\ln(1+\cot(x)+2^{1/2}+(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2})-1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})*2^{1/2}+2/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2}+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2\*(1+cot(x))^(3/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [B] time = 1.00, size = 254, normalized size = 1.83

$$\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}-\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}2i+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}2i\right)-\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}-\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}2i+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}2i\right)-\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}-\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}2i+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}2i\right)-\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}-\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}2i+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)^2*(cot(x) + 1)^(3/2),x)`

[Out] 
$$\begin{aligned} & \operatorname{atan}\left(\frac{2^{1/2} \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}{256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}\right) / \left(256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i\right) \\ & - \operatorname{atan}\left(\frac{2^{1/2} \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}{256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}\right) / \left(256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i\right) \\ & + \operatorname{atan}\left(\frac{2^{1/2} \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}{256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}\right) / \left(256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i\right) \\ & + \operatorname{atan}\left(\frac{2^{1/2} \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}{256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i}\right) / \left(256 \cdot (1/4 - 2^{1/2}/4)^{1/2} \cdot (2^{1/2}/4 + 1/4)^{1/2} \cdot (\cot(x) + 1)^{1/2} \cdot 64i\right) \\ & + 2 \cdot (\cot(x) + 1)^{1/2} - (2 \cdot (\cot(x) + 1)^{5/2}) / 5 \end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cot(x) + 1)^{\frac{3}{2}} \cot^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)**2*(1+cot(x))**(3/2),x)`

[Out] `Integral((cot(x) + 1)**(3/2)*cot(x)**2, x)`

### 3.44 $\int \cot(x)(1 + \cot(x))^{3/2} dx$

Optimal. Leaf size=221

$$-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}-\frac{\log\left(\cot(x)-\sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(\cot(x)+\sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

[Out]  $-2/3*(1+\cot(x))^{3/2}-2*(1+\cot(x))^{1/2}-1/2*\ln(1+\cot(x)+2^{1/2})-(1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2}/(1+2^{1/2})^{1/2}+1/2*\ln(1+\cot(x)+2^{1/2})+(1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2}/(1+2^{1/2})^{1/2}-\arctan((-2*(1+\cot(x))^{1/2}+(2+2*2^{1/2})^{1/2})/(-2+2*2^{1/2})^{1/2})*(1+2^{1/2})^{1/2}+\arctan((2*(1+\cot(x))^{1/2}+(2+2*2^{1/2})^{1/2})/(-2+2*2^{1/2})^{1/2})*(1+2^{1/2})^{1/2}$

Rubi [A] time = 0.21, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$ , Rules used = {3528, 12, 3485, 708, 1094, 634, 618, 204, 628}

$$-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}-\frac{\log\left(\cot(x)-\sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(\cot(x)+\sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]\*(1 + Cot[x])^(3/2), x]

[Out]  $-(\text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])]] - 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]) + \text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])]] + 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]) - 2*\text{Sqrt}[1 + \text{Cot}[x]] - (2*(1 + \text{Cot}[x])^{3/2})/3 - \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] - \text{Sqrt}[2*(1 + \text{Sqrt}[2])]]*\text{Sqrt}[1 + \text{Cot}[x]]/(2*\text{Sqrt}[1 + \text{Sqrt}[2]]) + \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] + \text{Sqrt}[2*(1 + \text{Sqrt}[2])]]*\text{Sqrt}[1 + \text{Cot}[x]]/(2*\text{Sqrt}[1 + \text{Sqrt}[2]])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2*e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```



0] &amp;&amp; GtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \cot(x)(1 + \cot(x))^{3/2} dx &= -\frac{2}{3}(1 + \cot(x))^{3/2} - \int (1 - \cot(x))\sqrt{1 + \cot(x)} dx \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} - \int \frac{2}{\sqrt{1 + \cot(x)}} dx \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} - 2 \int \frac{1}{\sqrt{1 + \cot(x)}} dx \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} + 2 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+x}(1+x^2)} dx, x, \cot(x) \right) \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} + 4 \operatorname{Subst} \left( \int \frac{1}{2-2x^2+x^4} dx, x, \sqrt{1 + \cot(x)} \right) \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} + \frac{\operatorname{Subst} \left( \int \frac{\sqrt{2(1+\sqrt{2})-x}}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1 + \cot(x)} \right)}{\sqrt{1 + \sqrt{2}}} \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} + \frac{\operatorname{Subst} \left( \int \frac{1}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1 + \cot(x)} \right)}{\sqrt{2}} \\
&= -2\sqrt{1 + \cot(x)} - \frac{2}{3}(1 + \cot(x))^{3/2} - \frac{\log \left( 1 + \sqrt{2} + \cot(x) - \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \cot(x)} \right)}{2\sqrt{1 + \sqrt{2}}} \\
&= -\frac{\tan^{-1} \left( \frac{\sqrt{2(1+\sqrt{2})-2\sqrt{1+\cot(x)}}}{\sqrt{2(-1+\sqrt{2})}} \right)}{\sqrt{-1 + \sqrt{2}}} + \frac{\tan^{-1} \left( \frac{\sqrt{2(1+\sqrt{2})+2\sqrt{1+\cot(x)}}}{\sqrt{2(-1+\sqrt{2})}} \right)}{\sqrt{-1 + \sqrt{2}}} - 2\sqrt{1 + \cot(x)} - \frac{2}{3}
\end{aligned}$$

**Mathematica [C]** time = 0.26, size = 98, normalized size = 0.44

$$\frac{\sin(x) \left( -\frac{2}{3}(\cot(x) + 1)^{3/2}(\cot(x) + 4)(\sin(x) + \cos(x)) + (1 + i) \sin(x)(\cot(x) + 1)^2 \left( \sqrt{1 + i} \tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}} \right) \right) \right)}{(\sin(x) + \cos(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]\*(1 + Cot[x])^(3/2),x]

[Out] (Sin[x]\*((1 + I)\*((-I)\*Sqrt[1 - I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + Sqrt[1 + I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])\*(1 + Cot[x])^2\*Sin[x] - (2\*(1 + Cot[x])^(3/2)\*(4 + Cot[x])\*(Cos[x] + Sin[x]))/3)/(Cos[x] + Sin[x])^2

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cot(x) + 1)^{\frac{3}{2}} \cot(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))^(3/2),x, algorithm="giac")

[Out] integrate((cot(x) + 1)^(3/2)\*cot(x), x)

**maple** [B] time = 0.12, size = 452, normalized size = 2.05

$$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3} - 2\sqrt{1+\cot(x)} + \frac{\sqrt{2\sqrt{2}+2}\sqrt{2}\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2\sqrt{2}+2}\right)}{4} - \frac{\sqrt{2\sqrt{2}+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)\*(1+cot(x))^(3/2),x)

[Out] -2/3\*(1+cot(x))^(3/2)-2\*(1+cot(x))^(1/2)+1/4\*(2\*2^(1/2)+2)^(1/2)\*2^(1/2)\*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)\*(2\*2^(1/2)+2)^(1/2))-1/2\*(2\*2^(1/2)+2)^(1/2)\*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)\*(2\*2^(1/2)+2)^(1/2))+1/2\*2^(1/2)\*(2\*2^(1/2)+2)/(-2+2\*2^(1/2))^(1/2)\*arctan((2\*(1+cot(x))^(1/2)-(2\*2^(1/2)+2)^(1/2))/(-2+2\*2^(1/2))^(1/2))-2\*(2^(1/2)+2)/(-2+2\*2^(1/2))^(1/2)\*arctan((2\*(1+cot(x))^(1/2)-(2\*2^(1/2)+2)^(1/2))/(-2+2\*2^(1/2))^(1/2))+2/(-2+2\*2^(1/2))^(1/2)\*arctan((2\*(1+cot(x))^(1/2)-(2\*2^(1/2)+2)^(1/2))/(-2+2\*2^(1/2))^(1/2))+2^(1/2)-1/4\*(2\*2^(1/2)+2)^(1/2)\*2^(1/2)\*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2))

$$\frac{1}{2}*(2*2^{(1/2)+2})^{(1/2)}+1/2*(2*2^{(1/2)+2})^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/2*2^{(1/2)}*(2*2^{(1/2)+2})/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})-(2*2^{(1/2)+2})/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cot(x) + 1)^{\frac{3}{2}} \cot(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))^(3/2),x, algorithm="maxima")

[Out] integrate((cot(x) + 1)^(3/2)\*cot(x), x)

**mupad** [B] time = 0.67, size = 254, normalized size = 1.15

$$-\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}-\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{\cot(x)+1}64i}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}}{\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}2i+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}2i\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)\*(cot(x) + 1)^(3/2),x)

[Out] atan((2^(1/2)\*(-2^(1/2)/4 - 1/4)^(1/2)\*(cot(x) + 1)^(1/2)\*64i)/(256\*(2^(1/2)/4 - 1/4)^(1/2)\*(-2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)\*(2^(1/2)/4 - 1/4)^(1/2)\*(cot(x) + 1)^(1/2)\*64i)/(256\*(2^(1/2)/4 - 1/4)^(1/2)\*(-2^(1/2)/4 - 1/4)^(1/2) + 64))\*((-2^(1/2)/4 - 1/4)^(1/2)\*2i - (2^(1/2)/4 - 1/4)^(1/2)\*2i) - atan((2^(1/2)\*(-2^(1/2)/4 - 1/4)^(1/2)\*(cot(x) + 1)^(1/2)\*64i)/(256\*(2^(1/2)/4 - 1/4)^(1/2)\*(-2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)\*(2^(1/2)/4 - 1/4)^(1/2)\*(cot(x) + 1)^(1/2)\*64i)/(256\*(2^(1/2)/4 - 1/4)^(1/2)\*(-2^(1/2)/4 - 1/4)^(1/2) - 64))\*((-2^(1/2)/4 - 1/4)^(1/2)\*2i + (2^(1/2)/4 - 1/4)^(1/2)\*2i) - 2\*(cot(x) + 1)^(1/2) - (2\*(cot(x) + 1)^(3/2))/3

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cot(x) + 1)^{\frac{3}{2}} \cot(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*(1+cot(x))\*\*(3/2),x)

[Out] Integral((cot(x) + 1)\*\*(3/2)\*cot(x), x)

$$3.45 \quad \int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}} dx$$

**Optimal.** Leaf size=214

$$-2\sqrt{\cot(x)+1} - \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{1+\sqrt{2}}}$$

[Out]  $-2*(1+\cot(x))^{(1/2)} - 1/4*\ln(1+\cot(x)+2^{(1/2)} - (1+\cot(x))^{(1/2)}*(2+2*2^{(1/2)})^{(1/2)})/(1+2^{(1/2)})^{(1/2)} + 1/4*\ln(1+\cot(x)+2^{(1/2)} + (1+\cot(x))^{(1/2)}*(2+2*2^{(1/2)})^{(1/2)})/(1+2^{(1/2)})^{(1/2)} - 1/2*\arctan((-2*(1+\cot(x))^{(1/2)} + (2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*(1+2^{(1/2)})^{(1/2)} + 1/2*\arctan((2*(1+\cot(x))^{(1/2)} + (2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*(1+2^{(1/2)})^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$ , Rules used = {3543, 3485, 708, 1094, 634, 618, 204, 628}

$$-2\sqrt{\cot(x)+1} - \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^2/Sqrt[1 + Cot[x]], x]

[Out]  $-(\text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])]) - 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])])]/2 + (\text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])]) + 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])])]/2 - 2*\text{Sqrt}[1 + \text{Cot}[x]] - \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] - \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*\text{Sqrt}[1 + \text{Cot}[x]]]/(4*\text{Sqrt}[1 + \text{Sqrt}[2]]) + \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] + \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*\text{Sqrt}[1 + \text{Cot}[x]]]/(4*\text{Sqrt}[1 + \text{Sqrt}[2]])$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 708

Int[1/(Sqrt[(d\_) + (e\_)\*(x\_)])\*((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[2\*e, Subst[Int[1/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 1094

Int[((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(r - x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(r + x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 3485

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

### Rule 3543

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[(d^2\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[c^2 - d^2 + 2\*c\*d\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}} dx &= -2\sqrt{1+\cot(x)} - \int \frac{1}{\sqrt{1+\cot(x)}} dx \\
&= -2\sqrt{1+\cot(x)} + \text{Subst} \left( \int \frac{1}{\sqrt{1+x}(1+x^2)} dx, x, \cot(x) \right) \\
&= -2\sqrt{1+\cot(x)} + 2 \text{Subst} \left( \int \frac{1}{2-2x^2+x^4} dx, x, \sqrt{1+\cot(x)} \right) \\
&= -2\sqrt{1+\cot(x)} + \frac{\text{Subst} \left( \int \frac{\sqrt{2(1+\sqrt{2})}-x}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+\cot(x)} \right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst} \left( \int \frac{\sqrt{2(1+\sqrt{2})}+x}{\sqrt{2}+\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+\cot(x)} \right)}{2\sqrt{1+\sqrt{2}}} \\
&= -2\sqrt{1+\cot(x)} + \frac{\text{Subst} \left( \int \frac{1}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+\cot(x)} \right)}{2\sqrt{2}} + \frac{\text{Subst} \left( \int \frac{1}{\sqrt{2}+\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+\cot(x)} \right)}{2\sqrt{2}} \\
&= -2\sqrt{1+\cot(x)} - \frac{\log \left( 1 + \sqrt{2} + \cot(x) - \sqrt{2(1+\sqrt{2})} \sqrt{1+\cot(x)} \right)}{4\sqrt{1+\sqrt{2}}} + \frac{\log \left( 1 + \sqrt{2} + \cot(x) + \sqrt{2(1+\sqrt{2})} \sqrt{1+\cot(x)} \right)}{4\sqrt{1+\sqrt{2}}} \\
&= -\frac{\tan^{-1} \left( \frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\cot(x)}}{\sqrt{2(-1+\sqrt{2})}} \right)}{2\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1} \left( \frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\cot(x)}}{\sqrt{2(-1+\sqrt{2})}} \right)}{2\sqrt{-1+\sqrt{2}}} - 2\sqrt{1+\cot(x)} - \frac{\log(1+\cot(x))}{2}
\end{aligned}$$

**Mathematica** [C] time = 0.17, size = 67, normalized size = 0.31

$$-2\sqrt{\cot(x)+1} + \frac{1}{2}(1-i)^{3/2} \tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}} \right) + \frac{1}{2}(1+i)^{3/2} \tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^2/Sqrt[1+Cot[x]],x]

[Out] ((1-I)^(3/2)\*ArcTanh[Sqrt[1+Cot[x]]/Sqrt[1-I]])/2 + ((1+I)^(3/2)\*ArcTanh[Sqrt[1+Cot[x]]/Sqrt[1+I]])/2 - 2\*Sqrt[1+Cot[x]]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{\sqrt{\cot(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="giac")

[Out] integrate(cot(x)^2/sqrt(cot(x) + 1), x)

**maple** [B] time = 0.22, size = 442, normalized size = 2.07

$$-2\sqrt{1+\cot(x)} - \frac{\sqrt{2\sqrt{2}+2} \ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\sqrt{2\sqrt{2}+2}\right)}{4} + \frac{\sqrt{2\sqrt{2}+2}\sqrt{2} \ln\left(1+\cot(x)+\sqrt{2}+\sqrt{1+\cot(x)}\sqrt{2\sqrt{2}+2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2/(1+cot(x))^(1/2),x)

[Out]  $-2*(1+\cot(x))^{1/2}-1/4*(2*2^{1/2}+2)^{1/2}*\ln(1+\cot(x)+2^{1/2})-(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2}+1/8*(2*2^{1/2}+2)^{1/2}*2^{1/2}*\ln(1+\cot(x)+2^{1/2})-(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2}+1/4*2^{1/2}*(2*2^{1/2}+2)/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2})-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})-1/2*(2*2^{1/2}+2)/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2})-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})+1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2})-(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})*2^{1/2}+1/4*(2*2^{1/2}+2)^{1/2}*\ln(1+\cot(x)+2^{1/2})+(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2})-1/8*(2*2^{1/2}+2)^{1/2}*2^{1/2}*\ln(1+\cot(x)+2^{1/2})+(1+\cot(x))^{1/2}*(2*2^{1/2}+2)^{1/2}+1/4*2^{1/2}*(2*2^{1/2}+2)/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2})+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})-1/2*(2*2^{1/2}+2)/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2})+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})+1/(-2+2*2^{1/2})^{1/2}*\arctan((2*(1+\cot(x))^{1/2})+(2*2^{1/2}+2)^{1/2})/(-2+2*2^{1/2})^{1/2})*2^{1/2}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{\sqrt{\cot(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="maxima")

[Out] integrate(cot(x)^2/sqrt(cot(x) + 1), x)

**mupad [B]** time = 0.44, size = 238, normalized size = 1.11

$$\operatorname{atanh} \left( \frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8} - \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8} \right) \left( 2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} + 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2/(cot(x) + 1)^(1/2),x)

[Out] atanh((16\*2^(1/2)\*(-2^(1/2)/16 - 1/16)^(1/2)\*(cot(x) + 1)^(1/2))/(128\*(2^(1/2)/16 - 1/16)^(1/2)\*(-2^(1/2)/16 - 1/16)^(1/2) - 8) - (16\*2^(1/2)\*(2^(1/2)/16 - 1/16)^(1/2)\*(cot(x) + 1)^(1/2))/(128\*(2^(1/2)/16 - 1/16)^(1/2)\*(-2^(1/2)/16 - 1/16)^(1/2) - 8))\*(2\*(-2^(1/2)/16 - 1/16)^(1/2) + 2\*(2^(1/2)/16 - 1/16)^(1/2)) - atanh((16\*2^(1/2)\*(-2^(1/2)/16 - 1/16)^(1/2)\*(cot(x) + 1)^(1/2))/(128\*(2^(1/2)/16 - 1/16)^(1/2)\*(-2^(1/2)/16 - 1/16)^(1/2) + 8) + (16\*2^(1/2)\*(2^(1/2)/16 - 1/16)^(1/2)\*(cot(x) + 1)^(1/2))/(128\*(2^(1/2)/16 - 1/16)^(1/2)\*(-2^(1/2)/16 - 1/16)^(1/2) + 8))\*(2\*(-2^(1/2)/16 - 1/16)^(1/2) - 2\*(2^(1/2)/16 - 1/16)^(1/2)) - 2\*(cot(x) + 1)^(1/2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{\cot(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)\*\*2/(1+cot(x))\*\*(1/2),x)

[Out] Integral(cot(x)\*\*2/sqrt(cot(x) + 1), x)



$$3.46 \quad \int \frac{\cot(x)}{\sqrt{1+\cot(x)}} dx$$

**Optimal.** Leaf size=121

$$\frac{1}{2}\sqrt{\sqrt{2}-1} \tan^{-1}\left(\frac{(1-\sqrt{2})\cot(x)-2\sqrt{2}+3}{\sqrt{2(5\sqrt{2}-7)}\sqrt{\cot(x)+1}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}} \tanh^{-1}\left(\frac{(1+\sqrt{2})\cot(x)+2\sqrt{2}+3}{\sqrt{2(7+5\sqrt{2})}\sqrt{\cot(x)+1}}\right)$$

[Out] 1/2\*arctan((3+cot(x)\*(1-2^(1/2))-2\*2^(1/2))/(1+cot(x))^(1/2)/(-14+10\*2^(1/2))^(1/2))\*(2^(1/2)-1)^(1/2)+1/2\*arctanh((3+2\*2^(1/2)+cot(x)\*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10\*2^(1/2))^(1/2))\*(1+2^(1/2))^(1/2)

**Rubi [A]** time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3536, 3535, 203, 207}

$$\frac{1}{2}\sqrt{\sqrt{2}-1} \tan^{-1}\left(\frac{(1-\sqrt{2})\cot(x)-2\sqrt{2}+3}{\sqrt{2(5\sqrt{2}-7)}\sqrt{\cot(x)+1}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}} \tanh^{-1}\left(\frac{(1+\sqrt{2})\cot(x)+2\sqrt{2}+3}{\sqrt{2(7+5\sqrt{2})}\sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Sqrt[1 + Cot[x]], x]

[Out] (Sqrt[-1 + Sqrt[2]]\*ArcTan[(3 - 2\*Sqrt[2] + (1 - Sqrt[2])\*Cot[x])/(Sqrt[2\*(-7 + 5\*Sqrt[2]])\*Sqrt[1 + Cot[x]])])/2 + (Sqrt[1 + Sqrt[2]]\*ArcTanh[(3 + 2\*Sqrt[2] + (1 + Sqrt[2])\*Cot[x])/(Sqrt[2\*(7 + 5\*Sqrt[2]])\*Sqrt[1 + Cot[x]])])/2

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rule 3535

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(a\_) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*b\*c\*d - 4\*a\*d^2 +

$x^2$ ),  $x$ ],  $x$ ,  $(b*c - 2*a*d - b*d*\text{Tan}[e + f*x])/ \text{Sqrt}[a + b*\text{Tan}[e + f*x]]$ ],  $x$ ]  
 ] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0]  
 && NeQ[c^2 + d^2, 0] && EqQ[2\*a\*c\*d - b\*(c^2 - d^2), 0]

### Rule 3536

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])/Sqrt[(a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]]], x\_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2\*q), Int[(a\*c + b\*d + c\*q + (b\*c - a\*d + d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] - Dist[1/(2\*q), Int[(a\*c + b\*d - c\*q + (b\*c - a\*d - d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2\*a\*c\*d - b\*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])

### Rubi steps

$$\int \frac{\cot(x)}{\sqrt{1 + \cot(x)}} dx = \frac{\int \frac{-1 - (-1 - \sqrt{2})\cot(x)}{\sqrt{1 + \cot(x)}} dx}{2\sqrt{2}} - \frac{\int \frac{-1 - (-1 + \sqrt{2})\cot(x)}{\sqrt{1 + \cot(x)}} dx}{2\sqrt{2}}$$

$$= \frac{1}{2}(-4 + 3\sqrt{2}) \text{Subst} \left( \int \frac{1}{2(-1 + \sqrt{2}) - 4(-1 + \sqrt{2})^2 + x^2} dx, x, \frac{1 - 2(-1 + \sqrt{2}) - (-1 + \sqrt{2})\cot(x)}{\sqrt{1 + \cot(x)}} \right)$$

$$= \frac{1}{2}\sqrt{-1 + \sqrt{2}} \tan^{-1} \left( \frac{3 - 2\sqrt{2} + (1 - \sqrt{2})\cot(x)}{\sqrt{2}(-7 + 5\sqrt{2})\sqrt{1 + \cot(x)}} \right) + \frac{1}{2}\sqrt{1 + \sqrt{2}} \tanh^{-1} \left( \frac{3 + 2\sqrt{2} + (1 + \sqrt{2})\cot(x)}{\sqrt{2}(7 + 5\sqrt{2})\sqrt{1 + \cot(x)}} \right)$$

**Mathematica** [C] time = 0.08, size = 51, normalized size = 0.42

$$\frac{\tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}} \right)}{\sqrt{1-i}} + \frac{\tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}} \right)}{\sqrt{1+i}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]/Sqrt[1 + Cot[x]], x]

[Out] ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]]/Sqrt[1 + I]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{\cot(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="giac")

[Out] integrate(cot(x)/sqrt(cot(x) + 1), x)

**maple** [B] time = 0.16, size = 249, normalized size = 2.06

$$\frac{\sqrt{2\sqrt{2}+2} \sqrt{2} \ln\left(1 + \cot(x) + \sqrt{2} - \sqrt{1 + \cot(x)} \sqrt{2\sqrt{2}+2}\right)}{8} + \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)} - \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right) \sqrt{2}}{2\sqrt{-2+2\sqrt{2}}} + \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)} + \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right) \sqrt{2}}{2\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(1+cot(x))^(1/2),x)

[Out]  $-1/8*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}-1/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/8*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}-1/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{\cot(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="maxima")

[Out] integrate(cot(x)/sqrt(cot(x) + 1), x)

**mupad [B]** time = 0.41, size = 230, normalized size = 1.90

$$\operatorname{atanh} \left( \frac{16\sqrt{2} \sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} \sqrt{\cot(x)+1}}{128 \sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} \sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} - 8} - \frac{16\sqrt{2} \sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} \sqrt{\cot(x)+1}}{128 \sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} \sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} - 8} \right) \left( 2\sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} + 2\sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} \right) - \operatorname{atanh} \left( \frac{16\sqrt{2} \sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} \sqrt{\cot(x)+1}}{128 \sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} \sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} - 8} - \frac{16\sqrt{2} \sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} \sqrt{\cot(x)+1}}{128 \sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} \sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} - 8} \right) \left( 2\sqrt{\frac{1}{16} - \frac{\sqrt{2}}{16}} + 2\sqrt{\frac{\sqrt{2}}{16} + \frac{1}{16}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)/(cot(x) + 1)^(1/2), x)`

[Out] `atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/2)/16 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) - 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) + 2*(2^(1/2)/16 + 1/16)^(1/2)) - atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) + 8) + (16*2^(1/2)*(2^(1/2)/16 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) + 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) - 2*(2^(1/2)/16 + 1/16)^(1/2))`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{\cot(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)/(1+cot(x))**(1/2), x)`

[Out] `Integral(cot(x)/sqrt(cot(x) + 1), x)`

$$3.47 \quad \int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}} dx$$

Optimal. Leaf size=139

$$\frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{2}-1)} \tan^{-1} \left( \frac{(2-\sqrt{2})\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}} \right) + \frac{1}{2} \sqrt{\frac{1}{2}(1+\sqrt{2})} \tanh^{-1} \left( \frac{(2+\sqrt{2})\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}} \right)$$

[Out]  $1/(1+\cot(x))^{1/2} + 1/4 \cdot \arctan(1/2 \cdot (4+\cot(x) \cdot (2-2^{1/2})) - 3 \cdot 2^{1/2}) / (1+\cot(x))^{1/2} / (-7+5 \cdot 2^{1/2})^{1/2} \cdot (-2+2 \cdot 2^{1/2})^{1/2} + 1/4 \cdot \operatorname{arctanh}(1/2 \cdot (4+3 \cdot 2^{1/2} + \cot(x) \cdot (2+2^{1/2}))) / (1+\cot(x))^{1/2} / (7+5 \cdot 2^{1/2})^{1/2} \cdot (2+2 \cdot 2^{1/2})^{1/2}$

**Rubi [A]** time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {3542, 3536, 3535, 203, 207}

$$\frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{2}-1)} \tan^{-1} \left( \frac{(2-\sqrt{2})\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}} \right) + \frac{1}{2} \sqrt{\frac{1}{2}(1+\sqrt{2})} \tanh^{-1} \left( \frac{(2+\sqrt{2})\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^2/(1+Cot[x])^(3/2),x]

[Out]  $(\sqrt{(-1+\sqrt{2})/2} \cdot \operatorname{ArcTan}[(4-3\sqrt{2}+(2-\sqrt{2})\cot(x))/(2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)})]) / 2 + (\sqrt{(1+\sqrt{2})/2} \cdot \operatorname{ArcTanh}[(4+3\sqrt{2}+(2+\sqrt{2})\cot(x))/(2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)})]) / 2 + 1/\sqrt{1+\cot(x)}$

#### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 207

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rule 3535

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(-2*d^2)/f, Subst[Int[1/(2*b*c*d - 4*a*d^2 + x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

### Rule 3536

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

### Rule 3542

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cot^2(x)}{(1 + \cot(x))^{3/2}} dx &= \frac{1}{\sqrt{1 + \cot(x)}} + \frac{1}{2} \int \frac{-1 + \cot(x)}{\sqrt{1 + \cot(x)}} dx \\ &= \frac{1}{\sqrt{1 + \cot(x)}} + \frac{\int \frac{-\sqrt{2} - (2 - \sqrt{2}) \cot(x)}{\sqrt{1 + \cot(x)}} dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2} - (2 + \sqrt{2}) \cot(x)}{\sqrt{1 + \cot(x)}} dx}{4\sqrt{2}} \\ &= \frac{1}{\sqrt{1 + \cot(x)}} - \frac{1}{2} (-4 + 3\sqrt{2}) \text{Subst} \left( \int \frac{1}{2\sqrt{2} (2 - \sqrt{2}) - 4(2 - \sqrt{2})^2 + x^2} dx, x, \frac{\sqrt{2} - 2}{1 + \cot(x)} \right) \\ &= \frac{1}{2} \sqrt{\frac{1}{2} (-1 + \sqrt{2})} \tan^{-1} \left( \frac{4 - 3\sqrt{2} + (2 - \sqrt{2}) \cot(x)}{2\sqrt{-7 + 5\sqrt{2}} \sqrt{1 + \cot(x)}} \right) + \frac{1}{2} \sqrt{\frac{1}{2} (1 + \sqrt{2})} \tanh^{-1} \left( \frac{4 + 3\sqrt{2} + (2 + \sqrt{2}) \cot(x)}{2\sqrt{7 + 5\sqrt{2}} \sqrt{1 + \cot(x)}} \right) \end{aligned}$$

**Mathematica [C]** time = 0.13, size = 65, normalized size = 0.47

$$\frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{2}\sqrt{1-i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}}\right) + \frac{1}{2}\sqrt{1+i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^2/(1+Cot[x])^(3/2),x]

[Out] (Sqrt[1 - I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/2 + (Sqrt[1 + I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/2 + 1/Sqrt[1 + Cot[x]]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="giac")

[Out] integrate(cot(x)^2/(cot(x)+1)^(3/2),x)

**maple [B]** time = 0.19, size = 249, normalized size = 1.79

$$\frac{\sqrt{2\sqrt{2}+2} \ln\left(1 + \cot(x) + \sqrt{2} - \sqrt{1 + \cot(x)} \sqrt{2\sqrt{2}+2}\right)}{8} + \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)} - \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2}}{2\sqrt{-2+2\sqrt{2}}} - \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)} + \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right)\sqrt{2}}{2\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2/(1+cot(x))^(3/2),x)

[Out] -1/8\*(2\*2^(1/2)+2)^(1/2)\*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)\*(2\*2^(1/2)+2)^(1/2))+1/2/(-2+2\*2^(1/2))^(1/2)\*arctan((2\*(1+cot(x))^(1/2)-(2\*2^(1/2)+2)^(1/2))

$$\frac{1}{2})/(-2+2*2^{(1/2)})^{(1/2)}*2^{(1/2)}-1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)}+2)^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/8*(2*2^{(1/2)}+2)^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)}+2)^{(1/2)})-1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)}+2)^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)}+2)^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}+1/(1+\cot(x))^{(1/2)}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{(\cot(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="maxima")

[Out] integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)

**mupad** [B] time = 0.50, size = 208, normalized size = 1.50

$$\frac{1}{\sqrt{\cot(x) + 1}} - \operatorname{atanh} \left( \frac{\sqrt{\cot(x) + 1}}{8 \sqrt{\frac{\sqrt{2}}{32} + \frac{1}{32}}} - \frac{\sqrt{\cot(x) + 1}}{8 \sqrt{\frac{1}{32} - \frac{\sqrt{2}}{32}}} + \frac{\sqrt{2} \sqrt{\cot(x) + 1}}{16 \sqrt{\frac{1}{32} - \frac{\sqrt{2}}{32}}} + \frac{\sqrt{2} \sqrt{\cot(x) + 1}}{16 \sqrt{\frac{\sqrt{2}}{32} + \frac{1}{32}}} \right) \left( 2 \sqrt{\frac{1}{32} - \frac{\sqrt{2}}{32}} - 2 \sqrt{\frac{\sqrt{2}}{32} + \frac{1}{32}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2/(cot(x) + 1)^(3/2),x)

[Out]  $\frac{1}{(\cot(x) + 1)^{(1/2)} - \operatorname{atanh}((\cot(x) + 1)^{(1/2)}/(8*(2^{(1/2)}/32 + 1/32)^{(1/2)})) - (\cot(x) + 1)^{(1/2)}/(8*(1/32 - 2^{(1/2)}/32)^{(1/2)}) + (2^{(1/2)}*(\cot(x) + 1)^{(1/2)})/(16*(1/32 - 2^{(1/2)}/32)^{(1/2)}) + (2^{(1/2)}*(\cot(x) + 1)^{(1/2)})/(16*(2^{(1/2)}/32 + 1/32)^{(1/2)})} * (2*(1/32 - 2^{(1/2)}/32)^{(1/2)} - 2*(2^{(1/2)}/32 + 1/32)^{(1/2)}) + \operatorname{atanh}((\cot(x) + 1)^{(1/2)}/(8*(1/32 - 2^{(1/2)}/32)^{(1/2)}) + (\cot(x) + 1)^{(1/2)}/(8*(2^{(1/2)}/32 + 1/32)^{(1/2)}) - (2^{(1/2)}*(\cot(x) + 1)^{(1/2)})/(16*(1/32 - 2^{(1/2)}/32)^{(1/2)}) + (2^{(1/2)}*(\cot(x) + 1)^{(1/2)})/(16*(2^{(1/2)}/32 + 1/32)^{(1/2)})} * (2*(1/32 - 2^{(1/2)}/32)^{(1/2)} + 2*(2^{(1/2)}/32 + 1/32)^{(1/2)})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{(\cot(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cot(x)**2/(1+cot(x))**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(cot(x) + 1)**(3/2), x)
```

$$3.48 \quad \int \frac{\cot(x)}{(1+\cot(x))^{3/2}} dx$$

**Optimal.** Leaf size=226

$$\frac{1}{\sqrt{\cot(x)+1}} \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})} \sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{2(1+\sqrt{2})}} + \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})} \sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{2(1+\sqrt{2})}}$$

[Out]  $-1/(1+\cot(x))^{1/2}+1/4*\arctan((-2*(1+\cot(x))^{1/2}+(2+2*2^{1/2})^{1/2})/(-2+2*2^{1/2})^{1/2})*(2+2*2^{1/2})^{1/2}-1/4*\arctan((2*(1+\cot(x))^{1/2}+(2+2*2^{1/2})^{1/2})/(-2+2*2^{1/2})^{1/2})*(2+2*2^{1/2})^{1/2}-1/4*\ln(1+\cot(x)+2^{1/2}-(1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2})/(2+2*2^{1/2})^{1/2}+1/4*\ln(1+\cot(x)+2^{1/2}+(1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2})/(2+2*2^{1/2})^{1/2})$

**Rubi [A]** time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$ , Rules used = {3529, 21, 3485, 700, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{\sqrt{\cot(x)+1}} \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})} \sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{2(1+\sqrt{2})}} + \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})} \sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{4\sqrt{2(1+\sqrt{2})}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/(1 + Cot[x])^(3/2), x]

[Out]  $(\text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])] - 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]])/2 - (\text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])] + 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]])/2 - 1/\text{Sqrt}[1 + \text{Cot}[x]] - \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] - \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*\text{Sqrt}[1 + \text{Cot}[x]]]/(4*\text{Sqrt}[2*(1 + \text{Sqrt}[2])]) + \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] + \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*\text{Sqrt}[1 + \text{Cot}[x]]]/(4*\text{Sqrt}[2*(1 + \text{Sqrt}[2])])$

### Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 700

Int[Sqrt[(d\_) + (e\_.)\*(x\_)]/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[2\*e, Subst[Int[x^2/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 1127

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b\*x^2 + c\*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b\*x^2 + c\*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4\*a\*c, 0] && PosQ[a\*c]

### Rule 1161

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e - b/c, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && (GtQ[(2\*d)/e - b/c, 0] || (!LtQ[(2\*d)/e - b/c, 0] && EqQ[d - e\*Rt[a/c, 2], 0]))

### Rule 1164

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e - b/c, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - a\*e^2, 0] && !GtQ[b^2 - 4\*a\*c, 0]

Rule 3485

Int[((a\_) + (b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

Rule 3529

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\cot(x)}{(1 + \cot(x))^{3/2}} dx &= -\frac{1}{\sqrt{1 + \cot(x)}} - \frac{1}{2} \int \frac{-1 - \cot(x)}{\sqrt{1 + \cot(x)}} dx \\
 &= -\frac{1}{\sqrt{1 + \cot(x)}} + \frac{1}{2} \int \sqrt{1 + \cot(x)} dx \\
 &= -\frac{1}{\sqrt{1 + \cot(x)}} - \frac{1}{2} \text{Subst} \left( \int \frac{\sqrt{1+x}}{1+x^2} dx, x, \cot(x) \right) \\
 &= -\frac{1}{\sqrt{1 + \cot(x)}} - \text{Subst} \left( \int \frac{x^2}{2-2x^2+x^4} dx, x, \sqrt{1 + \cot(x)} \right) \\
 &= -\frac{1}{\sqrt{1 + \cot(x)}} + \frac{1}{2} \text{Subst} \left( \int \frac{\sqrt{2}-x^2}{2-2x^2+x^4} dx, x, \sqrt{1 + \cot(x)} \right) - \frac{1}{2} \text{Subst} \left( \int \frac{\sqrt{2}+x^2}{2-2x^2+x^4} dx, x, \sqrt{1 + \cot(x)} \right) \\
 &= -\frac{1}{\sqrt{1 + \cot(x)}} - \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1 + \cot(x)} \right) - \frac{1}{4} \text{Subst} \left( \int \frac{1}{\sqrt{2}+\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1 + \cot(x)} \right) \\
 &= -\frac{1}{\sqrt{1 + \cot(x)}} - \frac{\log \left( 1 + \sqrt{2} + \cot(x) - \sqrt{2(1+\sqrt{2})} \sqrt{1 + \cot(x)} \right)}{4\sqrt{2(1+\sqrt{2})}} + \frac{\log \left( 1 + \sqrt{2} + \cot(x) + \sqrt{2(1+\sqrt{2})} \sqrt{1 + \cot(x)} \right)}{4\sqrt{2(1+\sqrt{2})}} \\
 &= \frac{\tan^{-1} \left( \frac{\sqrt{2(1+\sqrt{2})} - 2\sqrt{1+\cot(x)}}{\sqrt{2(-1+\sqrt{2})}} \right)}{2\sqrt{2(-1+\sqrt{2})}} - \frac{\tan^{-1} \left( \frac{\sqrt{2(1+\sqrt{2})} + 2\sqrt{1+\cot(x)}}{\sqrt{2(-1+\sqrt{2})}} \right)}{2\sqrt{2(-1+\sqrt{2})}} - \frac{1}{\sqrt{1 + \cot(x)}} - \frac{\log \left( 1 + \sqrt{2} + \cot(x) - \sqrt{2(1+\sqrt{2})} \sqrt{1 + \cot(x)} \right)}{4\sqrt{2(1+\sqrt{2})}} + \frac{\log \left( 1 + \sqrt{2} + \cot(x) + \sqrt{2(1+\sqrt{2})} \sqrt{1 + \cot(x)} \right)}{4\sqrt{2(1+\sqrt{2})}}
 \end{aligned}$$

**Mathematica [C]** time = 0.14, size = 71, normalized size = 0.31

$$-\frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{2}i\sqrt{1-i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}}\right) - \frac{1}{2}i\sqrt{1+i} \tanh^{-1}\left(\frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]/(1 + Cot[x])^(3/2), x]

[Out] (I/2)\*Sqrt[1 - I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] - (I/2)\*Sqrt[1 + I]\*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - 1/Sqrt[1 + Cot[x]]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(3/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(3/2), x, algorithm="giac")

[Out] integrate(cot(x)/(cot(x) + 1)^(3/2), x)

**maple [B]** time = 0.13, size = 356, normalized size = 1.58

$$\frac{\sqrt{2\sqrt{2}+2} \sqrt{2} \ln\left(1 + \cot(x) + \sqrt{2} - \sqrt{1 + \cot(x)} \sqrt{2\sqrt{2}+2}\right) + \sqrt{2} (2\sqrt{2}+2) \arctan\left(\frac{2\sqrt{1+\cot(x)} - \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right)}{8 \sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(1+cot(x))^(3/2), x)

[Out] -1/8\*(2\*2^(1/2)+2)^(1/2)\*2^(1/2)\*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)\*(2\*2^(1/2)+2)^(1/2))-1/4\*2^(1/2)\*(2\*2^(1/2)+2)/(-2+2\*2^(1/2))^(1/2)\*arctan((2\*(1

$$\begin{aligned}
& +\cot(x)^{(1/2)}-(2*2^{(1/2)}+2)^{(1/2))/(-2+2*2^{(1/2)})^{(1/2)}+1/8*(2*2^{(1/2)}+2)^{(1/2)} \\
& * \ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)}+2)^{(1/2)}+1/4*(2*2^{(1/2)}+2)^{(1/2)} \\
& /(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)}+2)^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)} \\
& +1/8*(2*2^{(1/2)}+2)^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)}+2)^{(1/2)} \\
& -1/4*2^{(1/2)}*(2*2^{(1/2)}+2)/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)}+2)^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)} \\
& -1/8*(2*2^{(1/2)}+2)^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)}+2)^{(1/2)}+1/4*(2*2^{(1/2)}+2)^{(1/2)} \\
& /(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)}+2)^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)}-1/(1+\cot(x))^{(1/2)}
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="maxima")

[Out] integrate(cot(x)/(cot(x) + 1)^(3/2), x)

**mupad** [B] time = 0.40, size = 121, normalized size = 0.54

$$-\operatorname{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)-\frac{1}{\sqrt{\cot(x)+1}}-\operatorname{atanh}\left(\frac{1}{\sqrt{\cot(x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(cot(x) + 1)^(3/2),x)

[Out] - atanh(32\*(cot(x) + 1)^(1/2)\*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)\*(2\*(- 2^(1/2)/32 - 1/32)^(1/2) + 2\*(2^(1/2)/32 - 1/32)^(1/2)) - 1/(cot(x) + 1)^(1/2) - atanh(32\*(cot(x) + 1)^(1/2)\*((- 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)\*(2\*(- 2^(1/2)/32 - 1/32)^(1/2) - 2\*(2^(1/2)/32 - 1/32)^(1/2))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))\*\*(3/2),x)

[Out] Integral(cot(x)/(cot(x) + 1)\*\*(3/2), x)

$$3.49 \quad \int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}} dx$$

Optimal. Leaf size=143

$$-\frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{3(\cot(x)+1)^{3/2}} + \frac{1}{4}\sqrt{\sqrt{2}-1} \tan^{-1}\left(\frac{(1-\sqrt{2})\cot(x)-2\sqrt{2}+3}{\sqrt{2}(5\sqrt{2}-7)\sqrt{\cot(x)+1}}\right) + \frac{1}{4}\sqrt{1+\sqrt{2}} \tanh^{-1}\left(\frac{(1+\sqrt{2})\cot(x)+2\sqrt{2}-3}{\sqrt{2}(7+5\sqrt{2})\sqrt{\cot(x)+1}}\right)$$

[Out] 1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)+1/4\*arctan((3+cot(x)\*(1-2^(1/2))-2\*2^(1/2))/(1+cot(x))^(1/2)/(-14+10\*2^(1/2))^(1/2))\*(2^(1/2)-1)^(1/2)+1/4\*arc tanh((3+2\*2^(1/2)+cot(x)\*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10\*2^(1/2))^(1/2))\* (1+2^(1/2))^(1/2)

**Rubi [A]** time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$ , Rules used = {3542, 3529, 12, 3536, 3535, 203, 207}

$$-\frac{1}{\sqrt{\cot(x)+1}} + \frac{1}{3(\cot(x)+1)^{3/2}} + \frac{1}{4}\sqrt{\sqrt{2}-1} \tan^{-1}\left(\frac{(1-\sqrt{2})\cot(x)-2\sqrt{2}+3}{\sqrt{2}(5\sqrt{2}-7)\sqrt{\cot(x)+1}}\right) + \frac{1}{4}\sqrt{1+\sqrt{2}} \tanh^{-1}\left(\frac{(1+\sqrt{2})\cot(x)+2\sqrt{2}-3}{\sqrt{2}(7+5\sqrt{2})\sqrt{\cot(x)+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^2/(1 + Cot[x])^(5/2), x]

[Out] (Sqrt[-1 + Sqrt[2]]\*ArcTan[(3 - 2\*Sqrt[2] + (1 - Sqrt[2])\*Cot[x])/(Sqrt[2\*(-7 + 5\*Sqrt[2]])\*Sqrt[1 + Cot[x]])])/4 + (Sqrt[1 + Sqrt[2]]\*ArcTanh[(3 + 2\*Sqrt[2] + (1 + Sqrt[2])\*Cot[x])/(Sqrt[2\*(7 + 5\*Sqrt[2]])\*Sqrt[1 + Cot[x]])])/4 + 1/(3\*(1 + Cot[x])^(3/2)) - 1/Sqrt[1 + Cot[x]]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

, 0] || GtQ[b, 0])

### Rule 3529

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

### Rule 3535

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(-2\*d^2)/f, Subst[Int[1/(2\*b\*c\*d - 4\*a\*d^2 + x^2), x], x, (b\*c - 2\*a\*d - b\*d\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && EqQ[2\*a\*c\*d - b\*(c^2 - d^2), 0]

### Rule 3536

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/Sqrt[(a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2\*q), Int[(a\*c + b\*d + c\*q + (b\*c - a\*d + d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] - Dist[1/(2\*q), Int[(a\*c + b\*d - c\*q + (b\*c - a\*d - d\*q)\*Tan[e + f\*x])/Sqrt[a + b\*Tan[e + f\*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2\*a\*c\*d - b\*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])

### Rule 3542

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[((b\*c - a\*d)^2\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c^2 + 2\*b\*c\*d - a\*d^2 - (b\*c^2 - 2\*a\*c\*d - b\*d^2)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{\cot^2(x)}{(1 + \cot(x))^{5/2}} dx &= \frac{1}{3(1 + \cot(x))^{3/2}} + \frac{1}{2} \int \frac{-1 + \cot(x)}{(1 + \cot(x))^{3/2}} dx \\
&= \frac{1}{3(1 + \cot(x))^{3/2}} - \frac{1}{\sqrt{1 + \cot(x)}} + \frac{1}{4} \int \frac{2 \cot(x)}{\sqrt{1 + \cot(x)}} dx \\
&= \frac{1}{3(1 + \cot(x))^{3/2}} - \frac{1}{\sqrt{1 + \cot(x)}} + \frac{1}{2} \int \frac{\cot(x)}{\sqrt{1 + \cot(x)}} dx \\
&= \frac{1}{3(1 + \cot(x))^{3/2}} - \frac{1}{\sqrt{1 + \cot(x)}} + \frac{\int \frac{-1 - (-1 - \sqrt{2}) \cot(x)}{\sqrt{1 + \cot(x)}} dx}{4\sqrt{2}} - \frac{\int \frac{-1 - (-1 + \sqrt{2}) \cot(x)}{\sqrt{1 + \cot(x)}} dx}{4\sqrt{2}} \\
&= \frac{1}{3(1 + \cot(x))^{3/2}} - \frac{1}{\sqrt{1 + \cot(x)}} + \frac{1}{4} (-4 + 3\sqrt{2}) \text{Subst} \left( \int \frac{1}{2(-1 + \sqrt{2}) - 4(-1 + \sqrt{2})} \right) \\
&= \frac{1}{4} \sqrt{-1 + \sqrt{2}} \tan^{-1} \left( \frac{3 - 2\sqrt{2} + (1 - \sqrt{2}) \cot(x)}{\sqrt{2}(-7 + 5\sqrt{2}) \sqrt{1 + \cot(x)}} \right) + \frac{1}{4} \sqrt{1 + \sqrt{2}} \tanh^{-1} \left( \frac{3 + 2\sqrt{2} + (1 + \sqrt{2}) \cot(x)}{\sqrt{2}(7 + 5\sqrt{2}) \sqrt{1 + \cot(x)}} \right)
\end{aligned}$$

**Mathematica [C]** time = 0.41, size = 75, normalized size = 0.52

$$\frac{-3 \cot(x) - 2}{3(\cot(x) + 1)^{3/2}} + \frac{\tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1-i}} \right)}{2\sqrt{1-i}} + \frac{\tanh^{-1} \left( \frac{\sqrt{\cot(x)+1}}{\sqrt{1+i}} \right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^2/(1 + Cot[x])^(5/2), x]

[Out] ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/(2\*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]]/(2\*Sqrt[1 + I]) + (-2 - 3\*Cot[x])/(3\*(1 + Cot[x])^(3/2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(5/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{(\cot(x) + 1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="giac")

[Out] integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)

**maple [B]** time = 0.19, size = 265, normalized size = 1.85

$$\frac{\sqrt{2\sqrt{2}+2} \sqrt{2} \ln\left(1 + \cot(x) + \sqrt{2} - \sqrt{1 + \cot(x)} \sqrt{2\sqrt{2}+2}\right)}{16} + \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)} - \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right) \sqrt{2}}{4\sqrt{-2+2\sqrt{2}}} - \frac{\arctan\left(\frac{2\sqrt{1+\cot(x)} + \sqrt{2\sqrt{2}+2}}{\sqrt{-2+2\sqrt{2}}}\right) \sqrt{2}}{2\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2/(1+cot(x))^(5/2),x)

[Out]  $-1/16*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/4/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}-1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/16*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/4/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}-1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/3/(1+\cot(x))^{(3/2)}-1/(1+\cot(x))^{(1/2)}$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)^2}{(\cot(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="maxima")

[Out] integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)

**mupad [B]** time = 0.80, size = 242, normalized size = 1.69

$$\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}-\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}-\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}-\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}-\frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)^2/(cot(x) + 1)^(5/2),x)`

[Out] 
$$\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{1/64 - 2^{1/2}/64}^{1/2}(\cot(x) + 1)^{1/2}}{64\sqrt{1/64 - 2^{1/2}/64}^{1/2}\sqrt{2^{1/2}/64 + 1/64}^{1/2} - 1} - \frac{4\sqrt{2}\sqrt{2^{1/2}/64 + 1/64}^{1/2}(\cot(x) + 1)^{1/2}}{64\sqrt{1/64 - 2^{1/2}/64}^{1/2}\sqrt{2^{1/2}/64 + 1/64}^{1/2} - 1}\right) \cdot \left(2\sqrt{1/64 - 2^{1/2}/64}^{1/2} + 2\sqrt{2^{1/2}/64 + 1/64}^{1/2}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{1/64 - 2^{1/2}/64}^{1/2}(\cot(x) + 1)^{1/2}}{64\sqrt{1/64 - 2^{1/2}/64}^{1/2}\sqrt{2^{1/2}/64 + 1/64}^{1/2} + 1} + \frac{4\sqrt{2}\sqrt{2^{1/2}/64 + 1/64}^{1/2}(\cot(x) + 1)^{1/2}}{64\sqrt{1/64 - 2^{1/2}/64}^{1/2}\sqrt{2^{1/2}/64 + 1/64}^{1/2} + 1}\right) \cdot \left(2\sqrt{1/64 - 2^{1/2}/64}^{1/2} - 2\sqrt{2^{1/2}/64 + 1/64}^{1/2}\right) - (\cot(x) + 2/3)/(\cot(x) + 1)^{3/2}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{(\cot(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)**2/(1+cot(x))**(5/2),x)`

[Out] `Integral(cot(x)**2/(cot(x) + 1)**(5/2), x)`

$$3.50 \quad \int \frac{\cot(x)}{(1+\cot(x))^{5/2}} dx$$

**Optimal.** Leaf size=216

$$-\frac{1}{3(\cot(x)+1)^{3/2}} + \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{8\sqrt{1+\sqrt{2}}} - \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}\right)}{8\sqrt{1+\sqrt{2}}}$$

[Out]  $-1/3/(1+\cot(x))^{3/2} + 1/8*\ln(1+\cot(x)+2^{1/2}) - (1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2} / (1+2^{1/2})^{1/2} - 1/8*\ln(1+\cot(x)+2^{1/2}) + (1+\cot(x))^{1/2}*(2+2*2^{1/2})^{1/2} / (1+2^{1/2})^{1/2} + 1/4*\arctan((-2*(1+\cot(x))^{1/2} + (2+2*2^{1/2})^{1/2}) / (-2*2^{1/2})^{1/2}) * (1+2^{1/2})^{1/2} - 1/4*\arctan((2*(1+\cot(x))^{1/2} + (2+2*2^{1/2})^{1/2}) / (-2*2^{1/2})^{1/2}) * (1+2^{1/2})^{1/2}$

**Rubi [A]** time = 0.18, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$ , Rules used = {3529, 21, 3485, 708, 1094, 634, 618, 204, 628}

$$-\frac{1}{3(\cot(x)+1)^{3/2}} + \frac{\log\left(\cot(x) - \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1} + \sqrt{2} + 1\right)}{8\sqrt{1+\sqrt{2}}} - \frac{\log\left(\cot(x) + \sqrt{2(1+\sqrt{2})}\sqrt{\cot(x)+1}\right)}{8\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/(1 + Cot[x])^(5/2), x]

[Out]  $(\text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])]] - 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])])]/4 - (\text{Sqrt}[1 + \text{Sqrt}[2]]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])]] + 2*\text{Sqrt}[1 + \text{Cot}[x]])/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])])]/4 - 1/(3*(1 + \text{Cot}[x])^{3/2}) + \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] - \text{Sqrt}[2*(1 + \text{Sqrt}[2])]]*\text{Sqrt}[1 + \text{Cot}[x]]/(8*\text{Sqrt}[1 + \text{Sqrt}[2]]) - \text{Log}[1 + \text{Sqrt}[2] + \text{Cot}[x] + \text{Sqrt}[2*(1 + \text{Sqrt}[2])]]*\text{Sqrt}[1 + \text{Cot}[x]]/(8*\text{Sqrt}[1 + \text{Sqrt}[2]])$

#### Rule 21

Int[(u\_.)\*((a\_.) + (b\_.)\*(v\_))^(m\_.)\*((c\_.) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

#### Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 708

Int[1/(Sqrt[(d\_) + (e\_.)\*(x\_)])\*((a\_.) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[2\*e, Subst[Int[1/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 1094

Int[((a\_.) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(r - x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(r + x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 3485

Int[((a\_.) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

### Rule 3529

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])

$(m + 1) \cdot \text{Simp}[a \cdot c + b \cdot d - (b \cdot c - a \cdot d) \cdot \text{Tan}[e + f \cdot x], x], x] / ; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

### Rubi steps

$$\begin{aligned}
 \int \frac{\cot(x)}{(1 + \cot(x))^{5/2}} dx &= -\frac{1}{3(1 + \cot(x))^{3/2}} - \frac{1}{2} \int \frac{-1 - \cot(x)}{(1 + \cot(x))^{3/2}} dx \\
 &= -\frac{1}{3(1 + \cot(x))^{3/2}} + \frac{1}{2} \int \frac{1}{\sqrt{1 + \cot(x)}} dx \\
 &= -\frac{1}{3(1 + \cot(x))^{3/2}} - \frac{1}{2} \text{Subst} \left( \int \frac{1}{\sqrt{1 + x} (1 + x^2)} dx, x, \cot(x) \right) \\
 &= -\frac{1}{3(1 + \cot(x))^{3/2}} - \text{Subst} \left( \int \frac{1}{2 - 2x^2 + x^4} dx, x, \sqrt{1 + \cot(x)} \right) \\
 &= -\frac{1}{3(1 + \cot(x))^{3/2}} - \frac{\text{Subst} \left( \int \frac{\sqrt{2(1 + \sqrt{2}) - x}}{\sqrt{2} - \sqrt{2(1 + \sqrt{2})} x + x^2} dx, x, \sqrt{1 + \cot(x)} \right)}{4\sqrt{1 + \sqrt{2}}} - \frac{\text{Subst} \left( \int \frac{\sqrt{2(1 + \sqrt{2}) + x}}{\sqrt{2} + \sqrt{2(1 + \sqrt{2})} x + x^2} dx, x, \sqrt{1 + \cot(x)} \right)}{4\sqrt{1 + \sqrt{2}}} \\
 &= -\frac{1}{3(1 + \cot(x))^{3/2}} - \frac{\text{Subst} \left( \int \frac{1}{\sqrt{2} - \sqrt{2(1 + \sqrt{2})} x + x^2} dx, x, \sqrt{1 + \cot(x)} \right)}{4\sqrt{2}} - \frac{\text{Subst} \left( \int \frac{1}{\sqrt{2} + \sqrt{2(1 + \sqrt{2})} x + x^2} dx, x, \sqrt{1 + \cot(x)} \right)}{4\sqrt{2}} \\
 &= -\frac{1}{3(1 + \cot(x))^{3/2}} + \frac{\log \left( 1 + \sqrt{2} + \cot(x) - \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \cot(x)} \right)}{8\sqrt{1 + \sqrt{2}}} - \frac{\log \left( 1 + \sqrt{2} + \cot(x) + \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \cot(x)} \right)}{8\sqrt{1 + \sqrt{2}}} \\
 &= \frac{\tan^{-1} \left( \frac{\sqrt{2(1 + \sqrt{2}) - 2\sqrt{1 + \cot(x)}}}{\sqrt{2(-1 + \sqrt{2})}} \right)}{4\sqrt{-1 + \sqrt{2}}} - \frac{\tan^{-1} \left( \frac{\sqrt{2(1 + \sqrt{2}) + 2\sqrt{1 + \cot(x)}}}{\sqrt{2(-1 + \sqrt{2})}} \right)}{4\sqrt{-1 + \sqrt{2}}} - \frac{1}{3(1 + \cot(x))^{3/2}} + \frac{\log \left( 1 + \sqrt{2} + \cot(x) - \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \cot(x)} \right)}{8\sqrt{1 + \sqrt{2}}} - \frac{\log \left( 1 + \sqrt{2} + \cot(x) + \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \cot(x)} \right)}{8\sqrt{1 + \sqrt{2}}}
 \end{aligned}$$

**Mathematica** [C] time = 0.24, size = 69, normalized size = 0.32

$$-\frac{1}{3(\cot(x) + 1)^{3/2}} - \frac{1}{4}(1 - i)^{3/2} \tanh^{-1} \left( \frac{\sqrt{\cot(x) + 1}}{\sqrt{1 - i}} \right) - \frac{1}{4}(1 + i)^{3/2} \tanh^{-1} \left( \frac{\sqrt{\cot(x) + 1}}{\sqrt{1 + i}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]/(1 + Cot[x])^(5/2), x]

[Out]  $-1/4*((1 - I)^{(3/2)}*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]) - ((1 + I)^{(3/2)}*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/4 - 1/(3*(1 + Cot[x])^{(3/2)})$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="fricas")`

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="giac")`

[Out] `integrate(cot(x)/(cot(x) + 1)^(5/2), x)`

**maple** [B] time = 0.13, size = 444, normalized size = 2.06

$$\frac{\sqrt{2\sqrt{2} + 2} \sqrt{2} \ln\left(1 + \cot(x) + \sqrt{2} - \sqrt{1 + \cot(x)} \sqrt{2\sqrt{2} + 2}\right)}{16} + \frac{\sqrt{2\sqrt{2} + 2} \ln\left(1 + \cot(x) + \sqrt{2} - \sqrt{1 + \cot(x)}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(x)/(1+cot(x))^(5/2),x)`

[Out]  $-1/16*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})+1/8*(2*2^{(1/2)+2})^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}-(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})-1/8*2^{(1/2)}*(2*2^{(1/2)+2})/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/4*(2*2^{(1/2)+2})/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})-1/2/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}-(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/16*(2*2^{(1/2)+2})^{(1/2)}*2^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})-1/8*(2*2^{(1/2)+2})^{(1/2)}*\ln(1+\cot(x)+2^{(1/2)}+(1+\cot(x))^{(1/2)}*(2*2^{(1/2)+2})^{(1/2)})-1/8*2^{(1/2)}*(2*2^{(1/2)+2})/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+\cot(x))^{(1/2)}+(2*2^{(1/2)+2})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})+1/4*(2*2^{(1/2)+2})/(-2+2*2^{(1/2)})$

$$\frac{\sqrt{\frac{1}{2}} \arctan\left(\frac{(2\sqrt{1+\cot(x)})^{\frac{1}{2}} + (2\sqrt{2^{\frac{1}{2}}+2})^{\frac{1}{2}}}{(-2+2\sqrt{2^{\frac{1}{2}}})^{\frac{1}{2}}}\right) - \frac{1}{2} \sqrt{\frac{1}{2}} \arctan\left(\frac{(2\sqrt{1+\cot(x)})^{\frac{1}{2}} + (2\sqrt{2^{\frac{1}{2}}+2})^{\frac{1}{2}}}{(-2+2\sqrt{2^{\frac{1}{2}}})^{\frac{1}{2}}}\right) \sqrt{2^{\frac{1}{2}} - 1/3} \sqrt{1+\cot(x)}^{\frac{3}{2}}}{\dots}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="maxima")

[Out] integrate(cot(x)/(cot(x) + 1)^(5/2), x)

**mupad** [B] time = 0.71, size = 238, normalized size = 1.10

$$\operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1} + \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}} - 2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \operatorname{atanh}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(cot(x) + 1)^(5/2),x)

[Out]  $\operatorname{atanh}\left(\frac{(4\sqrt{2}(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(\cot(x) + 1)^{\frac{1}{2}})/(64(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}} + 1) + (4\sqrt{2}(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(\cot(x) + 1)^{\frac{1}{2}})/(64(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}} + 1)}{(2(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}} - 2(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}})} - \operatorname{atanh}\left(\frac{(4\sqrt{2}(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(\cot(x) + 1)^{\frac{1}{2}})/(64(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}} - 1) - (4\sqrt{2}(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(\cot(x) + 1)^{\frac{1}{2}})/(64(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}}(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}} - 1)}{(2(-2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}} + 2(2^{\frac{1}{2}}/64 - 1/64)^{\frac{1}{2}})} - 1/(3(\cot(x) + 1)^{\frac{3}{2}})\right)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(\cot(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)/(1+cot(x))\*\*(5/2),x)

[Out] Integral(cot(x)/(cot(x) + 1)\*\*(5/2), x)



### 3.51 $\int (e \cot(c + dx))^{3/2} (a + b \cot(c + dx)) dx$

**Optimal.** Leaf size=247

$$\frac{e^{3/2}(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{e^{3/2}(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2}d}$$

[Out]  $-2/3*b*(e*\cot(d*x+c))^{3/2}/d-1/2*(a+b)*e^{3/2}*arctan(1-2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/d*2^{1/2}+1/2*(a+b)*e^{3/2}*arctan(1+2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/d*2^{1/2}-1/4*(a-b)*e^{3/2}*ln(e^{1/2}+\cot(d*x+c)*e^{1/2})-2^{1/2}*(e*\cot(d*x+c))^{1/2}/d*2^{1/2}+1/4*(a-b)*e^{3/2}*ln(e^{1/2}+\cot(d*x+c)*e^{1/2})+2^{1/2}*(e*\cot(d*x+c))^{1/2}/d*2^{1/2}-2*a*e*(e*\cot(d*x+c))^{1/2}/d$

**Rubi [A]** time = 0.21, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {3528, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{e^{3/2}(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{e^{3/2}(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x]),x]

[Out]  $-(((a+b)*e^{3/2}*ArcTan[1-(Sqrt[2]*Sqrt[e*Cot[c+d*x]])/Sqrt[e]])/(Sqrt[2]*d))+((a+b)*e^{3/2}*ArcTan[1+(Sqrt[2]*Sqrt[e*Cot[c+d*x]])/Sqrt[e]])/(Sqrt[2]*d)-(2*a*e*Sqrt[e*Cot[c+d*x]])/d-(2*b*(e*Cot[c+d*x])^{3/2})/(3*d)-((a-b)*e^{3/2}*Log[Sqrt[e]+Sqrt[e]*Cot[c+d*x]-Sqrt[2]*Sqrt[e*Cot[c+d*x]])/(2*Sqrt[2]*d)+((a-b)*e^{3/2}*Log[Sqrt[e]+Sqrt[e]*Cot[c+d*x]+Sqrt[2]*Sqrt[e*Cot[c+d*x]])/(2*Sqrt[2]*d)$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

### Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

### Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

### Rule 3528

$\text{Int}[\frac{((a_.) + (b_.)\tan[(e_.) + (f_.)x])^{(m_.)}((c_.) + (d_.)\tan[(e_.) + (f_.)x])}{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}, x\_Symbol] \rightarrow \text{Simp}[\frac{d(a + b \tan[e + fx])^m}{f^m}, x] + \text{Int}[(a + b \tan[e + fx])^{(m-1)} \text{Simp}[ac - bd + (bc + ad)\tan[e + fx], x], x] \ /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[bc - ad, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3534

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\sqrt{(b_.)\tan[(e_.) + (f_.)x]}}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(bc + dx^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \tan[e + fx]]], x] \ /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{3/2} (a + b \cot(c + dx)) dx &= -\frac{2b(e \cot(c + dx))^{3/2}}{3d} + \int \sqrt{e \cot(c + dx)} (-be + ae \cot(c + dx)) dx \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2b(e \cot(c + dx))^{3/2}}{3d} + \int \frac{-ae^2 - be^2 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2b(e \cot(c + dx))^{3/2}}{3d} + \frac{2 \operatorname{Subst}\left(\int \frac{ae^3 + be^2 x^2}{e^2 + x^4} dx\right)}{d} \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2b(e \cot(c + dx))^{3/2}}{3d} + \frac{((a - b)e^2) \operatorname{Subst}\left(\int \frac{e}{e^2 + x^4} dx\right)}{d} \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2b(e \cot(c + dx))^{3/2}}{3d} - \frac{((a - b)e^{3/2}) \operatorname{Subst}\left(\int \frac{e}{e^2 + x^4} dx\right)}{d} \\
&= -\frac{2ae\sqrt{e \cot(c + dx)}}{d} - \frac{2b(e \cot(c + dx))^{3/2}}{3d} - \frac{(a - b)e^{3/2} \log(\sqrt{e} + \sqrt{e \cot(c + dx)})}{\sqrt{2}d} \\
&= -\frac{(a + b)e^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a + b)e^{3/2} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** time = 0.13, size = 68, normalized size = 0.28

$$\frac{2e\sqrt{e \cot(c + dx)} \left(3a {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\tan^2(c + dx)\right) + b \cot(c + dx) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\tan^2(c + dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x]),x]

[Out] (-2\*e\*Sqrt[e\*Cot[c + d\*x]]\*(b\*Cot[c + d\*x]\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d\*x]^2] + 3\*a\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2]))/(3\*d)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a) (e \cot(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(3/2), x)

**maple** [A] time = 0.37, size = 363, normalized size = 1.47

$$\frac{2b(e \cot(dx + c))^{\frac{3}{2}}}{3d} - \frac{2ae\sqrt{e \cot(dx + c)}}{d} + \frac{ae(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{4d} + \frac{ae(e^2)^{\frac{1}{4}}\sqrt{2} \arctan\left(\frac{e \cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c)),x)

[Out] 
$$-2/3*b*(e \cot(dx+c))^{3/2}/d - 2*a*e*(e \cot(dx+c))^{1/2}/d + 1/4*a/d*e*(e^2)^{1/4}*2^{1/2}*\ln((e \cot(dx+c)+(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2})/(e \cot(dx+c)-(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))) + 1/2*a/d*e*(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1) - 1/2*a/d*e*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1) + 1/4/d*e^2*b/(e^2)^{1/4}*2^{1/2}*\ln((e \cot(dx+c)-(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2})/(e \cot(dx+c)+(e^2)^{1/4}*(e \cot(dx+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))) + 1/2/d*e^2*b/(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1) - 1/2/d*e^2*b/(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e \cot(dx+c))^{1/2}+1)$$

**maxima** [A] time = 0.85, size = 221, normalized size = 0.89

$$3 \left( \frac{2\sqrt{2}(a+b) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a+b) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2}(a-b) \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right)$$

12d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c)),x, algorithm="maxima")

[Out] 1/12\*(3\*(2\*sqrt(2)\*(a + b)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a + b)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + sqrt(2)\*(a - b)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*(a - b)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))\*e - 8\*(3\*a\*e\*sqrt(e/tan(d\*x + c)) + b\*(e/tan(d\*x + c))^(3/2))/e)\*e/d

**mupad [B]** time = 1.40, size = 153, normalized size = 0.62

$$\frac{(-1)^{1/4} b e^{3/2} \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{2 a e \sqrt{e \cot(c+dx)}}{d} - \frac{2 b (e \cot(c+dx))^{3/2}}{3 d} - \frac{(-1)^{1/4} b e^{3/2} \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(3/2)\*(a + b\*cot(c + d\*x)),x)

[Out] ((-1)^(1/4)\*b\*e^(3/2)\*atan(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/d - (2\*a\*e\*(e\*cot(c + d\*x))^(1/2))/d - ((-1)^(1/4)\*a\*e^(3/2)\*atan(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*1i)/d - ((-1)^(1/4)\*a\*e^(3/2)\*atanh(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*1i)/d - (2\*b\*(e\*cot(c + d\*x))^(3/2))/(3\*d) - ((-1)^(1/4)\*b\*e^(3/2)\*atanh(((1/4)\*(-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/d

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (e \cot(c + dx))^{\frac{3}{2}} (a + b \cot(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(3/2)\*(a+b\*cot(d\*x+c)),x)

[Out] Integral((e\*cot(c + d\*x))\*\*(3/2)\*(a + b\*cot(c + d\*x)), x)

### 3.52 $\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx)) dx$

Optimal. Leaf size=226

$$\frac{\sqrt{e}(a+b) \log\left(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}\right)}{2\sqrt{2}d} + \frac{\sqrt{e}(a+b) \log\left(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)}\right)}{2\sqrt{2}d}$$

[Out]  $1/2*(a-b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}-1/2*(a-b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}-1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}+1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}-2*b*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.17, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {3528, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{e}(a+b) \log\left(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}\right)}{2\sqrt{2}d} + \frac{\sqrt{e}(a+b) \log\left(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)}\right)}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x]),x]

[Out]  $((a - b)*\text{Sqrt}[e]*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d) - ((a - b)*\text{Sqrt}[e]*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d) - (2*b*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - ((a + b)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d) + ((a + b)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d)$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*c/b}, simplify[(a\*c)/b^2], Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 3528

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx)) dx &= -\frac{2b\sqrt{e \cot(c + dx)}}{d} + \int \frac{-be + ae \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{2b\sqrt{e \cot(c + dx)}}{d} + \frac{2 \operatorname{Subst}\left(\int \frac{be^2 - aex^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{2b\sqrt{e \cot(c + dx)}}{d} - \frac{((a - b)e) \operatorname{Subst}\left(\int \frac{e + x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} + \dots \\
&= -\frac{2b\sqrt{e \cot(c + dx)}}{d} - \frac{((a + b)\sqrt{e}) \operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e} + 2x}{-e - \sqrt{2}\sqrt{e}x - x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2}d} \\
&= -\frac{2b\sqrt{e \cot(c + dx)}}{d} - \frac{(a + b)\sqrt{e} \log\left(\sqrt{e} + \sqrt{e} \cot(c + dx) - \sqrt{2}\sqrt{e} \cot(c + dx)\right)}{2\sqrt{2}d} \\
&= \frac{(a - b)\sqrt{e} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a - b)\sqrt{e} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica** [C] time = 0.28, size = 155, normalized size = 0.69

$$\frac{\sqrt{e \cot(c + dx)} \left( \sqrt{2} a \sqrt{\tan(c + dx)} \left( 2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} \right) - 2 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(c + dx)} + 1 \right) + \log \left( \tan \left( \sqrt{2} \sqrt{\tan(c + dx)} + 1 \right) \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x]),x]

[Out] -1/4\*(Sqrt[e\*Cot[c + d\*x]]\*(8\*b\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d\*x]^2] + Sqrt[2]\*a\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]])\*Sqrt[Tan[c + d\*x]]))/d

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a) \sqrt{e \cot(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)\*sqrt(e\*cot(d\*x + c)), x)

**maple** [A] time = 0.36, size = 337, normalized size = 1.49

$$-\frac{2b\sqrt{e \cot(dx + c)}}{d} + \frac{b(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d} + \frac{b(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c)),x)

[Out]  $-2*b*(e*\cot(d*x+c))^{1/2}/d+1/4/d*b*(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))+1/2/d*b*(e^2)^{1/4}*2^{1/2}*a*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/2/d*b*(e^2)^{1/4}*2^{1/2}*a*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-1/4/d*e*a/(e^2)^{1/4})*2^{1/2}*\ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))-1/2/d*e*a/(e^2)^{1/4})*2^{1/2}*a*\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+1/2/d*e*a/(e^2)^{1/4})*2^{1/2}*a*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)$

**maxima** [A] time = 0.73, size = 199, normalized size = 0.88

$$\frac{\left( \frac{2\sqrt{2}(a-b)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a-b)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}+e+\frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/4*(2*\sqrt{2}*(a - b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(dx + c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a - b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(dx + c)}))/\sqrt{e})/\sqrt{e} - \sqrt{2}*(a + b)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx + c)} + e + e/\tan(dx + c))/\sqrt{e} + \sqrt{2}*(a + b)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx + c)} + e + e/\tan(dx + c))/\sqrt{e} + 8*b*\sqrt{e/\tan(dx + c)}/e)*e/d$

**mupad [B]** time = 0.73, size = 128, normalized size = 0.57

$$\frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{(-1)^{1/4}a\sqrt{e}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) - \operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\right)}{d} - \frac{(-1)^{1/4}b\sqrt{e}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x)), x)`

[Out]  $-(2*b*(e*\cot(c + d*x))^{1/2})/d - ((-1)^{1/4}*b*e^{1/2}*\operatorname{atan}((( -1)^{1/4}*(e*\cot(c + d*x))^{1/2})/e^{1/2})*i)/d - ((-1)^{1/4}*b*e^{1/2}*\operatorname{atanh}((( -1)^{1/4}*(e*\cot(c + d*x))^{1/2})/e^{1/2})*i)/d - ((-1)^{1/4}*a*e^{1/2}*(\operatorname{atan}((( -1)^{1/4}*(e*\cot(c + d*x))^{1/2})/e^{1/2}) - \operatorname{atanh}((( -1)^{1/4}*(e*\cot(c + d*x))^{1/2})/e^{1/2}))/d$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c)), x)`

[Out] `Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x)), x)`

$$3.53 \quad \int \frac{a+b \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx$$

**Optimal.** Leaf size=208

$$\frac{(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}}$$

[Out]  $1/2*(a+b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d*2^{(1/2)}/e^{(1/2)} - 1/2*(a+b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d*2^{(1/2)}/e^{(1/2)} + 1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d*2^{(1/2)}/e^{(1/2)} - 1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d*2^{(1/2)}/e^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])/Sqrt[e\*Cot[c + d\*x]],x]

[Out]  $((a+b)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/\text{Sqrt}[2]*d*\text{Sqrt}[e] - ((a+b)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/\text{Sqrt}[2]*d*\text{Sqrt}[e] + ((a-b)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*\text{Sqrt}[e]) - ((a-b)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*\text{Sqrt}[e])$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 3534

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{-ae - bx^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{(a - b) \operatorname{Subst}\left(\int \frac{e - x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} - \frac{(a + b) \operatorname{Subst}\left(\int \frac{e + x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{(a + b) \operatorname{Subst}\left(\int \frac{1}{e - \sqrt{2} \sqrt{e} x + x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2d} - \frac{(a + b) \operatorname{Subst}\left(\int \frac{1}{e + \sqrt{2} \sqrt{e} x + x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2d} \\
&= \frac{(a - b) \log\left(\sqrt{e} + \sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} d \sqrt{e}} - \frac{(a - b) \log\left(\sqrt{e} + \sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} d \sqrt{e}} \\
&= \frac{(a + b) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} d \sqrt{e}} - \frac{(a + b) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} d \sqrt{e}} + \frac{(a - b) \log\left(\sqrt{e} + \sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{\sqrt{2} d \sqrt{e}}
\end{aligned}$$

**Mathematica [C]** time = 0.22, size = 166, normalized size = 0.80

$$\frac{8a \tan^{\frac{3}{2}}(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\tan^2(c + dx)\right) + 3\sqrt{2}b\left(-2 \tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(c + dx)}\right) + 2 \tan^{-1}\left(\sqrt{2} \sqrt{\tan(c + dx)}\right)\right)}{12d \sqrt{\tan(c + dx)} \sqrt{e \cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])/Sqrt[e\*Cot[c + d\*x]], x]

[Out] (3\*Sqrt[2]\*b\*(-2\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]])] + 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]) - Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] + Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + 8\*a\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(3/2)/(12\*d\*Sqrt[e\*Cot[c + d\*x]]\*Sqrt[Tan[c + d\*x]])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) + a}{\sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)/sqrt(e\*cot(d\*x + c)), x)

**maple** [B] time = 0.41, size = 327, normalized size = 1.57

$$\frac{a \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + \left( e^2 \right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - \left( e^2 \right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{4de} - \frac{a \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{\left( e^2 \right)^{\frac{1}{4}}} + 1 \right)}{2de} + \frac{a \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \arctan \left( \dots \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2),x)

[Out]  $-1/4*a/d/e*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/2*a/d/e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2*a/d/e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/4/d*b/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/2/d*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2/d*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)$

**maxima** [A] time = 0.75, size = 180, normalized size = 0.87

$$\frac{2 \sqrt{2} (a+b) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e+2} \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{2 \sqrt{2} (a+b) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{e-2} \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{\sqrt{2} (a-b) \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)} \right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out]  $-1/4*(2*\sqrt{2}*(a + b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e}))/\sqrt{e} + 2*\sqrt{2}*(a + b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*$

) $\sqrt{e} - 2\sqrt{e/\tan(dx + c)}/\sqrt{e})/\sqrt{e} + \sqrt{2}(a - b)\log(\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx + c)} + e + e/\tan(dx + c))/\sqrt{e} - \sqrt{2}(a - b)\log(-\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx + c)} + e + e/\tan(dx + c))/\sqrt{e})/d$

**mupad** [B] time = 0.65, size = 118, normalized size = 0.57

$$\frac{(-1)^{1/4} b \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} - \frac{(-1)^{1/4} b \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} + \frac{(-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) \operatorname{li}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{d \sqrt{e}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(1/2), x)`

[Out]  $((-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} (e \cot(c + dx))^{1/2}}{e^{1/2}}\right) \operatorname{li}\left(\frac{(-1)^{1/4} (e \cot(c + dx))^{1/2}}{e^{1/2}}\right) / (d e^{1/2}) + ((-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} (e \cot(c + dx))^{1/2}}{e^{1/2}}\right) \operatorname{li}\left(\frac{(-1)^{1/4} (e \cot(c + dx))^{1/2}}{e^{1/2}}\right) / (d e^{1/2}) - ((-1)^{1/4} b \operatorname{atan}\left(\frac{(-1)^{1/4} (e \cot(c + dx))^{1/2}}{e^{1/2}}\right) / (d e^{1/2}) + ((-1)^{1/4} b \operatorname{atanh}\left(\frac{(-1)^{1/4} (e \cot(c + dx))^{1/2}}{e^{1/2}}\right) / (d e^{1/2})) / (d e^{1/2})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(1/2), x)`

[Out] `Integral((a + b*cot(c + d*x))/sqrt(e*cot(c + d*x)), x)`

$$3.54 \quad \int \frac{a+b \cot(c+dx)}{(e \cot(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=229

$$\frac{(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}} - \frac{(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}}$$

[Out]  $-1/2*(a-b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(3/2)*2^{(1/2)}} + 1/2*(a-b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(3/2)*2^{(1/2)}} + 1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(3/2)*2^{(1/2)}} - 1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(3/2)*2^{(1/2)}} + 2*a/d/e/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {3529, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}} - \frac{(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2), x]`

[Out]  $-(((a-b)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(3/2)})) + ((a-b)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(3/2)}) + (2*a)/(d*e*\text{Sqrt}[e*\text{Cot}[c + d*x]]) + ((a+b)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(3/2)}) - ((a+b)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(3/2)})$

#### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

#### Rule 617

`Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`



Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 3529

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx &= \frac{2a}{de\sqrt{e \cot(c + dx)}} + \frac{\int \frac{be - ae \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{e^2} \\
&= \frac{2a}{de\sqrt{e \cot(c + dx)}} + \frac{2 \operatorname{Subst}\left(\int \frac{-be^2 + aex^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de^2} \\
&= \frac{2a}{de\sqrt{e \cot(c + dx)}} + \frac{(a - b) \operatorname{Subst}\left(\int \frac{e + x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de} - \frac{(a + b) \operatorname{Subst}\left(\int \frac{e - x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de} \\
&= \frac{2a}{de\sqrt{e \cot(c + dx)}} + \frac{(a + b) \operatorname{Subst}\left(\int \frac{\sqrt{2} \sqrt{e} + 2x}{-e - \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} de^{3/2}} + \frac{(a + b) \operatorname{Subst}\left(\int \frac{\sqrt{2} \sqrt{e} - 2x}{-e + \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} de^{3/2}} \\
&= \frac{2a}{de\sqrt{e \cot(c + dx)}} + \frac{(a + b) \log\left(\sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} de^{3/2}} - \frac{(a + b) \log\left(\sqrt{e} - \sqrt{e \cot(c + dx)} + \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} de^{3/2}} \\
&= -\frac{(a - b) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{3/2}} + \frac{(a - b) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{3/2}} + \frac{2a}{de\sqrt{e \cot(c + dx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.36, size = 196, normalized size = 0.86

$$\frac{3a \left( 2 \left( \sqrt{2} \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} \right) - \sqrt{2} \tan^{-1} \left( \sqrt{2} \sqrt{\tan(c + dx)} + 1 \right) \right) + 8 \sqrt{\tan(c + dx)} + \sqrt{2} \log \left( \tan(c + dx) - \sqrt{2} \sqrt{\tan(c + dx)} + 1 \right) - \sqrt{2} \log \left( \tan(c + dx) + \sqrt{2} \sqrt{\tan(c + dx)} + 1 \right) \right)}{12d \tan^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])/(e\*Cot[c + d\*x])^(3/2), x]

[Out] (3\*a\*(2\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]]) + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + 8\*Sqrt[Tan[c + d\*x]]) + 8\*b\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2]\*Tan[c + d\*x]^(3/2))/(12\*d\*(e\*Cot[c + d\*x])^(3/2)\*Tan[c + d\*x]^(3/2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx+c) + a}{(e \cot(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)/(e\*cot(d\*x + c))^(3/2), x)

**maple** [A] time = 0.37, size = 355, normalized size = 1.55

$$\frac{b(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d e^2} + \frac{b(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{2d e^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2),x)

[Out] 
$$-1/4/d/e^2*b*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2))})-1/2/d/e^2*b*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/2/d/e^2*b*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/4*a/d/e^2*(1/2)/(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2))})+1/2*a/d/e^2*(1/2)/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/2*a/d/e^2*(1/2)/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+2*a/d/e/(e*\cot(d*x+c))^{(1/2)}$$

**maxima** [A] time = 0.69, size = 204, normalized size = 0.89

$$e \left( \frac{2 \sqrt{2} (a-b) \arctan\left(\frac{\sqrt{2} (\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2} (a-b) \arctan\left(-\frac{\sqrt{2} (\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2} (a+b) \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} + \frac{\sqrt{2} (a+b) \log\left(\sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} - e - \frac{e}{\tan(dx+c)}\right)}{\sqrt{e}} \right) / e^2$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2),x, algorithm="maxima")

[Out]  $\frac{1}{4}e^{1/2} \left( \frac{2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{e}\sqrt{\cot(d*x+c)}\right)}{\sqrt{e}} + 2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\sqrt{e}\sqrt{\cot(d*x+c)}\right)}{\sqrt{e}} - \sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{e}\sqrt{\cot(d*x+c)}\right) + e + \frac{e}{\tan(d*x+c)}\right) + \sqrt{2}(a+b)\log\left(-\sqrt{2}\sqrt{e}\sqrt{\cot(d*x+c)}\right) + e + \frac{e}{\tan(d*x+c)}\right) / \sqrt{e} + \frac{8a}{e^{3/2}\sqrt{e}\sqrt{\cot(d*x+c)}} / d$

mupad [B] time = 0.80, size = 137, normalized size = 0.60

$$\frac{2a}{de\sqrt{e\cot(c+dx)}} + \frac{(-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{(-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{(-1)^{1/4} b \operatorname{atan}\left(\frac{(-1)^{1/4}}{\sqrt{e}}\right)}{de^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))/(e\*cot(c + d\*x))^(3/2),x)

[Out]  $(2a)/(d e^{3/2} \sqrt{e \cot(c+dx)}) + ((-1)^{1/4} a \operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right) - (-1)^{1/4} a \operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)) / (d e^{3/2}) + ((-1)^{1/4} b \operatorname{atan}\left(\frac{(-1)^{1/4}}{\sqrt{e}}\right) + (-1)^{1/4} b \operatorname{atanh}\left(\frac{(-1)^{1/4}}{\sqrt{e}}\right)) / (d e^{3/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(3/2),x)

[Out] Integral((a + b\*cot(c + d\*x))/(e\*cot(c + d\*x))^(3/2), x)

$$3.55 \quad \int \frac{a+b \cot(c+dx)}{(e \cot(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=252

$$\frac{(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}} + \frac{(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}}$$

[Out]  $2/3*a/d/e/(e*\cot(d*x+c))^{(3/2)}-1/2*(a+b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+1/2*(a+b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}-1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d/e^{(5/2)}*2^{(1/2)}+1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d/e^{(5/2)}*2^{(1/2)}+2*b/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.26, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {3529, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}} + \frac{(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])/(e\*Cot[c + d\*x])^(5/2), x]

[Out]  $-(((a+b)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(5/2)})) + ((a+b)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(5/2)}) + (2*a)/(3*d*e*(e*\text{Cot}[c+d*x])^{(3/2)}) + (2*b)/(d*e^2*\text{Sqrt}[e*\text{Cot}[c+d*x]]) - ((a-b)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c+d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(2*\text{Sqrt}[2]*d*e^{(5/2)}) + ((a-b)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c+d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(2*\text{Sqrt}[2]*d*e^{(5/2)})$

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \ \text{Dist}[(d*q + a*e)/(2*a*c), \ \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \ \text{Dist}[(d*q - a*e)/(2*a*c), \ \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

### Rule 3529

$\text{Int}[\frac{((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}{(f_.)*(x_.)}, x\_Symbol] \ :> \ \text{Simp}[\frac{(b*c - a*d)*(a + b*\text{Tan}[e + f*x])^{(m + 1)}}{(f*(m + 1)*(a^2 + b^2)}, x] + \ \text{Dist}[1/(a^2 + b^2), \ \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

### Rule 3534

$\text{Int}[\frac{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}], x\_Symbol] \ :> \ \text{Dist}[2/f, \ \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \ \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{a + b \cot(c + dx)}{(e \cot(c + dx))^{5/2}} dx &= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{\int \frac{be - ae \cot(c+dx)}{(e \cot(c+dx))^{3/2}} dx}{e^2} \\
&= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2b}{de^2 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{-ae^2 - be^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{e^4} \\
&= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2b}{de^2 \sqrt{e \cot(c + dx)}} + \frac{2 \text{Subst} \left( \int \frac{ae^3 + be^2 x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{de^4} \\
&= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2b}{de^2 \sqrt{e \cot(c + dx)}} + \frac{(a - b) \text{Subst} \left( \int \frac{e - x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{de^2} \\
&= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2b}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(a - b) \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{e} + 2x}{-e - \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)} \right)}{2\sqrt{2} de^{5/2}} \\
&= \frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2b}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(a - b) \log \left( \sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2} \sqrt{e \cot(c + dx)} \right)}{2\sqrt{2} de^{5/2}} \\
&= -\frac{(a + b) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{5/2}} + \frac{(a + b) \tan^{-1} \left( 1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{5/2}} + \frac{2a}{3de(e \cot(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica** [C] time = 0.76, size = 196, normalized size = 0.78

$$3b \left( 2\sqrt{2} \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(c + dx)} \right) - 2\sqrt{2} \tan^{-1} \left( \sqrt{2} \sqrt{\tan(c + dx)} + 1 \right) + 8\sqrt{\tan(c + dx)} + \sqrt{2} \log \left( \tan(c + dx) \right) \right)$$

12d tan

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])/(e\*Cot[c + d\*x])^(5/2), x]

[Out] (3\*b\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Tan[c + d\*x]] + Tan[c + d\*x]]) + 8\*Sqrt[Tan[c + d\*x]] - 8\*a\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d\*x]^2])\*Tan[c + d\*x]^(3/2))/(12\*d\*(e\*Cot[c + d\*x])^(5/2)\*Tan[c + d\*x]^(5/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) + a}{(e \cot(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)/(e\*cot(d\*x + c))^(5/2), x)

**maple** [A] time = 0.48, size = 374, normalized size = 1.48

$$\frac{a(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d e^3} + \frac{a(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{2d e^3} - \frac{a(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} - 1\right)}{2d e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(5/2),x)

[Out] 1/4/d/e^3\*a\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/2/d/e^3\*a\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/2/d/e^3\*a\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+1/4/d/e^2\*b/(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)-(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2)))+1/2/d/e^2\*b/(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)-1/2/d/e^2\*b/(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)+2/3\*a/d/e/(e\*cot(d\*x+c))^(3/2)+2\*b/d/e^2/(e\*cot(d\*x+c))^(1/2)



**maxima [A]** time = 0.70, size = 220, normalized size = 0.87

$$e \left( \frac{2 \sqrt{2} (a+b) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{2 \sqrt{2} (a+b) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{\sqrt{2} (a-b) \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e + \frac{e}{\tan(dx+c)} \right)}{\sqrt{e}} - \frac{\sqrt{2} (a-b)}{\sqrt{e}} \right) \frac{1}{e^3} \frac{1}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))/(e\*cot(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/12\*e\*(3\*(2\*sqrt(2)\*(a + b)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a + b)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + sqrt(2)\*(a - b)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*(a - b)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/e^3 + 8\*(a\*e + 3\*b\*e/tan(d\*x + c))/(e^3\*(e/tan(d\*x + c))^(3/2))/d

**mupad [B]** time = 1.24, size = 158, normalized size = 0.63

$$\frac{2a}{3de(e \cot(c + dx))^{3/2}} + \frac{2b}{de^2 \sqrt{e \cot(c + dx)}} + \frac{(-1)^{1/4} b \operatorname{atan} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{de^{5/2}} - \frac{(-1)^{1/4} b \operatorname{atanh} \left( \frac{(-1)^{1/4} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{de^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))/(e\*cot(c + d\*x))^(5/2),x)

[Out] (2\*a)/(3\*d\*e\*(e\*cot(c + d\*x))^(3/2)) + (2\*b)/(d\*e^2\*(e\*cot(c + d\*x))^(1/2)) - ((-1)^(1/4)\*a\*atan((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*1i)/(d\*e^(5/2)) - ((-1)^(1/4)\*a\*atanh((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2))\*1i)/(d\*e^(5/2)) + ((-1)^(1/4)\*b\*atan((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/(d\*e^(5/2)) - ((-1)^(1/4)\*b\*atanh((-1)^(1/4)\*(e\*cot(c + d\*x))^(1/2))/e^(1/2)))/(d\*e^(5/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \cot(c + dx)}{(e \cot(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(5/2), x)
```

### 3.56 $\int (e \cot(c + dx))^{3/2} (a + b \cot(c + dx))^2 dx$

**Optimal.** Leaf size=317

$$\frac{e^{3/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{e^{3/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx))}{2\sqrt{2}d}$$

[Out]  $-4/3*a*b*(e*\cot(d*x+c))^{(3/2)}/d-2/5*b^2*(e*\cot(d*x+c))^{(5/2)}/d/e-1/2*(a^2+2*a*b-b^2)*e^{(3/2)}*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d*2^{(1/2)}+1/2*(a^2+2*a*b-b^2)*e^{(3/2)}*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d*2^{(1/2)}-1/4*(a^2-2*a*b-b^2)*e^{(3/2)}*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d*2^{(1/2)}+1/4*(a^2-2*a*b-b^2)*e^{(3/2)}*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d*2^{(1/2)}-2*(a^2-b^2)*e*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3543, 3528, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{e^{3/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{e^{3/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx))}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*\text{Cot}[c + d*x])^{(3/2)}*(a + b*\text{Cot}[c + d*x])^2, x]$

[Out]  $-(((a^2 + 2*a*b - b^2)*e^{(3/2)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d) + ((a^2 + 2*a*b - b^2)*e^{(3/2)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d) - (2*(a^2 - b^2)*e*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (4*a*b*(e*\text{Cot}[c + d*x])^{(3/2)})/(3*d) - (2*b^2*(e*\text{Cot}[c + d*x])^{(5/2)})/(5*d*e) - ((a^2 - 2*a*b - b^2)*e^{(3/2)}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*d) + ((a^2 - 2*a*b - b^2)*e^{(3/2)}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*d)$

#### Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b]$

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

### Rule 3534

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

NeQ[c^2 + d^2, 0]

### Rule 3543

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] :> Simp[(d^2\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[c^2 - d^2 + 2\*c\*d\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])

### Rubi steps

$$\begin{aligned}
 \int (e \cot(c + dx))^{3/2} (a + b \cot(c + dx))^2 dx &= -\frac{2b^2(e \cot(c + dx))^{5/2}}{5de} + \int (e \cot(c + dx))^{3/2} (a^2 - b^2 + 2ab \cot(c + dx)) dx \\
 &= -\frac{4ab(e \cot(c + dx))^{3/2}}{3d} - \frac{2b^2(e \cot(c + dx))^{5/2}}{5de} + \int \sqrt{e \cot(c + dx)} (a^2 - b^2 + 2ab \cot(c + dx)) dx \\
 &= -\frac{2(a^2 - b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{4ab(e \cot(c + dx))^{3/2}}{3d} - \frac{2b^2(e \cot(c + dx))^{5/2}}{5de} \\
 &= -\frac{2(a^2 - b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{4ab(e \cot(c + dx))^{3/2}}{3d} - \frac{2b^2(e \cot(c + dx))^{5/2}}{5de} \\
 &= -\frac{2(a^2 - b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{4ab(e \cot(c + dx))^{3/2}}{3d} - \frac{2b^2(e \cot(c + dx))^{5/2}}{5de} \\
 &= -\frac{2(a^2 - b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{4ab(e \cot(c + dx))^{3/2}}{3d} - \frac{2b^2(e \cot(c + dx))^{5/2}}{5de} \\
 &= -\frac{2(a^2 - b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{4ab(e \cot(c + dx))^{3/2}}{3d} - \frac{2b^2(e \cot(c + dx))^{5/2}}{5de} \\
 &= -\frac{(a^2 + 2ab - b^2)e^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a^2 + 2ab - b^2)e^{3/2}}{\sqrt{2}d}
 \end{aligned}$$

**Mathematica [C]** time = 1.98, size = 224, normalized size = 0.71

---


$$(e \cot(c + dx))^{3/2} \left( \frac{1}{4} (a^2 - b^2) (8\sqrt{\cot(c + dx)} + \sqrt{2} \log(\cot(c + dx) - \sqrt{2}\sqrt{\cot(c + dx)} + 1)) - \sqrt{2} \log(\cot(c + dx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])^2,x]

[Out] -(((e\*Cot[c + d\*x])^(3/2)\*((2\*b^2\*Cot[c + d\*x]^(5/2))/5 - (4\*a\*b\*Cot[c + d\*x]^(3/2)\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2])))/3 + ((a^2 - b^2)\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 8\*Sqrt[Cot[c + d\*x]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/4))/(d\*Cot[c + d\*x]^(3/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^2 (e \cot(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^2\*(e\*cot(d\*x + c))^(3/2), x)

**maple** [B] time = 0.65, size = 581, normalized size = 1.83

$$\frac{2b^2 (e \cot(dx + c))^{\frac{5}{2}}}{5de} - \frac{4ab (e \cot(dx + c))^{\frac{3}{2}}}{3d} - \frac{2e a^2 \sqrt{e \cot(dx + c)}}{d} + \frac{2e b^2 \sqrt{e \cot(dx + c)}}{d} - \frac{e (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{y}{x}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c))^2,x)

[Out] -2/5\*b^2\*(e\*cot(d\*x+c))^(5/2)/d/e-4/3\*a\*b\*(e\*cot(d\*x+c))^(3/2)/d-2\*e/d\*a^2\*(e\*cot(d\*x+c))^(1/2)+2\*e/d\*b^2\*(e\*cot(d\*x+c))^(1/2)-1/2\*e/d\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)\*a^2+1/2\*e/d\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)\*b^2+1/4\*e

$$\begin{aligned} & /d*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+ \\ & (e^2)^{(1/2)))/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+( \\ & e^2)^{(1/2)))*a^2-1/4*e/d*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*( \\ & e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d \\ & *x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))*b^2+1/2*e/d*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2 \\ & ^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^2-1/2*e/d*(e^2)^{(1/4)}*2^{(1/2)}* \\ & \arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*b^2+1/2*e^2/d*a*b/(e^2)^{(1/4)}*2^{(1/2)} \\ & *\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))+e^2/d*a*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-e^2/d*a*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1) \end{aligned}$$

**maxima [A]** time = 0.58, size = 287, normalized size = 0.91

$$15 \left( \frac{2\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2+2ab-b^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2}(a^2-2ab-b^2)\log\left(\sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/60\*(15\*(2\*sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) - sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))\*e - 8\*(10\*a\*b\*e\*(e/tan(d\*x + c))^(3/2) + 3\*b^2\*(e/tan(d\*x + c))^(5/2) + 15\*(a^2 - b^2)\*e^2\*sqrt(e/tan(d\*x + c)))/e^2)\*e/d

**mupad [B]** time = 2.47, size = 1274, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(3/2)\*(a + b\*cot(c + d\*x))^2,x)

[Out] atan((a^4\*e^6\*(e\*cot(c + d\*x))^(1/2)\*((a\*b^3\*e^3)/d^2 - (b^4\*e^3\*1i)/(4\*d^2) - (a^4\*e^3\*1i)/(4\*d^2) - (a^3\*b\*e^3)/d^2 + (a^2\*b^2\*e^3\*3i)/(2\*d^2))^(1/2)\*32i)/((a^6\*e^8\*16i)/d - (b^6\*e^8\*16i)/d + (32\*a\*b^5\*e^8)/d + (32\*a^5\*b\*e^8

$$\begin{aligned}
& 8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d + \\
& (b^4*e^6*(e*\cot(c + d*x))^{(1/2)}*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2) - \\
& (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^{(1/2)}*32 \\
& i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d \\
& + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d - (a^ \\
& 2*b^2*e^6*(e*\cot(c + d*x))^{(1/2)}*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^2) - \\
& (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^{(1/2)}*19 \\
& 2i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b*e^8)/ \\
& d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/d)*(- \\
& (a^4*e^3*1i + b^4*e^3*1i - 4*a*b^3*e^3 + 4*a^3*b*e^3 - a^2*b^2*e^3*6i)/(4*d^ \\
& 2))^{(1/2)}*2i + \operatorname{atan}((a^4*e^6*(e*\cot(c + d*x))^{(1/2)}*((a^4*e^3*1i)/(4*d^2) + \\
& (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i) \\
& )/(2*d^2))^{(1/2)}*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d \\
& + (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2 \\
& *e^8*112i)/d) + (b^4*e^6*(e*\cot(c + d*x))^{(1/2)}*((a^4*e^3*1i)/(4*d^2) + (b^ \\
& 4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2 \\
& *d^2))^{(1/2)}*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d + ( \\
& 32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^8 \\
& *112i)/d) - (a^2*b^2*e^6*(e*\cot(c + d*x))^{(1/2)}*((a^4*e^3*1i)/(4*d^2) + (b^ \\
& 4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 - (a^2*b^2*e^3*3i)/(2 \\
& *d^2))^{(1/2)}*192i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d + (32*a*b^5*e^8)/d + \\
& (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d + (a^4*b^2*e^ \\
& 8*112i)/d)*((a^4*e^3*1i + b^4*e^3*1i + 4*a*b^3*e^3 - 4*a^3*b*e^3 - a^2*b^2 \\
& *e^3*6i)/(4*d^2))^{(1/2)}*2i - (e*\cot(c + d*x))^{(1/2)}*((2*a^2*e)/d - (2*b^2*e \\
& )/d - (2*b^2*(e*\cot(c + d*x))^{(5/2)})/(5*d*e) - (4*a*b*(e*\cot(c + d*x))^{(3/ \\
& 2)})/(3*d)
\end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e \cot(c + dx))^{\frac{3}{2}} (a + b \cot(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**2,x)`

[Out] `Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2, x)`



### 3.57 $\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2 dx$

**Optimal.** Leaf size=288

$$\frac{\sqrt{e} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{\sqrt{e} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx))}{2\sqrt{2}d}$$

[Out]  $-2/3*b^2*(e*\cot(d*x+c))^(3/2)/d/e+1/2*(a^2-2*a*b-b^2)*\arctan(1-2^(1/2)*(e*\cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*\arctan(1+2^(1/2)*(e*\cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*\ln(e^(1/2)+\cot(d*x+c)*e^(1/2)-2^(1/2)*(e*\cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/4*(a^2+2*a*b-b^2)*\ln(e^(1/2)+\cot(d*x+c)*e^(1/2)+2^(1/2)*(e*\cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-4*a*b*(e*\cot(d*x+c))^(1/2)/d$

**Rubi [A]** time = 0.27, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3543, 3528, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{e} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{\sqrt{e} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx))}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^2,x]

[Out]  $((a^2 - 2*a*b - b^2)*\text{Sqrt}[e]*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/\text{Sqrt}[e])/(\text{Sqrt}[2]*d) - ((a^2 - 2*a*b - b^2)*\text{Sqrt}[e]*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/\text{Sqrt}[e])/(\text{Sqrt}[2]*d) - (4*a*b*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (2*b^2*(e*\text{Cot}[c + d*x])^(3/2))/(3*d*e) - ((a^2 + 2*a*b - b^2)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*d) + ((a^2 + 2*a*b - b^2)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*d)$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

### Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

### Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

### Rule 3528

$\text{Int}[\frac{((a_.) + (b_.)\tan[(e_.) + (f_.)x])^{(m_.)}((c_.) + (d_.)\tan[(e_.) + (f_.)x])}{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}, x\_Symbol] \rightarrow \text{Simp}[\frac{d(a + b \tan[e + fx])^m}{f^m}, x] + \text{Int}[(a + b \tan[e + fx])^{(m-1)} \text{Simp}[ac - bd + (bc + ad)\tan[e + fx], x], x] \ /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[bc - ad, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3534

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\sqrt{(b_.)\tan[(e_.) + (f_.)x]}}, x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(bc + dx^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \tan[e + fx]]], x] \ /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] := Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2 dx &= -\frac{2b^2(e \cot(c + dx))^{3/2}}{3de} + \int \sqrt{e \cot(c + dx)} (a^2 - b^2 + 2ab \cot(c + dx)) dx \\
&= -\frac{4ab\sqrt{e \cot(c + dx)}}{d} - \frac{2b^2(e \cot(c + dx))^{3/2}}{3de} + \int \frac{-2abe + (a^2 - b^2)\sqrt{e \cot(c + dx)}}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{4ab\sqrt{e \cot(c + dx)}}{d} - \frac{2b^2(e \cot(c + dx))^{3/2}}{3de} + \frac{2 \operatorname{Subst}\left(\int \frac{2abe^2 - (a^2 - b^2)\sqrt{e \cot(c + dx)}}{e^2 + x^4} dx\right)}{\sqrt{e \cot(c + dx)}} \\
&= -\frac{4ab\sqrt{e \cot(c + dx)}}{d} - \frac{2b^2(e \cot(c + dx))^{3/2}}{3de} - \frac{((a^2 - 2ab - b^2)e)\sqrt{e \cot(c + dx)}}{\sqrt{e \cot(c + dx)}} \\
&= -\frac{4ab\sqrt{e \cot(c + dx)}}{d} - \frac{2b^2(e \cot(c + dx))^{3/2}}{3de} - \frac{((a^2 + 2ab - b^2)\sqrt{e})\sqrt{e \cot(c + dx)}}{\sqrt{e \cot(c + dx)}} \\
&= -\frac{4ab\sqrt{e \cot(c + dx)}}{d} - \frac{2b^2(e \cot(c + dx))^{3/2}}{3de} - \frac{(a^2 + 2ab - b^2)\sqrt{e} \log(\cot(c + dx))}{\sqrt{e \cot(c + dx)}} \\
&= \frac{(a^2 - 2ab - b^2)\sqrt{e} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a^2 - 2ab - b^2)\sqrt{e} \log(\cot(c + dx))}{\sqrt{e \cot(c + dx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.57, size = 220, normalized size = 0.76

$$\frac{\sqrt{e \cot(c + dx)} \left( 4(a^2 - b^2) \cot^{\frac{3}{2}}(c + dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right) + b \left( 24a\sqrt{\cot(c + dx)} + 3\sqrt{2}a \log(\cot(c + dx)) \right) \right)}{\sqrt{e \cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^2,x]

[Out] -1/6\*(Sqrt[e\*Cot[c + d\*x]]\*(4\*(a^2 - b^2)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] + b\*(6\*Sqrt[2]\*a\*ArcTan[1 - Sqrt[2]\*Sqrt[

$\text{Cot}[c + d*x]] - 6*\text{Sqrt}[2]*a*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]]] + 24*a*\text{Sqrt}[\text{Cot}[c + d*x]] + 4*b*\text{Cot}[c + d*x]^{(3/2)} + 3*\text{Sqrt}[2]*a*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]] - 3*\text{Sqrt}[2]*a*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[c + d*x]] + \text{Cot}[c + d*x]])))/(d*\text{Sqrt}[\text{Cot}[c + d*x]])$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^2 \sqrt{e \cot(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^2\*sqrt(e\*cot(d\*x + c)), x)

**maple** [B] time = 0.54, size = 534, normalized size = 1.85

$$\frac{2b^2 (e \cot(dx + c))^{\frac{3}{2}}}{3de} - \frac{4ab\sqrt{e \cot(dx + c)}}{d} + \frac{ab(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{2d} + \frac{ab(e^2)^{\frac{1}{4}}\sqrt{2} \arcsin\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c))^2,x)

[Out]  $-2/3*b^2*(e*\cot(d*x+c))^{(3/2)}/d/e-4*a*b*(e*\cot(d*x+c))^{(1/2)}/d+1/2/d*a*b*(e^{(2)}^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))/(e*\cot(d*x+c)-(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))$   
 $+1/d*a*b*(e^{(2)}^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/d*a*b*(e^{(2)}^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/4*e/d*2^{(1/2)}/(e^{(2)}^{(1/4)}*\ln((e*\cot(d*x+c)-(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))/(e*\cot(d*x+c)+(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))$   
 $*a^2+1/4*e/d*2^{(1/2)}/(e^{(2)}^{(1/4)}*\ln((e*\cot(d*x+c)-(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))/(e*\cot(d*x+c)+(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))$   
 $*b^2+1/2*e/d*2^{(1/2)}/(e^{(2)}^{(1/4)}*\ln((e*\cot(d*x+c)-(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))/(e*\cot(d*x+c)+(e^{(2)}^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^{(2)}^{(1/2)})))$

$$\frac{1}{(e^2)^{1/4}} \arctan(-2^{1/2}/(e^2)^{1/4} * (e \cot(dx+c))^{1/2} + 1) * a^{2-1/2} * e/d * 2^{1/2} / (e^2)^{1/4} \arctan(-2^{1/2}/(e^2)^{1/4} * (e \cot(dx+c))^{1/2} + 1) * b^{2-1/2} * e/d * 2^{1/2} / (e^2)^{1/4} \arctan(2^{1/2}/(e^2)^{1/4} * (e \cot(dx+c))^{1/2} + 1) * a^{2+1/2} * e/d * 2^{1/2} / (e^2)^{1/4} \arctan(2^{1/2}/(e^2)^{1/4} * (e \cot(dx+c))^{1/2} + 1) * b^2$$

**maxima [A]** time = 0.45, size = 257, normalized size = 0.89

$$\frac{\left( \frac{6 \sqrt{2} (a^2 - 2ab - b^2) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} + \frac{6 \sqrt{2} (a^2 - 2ab - b^2) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} - \frac{3 \sqrt{2} (a^2 + 2ab - b^2) \log\left(\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}} \right)}{12d}$$

12d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)\*(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $-1/12 * (6 * \sqrt{2} * (a^2 - 2 * a * b - b^2) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e / \tan(dx + c)}) / \sqrt{e})) / \sqrt{e} + 6 * \sqrt{2} * (a^2 - 2 * a * b - b^2) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e / \tan(dx + c)}) / \sqrt{e})) / \sqrt{e} - 3 * \sqrt{2} * (a^2 + 2 * a * b - b^2) * \log(\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx + c)} + e + e / \tan(dx + c)) / \sqrt{e} + 3 * \sqrt{2} * (a^2 + 2 * a * b - b^2) * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx + c)} + e + e / \tan(dx + c)) / \sqrt{e} + 8 * (6 * a * b * e * \sqrt{e / \tan(dx + c)} + b^2 * (e / \tan(dx + c))^{3/2}) / e^2 * e / d$

**mupad [B]** time = 1.21, size = 1157, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(1/2)\*(a + b\*cot(c + d\*x))^2,x)

[Out]  $\operatorname{atan}\left(\frac{a^4 * e^4 * (e \cot(c + d * x))^{1/2} * ((a^4 * e * 1i) / (4 * d^2) + (b^4 * e * 1i) / (4 * d^2) - (a^2 * b^2 * e * 3i) / (2 * d^2) - (a * b^3 * e) / d^2 + (a^3 * b * e) / d^2)^{1/2} * 32i}{((16 * b^6 * e^5) / d - (16 * a^6 * e^5) / d + (a * b^5 * e^5 * 32i) / d + (a^5 * b * e^5 * 32i) / d - (112 * a^2 * b^4 * e^5) / d - (a^3 * b^3 * e^5 * 192i) / d + (112 * a^4 * b^2 * e^5) / d) + (b^4 * e^4 * (e \cot(c + d * x))^{1/2} * ((a^4 * e * 1i) / (4 * d^2) + (b^4 * e * 1i) / (4 * d^2) - (a^2 * b^2 * e * 3i) / (2 * d^2) - (a * b^3 * e) / d^2 + (a^3 * b * e) / d^2)^{1/2} * 32i}{((16 * b^6 * e^5) / d - (16 * a^6 * e^5) / d + (a * b^5 * e^5 * 32i) / d + (a^5 * b * e^5 * 32i) / d - (112 * a^2 * b^4 * e^5) / d - (a^3 * b^3 * e^5 * 192i) / d + (112 * a^4 * b^2 * e^5) / d) - (a^2 * b^2 * e^4 * (e \cot(c + d * x))^{1/2} * ((a^4 * e * 1i) / (4 * d^2) + (b^4 * e * 1i) / (4 * d^2) - (a^2 * b^2 * e * 3i) / (2 * d^2) - (a * b^3 * e) / d^2 + (a^3 * b * e) / d^2)^{1/2} * 192i}{((16 * b^6 * e^5) / d - (16 * a^6 * e^5) / d + (a * b^5 * e^5 * 32i) / d + (a^5 * b * e^5 * 32i) / d - (112 * a^2 * b^4 * e^5) / d - (a^3 * b^3 * e^5 * 192i) / d + (112 * a^4 * b^2 * e^5) / d) - (a^3 * b^3 * e^5 * 192i) / d + (112 * a^4 * b^2 * e^5) / d} \right)$

```

^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d))*((a^4*e*1i + b^4*e*1i - a^2*b^2*e*6i
- 4*a*b^3*e + 4*a^3*b*e)/(4*d^2))^(1/2)*2i - atan((a^4*e^4*(e*cot(c + d*x)
)^(1/2)*((a^2*b^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) -
(a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d
+ (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e
^5*192i)/d - (112*a^4*b^2*e^5)/d) + (b^4*e^4*(e*cot(c + d*x))^(1/2)*((a^2*b
^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) - (a*b^3*e)/d^2
+ (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d + (a*b^5*e^5*3
2i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d - (1
12*a^4*b^2*e^5)/d) - (a^2*b^2*e^4*(e*cot(c + d*x))^(1/2)*((a^2*b^2*e*3i)/(2
*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)
/d^2)^(1/2)*192i)/((16*a^6*e^5)/d - (16*b^6*e^5)/d + (a*b^5*e^5*32i)/d + (a
^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d - (112*a^4*b^2
*e^5)/d))*(-(a^4*e*1i + b^4*e*1i - a^2*b^2*e*6i + 4*a*b^3*e - 4*a^3*b*e)/(4
*d^2))^(1/2)*2i - (2*b^2*(e*cot(c + d*x))^(3/2))/(3*d*e) - (4*a*b*(e*cot(c
+ d*x))^(1/2))/d

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)\*(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Integral(sqrt(e\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))\*\*2, x)

$$3.58 \quad \int \frac{(a+b \cot(c+dx))^2}{\sqrt{e \cot(c+dx)}} dx$$

Optimal. Leaf size=267

$$\frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e}} - \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} d \sqrt{e}}$$

[Out]  $1/2*(a^2+2*a*b-b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d*2^{(1/2)}/e^{(1/2)}-1/2*(a^2+2*a*b-b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d*2^{(1/2)}/e^{(1/2)}+1/4*(a^2-2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d*2^{(1/2)}/e^{(1/2)}-1/4*(a^2-2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d*2^{(1/2)}/e^{(1/2)}-2*b^2*(e*\cot(d*x+c))^{(1/2)}/d/e$

Rubi [A] time = 0.25, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {3543, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e}} - \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} d \sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^2/Sqrt[e\*Cot[c + d\*x]],x]

[Out]  $((a^2 + 2*a*b - b^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*\text{Sqrt}[e]) - ((a^2 + 2*a*b - b^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*\text{Sqrt}[e]) - (2*b^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(d*e) + ((a^2 - 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*\text{Sqrt}[e]) - ((a^2 - 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*\text{Sqrt}[e])$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \ \text{Dist}[e/(2*c), \ \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \ \text{Dist}[e/(2*c*q), \ \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \ \text{Dist}[(d*q + a*e)/(2*a*c), \ \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \ \text{Dist}[(d*q - a*e)/(2*a*c), \ \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

### Rule 3534

$\text{Int}[\frac{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{\text{Sqrt}[(b_.)*\text{tan}[(e_.) + (f_.)*(x_.)] ]}, x\_Symbol] \ :> \ \text{Dist}[2/f, \ \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \ \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] \ /; \ \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3543

$\text{Int}[\frac{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}]^m, x\_Symbol] \ :> \ \text{Simp}[(d^2*(a + b*\text{Tan}[e + f*x])^{m+1})/(b*f*(m+1)), x] + \ \text{Int}[(a + b*\text{Tan}[e + f*x])^m*\text{Simp}[c^2 - d^2 + 2*c*d*\text{Tan}[e + f*x], x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ !\text{LeQ}[m, -1] \ \&\& \ !(\text{EqQ}[m, 2] \ \&\& \ \text{EqQ}[a, 0])$



Rubi steps

$$\begin{aligned}
\int \frac{(a + b \cot(c + dx))^2}{\sqrt{e \cot(c + dx)}} dx &= -\frac{2b^2 \sqrt{e \cot(c + dx)}}{de} + \int \frac{a^2 - b^2 + 2ab \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx \\
&= -\frac{2b^2 \sqrt{e \cot(c + dx)}}{de} + \frac{2 \operatorname{Subst}\left(\int \frac{-(a^2 - b^2)e - 2abx^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} \\
&= -\frac{2b^2 \sqrt{e \cot(c + dx)}}{de} - \frac{(a^2 - 2ab - b^2) \operatorname{Subst}\left(\int \frac{e^{-x^2}}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{d} - \frac{(a^2 + 2ab - b^2) \operatorname{Subst}\left(\int \frac{1}{e - \sqrt{2} \sqrt{e} x + x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2d} \\
&= -\frac{2b^2 \sqrt{e \cot(c + dx)}}{de} + \frac{(a^2 - 2ab - b^2) \log\left(\sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2} \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} d \sqrt{e}} \\
&= \frac{(a^2 + 2ab - b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} d \sqrt{e}} - \frac{(a^2 + 2ab - b^2) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} d \sqrt{e}}
\end{aligned}$$

**Mathematica [C]** time = 0.89, size = 192, normalized size = 0.72

$$\frac{\sqrt{\cot(c + dx)} \left( -\frac{(a^2 - b^2)(\log(\cot(c + dx) - \sqrt{2} \sqrt{\cot(c + dx)} + 1) - \log(\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)} + 1)) + 2 \tan^{-1}(1 - \sqrt{2} \sqrt{\cot(c + dx)}) - 2 \tan^{-1}(\sqrt{2} \sqrt{\cot(c + dx)})}{2\sqrt{2}} \right)}{d \sqrt{e \cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^2/Sqrt[e\*Cot[c + d\*x]],x]

[Out] -((Sqrt[Cot[c + d\*x]]\*(2\*b^2\*Sqrt[Cot[c + d\*x]] + (4\*a\*b\*Cot[c + d\*x])^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2])/3 - ((a^2 - b^2)\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/(2\*Sqrt[2]))/(d\*Sqrt[e\*Cot[c + d\*x]]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^2}{\sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^2/sqrt(e\*cot(d\*x + c)), x)

**maple** [B] time = 0.52, size = 529, normalized size = 1.98

$$-\frac{2b^2\sqrt{e \cot(dx + c)}}{de} + \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right) a^2}{2ed} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right) b^2}{2ed} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right) b^2}{2ed} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right) b^2}{2ed}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x)

[Out] 
$$-2*b^2*(e*\cot(d*x+c))^{1/2}/d/e+1/2/e/d*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2})/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)*a^2-1/2/e/d*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2})/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)*b^2-1/4/e/d*(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))*a^2+1/4/e/d*(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))*b^2-1/2/e/d*(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2})/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)*a^2+1/2/e/d*(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2})/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)*b^2-1/2/d*a*b/(e^2)^{1/4}*2^{1/2}*\ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))-1/d*a*b/(e^2)^{1/4}*2^{1/2}*\arctan(2^{1/2})/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+1/d*a*b/(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2})/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)$$

**maxima [A]** time = 0.80, size = 242, normalized size = 0.91

$$e \left[ \frac{8b^2 \sqrt{\frac{e}{\tan(dx+c)}}}{e^2} + \frac{2\sqrt{2}(a^2+2ab-b^2) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2+2ab-b^2) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2}(a^2-2ab-b^2) \log\left(\frac{e}{e}\right)}{e} \right] 4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out]  $-1/4*e*(8*b^2*\sqrt{e/\tan(d*x+c)})/e^2 + (2*\sqrt{2}*(a^2+2*a*b-b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a^2+2*a*b-b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e})/\sqrt{e} + \sqrt{2}*(a^2-2*a*b-b^2)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e} - \sqrt{2}*(a^2-2*a*b-b^2)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e})/e)/d$

**mupad [B]** time = 1.01, size = 1234, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^2/(e\*cot(c + d\*x))^(1/2),x)

[Out]  $2*\operatorname{atanh}\left(\frac{32*a^4*e^2*(e*\cot(c+d*x))^{1/2}*((a*b^3)/(d^2*e)-(b^4*1i)/(4*d^2*e)-(a^4*1i)/(4*d^2*e)-(a^3*b)/(d^2*e)+(a^2*b^2*3i)/(2*d^2*e))^{1/2}}{(a^6*e^2*16i)/d-(b^6*e^2*16i)/d+(32*a*b^5*e^2)/d+(32*a^5*b*e^2)/d+(a^2*b^4*e^2*112i)/d-(192*a^3*b^3*e^2)/d-(a^4*b^2*e^2*112i)/d+(32*b^4*e^2*(e*\cot(c+d*x))^{1/2}*((a*b^3)/(d^2*e)-(b^4*1i)/(4*d^2*e)-(a^4*1i)/(4*d^2*e)-(a^3*b)/(d^2*e)+(a^2*b^2*3i)/(2*d^2*e))^{1/2}}{(a^6*e^2*16i)/d-(b^6*e^2*16i)/d+(32*a*b^5*e^2)/d+(32*a^5*b*e^2)/d+(a^2*b^4*e^2*112i)/d-(192*a^3*b^3*e^2)/d-(a^4*b^2*e^2*112i)/d-(192*a^2*b^2*e^2*(e*\cot(c+d*x))^{1/2}*((a*b^3)/(d^2*e)-(b^4*1i)/(4*d^2*e)-(a^4*1i)/(4*d^2*e)-(a^3*b)/(d^2*e)+(a^2*b^2*3i)/(2*d^2*e))^{1/2}}{(a^6*e^2*16i)/d-(b^6*e^2*16i)/d+(32*a*b^5*e^2)/d+(32*a^5*b*e^2)/d+(a^2*b^4*e^2*112i)/d-(192*a^3*b^3*e^2)/d-(a^4*b^2*e^2*112i)/d}\right)*((a*b^3)/(d^2*e)-(b^4*1i)/(4*d^2*e)-(a^4*1i)/(4*d^2*e)-(a^3*b)/(d^2*e)+(a^2*b^2*3i)/(2*d^2*e))^{1/2} + 2*\operatorname{atanh}\left(\frac{32*a^4*e^2*(e*\cot(c+d*x))^{1/2}*((a^4*1i)/(4*d^2*e)+(b^4*1i)/(4*d^2*e)+(a*b^3)/(d^2*e)-(a^3*b)/(d^2*e)-(a^2*b^2*3i)/(2*d^2*e))^{1/2}}{(a^6*e^2*16i)/d-(b^6*e^2*16i)/d+(32*a*b^5*e^2)/d+(32*a^5*b*e^2)/d+(a^2*b^4*e^2*112i)/d-(192*a^3*b^3*e^2)/d-(a^4*b^2*e^2*112i)/d}\right)*((a^4*1i)/(4*d^2*e)+(b^4*1i)/(4*d^2*e)+(a*b^3)/(d^2*e)-(a^3*b)/(d^2*e)-(a^2*b^2*3i)/(2*d^2*e))^{1/2}$

$$\begin{aligned} & (2*d^2*e)^{(1/2)} / ((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d) + (32*b^4*e^2*(e*\cot(c + d*x))^{(1/2)}*((a^4*1i)/(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^{(1/2)}) / ((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d - (192*a^2*b^2*e^2*(e*\cot(c + d*x))^{(1/2)}*((a^4*1i)/(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^{(1/2)}) / ((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d + (a^4*b^2*e^2*112i)/d) * ((a^4*1i)/(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*b^2*3i)/(2*d^2*e))^{(1/2)} - (2*b^2*(e*\cot(c + d*x))^{(1/2)})/(d*e) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^2}{\sqrt{e \cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*2/(e\*cot(d\*x+c))\*\*(1/2), x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*2/sqrt(e\*cot(c + d\*x)), x)

$$3.59 \quad \int \frac{(a+b \cot(c+dx))^2}{(e \cot(c+dx))^{3/2}} dx$$

Optimal. Leaf size=267

$$\frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}} - \frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{3/2}}$$

[Out]  $-1/2*(a^2-2*a*b-b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(3/2)}*2^{(1/2)}+1/2*(a^2-2*a*b-b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(3/2)}*2^{(1/2)}+1/4*(a^2+2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)})-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d/e^{(3/2)}*2^{(1/2)}-1/4*(a^2+2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)})+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d/e^{(3/2)}*2^{(1/2)}+2*a^2/d/e^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {3542, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}} - \frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(3/2), x]

[Out]  $-(((a^2 - 2*a*b - b^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(3/2)})) + ((a^2 - 2*a*b - b^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(3/2)}) + (2*a^2)/(d*e*\text{Sqrt}[e*\text{Cot}[c + d*x]]) + ((a^2 + 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(3/2)}) - ((a^2 + 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(3/2)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3542

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

&& LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cot(c + dx))^2}{(e \cot(c + dx))^{3/2}} dx &= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{\int \frac{2abe - (a^2 - b^2)e \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{e^2} \\
 &= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{2 \operatorname{Subst}\left(\int \frac{-2abe^2 + (a^2 - b^2)ex^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de^2} \\
 &= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{(a^2 - 2ab - b^2) \operatorname{Subst}\left(\int \frac{e + x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de} - \frac{(a^2 + 2ab - b^2) \operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e} + 2x}{-e - \sqrt{2}\sqrt{e}x - x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2}de^{3/2}} \\
 &= \frac{2a^2}{de\sqrt{e \cot(c + dx)}} + \frac{(a^2 + 2ab - b^2) \operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e} + 2x}{-e - \sqrt{2}\sqrt{e}x - x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{(a^2 - 2ab - b^2) \log\left(\sqrt{e} + \sqrt{e \cot(c + dx)} - \sqrt{2}\sqrt{e \cot(c + dx)}\right)}{2\sqrt{2}de^{3/2}} \\
 &= -\frac{(a^2 - 2ab - b^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a^2 - 2ab - b^2) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.33, size = 218, normalized size = 0.82

$$\frac{\cot^{\frac{3}{2}}(c + dx) \left( -\frac{2(a^2 - b^2) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right)}{\sqrt{\cot(c + dx)}} + 4ab \left( \frac{1}{2} \left( \frac{\log(\cot(c + dx) + \sqrt{2}\sqrt{\cot(c + dx)} + 1)}{2\sqrt{2}} - \frac{\log(\cot(c + dx) - \sqrt{2}\sqrt{\cot(c + dx)} + 1)}{2\sqrt{2}} \right) \right)}{d(e \cot(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(3/2), x]

[Out] -((Cot[c + d\*x])^(3/2)\*((-2\*b^2)/Sqrt[Cot[c + d\*x]] - (2\*(a^2 - b^2)\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2])/Sqrt[Cot[c + d\*x]] + 4\*a\*b\*((-ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]/Sqrt[2])/2 + (-1/2\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/Sqrt[2] + Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]/(2\*Sqrt[2]))/2))/(d\*(e\*Cot[c + d\*x])^(3/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^2}{(e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^2/(e\*cot(d\*x + c))^(3/2), x)

**maple** [B] time = 0.46, size = 538, normalized size = 2.01

$$\frac{ab \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \ln \left( \frac{e \cot(dx+c) + \left( e^2 \right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - \left( e^2 \right)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}} \right)}{2e^2 d} + \frac{ab \left( e^2 \right)^{\frac{1}{4}} \sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{\left( e^2 \right)^{\frac{1}{4}}} + 1 \right)}{e^2 d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(3/2),x)

[Out] 
$$\begin{aligned} & -1/2/e^2/d*a*b*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2))}) \\ & -1/e^2/d*a*b*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/e^2/d*a*b*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1) \\ & +1/2/e/d*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a^2-1/2/e/d*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*b^2-1/2/e/d*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a^2+1/2/e/d*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*b^2 \\ & +1/4/e/d*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2))}) \\ & *a^2-1/4/e/d*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)))/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2))}) \\ & *b^2+2*a^2/d/e/(e*cot(d*x+c))^{(1/2)} \end{aligned}$$



**maxima [A]** time = 0.90, size = 242, normalized size = 0.91

$$e \left( \frac{8a^2}{e^2 \sqrt{\frac{e}{\tan(dx+c)}}} + \frac{2\sqrt{2}(a^2-2ab-b^2) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2-2ab-b^2) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2}(a^2+2ab-b^2) \log\left(\sqrt{2}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{e^2} \right) / 4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/4\*e\*(8\*a^2/(e^2\*sqrt(e/tan(d\*x + c)))) + (2\*sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) + sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/e^2/d

**mupad [B]** time = 0.94, size = 1196, normalized size = 4.48

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}} + 2 \operatorname{atanh} \left( \frac{32a^4d^3e^5\sqrt{e\cot(c+dx)}\sqrt{\frac{a^41i}{4d^2e^3} + \frac{b^41i}{4d^2e^3} - \frac{ab^3}{d^2e^3} + \frac{a^3b}{d^2e^3} - \frac{a^2}{2a}}}{-16a^6d^2e^4 + a^5bd^2e^432i + 112a^4b^2d^2e^4 - a^3b^3d^2e^4192i - 112a^2b^4d^2e^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^2/(e\*cot(c + d\*x))^(3/2),x)

[Out] 2\*atanh((32\*a^4\*d^3\*e^5\*(e\*cot(c + d\*x))^(1/2)\*((a^4\*1i)/(4\*d^2\*e^3) + (b^4\*1i)/(4\*d^2\*e^3) - (a\*b^3)/(d^2\*e^3) + (a^3\*b)/(d^2\*e^3) - (a^2\*b^2\*3i)/(2\*d^2\*e^3))^(1/2))/(16\*b^6\*d^2\*e^4 - 16\*a^6\*d^2\*e^4 + a\*b^5\*d^2\*e^4\*32i + a^5\*b\*d^2\*e^4\*32i - 112\*a^2\*b^4\*d^2\*e^4 - a^3\*b^3\*d^2\*e^4\*192i + 112\*a^4\*b^2\*d^2\*e^4) + (32\*b^4\*d^3\*e^5\*(e\*cot(c + d\*x))^(1/2)\*((a^4\*1i)/(4\*d^2\*e^3) + (b^4\*1i)/(4\*d^2\*e^3) - (a\*b^3)/(d^2\*e^3) + (a^3\*b)/(d^2\*e^3) - (a^2\*b^2\*3i)/(2\*d^2\*e^3))^(1/2))/(16\*b^6\*d^2\*e^4 - 16\*a^6\*d^2\*e^4 + a\*b^5\*d^2\*e^4\*32i + a^5\*b\*d^2\*e^4\*32i - 112\*a^2\*b^4\*d^2\*e^4 - a^3\*b^3\*d^2\*e^4\*192i + 112\*a^4\*b^2\*d^2\*e^4) - (192\*a^2\*b^2\*d^3\*e^5\*(e\*cot(c + d\*x))^(1/2)\*((a^4\*1i)/(4\*d^2\*e^3) + (b^4\*1i)/(4\*d^2\*e^3) - (a\*b^3)/(d^2\*e^3) + (a^3\*b)/(d^2\*e^3) - (a^2\*b^2\*3i)/(2\*d^2\*e^3))^(1/2))/(16\*b^6\*d^2\*e^4 - 16\*a^6\*d^2\*e^4 + a\*b^5\*d^2\*e^4\*32i + a^5\*b\*d^2\*e^4\*32i - 112\*a^2\*b^4\*d^2\*e^4 - a^3\*b^3\*d^2\*e^4\*192i + 112\*a^4\*b^2\*d^2\*e^4))\*(((a\*b^3\*4i - a^3\*b\*4i + a^4 + b^4 - 6\*a^2\*b^2)\*1i)/(4\*d^2

```

2*e^3))^(1/2) - 2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^
2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3) +
(a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5*d
^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i
- 112*a^4*b^2*d^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^3*b)/(
d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^2*e^3)
+ (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4 + a*b^5
*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*19
2i - 112*a^4*b^2*d^2*e^4) - (192*a^2*b^2*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a
^3*b)/(d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)/(4*d^
2*e^3) + (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d^2*e^4
+ a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2
*e^4*192i - 112*a^4*b^2*d^2*e^4))*(-(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a
^2*b^2)*1i)/(4*d^2*e^3))^(1/2) + (2*a^2)/(d*e*(e*cot(c + d*x))^(1/2))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^2}{(e \cot(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*2/(e\*cot(d\*x+c))\*\*(3/2), x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*2/(e\*cot(c + d\*x))\*\*(3/2), x)

$$3.60 \quad \int \frac{(a+b \cot(c+dx))^2}{(e \cot(c+dx))^{5/2}} dx$$

Optimal. Leaf size=291

$$\frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}} + \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{5/2}}$$

[Out]  $2/3*a^2/d/e/(e*\cot(d*x+c))^{(3/2)}-1/2*(a^2+2*a*b-b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+1/2*(a^2+2*a*b-b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}-1/4*(a^2-2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+1/4*(a^2-2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+4*a*b/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3542, 3529, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}} + \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(5/2), x]

[Out]  $-(((a^2 + 2*a*b - b^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(5/2)})) + ((a^2 + 2*a*b - b^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(5/2)}) + (2*a^2)/(3*d*e*(e*\text{Cot}[c + d*x])^{(3/2)}) + (4*a*b)/(d*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]]) - ((a^2 - 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(5/2)}) + ((a^2 - 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(5/2)})$

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

### Rule 3534

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```

NeQ[c^2 + d^2, 0]

### Rule 3542

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] :> Simp[((b\*c - a\*d)^2\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c^2 + 2\*b\*c\*d - a\*d^2 - (b\*c^2 - 2\*a\*c\*d - b\*d^2)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cot(c + dx))^2}{(e \cot(c + dx))^{5/2}} dx &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{\int \frac{2abe - (a^2 - b^2)e \cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx}{e^2} \\
 &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4ab}{de^2 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{-(a^2 - b^2)e^2 - 2abe^2 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{e^4} \\
 &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4ab}{de^2 \sqrt{e \cot(c + dx)}} + \frac{2 \operatorname{Subst}\left(\int \frac{(a^2 - b^2)e^3 + 2abe^2 x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de^4} \\
 &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4ab}{de^2 \sqrt{e \cot(c + dx)}} + \frac{(a^2 - 2ab - b^2) \operatorname{Subst}\left(\int \frac{e - x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)}\right)}{de^2} \\
 &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4ab}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(a^2 - 2ab - b^2) \operatorname{Subst}\left(\int \frac{\sqrt{2} \sqrt{e} + 2x}{-e - \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)}\right)}{2\sqrt{2} de^{5/2}} \\
 &= \frac{2a^2}{3de(e \cot(c + dx))^{3/2}} + \frac{4ab}{de^2 \sqrt{e \cot(c + dx)}} - \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} + \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{5/2}} \\
 &= -\frac{(a^2 + 2ab - b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{5/2}} + \frac{(a^2 + 2ab - b^2) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{5/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.30, size = 82, normalized size = 0.28

$$\frac{2 \left( (a^2 - b^2) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx)\right) + b \left( 6a \cot(c + dx) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right) + b \right) \right)}{3de(e \cot(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(5/2),x]

[Out] (2\*((a^2 - b^2)\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2] + b\*(b + 6\*a\*Cot[c + d\*x]\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2]))/(3\*d\*e\*(e\*Cot[c + d\*x])^(3/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^2}{(e \cot(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^2/(e\*cot(d\*x + c))^(5/2), x)

**maple** [B] time = 0.47, size = 558, normalized size = 1.92

$$\frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right) a^2}{2e^3 d} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right) b^2}{2e^3 d} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{2e^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x)

[Out] 1/2/e^3/d\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)\*a^2-1/2/e^3/d\*(e^2)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)\*b^2-1/2/e^3/d\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)\*a^2+1/2/e^3/d\*(e^2)^(1/4)\*2^(1/2)\*arctan(-2^(1/2)/(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)+1)\*b^2+1/4/e^3/d\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x+c))^(1/2)\*2^(1/2)+(e^2)^(1/2))/(e\*cot

$d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2))}*a^2-1/4/e^3/d$   
 $* (e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2))})$   
 $/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2))})$   
 $*b^2+1/2/e^2/d*a*b/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*$   
 $\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2))})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*$   
 $\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2))})+1/e^2/d*a*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan$   
 $(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}-1/e^2/d*a*b/(e^2)^{(1/4)}*2^{(1/2)}$   
 $*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1})+2/3*a^2/d/e/(e*\cot(d*x+c))^{(3/2)}$   
 $+4*a*b/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

**maxima [A]** time = 0.63, size = 259, normalized size = 0.89

$$\frac{3 \left( \frac{2 \sqrt{2} (a^2 + 2ab - b^2) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} \right) + \frac{2 \sqrt{2} (a^2 + 2ab - b^2) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}} \right)}{2 \sqrt{e}} \right)}{\sqrt{e}} \right) + \frac{\sqrt{2} (a^2 - 2ab - b^2) \log \left( \sqrt{2} \sqrt{e} \sqrt{\frac{e}{\tan(dx+c)}} + e \right)}{\sqrt{e}}}{e^3}$$

$12d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(5/2),x, algorithm="maxima")

[Out]  $1/12*e*(3*(2*\sqrt{2})*(a^2 + 2*a*b - b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)})/\sqrt{e}))/\sqrt{e} + 2*\sqrt{2}*(a^2 + 2*a*b - b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x + c)})/\sqrt{e}))/\sqrt{e} + \sqrt{2}*(a^2 - 2*a*b - b^2)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)}) + e + e/\tan(d*x + c))/\sqrt{e} - \sqrt{2}*(a^2 - 2*a*b - b^2)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)}) + e + e/\tan(d*x + c))/\sqrt{e}/e^3 + 8*(a^2*e + 6*a*b*e/\tan(d*x + c))/(e^3*(e/\tan(d*x + c))^{(3/2)})/d$

**mupad [B]** time = 1.51, size = 1214, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^2/(e\*cot(c + d\*x))^(5/2),x)

[Out]  $((2*a^2)/3 + 4*a*b*\cot(c + d*x))/(d*e*(e*\cot(c + d*x))^{(3/2)}) - 2*atanh((32*a^4*d^3*e^8*(e*\cot(c + d*x))^{(1/2)}*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*d^2$

```

*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5))^(
1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d^2*
e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112i) +
(32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*
d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5)
)^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d
^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112i)
- (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4
*1i)/(4*d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*
d^2*e^5))^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32
*a^5*b*d^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e
^6*112i))*(((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)/(4*d^2*e^5))^(
1/2) - 2*atanh((32*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e^5) -
(b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5) + (a^2*b^2
*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32*a*b^5*d^2*
e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 - a^4*b
^2*d^2*e^6*112i) + (32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e^5
) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5) + (a^2*
b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32*a*b^5*d
^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 - a^
4*b^2*d^2*e^6*112i) - (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b^3)/
(d^2*e^5) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2*e^5)
+ (a^2*b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16i + 32
*a*b^5*d^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*
e^6 - a^4*b^2*d^2*e^6*112i))*(-(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^
2)*1i)/(4*d^2*e^5))^(1/2)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^2}{(e \cot(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*2/(e\*cot(d\*x+c))\*\*(5/2), x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*2/(e\*cot(c + d\*x))\*\*(5/2), x)



$$3.61 \quad \int \frac{(a+b \cot(c+dx))^2}{(e \cot(c+dx))^{7/2}} dx$$

Optimal. Leaf size=322

$$\frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} de^{7/2}} + \frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{7/2}}$$

[Out]  $2/5*a^2/d/e/(e*\cot(d*x+c))^{(5/2)}+4/3*a*b/d/e^2/(e*\cot(d*x+c))^{(3/2)}+1/2*(a^2-2*a*b-b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(7/2)}*2^{(1/2)}-1/2*(a^2-2*a*b-b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(7/2)}*2^{(1/2)}-1/4*(a^2+2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(7/2)}*2^{(1/2)}+1/4*(a^2+2*a*b-b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(7/2)}*2^{(1/2)}-2*(a^2-b^2)/d/e^{3/(e*\cot(d*x+c))^{(1/2)}}$

**Rubi [A]** time = 0.43, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3542, 3529, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{2(a^2 - b^2)}{de^3 \sqrt{e \cot(c + dx)}} - \frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} de^{7/2}} + \frac{(a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)})}{2\sqrt{2} de^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(7/2), x]

[Out]  $((a^2 - 2*a*b - b^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(7/2)}) - ((a^2 - 2*a*b - b^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(7/2)}) + (2*a^2)/(5*d*e*(e*\text{Cot}[c + d*x])^{(5/2)}) + (4*a*b)/(3*d*e^2*(e*\text{Cot}[c + d*x])^{(3/2)}) - (2*(a^2 - b^2))/(d*e^3*\text{Sqrt}[e*\text{Cot}[c + d*x]]) - ((a^2 + 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(7/2)}) + ((a^2 + 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*d*e^{(7/2)})$

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

### Rule 3534

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

$\text{t}[b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3542

$\text{Int}[\left((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_)]\right)^{(m_)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_)]), x\_Symbol] \text{:>} \text{Simp}[\left((b*c - a*d)^2*(a + b*\text{Tan}[e + f*x])^{(m + 1)}\right)/(b*f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cot(c + dx))^2}{(e \cot(c + dx))^{7/2}} dx &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{\int \frac{2abe - (a^2 - b^2)e \cot(c + dx)}{(e \cot(c + dx))^{5/2}} dx}{e^2} \\
 &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4ab}{3de^2(e \cot(c + dx))^{3/2}} + \frac{\int \frac{-(a^2 - b^2)e^2 - 2abe^2 \cot(c + dx)}{(e \cot(c + dx))^{3/2}} dx}{e^4} \\
 &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4ab}{3de^2(e \cot(c + dx))^{3/2}} - \frac{2(a^2 - b^2)}{de^3 \sqrt{e \cot(c + dx)}} + \frac{\int \frac{-2abe^3 + (a^2 - b^2)e^4}{\sqrt{e \cot(c + dx)}} dx}{e^4} \\
 &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4ab}{3de^2(e \cot(c + dx))^{3/2}} - \frac{2(a^2 - b^2)}{de^3 \sqrt{e \cot(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{2a^2 - 2abx - bx^2}{\sqrt{e \cot(c + dx)}} dx\right)}{e^4} \\
 &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4ab}{3de^2(e \cot(c + dx))^{3/2}} - \frac{2(a^2 - b^2)}{de^3 \sqrt{e \cot(c + dx)}} - \frac{(a^2 - 2ab - b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{e^4} \\
 &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4ab}{3de^2(e \cot(c + dx))^{3/2}} - \frac{2(a^2 - b^2)}{de^3 \sqrt{e \cot(c + dx)}} - \frac{(a^2 + 2ab - b^2) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{e^4} \\
 &= \frac{2a^2}{5de(e \cot(c + dx))^{5/2}} + \frac{4ab}{3de^2(e \cot(c + dx))^{3/2}} - \frac{2(a^2 - b^2)}{de^3 \sqrt{e \cot(c + dx)}} - \frac{(a^2 + 2ab - b^2) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{e^4} \\
 &= \frac{(a^2 - 2ab - b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{7/2}} - \frac{(a^2 - 2ab - b^2) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{7/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.36, size = 85, normalized size = 0.26

$$\frac{2 \left( 3 (a^2 - b^2) {}_2F_1 \left( -\frac{5}{4}, 1; -\frac{1}{4}; -\cot^2(c + dx) \right) + b \left( 10a \cot(c + dx) {}_2F_1 \left( -\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx) \right) + 3b \right) \right)}{15de(e \cot(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^2/(e\*Cot[c + d\*x])^(7/2),x]

[Out] (2\*(3\*(a^2 - b^2)\*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d\*x]^2] + b\*(3\*b + 10\*a\*Cot[c + d\*x]\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2]))/(15\*d\*e\*(e\*Cot[c + d\*x])^(5/2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^2}{(e \cot(dx + c))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^2/(e\*cot(d\*x + c))^(7/2), x)

**maple [B]** time = 0.45, size = 600, normalized size = 1.86

$$\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2+\sqrt{e^2}}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2+\sqrt{e^2}}}\right)}{2e^4d} + \frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{e^4d} - \frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}} - 1\right)}{e^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x)

```
[Out] 1/2/e^4/d*a*b*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+1/e^4/d*a*b*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/e^4/d*a*b*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/4/e^3/d*2^(1/2)/(e^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*a^2+1/4/e^3/d*2^(1/2)/(e^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*b^2-1/2/e^3/d*2^(1/2)/(e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^2+1/2/e^3/d*2^(1/2)/(e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*b^2+1/2/e^3/d*2^(1/2)/(e^2)^(1/4)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^2-1/2/e^3/d*2^(1/2)/(e^2)^(1/4)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*b^2+2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)-2/e^3/d/(e*cot(d*x+c))^(1/2)*a^2+2/e^3/d/(e*cot(d*x+c))^(1/2)*b^2+4/3*a*b/d/e^2/(e*cot(d*x+c))^(3/2)
```

**maxima [A]** time = 0.53, size = 286, normalized size = 0.89

$$e \left[ \frac{15 \left( \frac{2\sqrt{2}(a^2-2ab-b^2) \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right) + \frac{2\sqrt{2}(a^2-2ab-b^2) \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right) - \frac{\sqrt{2}(a^2+2ab-b^2) \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}}}{e^4} \right] 60d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^2/(e\*cot(d\*x+c))^(7/2),x, algorithm="maxima")

```
[Out] -1/60*e*(15*(2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(e) + 2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) + 2*sqrt(2)*(a^2 - 2*a*b - b^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(e) - 2*sqrt(e/tan(d*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)*(a^2 + 2*a*b - b^2)*log(sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e) + sqrt(2)*(a^2 + 2*a*b - b^2)*log(-sqrt(2)*sqrt(e)*sqrt(e/tan(d*x + c)) + e + e/tan(d*x + c))/sqrt(e))/e^4 - 8*(3*a^2*e^2 + 10*a*b*e^2/tan(d*x + c) - 15*(a^2 - b^2)*e^2/tan(d*x + c)^2)/(e^4*(e/tan(d*x + c))^(5/2))/d
```

**mupad [B]** time = 2.32, size = 1227, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)`

[Out]  $2*\operatorname{atanh}\left(\frac{32*a^4*d^3*e^{11}*(e*\cot(c + d*x))^{1/2}*((a^3*b)/(d^2*e^7) - (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)/(2*d^2*e^7))^{1/2}}{16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b^2*d^2*e^8} + \frac{32*b^4*d^3*e^{11}*(e*\cot(c + d*x))^{1/2}*((a^3*b)/(d^2*e^7) - (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)/(2*d^2*e^7))^{1/2}}{16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b^2*d^2*e^8} - \frac{192*a^2*b^2*d^3*e^{11}*(e*\cot(c + d*x))^{1/2}*((a^3*b)/(d^2*e^7) - (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)/(2*d^2*e^7))^{1/2}}{16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*b^2*d^2*e^8}\right) * \left(-\frac{(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i}{(4*d^2*e^7)^{1/2}} - 2*\operatorname{atanh}\left(\frac{32*a^4*d^3*e^{11}*(e*\cot(c + d*x))^{1/2}*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^{1/2}}{16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i + 112*a^4*b^2*d^2*e^8} + \frac{32*b^4*d^3*e^{11}*(e*\cot(c + d*x))^{1/2}*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^{1/2}}{16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i + 112*a^4*b^2*d^2*e^8} - \frac{192*a^2*b^2*d^3*e^{11}*(e*\cot(c + d*x))^{1/2}*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) + (a^3*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^{1/2}}{16*b^6*d^2*e^8 - 16*a^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i + 112*a^4*b^2*d^2*e^8}\right) * \left(\frac{(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)*1i}{(4*d^2*e^7)^{1/2}} + \frac{(2*a^2)/5 - 2*\cot(c + d*x)^2*(a^2 - b^2) + (4*a*b*\cot(c + d*x))/3}{d*e*(e*\cot(c + d*x))^{5/2}}\right)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^2}{(e \cot(c + dx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)`

[Out] `Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(7/2), x)`

### 3.62 $\int (e \cot(c + dx))^{3/2} (a + b \cot(c + dx))^3 dx$

Optimal. Leaf size=372

$$\frac{e^{3/2}(a+b)(a^2-4ab+b^2)\log(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e})}{2\sqrt{2}d} + \frac{e^{3/2}(a+b)(a^2-4ab+b^2)\log(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e})}{2\sqrt{2}d}$$

[Out]  $-2/3*b*(3*a^2-b^2)*(e*\cot(d*x+c))^{3/2}/d-32/35*a*b^2*(e*\cot(d*x+c))^{5/2}/d/e-2/7*b^2*(e*\cot(d*x+c))^{5/2}*(a+b*\cot(d*x+c))/d/e-1/2*(a-b)*(a^2+4*a*b+b^2)*e^{3/2}*\arctan(1-2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/d*2^{1/2}+1/2*(a-b)*(a^2+4*a*b+b^2)*e^{3/2}*\arctan(1+2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/d*2^{1/2}-1/4*(a+b)*(a^2-4*a*b+b^2)*e^{3/2}*\ln(e^{1/2}+\cot(d*x+c)*e^{1/2})-2^{1/2}*(e*\cot(d*x+c))^{1/2}/d*2^{1/2}+1/4*(a+b)*(a^2-4*a*b+b^2)*e^{3/2}*\ln(e^{1/2}+\cot(d*x+c)*e^{1/2})+2^{1/2}*(e*\cot(d*x+c))^{1/2}/d*2^{1/2}-2*a*(a^2-3*b^2)*e*(e*\cot(d*x+c))^{1/2}/d$

**Rubi [A]** time = 0.56, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3566, 3630, 3528, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{e^{3/2}(a+b)(a^2-4ab+b^2)\log(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e})}{2\sqrt{2}d} + \frac{e^{3/2}(a+b)(a^2-4ab+b^2)\log(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e})}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])^3,x]

[Out]  $-(((a-b)*(a^2+4*a*b+b^2)*e^{3/2}*\text{ArcTan}[1-(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d))+((a-b)*(a^2+4*a*b+b^2)*e^{3/2}*\text{ArcTan}[1+(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d)-(2*a*(a^2-3*b^2)*e*\text{Sqrt}[e*\text{Cot}[c+d*x]])/d-(2*b*(3*a^2-b^2)*(e*\text{Cot}[c+d*x])^{3/2})/(3*d)-(32*a*b^2*(e*\text{Cot}[c+d*x])^{5/2})/(35*d*e)-(2*b^2*(e*\text{Cot}[c+d*x])^{5/2}*(a+b*\text{Cot}[c+d*x]))/(7*d*e)-((a+b)*(a^2-4*a*b+b^2)*e^{3/2}*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]-\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*d)+((a+b)*(a^2-4*a*b+b^2)*e^{3/2}*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]+\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*d)$

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3528

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_) +
(f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rule 3534

```
Int[((c_) + (d_.)*tan[(e_) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_) + (f_.)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```



$\int [b \tan[e + f x]]^n dx$  /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3566

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m - 2)\*(c + d\*Tan[e + f\*x])^(n + 1))/(d\*f\*(m + n - 1)), x] + Dist[1/(d\*(m + n - 1)), Int[(a + b\*Tan[e + f\*x])^(m - 3)\*(c + d\*Tan[e + f\*x])^n\*Simp[a^3\*d\*(m + n - 1) - b^2\*(b\*c\*(m - 2) + a\*d\*(1 + n)) + b\*d\*(m + n - 1)\*(3\*a^2 - b^2)\*Tan[e + f\*x] - b^2\*(b\*c\*(m - 2) - a\*d\*(3\*m + 2\*n - 4))\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3630

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] :> Simp[(C\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int (e \cot(c + dx))^{3/2} (a + b \cot(c + dx))^3 dx &= -\frac{2b^2(e \cot(c + dx))^{5/2}(a + b \cot(c + dx))}{7de} - \frac{2 \int (e \cot(c + dx))^{3/2} \left(-\frac{1}{2}\right)}{7de} \\
&= -\frac{32ab^2(e \cot(c + dx))^{5/2}}{35de} - \frac{2b^2(e \cot(c + dx))^{5/2}(a + b \cot(c + dx))}{7de} \\
&= -\frac{2b(3a^2 - b^2)(e \cot(c + dx))^{3/2}}{3d} - \frac{32ab^2(e \cot(c + dx))^{5/2}}{35de} - \frac{2b^2(e \cot(c + dx))^{5/2}}{7de} \\
&= -\frac{2a(a^2 - 3b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{2b(3a^2 - b^2)(e \cot(c + dx))^{3/2}}{3d} \\
&= -\frac{2a(a^2 - 3b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{2b(3a^2 - b^2)(e \cot(c + dx))^{3/2}}{3d} \\
&= -\frac{2a(a^2 - 3b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{2b(3a^2 - b^2)(e \cot(c + dx))^{3/2}}{3d} \\
&= -\frac{2a(a^2 - 3b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{2b(3a^2 - b^2)(e \cot(c + dx))^{3/2}}{3d} \\
&= -\frac{2a(a^2 - 3b^2)e\sqrt{e \cot(c + dx)}}{d} - \frac{2b(3a^2 - b^2)(e \cot(c + dx))^{3/2}}{3d} \\
&= -\frac{(a - b)(a^2 + 4ab + b^2)e^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a - b)(a^2 - 3b^2)}{d}
\end{aligned}$$

**Mathematica [C]** time = 3.06, size = 251, normalized size = 0.67

$$\frac{(e \cot(c + dx))^{3/2} \left( \frac{2}{3}b(b^2 - 3a^2) \cot^{\frac{3}{2}}(c + dx) \left( {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx)\right) - 1 \right) + \frac{1}{4}a(a^2 - 3b^2) (8\sqrt{\cot(c + dx)}) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])^3,x]

[Out] -(((e\*Cot[c + d\*x])^(3/2))\*((6\*a\*b^2\*Cot[c + d\*x]^(5/2))/5 + (2\*b^3\*Cot[c + d\*x]^(7/2))/7 + (2\*b\*(-3\*a^2 + b^2)\*Cot[c + d\*x]^(3/2)\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/3 + (a\*(a^2 - 3\*b^2)\*(2\*sqrt[2]\*ArcTan[

$$\frac{1 - \sqrt{2} \sqrt{\cot[c + dx]} - 2\sqrt{2} \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\cot[c + dx]}] + 8\sqrt{\cot[c + dx]} + \sqrt{2} \operatorname{Log}[1 - \sqrt{2} \sqrt{\cot[c + dx]} + \cot[c + dx]] - \sqrt{2} \operatorname{Log}[1 + \sqrt{2} \sqrt{\cot[c + dx]} + \cot[c + dx]]}{4} / (d \cot[c + dx]^{3/2})$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(dx+c))^(3/2)\*(a+b\*cot(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^3 (e \cot(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(dx+c))^(3/2)\*(a+b\*cot(dx+c))^3,x, algorithm="giac")

[Out] integrate((b\*cot(dx + c) + a)^3\*(e\*cot(dx + c))^(3/2), x)

**maple** [B] time = 0.75, size = 807, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(dx+c))^(3/2)\*(a+b\*cot(dx+c))^3,x)

[Out] 
$$\begin{aligned} & -2/7/d/e^2*(e*\cot(dx+c))^{(7/2)}*b^3-6/5*a*b^2*(e*\cot(dx+c))^{(5/2)}/d/e-2/d* \\ & a^2*b*(e*\cot(dx+c))^{(3/2)}+2/3/d*(e*\cot(dx+c))^{(3/2)}*b^3-2*a^3*e*(e*\cot(dx+c))^{(1/2)}/d+6/d*e*a*b^2*(e*\cot(dx+c))^{(1/2)}+1/4/d*e*(e^2)^{(1/4)}*2^{(1/2)}* \\ & \ln((e*\cot(dx+c)+(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(dx+c)-(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) * a^3-3/4/d* \\ & e*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(dx+c)+(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(dx+c)-(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) * a*b^2-1/2/d*e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}+1) * a^3+3/2/d*e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}+1) * a*b^2+1/2/d*e*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}+1) * a^3-3/2/d*e*(e^2)^{(1/4)}*2^{(1/2)}*a \\ & \arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}+1) * a*b^2+3/4/d*e^2*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot(dx+c)-(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(dx+c)+(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \end{aligned}$$

$a^2 b^{-1/4} d e^{2x} (e^{1/2}) / (e^2)^{1/4} \ln((e \cot(dx+c) - (e^2)^{1/4}) (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/4}) / (e \cot(dx+c) + (e^2)^{1/4}) (e \cot(dx+c))^{1/2} 2^{1/2} + (e^2)^{1/4}) + b^3 - 3/2 d e^{2x} (e^{1/2}) / (e^2)^{1/4} \arctan(-2^{1/2} / (e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1) + a^2 b + 1/2 d e^{2x} (e^{1/2}) / (e^2)^{1/4} \arctan(-2^{1/2} / (e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1) + b^3 + 3/2 d e^{2x} (e^{1/2}) / (e^2)^{1/4} \arctan(2^{1/2} / (e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1) + a^2 b - 1/2 d e^{2x} (e^{1/2}) / (e^2)^{1/4} \arctan(2^{1/2} / (e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1) + b^3$

**maxima** [A] time = 0.44, size = 347, normalized size = 0.93

$$105 \left( \frac{2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan\left(\frac{\sqrt{2} (\sqrt{2} \sqrt{e+2} \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan\left(-\frac{\sqrt{2} (\sqrt{2} \sqrt{e-2} \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2} (a^3 - 3 a^2 b + 3 a b^2 - b^3)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)\*(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{420} (105 (2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan(1/2 \sqrt{2} (\sqrt{2} \sqrt{e} + 2 \sqrt{e/\tan(dx+c)})/\sqrt{e}))/\sqrt{e} + 2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan(-1/2 \sqrt{2} (\sqrt{2} \sqrt{e} - 2 \sqrt{e/\tan(dx+c)})/\sqrt{e}))/\sqrt{e} + \sqrt{2} (a^3 - 3 a^2 b + 3 a b^2 - b^3) \log(\sqrt{2} \sqrt{e} \sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))/\sqrt{e} - \sqrt{2} (a^3 - 3 a^2 b - 3 a b^2 + b^3) \log(-\sqrt{2} \sqrt{e} \sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))/\sqrt{e}) e - 8 (63 a^2 b^2 e (e/\tan(dx+c))^{5/2} + 15 b^3 (e/\tan(dx+c))^{7/2} + 105 (a^3 - 3 a^2 b^2) e^3 \sqrt{e/\tan(dx+c)} + 35 (3 a^2 b - b^3) e^2 (e/\tan(dx+c))^{3/2})/e^3) e/d$

**mupad** [B] time = 5.47, size = 2317, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(3/2)\*(a + b\*cot(c + d\*x))^3,x)

[Out]  $(e \cot(c + d x))^{3/2} \left( \frac{2 b^3}{3 d} - \frac{2 a^2 b}{d} - (e \cot(c + d x))^{1/2} \left( \frac{2 a^3 e}{d} - \frac{6 a^2 b^2 e}{d} - \operatorname{atan}\left(\frac{(16 (e \cot(c + d x))^{1/2} (a^6 e^6 - b^6 e^6 + 15 a^2 b^4 e^6 - 15 a^4 b^2 e^6))}{d^2} - \frac{8 (4 a^3 d^2 e^5 - 12 a^2 b^2 d^2 e^5) (-b^6 e^3 i - a^6 e^3 i + 6 a^2 b^5 e^3 + 6 a^5 b e^3 - a^2 b^4 e^3 i - 20 a^3 b^3 e^3 + a^4 b^2 e^3 i)}{(4 d^2)^{1/2}} \right) / d^3 \right) (-b^6 e^3 i - a^6 e^3 i + 6 a^2 b^5 e^3 + 6 a^5 b e^3 - a^2 b^4 e^3 i - 20$

$$\begin{aligned}
& *a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}*1i + ((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}*1i)/(((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)} - ((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)} + (16*(3*a^8*b*e^8 - b^9*e^8 + 6*a^4*b^5*e^8 + 8*a^6*b^3*e^8))/d^3))*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}*2i - \operatorname{atan}(((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}*1i + ((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}*1i)/(((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)} - ((16*(e*\cot(c + d*x))^{(1/2)}*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}))/d^3)*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)} + (16*(3*a^8*b*e^8 - b^9*e^8 + 6*a^4*b^5*e^8 + 8*a^6*b^3*e^8))/d^3))*(-(a^6*e^3*1i - b^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 + a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 - a^4*b^2*e^3*15i)/(4*d^2))^{(1/2)}*2i - (2*b^3*(e*\cot(c + d*x))^{(7/2)})/(7*d*e^2) - (6*a*b^2*(e*\cot(c + d*x))^{(5/2)})/(5*d*e)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e \cot(c + dx))^{\frac{3}{2}} (a + b \cot(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(3/2)\*(a+b\*cot(d\*x+c))\*\*3,x)

[Out] Integral((e\*cot(c + d\*x))\*\*(3/2)\*(a + b\*cot(c + d\*x))\*\*3, x)

### 3.63 $\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^3 dx$

**Optimal.** Leaf size=342

$$\frac{2b(3a^2 - b^2) \sqrt{e \cot(c + dx)}}{d} - \frac{\sqrt{e}(a - b)(a^2 + 4ab + b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{\sqrt{e}}{2\sqrt{2}d}$$

[Out]  $-8/5*a*b^2*(e*\cot(d*x+c))^{(3/2)}/d/e-2/5*b^2*(e*\cot(d*x+c))^{(3/2)}*(a+b*\cot(d*x+c))/d/e+1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}-1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}-1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^{(1/2)+\cot(d*x+c)}*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}+1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^{(1/2)+\cot(d*x+c)}*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})*e^{(1/2)}/d*2^{(1/2)}-2*b*(3*a^2-b^2)*(e*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.48, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3566, 3630, 3528, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{2b(3a^2 - b^2) \sqrt{e \cot(c + dx)}}{d} - \frac{\sqrt{e}(a - b)(a^2 + 4ab + b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2}d} + \frac{\sqrt{e}}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^3,x]

[Out]  $((a + b)*(a^2 - 4*a*b + b^2)*\text{Sqrt}[e]*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d) - ((a + b)*(a^2 - 4*a*b + b^2)*\text{Sqrt}[e]*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d) - (2*b*(3*a^2 - b^2)*\text{Sqrt}[e*\text{Cot}[c + d*x]])/d - (8*a*b^2*(e*\text{Cot}[c + d*x])^{(3/2)})/(5*d*e) - (2*b^2*(e*\text{Cot}[c + d*x])^{(3/2)}*(a + b*\text{Cot}[c + d*x]))/(5*d*e) - ((a - b)*(a^2 + 4*a*b + b^2)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*d) + ((a - b)*(a^2 + 4*a*b + b^2)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(2*\text{Sqrt}[2]*d)$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]
```

### Rule 3534

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
```



NeQ[c^2 + d^2, 0]

### Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
+ f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(C*(a +
b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))^3 dx &= -\frac{2b^2(e \cot(c+dx))^{3/2}(a+b \cot(c+dx))}{5de} - \frac{2 \int \sqrt{e \cot(c+dx)} \left(-\frac{1}{2}a \left(5\right.\right.}{5de} \\
&= -\frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}(a+b \cot(c+dx))}{5de} - \frac{2}{5de} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e \cot(c+dx)}}{d} - \frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}}{5de} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e \cot(c+dx)}}{d} - \frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}}{5de} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e \cot(c+dx)}}{d} - \frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}}{5de} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e \cot(c+dx)}}{d} - \frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}}{5de} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e \cot(c+dx)}}{d} - \frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}}{5de} \\
&= -\frac{2b(3a^2-b^2)\sqrt{e \cot(c+dx)}}{d} - \frac{8ab^2(e \cot(c+dx))^{3/2}}{5de} - \frac{2b^2(e \cot(c+dx))^{3/2}}{5de} \\
&= \frac{(a+b)(a^2-4ab+b^2)\sqrt{e} \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a+b)(a^2-4ab+b^2)\sqrt{e} \tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}
\end{aligned}$$

**Mathematica [C]** time = 2.59, size = 247, normalized size = 0.72

$$\sqrt{e \cot(c+dx)} \left( \frac{2}{3}a(a^2-3b^2) \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right) - \frac{1}{4}b(b^2-3a^2) \left(8\sqrt{\cot(c+dx)} + \sqrt{2} \log\left(\frac{1-\sqrt{2}\sqrt{\cot(c+dx)}}{1+\sqrt{2}\sqrt{\cot(c+dx)}}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^3,x]

[Out] -((Sqrt[e\*Cot[c + d\*x]]\*(2\*a\*b^2\*Cot[c + d\*x]^(3/2) + (2\*b^3\*Cot[c + d\*x]^(5/2))/5 + (2\*a\*(a^2 - 3\*b^2)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2])/3 - (b\*(-3\*a^2 + b^2)\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]) + 8\*Sqrt[Cot[c + d\*x]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/4))/(d\*Sqrt[Cot[c + d\*x]])





```

6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*
e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i -
20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) - ((1
6*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e
^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4))*((b^6*e*1i - a^6*e*1i - a^
2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2)
)^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^
2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) + (16*(3*a*b^8*e^5 - a^9*e^
5 + 8*a^3*b^6*e^5 + 6*a^5*b^4*e^5))/d^3))*((b^6*e*1i - a^6*e*1i - a^2*b^4*e
*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)
*2i + atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4
- 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4))*((a^6*e*1i
- b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a
^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3
*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i + ((16*(e
*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))
/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4))*((a^6*e*1i - b^6*e*1i + a^2*b^
4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1
/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*
15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i)/((((16*(e*cot(c + d*x))^(1/2)
*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^
2*e^4 - 12*a^2*b*d^2*e^4))*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^
3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1
i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6
*a^5*b*e)/(4*d^2))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 +
15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e
^4))*((a^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i +
6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((a^6*e*1i - b^6*e*1i + a^2*b^4
*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/
2) + (16*(3*a*b^8*e^5 - a^9*e^5 + 8*a^3*b^6*e^5 + 6*a^5*b^4*e^5))/d^3))*((a
^6*e*1i - b^6*e*1i + a^2*b^4*e*15i - 20*a^3*b^3*e - a^4*b^2*e*15i + 6*a*b^5
*e + 6*a^5*b*e)/(4*d^2))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2
) - (2*a*b^2*(e*cot(c + d*x))^(3/2))/(d*e)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)\*(a+b\*cot(d\*x+c))\*\*3,x)

[Out] Integral(sqrt(e\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))\*\*3, x)

$$3.64 \quad \int \frac{(a+b \cot(c+dx))^3}{\sqrt{e \cot(c+dx)}} dx$$

Optimal. Leaf size=313

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e \cot(c+dx)} - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}} - \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e \cot(c+dx)} + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}}$$

```
[Out] 1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*
2^(1/2)/e^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(
1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a+b)*(a^2
-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2
^(1/2)/e^(1/2)-16/3*a*b^2*(e*cot(d*x+c))^(1/2)/d/e-2/3*b^2*(a+b*cot(d*x+c))
*(e*cot(d*x+c))^(1/2)/d/e
```

**Rubi [A]** time = 0.43, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3566, 3630, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e \cot(c+dx)} - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}} - \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e \cot(c+dx)} + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]], x]
```

```
[Out] ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]]/(Sqrt[2]*d*Sqrt[e]) - ((a - b)*(a^2 + 4*a*b + b^2)*ArcTan[1 + (Sqrt[2]
)*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*d*Sqrt[e]) - (16*a*b^2*Sqrt[e*Co
t[c + d*x]])/(3*d*e) - (2*b^2*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]))/(3
*d*e) + ((a + b)*(a^2 - 4*a*b + b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e]) - ((a + b)*(a^2 - 4*a*b
+ b^2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]])
/(2*Sqrt[2]*d*Sqrt[e])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3566

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
```

```
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n - 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3630

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_.)] + (C_.)*tan[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cot(c + dx))^3}{\sqrt{e \cot(c + dx)}} dx &= -\frac{2b^2 \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))}{3de} - \frac{2 \int \frac{-\frac{1}{2}a(3a^2 - b^2)e - \frac{3}{2}b(3a^2 - b^2)e \cot(c + dx) - 4ab^2 e \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{3e} \\
 &= -\frac{16ab^2 \sqrt{e \cot(c + dx)}}{3de} - \frac{2b^2 \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))}{3de} - \frac{2 \int \frac{-\frac{3}{2}a(a^2 - 3b^2)e - \frac{3}{2}b(a^2 - 3b^2)e \cot(c + dx) - 4ab^2 e \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{3e} \\
 &= -\frac{16ab^2 \sqrt{e \cot(c + dx)}}{3de} - \frac{2b^2 \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))}{3de} - \frac{4 \text{Subst} \left( \int \frac{\frac{3}{2}a(a^2 - 3b^2) - \frac{3}{2}b(a^2 - 3b^2) \cot(u)}{\sqrt{e \cot(u)}} du \right)}{3e} \\
 &= -\frac{16ab^2 \sqrt{e \cot(c + dx)}}{3de} - \frac{2b^2 \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))}{3de} - \frac{((a + b)(a^2 - 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right))}{\sqrt{2} d \sqrt{e}} \\
 &= -\frac{16ab^2 \sqrt{e \cot(c + dx)}}{3de} - \frac{2b^2 \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))}{3de} - \frac{((a - b)(a^2 + 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right))}{\sqrt{2} d \sqrt{e}} + \frac{(a + b)(a^2 - 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} d \sqrt{e}} \\
 &= \frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} d \sqrt{e}} - \frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} d \sqrt{e}} + \frac{(a + b)(a^2 - 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} d \sqrt{e}}
 \end{aligned}$$



**Mathematica [C]** time = 1.03, size = 216, normalized size = 0.69

$$2\sqrt{\cot(c+dx)} \left( -b(b^2-3a^2) \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right) - \frac{3a(a^2-3b^2)(\log(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)+1}) - \frac{3d\sqrt{e}\cot(c+dx)}{2de} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^3/Sqrt[e\*Cot[c + d\*x]],x]

[Out] (-2\*Sqrt[Cot[c + d\*x]]\*(9\*a\*b^2\*Sqrt[Cot[c + d\*x]] + b^3\*Cot[c + d\*x]^(3/2) - b\*(-3\*a^2 + b^2)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] - (3\*a\*(a^2 - 3\*b^2)\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])))/(4\*Sqrt[2]))/(3\*d\*Sqrt[e\*Cot[c + d\*x]])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^3}{\sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^3/sqrt(e\*cot(d\*x + c)), x)

**maple [B]** time = 0.61, size = 725, normalized size = 2.32

$$\frac{2b^3 (e \cot(dx + c))^{\frac{3}{2}}}{3de^2} - \frac{6ab^2 \sqrt{e \cot(dx + c)}}{de} + \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{2de} - \frac{3 (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}} + 1\right)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(1/2),x)

[Out] 
$$-2/3/d/e^2*b^3*(e*cot(d*x+c))^{3/2}-6*a*b^2*(e*cot(d*x+c))^{1/2}/d/e+1/2/d*a^3/e*(e^2)^{1/4}*2^{1/2}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)-3/2/d/e*(e^2)^{1/4}*2^{1/2}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)*a*b^2-1/2/d*a^3/e*(e^2)^{1/4}*2^{1/2}*arctan(2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)+3/2/d/e*(e^2)^{1/4}*2^{1/2}*arctan(2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)*a*b^2-1/4/d*a^3/e*(e^2)^{1/4}*2^{1/2}*ln((e*cot(d*x+c)+(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e*cot(d*x+c)-(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))+3/4/d/e*(e^2)^{1/4}*2^{1/2}*ln((e*cot(d*x+c)+(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e*cot(d*x+c)-(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))*a*b^2+3/2/d*2^{1/2}/(e^2)^{1/4}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)*a^2*b-1/2/d*2^{1/2}/(e^2)^{1/4}*arctan(-2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)*b^3-3/4/d*2^{1/2}/(e^2)^{1/4}*ln((e*cot(d*x+c)-(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e*cot(d*x+c)+(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))*a^2*b+1/4/d*2^{1/2}/(e^2)^{1/4}*ln((e*cot(d*x+c)-(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))/((e*cot(d*x+c)+(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}*2^{1/2}+(e^2)^{1/2}))*b^3-3/2/d*2^{1/2}/(e^2)^{1/4}*arctan(2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)*a^2*b+1/2/d*2^{1/2}/(e^2)^{1/4}*arctan(2^{1/2}/(e^2)^{1/4}*(e*cot(d*x+c))^{1/2}+1)*b^3$$

**maxima** [A] time = 0.52, size = 292, normalized size = 0.93

$$e^3 \left( \frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2}(a^3-3a^2b-3ab^2+b^3) \log\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{e} \right)$$

12 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 
$$-1/12*e*(3*(2*\sqrt{2}*(a^3+3*a^2*b-3*a*b^2-b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(dx+c)}))/\sqrt{e}))/\sqrt{e}+2*\sqrt{2}*(a^3+3*a^2*b-3*a*b^2-b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(dx+c)}))/\sqrt{e}))/\sqrt{e}+\sqrt{2}*(a^3-3*a^2*b-3*a*b^2+b^3)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)}+e+e/\tan(dx+c))/\sqrt{e}-\sqrt{2}*(a^3-3*a^2*b-3*a*b^2+b^3)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)}+e+e/\tan(dx+c))/\sqrt{e}$$

c)) + e + e/tan(d\*x + c))/sqrt(e))/e + 8\*(9\*a\*b^2\*e\*sqrt(e/tan(d\*x + c)) + b^3\*(e/tan(d\*x + c))^(3/2))/e^3)/d

mupad [B] time = 1.41, size = 1896, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^3/(e\*cot(c + d\*x))^(1/2),x)

[Out] atan((((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 - (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2))/d^3)\*(((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2)\*1i + ((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 + (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2))/d^3)\*(((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2)\*1i)/(((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 + (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2))/d^3)\*(((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2) - ((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 - (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2))/d^3)\*(((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2) + (16\*(3\*a^8\*b\*e^2 - b^9\*e^2 + 6\*a^4\*b^5\*e^2 + 8\*a^6\*b^3\*e^2))/d^3))\*(((a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2)\*2i + atan((((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 - (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((a\*b^5\*6i + a^5\*b\*6i + a^6 - b^6 + 15\*a^2\*b^4 - a^3\*b^3\*20i - 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2))/d^3)\*(((a\*b^5\*6i + a^5\*b\*6i + a^6 - b^6 + 15\*a^2\*b^4 - a^3\*b^3\*20i - 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2)\*1i + ((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 + (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((a\*b^5\*6i + a^5\*b\*6i + a^6 - b^6 + 15\*a^2\*b^4 - a^3\*b^3\*20i - 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2))/d^3)\*(((a\*b^5\*6i + a^5\*b\*6i + a^6 - b^6 + 15\*a^2\*b^4 - a^3\*b^3\*20i - 15\*a^4\*b^2)\*1i)/(4\*d^2\*e))^(1/2) - ((16\*(e\*cot(c + d\*x))^(1/2)\*(a^6\*e^2 - b^6\*e^2 + 15\*a^2\*b^4\*e^2 - 15\*a^4\*b^2\*e^2))/d^2 - (8\*(4\*a^3\*d^2\*e^3 - 12\*a\*b^2\*d^2\*e^3)\*((

```
(a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1
i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 -
a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) + (16*(3*a^8*b*e^2 - b^9*e^
2 + 6*a^4*b^5*e^2 + 8*a^6*b^3*e^2))/d^3))*(((a*b^5*6i + a^5*b*6i + a^6 - b^
6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*2i - (2*b^3
*(e*cot(c + d*x))^(3/2))/(3*d*e^2) - (6*a*b^2*(e*cot(c + d*x))^(1/2))/(d*e)
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(a + b \cot(c + dx))^3}{\sqrt{e \cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x))**3/sqrt(e*cot(c + d*x)), x)
```

$$3.65 \quad \int \frac{(a+b \cot(c+dx))^3}{(e \cot(c+dx))^{3/2}} dx$$

Optimal. Leaf size=313

$$\frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}} - \frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{3/2}}$$

[Out]  $-1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d$   
 $/e^{(3/2)}*2^{(1/2)}+1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(3/2)}*2^{(1/2)}+1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d/e^{(3/2)}*2^{(1/2)}-1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/d/e^{(3/2)}*2^{(1/2)}+2*a^2*(a+b*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^{(1/2)}-2*b*(a^2+b^2)*(e*\cot(d*x+c))^{(1/2)}/d/e^2$

**Rubi [A]** time = 0.42, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3565, 3630, 3534, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{2b(a^2+b^2)\sqrt{e \cot(c+dx)}}{de^2} + \frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2}} - \frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(3/2), x]

[Out]  $-(((a+b)*(a^2-4*a*b+b^2)*\text{ArcTan}[1-(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(3/2)})) + ((a+b)*(a^2-4*a*b+b^2)*\text{ArcTan}[1+(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(3/2)}) - (2*b*(a^2+b^2)*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(d*e^2) + (2*a^2*(a+b*\text{Cot}[c+d*x]))/(d*e*\text{Sqrt}[e*\text{Cot}[c+d*x]]) + ((a-b)*(a^2+4*a*b+b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c+d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(2*\text{Sqrt}[2]*d*e^{(3/2)}) - ((a-b)*(a^2+4*a*b+b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c+d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(2*\text{Sqrt}[2]*d*e^{(3/2)})$

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1
```

/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 3)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a^2\*d\*(b\*d\*(m - 2) - a\*c\*(n + 1)) + b\*(b\*c - 2\*a\*d)\*(b\*c\*(m - 2) + a\*d\*(n + 1)) - d\*(n + 1)\*(3\*a^2\*b\*c - b^3\*c - a^3\*d + 3\*a\*b^2\*d)\*Tan[e + f\*x] - b\*(a\*d\*(2\*b\*c - a\*d)\*(m + n - 1) - b^2\*(c^2\*(m - 2) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && IntegerQ[2\*m]

### Rule 3630

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^2, x\_Symbol] :> Simp[(C\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)), x] + Int[(a + b\*Tan[e + f\*x])^m\*Simp[A - C + B\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && !LeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{3/2}} dx &= \frac{2a^2(a + b \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} - \frac{2 \int \frac{-2a^2be^2 + \frac{1}{2}a(a^2 - 3b^2)e^2 \cot(c + dx) - \frac{1}{2}b(a^2 + b^2)e^2 \cot^2(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{e^3} \\
 &= -\frac{2b(a^2 + b^2)\sqrt{e \cot(c + dx)}}{de^2} + \frac{2a^2(a + b \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} - \frac{2 \int \frac{-\frac{1}{2}b(3a^2 - b^2)e^2 + \frac{1}{2}a(a^2 - 3b^2)}{\sqrt{e \cot(c + dx)}} dx}{e^3} \\
 &= -\frac{2b(a^2 + b^2)\sqrt{e \cot(c + dx)}}{de^2} + \frac{2a^2(a + b \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} - \frac{4 \operatorname{Subst}\left(\int \frac{\frac{1}{2}b(3a^2 - b^2)e^3 - \frac{1}{2}a}{e^2 + x^2} dx\right)}{e^3} \\
 &= -\frac{2b(a^2 + b^2)\sqrt{e \cot(c + dx)}}{de^2} + \frac{2a^2(a + b \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} + \frac{((a + b)(a^2 - 4ab + b^2))}{e^3} \\
 &= -\frac{2b(a^2 + b^2)\sqrt{e \cot(c + dx)}}{de^2} + \frac{2a^2(a + b \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} + \frac{((a - b)(a^2 + 4ab + b^2))}{e^3} \\
 &= -\frac{2b(a^2 + b^2)\sqrt{e \cot(c + dx)}}{de^2} + \frac{2a^2(a + b \cot(c + dx))}{de\sqrt{e \cot(c + dx)}} + \frac{(a - b)(a^2 + 4ab + b^2) \operatorname{lo}}{e^3} \\
 &= -\frac{(a + b)(a^2 - 4ab + b^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{(a + b)(a^2 - 4ab + b^2) \tan^{-1}}{\sqrt{2}de^{3/2}}
 \end{aligned}$$

**Mathematica [C]** time = 3.39, size = 193, normalized size = 0.62

$$\frac{2 \left( a (a^2 - 3b^2) {}_2F_1 \left( -\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx) \right) - \frac{b(b^2 - 3a^2) \sqrt{\cot(c + dx)} (\log(\cot(c + dx) - \sqrt{2} \sqrt{\cot(c + dx)} + 1) - \log(\cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)}))}{4\sqrt{2}} \right)}{de \sqrt{e \cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(3/2), x]

[Out] (2\*(3\*a\*b^2 - b^3\*Cot[c + d\*x] + a\*(a^2 - 3\*b^2)\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] - (b\*(-3\*a^2 + b^2)\*Sqrt[Cot[c + d\*x]]\*(2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])] + Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/(4\*Sqrt[2]))/(d\*e\*Sqrt[e\*Cot[c + d\*x]])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(3/2), x)

**maple [B]** time = 0.47, size = 742, normalized size = 2.37

$$-\frac{2b^3 \sqrt{e \cot(dx + c)}}{de^2} + \frac{3(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx + c)}}{(e^2)^{\frac{1}{4}}} + 1\right) a^2 b}{2de^2} - \frac{(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(-\frac{\sqrt{2} \sqrt{e \cot(dx + c)}}{(e^2)^{\frac{1}{4}}} + 1\right) b^3}{2de^2} -$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x)`

[Out] 
$$\begin{aligned} & -2/d/e^2*b^3*(e*cot(d*x+c))^{(1/2)}+3/2/d/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a^2*b-1/2/d/e^2*(e^2)^{(1/4)}*2^{(1/2)} \\ & *\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*b^3-3/4/d/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}) \\ & )/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & )*a^2*b+1/4/d/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}) \\ & )/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & )*b^3-3/2/d/e^2*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a^2*b+1/2/d/e^2*(e^2)^{(1/4)}*2^{(1/2)}* \\ & \arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*b^3+1/4/d*a^3/e*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}) \\ & )/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & )-3/4/d/e*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}) \\ & )/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & )*a*b^2-1/2/d*a^3/e*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+3/2/d/e*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a*b^2+1/2/d*a^3/e*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-3/2/d/e*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a*b^2+2/d*a^3/e/(e*cot(d*x+c))^{(1/2)} \end{aligned}$$

**maxima** [A] time = 0.65, size = 290, normalized size = 0.93

$$e \left( \frac{8a^3}{e^2 \sqrt{\frac{e}{\tan(dx+c)}}} - \frac{8b^3 \sqrt{\frac{e}{\tan(dx+c)}}}{e^3} + \frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right)$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 1/4*e*(8*a^3/(e^2*\sqrt{e/\tan(d*x+c)})) - 8*b^3*\sqrt{e/\tan(d*x+c)}/e^3 + \\ & (2*\sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e}))/\sqrt{e} + 2*\sqrt{2}*(a^3 - 3*a^2*b - \\ & 3*a*b^2 + b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e}))/\sqrt{e} - \sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)} + e + e/\tan(d*x+c))/\sqrt{e} + \sqrt{2}*(a^3 + \\ & 3*a^2*b - 3*a*b^2 - b^3)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)} + e + e/\tan(d*x+c))/\sqrt{e}))/e^2)/d \end{aligned}$$



$$2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2) - 16*a^9*d^2*e^4 + 48*a*b^8*d^2*e^4 + 128*a^3*b^6*d^2*e^4 + 96*a^5*b^4*d^2*e^4))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2)*2i - (2*b^3*(e*cot(c + d*x))^(1/2))/(d*e^2)$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*3/(e\*cot(d\*x+c))\*\*(3/2), x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*3/(e\*cot(c + d\*x))\*\*(3/2), x)

$$3.66 \quad \int \frac{(a+b \cot(c+dx))^3}{(e \cot(c+dx))^{5/2}} dx$$

Optimal. Leaf size=313

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}} + \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}}$$

[Out]  $2/3*a^2*(a+b*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^{(3/2)}-1/2*(a-b)*(a^2+4*a*b+b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+1/2*(a-b)*(a^2+4*a*b+b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}-1/4*(a+b)*(a^2-4*a*b+b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+1/4*(a+b)*(a^2-4*a*b+b^2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/d/e^{(5/2)}*2^{(1/2)}+16/3*a^2*b/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.46, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$ , Rules used = {3565, 3628, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}} + \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(5/2), x]

[Out]  $-(((a-b)*(a^2+4*a*b+b^2)*\text{ArcTan}[1-(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(5/2)})) + ((a-b)*(a^2+4*a*b+b^2)*\text{ArcTan}[1+(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e]])/(\text{Sqrt}[2]*d*e^{(5/2)}) + (16*a^2*b)/(3*d*e^2*\text{Sqrt}[e*\text{Cot}[c+d*x]]) + (2*a^2*(a+b*\text{Cot}[c+d*x]))/(3*d*e*(e*\text{Cot}[c+d*x])^{(3/2)}) - ((a+b)*(a^2-4*a*b+b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c+d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*d*e^{(5/2)}) + ((a+b)*(a^2-4*a*b+b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c+d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*d*e^{(5/2)})$

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(
m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1
```

/(d\*(n + 1)\*(c^2 + d^2)), Int[(a + b\*Tan[e + f\*x])^(m - 3)\*(c + d\*Tan[e + f\*x])^(n + 1)\*Simp[a^2\*d\*(b\*d\*(m - 2) - a\*c\*(n + 1)) + b\*(b\*c - 2\*a\*d)\*(b\*c\*(m - 2) + a\*d\*(n + 1)) - d\*(n + 1)\*(3\*a^2\*b\*c - b^3\*c - a^3\*d + 3\*a\*b^2\*d)\*Tan[e + f\*x] - b\*(a\*d\*(2\*b\*c - a\*d)\*(m + n - 1) - b^2\*(c^2\*(m - 2) - d^2\*(n + 1)))\*Tan[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && IntegerQ[2\*m]

### Rule 3628

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((A\_.) + (B\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)] + (C\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^2), x\_Symbol] :> Simp[((A\*b^2 - a\*b\*B + a^2\*C)\*(a + b\*Tan[e + f\*x])^(m + 1))/(b\*f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[b\*B + a\*(A - C) - (A\*b - a\*B - b\*C)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[A\*b^2 - a\*b\*B + a^2\*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{5/2}} dx &= \frac{2a^2(a + b \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{2 \int \frac{-4a^2be^2 + \frac{3}{2}a(a^2 - 3b^2)e^2 \cot(c + dx) + \frac{1}{2}b(a^2 - 3b^2)e^2 \cot^2(c + dx)}{(e \cot(c + dx))^{3/2}} dx}{3e^3} \\
 &= \frac{16a^2b}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{2 \int \frac{\frac{3}{2}a(a^2 - 3b^2)e^3 + \frac{3}{2}b(3a^2 - b^2)e^3 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{3e^5} \\
 &= \frac{16a^2b}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{4 \operatorname{Subst}\left(\int \frac{-\frac{3}{2}a(a^2 - 3b^2)e^4 - \frac{3}{2}b(3a^2 - b^2)e^3 x^2}{e^2 + x^4} dx\right)}{3de^5} \\
 &= \frac{16a^2b}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} + \frac{((a + b)(a^2 - 4ab + b^2)) \operatorname{Subst}\left(\int \frac{e}{e^2 - x^2} dx\right)}{de^2} \\
 &= \frac{16a^2b}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{((a + b)(a^2 - 4ab + b^2)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{2} dx}\right)}{2\sqrt{2} de} \\
 &= \frac{16a^2b}{3de^2\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{3de(e \cot(c + dx))^{3/2}} - \frac{(a + b)(a^2 - 4ab + b^2) \log(\sqrt{e} + \sqrt{e \cot(c + dx)})}{2\sqrt{2} de} \\
 &= -\frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{5/2}} + \frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{5/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.38, size = 104, normalized size = 0.33

$$\frac{-6b(b^2 - 3a^2) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c + dx)\right) + 2a(a^2 - 3b^2) \tan(c + dx) {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx)\right) + 6b^2(a \tan(c + dx) + \cot(c + dx))}{3de^2\sqrt{e \cot(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(5/2), x]

[Out] (-6\*b\*(-3\*a^2 + b^2)\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + 2\*a\*(a^2 - 3\*b^2)\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2]\*Tan[c + d\*x] + 6\*b^2\*(b + a\*Tan[c + d\*x]))/(3\*d\*e^2\*Sqrt[e\*Cot[c + d\*x]])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(5/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(5/2), x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(5/2), x)

**maple [B]** time = 0.46, size = 743, normalized size = 2.37

$$\frac{(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right) a^3}{4d e^3} - \frac{3 (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right) a b^2}{4d e^3} + \frac{a^3 (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d e^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(5/2), x)

[Out]  $\frac{1}{4} \frac{d}{e^3} (e^2)^{1/4} 2^{1/2} \ln((e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2})^{1/2} 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2}) 2^{1/2} + (e^2)^{1/2}) * a^3 - 3/4 \frac{d}{e^3} (e^2)^{1/4} 2^{1/2} \ln((e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2})^{1/2} 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2}) 2^{1/2} + (e^2)^{1/2}) * a * b^2 + 1/2 \frac{d}{e^3} (e^2)^{1/4} 2^{1/2} * \arctan(2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) - 3/2 \frac{d}{e^3} (e^2)^{1/4} 2^{1/2} * \arctan(2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * a * b^2 - 1/2 \frac{d}{e^3} (e^2)^{1/4} 2^{1/2} * \arctan(-2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * a^3 + 3/2 \frac{d}{e^3} (e^2)^{1/4} 2^{1/2} * \arctan(-2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * a * b^2 + 3/4 \frac{d}{e^2} 2^{1/2} / ((e^2)^{1/4} * \ln((e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2})^{1/2} 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2}) 2^{1/2} + (e^2)^{1/2})) * a^2 * b - 1/4 \frac{d}{e^2} 2^{1/2} / ((e^2)^{1/4} * \ln((e \cot(dx+c) - (e^2)^{1/4} (e \cot(dx+c))^{1/2})^{1/2} 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) + (e^2)^{1/4} (e \cot(dx+c))^{1/2}) 2^{1/2} + (e^2)^{1/2})) * b^3 + 3/2 \frac{d}{e^2} 2^{1/2} / ((e^2)^{1/4} * \arctan(2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * a^2 * b - 1/2 \frac{d}{e^2} 2^{1/2} / ((e^2)^{1/4} * \arctan(2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * b^3 - 3/2 \frac{d}{e^2} 2^{1/2} / ((e^2)^{1/4} * \arctan(-2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * (e \cot(dx+c))^{1/2} + 1) * a^2 * b + 1/2 \frac{d}{e^2} 2^{1/2} / ((e^2)^{1/4} * \arctan(-2^{1/2} / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) / ((e^2)^{1/4} (e \cot(dx+c))^{1/2} + 1)) * b^3 + 2/3 \frac{d}{e} a^3 / (e \cot(dx+c))^{3/2} + 6 a^2 * b / d / e^2 / (e \cot(dx+c))^{1/2}$

**maxima** [A] time = 0.69, size = 289, normalized size = 0.92

$$e \left( \frac{2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan \left( \frac{\sqrt{2} (\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan \left( -\frac{\sqrt{2} (\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}})}{2 \sqrt{e}} \right)}{\sqrt{e}} + \frac{\sqrt{2} (a^3 - 3 a^2 b - 3 a b^2 + b^3) \log(\sqrt{e})}{e^3} \right)$$

12 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(5/2), x, algorithm="maxima")

[Out]  $\frac{1}{12} e * (3 * (2 * \sqrt{2}) * (a^3 + 3 a^2 b - 3 a b^2 - b^3) * \arctan(1/2 * \sqrt{2}) * (\sqrt{2}) * \sqrt{e} + 2 * \sqrt{e} / \tan(dx+c)) / \sqrt{e} / \sqrt{e} + 2 * \sqrt{2} * (a^3 + 3 a^2 b - 3 a b^2 - b^3) * \arctan(-1/2 * \sqrt{2}) * (\sqrt{2}) * \sqrt{e} - 2 * \sqrt{e} / \tan(dx+c)) / \sqrt{e} / \sqrt{e} + \sqrt{2} * (a^3 - 3 a^2 b - 3 a b^2 + b^3) * \log(\sqrt{2}) * \sqrt{e} * \sqrt{e} / \tan(dx+c) + e + e / \tan(dx+c)) / \sqrt{e} - \sqrt{2} * (a^3 - 3 a^2 b - 3 a b^2 + b^3) * \log(-\sqrt{2}) * \sqrt{e} * \sqrt{e} / \tan(dx+c)$





$$4 - a^3 b^3 20i + 15 a^4 b^2) 1i) / (4 d^2 e^5)^{1/2} - ((e \cot(c + dx))^{1/2} * (16 a^6 d^3 e^8 - 16 b^6 d^3 e^8 + 240 a^2 b^4 d^3 e^8 - 240 a^4 b^2 d^3 e^8) - (32 a^3 d^4 e^{11} - 96 a^2 b^2 d^4 e^{11}) * ((a b^5 6i + a^5 b 6i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) 1i) / (4 d^2 e^5)^{1/2}) * ((a b^5 6i + a^5 b 6i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) 1i) / (4 d^2 e^5)^{1/2} - 16 b^9 d^2 e^6 + 48 a^8 b d^2 e^6 + 96 a^4 b^5 d^2 e^6 + 128 a^6 b^3 d^2 e^6) * ((a b^5 6i + a^5 b 6i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) 1i) / (4 d^2 e^5)^{1/2} * 2i$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*3/(e\*cot(d\*x+c))\*\*(5/2),x)

[Out] Integral((a + b\*cot(c + d\*x))\*3/(e\*cot(c + d\*x))\*\*(5/2), x)

$$3.67 \quad \int \frac{(a+b \cot(c+dx))^3}{(e \cot(c+dx))^{7/2}} dx$$

Optimal. Leaf size=343

$$\frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{7/2}} + \frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{7/2}}$$

[Out]  $8/5*a^2*b/d/e^2/(e*\cot(d*x+c))^(3/2)+2/5*a^2*(a+b*\cot(d*x+c))/d/e/(e*\cot(d*x+c))^(5/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1-2^(1/2)*(e*\cot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1+2^(1/2)*(e*\cot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^(1/2)+\cot(d*x+c)*e^(1/2)-2^(1/2)*(e*\cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^(1/2)+\cot(d*x+c)*e^(1/2)+2^(1/2)*(e*\cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)-2*a*(a^2-3*b^2)/d/e^3/(e*\cot(d*x+c))^(1/2)$

**Rubi [A]** time = 0.56, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3565, 3628, 3529, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{2a(a^2-3b^2)}{de^3\sqrt{e}\cot(c+dx)} - \frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{7/2}} + \frac{(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(7/2), x]

[Out]  $((a+b)*(a^2-4*a*b+b^2)*\text{ArcTan}[1-(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e])/(\text{Sqrt}[2]*d*e^(7/2))-((a+b)*(a^2-4*a*b+b^2)*\text{ArcTan}[1+(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e])/(\text{Sqrt}[2]*d*e^(7/2))+((8*a^2*b)/(5*d*e^2*(e*\text{Cot}[c+d*x])^(3/2))-(2*a*(a^2-3*b^2))/(d*e^3*\text{Sqrt}[e*\text{Cot}[c+d*x]])+(2*a^2*(a+b*\text{Cot}[c+d*x]))/(5*d*e*(e*\text{Cot}[c+d*x])^(5/2))-((a-b)*(a^2+4*a*b+b^2)*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]-\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*d*e^(7/2))+((a-b)*(a^2+4*a*b+b^2)*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]+\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(2*\text{Sqrt}[2]*d*e^(7/2)))$

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

### Rule 3534

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
```

$\text{t}[b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3565

$\text{Int}[\left((a_{.}) + (b_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(n_{.})}, x\_Symbol] := \text{Simp}[\left((b*c - a*d)^2*(a + b*\text{Tan}[e + f*x])^{(m - 2)}*(c + d*\text{Tan}[e + f*x])^{(n + 1)}\right)/(d*f*(n + 1)*(c^2 + d^2)), x] - \text{Dist}\left[1/(d*(n + 1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 3)}*(c + d*\text{Tan}[e + f*x])^{(n + 1)}*\text{Simp}[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*\text{Tan}[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n + 1)))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{GtQ}[m, 2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*m]$

### Rule 3628

$\text{Int}[\left((a_{.}) + (b_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]\right)^{(m_{.})}*\left((A_{.}) + (B_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})] + (C_{.})*\text{tan}[(e_{.}) + (f_{.})*(x_{.})]^2\right), x\_Symbol] := \text{Simp}[\left((A*b^2 - a*b*B + a^2*C)*(a + b*\text{Tan}[e + f*x])^{(m + 1)}\right)/(b*f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}\left[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[b*B + a*(A - C) - (A*b - a*B - b*C)*\text{Tan}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[A*b^2 - a*b*B + a^2*C, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{7/2}} dx &= \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{2 \int \frac{-6a^2be^2 + \frac{5}{2}a(a^2 - 3b^2)e^2 \cot(c + dx) + \frac{1}{2}b(3a^2 - 5b^2)e^2 \cot^2(c + dx)}{(e \cot(c + dx))^{5/2}} dx}{5e^3} \\
&= \frac{8a^2b}{5de^2(e \cot(c + dx))^{3/2}} + \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{2 \int \frac{\frac{5}{2}a(a^2 - 3b^2)e^3 + \frac{5}{2}b(3a^2 - b^2)e^3 \cot(c + dx)}{(e \cot(c + dx))^{3/2}}}{5e^5} \\
&= \frac{8a^2b}{5de^2(e \cot(c + dx))^{3/2}} - \frac{2a(a^2 - 3b^2)}{de^3\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{2 \int \frac{\frac{5}{2}b(3a^2 - b^2)}{}}{5e^5} \\
&= \frac{8a^2b}{5de^2(e \cot(c + dx))^{3/2}} - \frac{2a(a^2 - 3b^2)}{de^3\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{4 \text{Subst} \left( \int \frac{1}{\sqrt{e \cot(c + dx)}} \right)}{5e^5} \\
&= \frac{8a^2b}{5de^2(e \cot(c + dx))^{3/2}} - \frac{2a(a^2 - 3b^2)}{de^3\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{((a + b)(a^2 - 4ab + b^2)) \tan^{-1} \left( 1 - \frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{7/2}} \\
&= \frac{8a^2b}{5de^2(e \cot(c + dx))^{3/2}} - \frac{2a(a^2 - 3b^2)}{de^3\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{((a - b)(a^2 + 4ab + b^2)) \tan^{-1} \left( 1 + \frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{7/2}} \\
&= \frac{8a^2b}{5de^2(e \cot(c + dx))^{3/2}} - \frac{2a(a^2 - 3b^2)}{de^3\sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{5de(e \cot(c + dx))^{5/2}} - \frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1} \left( 1 + \frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{7/2}} \\
&= \frac{(a + b)(a^2 - 4ab + b^2) \tan^{-1} \left( 1 - \frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{7/2}} - \frac{(a + b)(a^2 - 4ab + b^2) \tan^{-1} \left( 1 + \frac{\sqrt{e \cot(c + dx)}}{\sqrt{e}} \right)}{\sqrt{2} de^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.61, size = 108, normalized size = 0.31

$$\frac{2 \left( 3a(a^2 - 3b^2) {}_2F_1 \left( -\frac{5}{4}, 1; -\frac{1}{4}; -\cot^2(c + dx) \right) + b \left( 5(3a^2 - b^2) \cot(c + dx) {}_2F_1 \left( -\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c + dx) \right) + b(9a^2 - 3ab + b^2) \right) \right)}{15de(e \cot(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(7/2), x]

[Out] (2\*(3\*a\*(a^2 - 3\*b^2)\*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d\*x]^2] + b\*(b\*(9\*a + 5\*b\*Cot[c + d\*x]) + 5\*(3\*a^2 - b^2)\*Cot[c + d\*x]\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2]))/(15\*d\*e\*(e\*Cot[c + d\*x])^(5/2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(7/2), x)

maple [B] time = 0.46, size = 786, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2),x)

[Out] 
$$\begin{aligned} & -3/2/d/e^4*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & *a^2*b+1/2/d/e^4*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & *b^3+3/4/d/e^4*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})) \\ & *a^2*b-1/4/d/e^4*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})) \\ & *b^3+3/2/d/e^4*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & *a^2*b-1/2/d/e^4*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & *b^3-1/4/d*a^3/e^3*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})) \\ & +3/4/d/e^3*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)*2^{(1/2)}+(e^2)^{(1/2)})) \\ & *a*b^2+1/2/d*a^3/e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & -3/2/d/e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & *a*b^2-1/2/d*a^3/e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & +3/2/d/e^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1}) \\ & *a*b^2+2/5/d*a^3/e/(e*\cot(d*x+c))^{(5/2)}+2*a^2*b/d/e^2/(e*\cot(d*x+c))^{(3/2)}-2*a^3/d/e^3/(e*\cot(d*x+c))^{(1/2)}+6/d/e^3*a/(e*\cot(d*x+c))^{(1/2)}*b^2 \end{aligned}$$

**maxima [A]** time = 0.71, size = 316, normalized size = 0.92

$$e^5 \left( \frac{2\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e} - 2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3) \log\left(\frac{\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{2}\sqrt{e} - 2\sqrt{\frac{e}{\tan(dx+c)}}}\right)}{e^4} \right)$$

20d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(7/2),x, algorithm="maxima")

[Out] -1/20\*e\*(5\*(2\*sqrt(2)\*(a^3 - 3\*a^2\*b - 3\*a\*b^2 + b^3)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a^3 - 3\*a^2\*b - 3\*a\*b^2 + b^3)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)\*(a^3 + 3\*a^2\*b - 3\*a\*b^2 - b^3)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) + sqrt(2)\*(a^3 + 3\*a^2\*b - 3\*a\*b^2 - b^3)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/e^4 - 8\*(a^3\*e^2 + 5\*a^2\*b\*e^2/tan(d\*x + c) - 5\*(a^3 - 3\*a\*b^2)\*e^2/tan(d\*x + c)^2)/(e^4\*(e/tan(d\*x + c))^(5/2))/d

**mupad [B]** time = 3.06, size = 1969, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^3/(e\*cot(c + d\*x))^(7/2),x)

[Out] atan((((e\*cot(c + d\*x))^(1/2)\*(16\*a^6\*d^3\*e^11 - 16\*b^6\*d^3\*e^11 + 240\*a^2\*b^4\*d^3\*e^11 - 240\*a^4\*b^2\*d^3\*e^11) + (32\*b^3\*d^4\*e^15 - 96\*a^2\*b\*d^4\*e^15)\*(-(a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e^7))^(1/2))\*(-(a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e^7))^(1/2)\*1i + ((e\*cot(c + d\*x))^(1/2)\*(16\*a^6\*d^3\*e^11 - 16\*b^6\*d^3\*e^11 + 240\*a^2\*b^4\*d^3\*e^11 - 240\*a^4\*b^2\*d^3\*e^11) - (32\*b^3\*d^4\*e^15 - 96\*a^2\*b\*d^4\*e^15)\*(-(a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e^7))^(1/2))\*(-(a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e^7))^(1/2)\*1i)/(((e\*cot(c + d\*x))^(1/2)\*(16\*a^6\*d^3\*e^11 - 16\*b^6\*d^3\*e^11 + 240\*a^2\*b^4\*d^3\*e^11 - 240\*a^4\*b^2\*d^3\*e^11) + (32\*b^3\*d^4\*e^15 - 96\*a^2\*b\*d^4\*e^15)\*(-(a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e^7))^(1/2))\*(-(a\*b^5\*6i + a^5\*b\*6i - a^6 + b^6 - 15\*a^2\*b^4 - a^3\*b^3\*20i + 15\*a^4\*b^2)\*1i)/(4\*d^2\*e^7))^(1/2)\*1i)



```

- 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) - (32*b^3
*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-(a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2
*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-(a*b^5*6i + a^5
*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))
^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a
^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e
^15)*(-(a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^
4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-(a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2
*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2) - 16*a^9*d^2*e^8 +
48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*d^2*e^8))*(-(a*b^5*6i
+ a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*
e^7))^(1/2)*2i + atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^
3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 -
96*a^2*b*d^4*e^15)*(-(a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b
^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-(a*b^5*6i + a^5*b*6i + a^6
- b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i +
((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^
3*e^11 - 240*a^4*b^2*d^3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-(
a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i
)/(4*d^2*e^7))^(1/2))*(-(a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^
3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)
*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^
3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-(a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-
((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)
*1i)/(4*d^2*e^7))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6
*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15
- 96*a^2*b*d^4*e^15)*(-(a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^
3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-(a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2) -
16*a^9*d^2*e^8 + 48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*d^2*e^
8))*(-(a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4
*b^2)*1i)/(4*d^2*e^7))^(1/2)*2i + ((2*a^3*e)/5 + 2*e*cot(c + d*x))^2*(3*a*b^
2 - a^3) + 2*a^2*b*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(5/2))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*3/(e\*cot(d\*x+c))\*\*(7/2), x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*3/(e\*cot(c + d\*x))\*\*(7/2), x)

$$3.68 \quad \int \frac{(a+b \cot(c+dx))^3}{(e \cot(c+dx))^{9/2}} dx$$

Optimal. Leaf size=377

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{9/2}} - \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{9/2}}$$

[Out] 32/35\*a^2\*b/d/e^2/(e\*cot(d\*x+c))^(5/2)-2/3\*a\*(a^2-3\*b^2)/d/e^3/(e\*cot(d\*x+c))^(3/2)+2/7\*a^2\*(a+b\*cot(d\*x+c))/d/e/(e\*cot(d\*x+c))^(7/2)+1/2\*(a-b)\*(a^2+4\*a\*b+b^2)\*arctan(1-2^(1/2)\*(e\*cot(d\*x+c))^(1/2)/e^(1/2))/d/e^(9/2)\*2^(1/2)-1/2\*(a-b)\*(a^2+4\*a\*b+b^2)\*arctan(1+2^(1/2)\*(e\*cot(d\*x+c))^(1/2)/e^(1/2))/d/e^(9/2)\*2^(1/2)+1/4\*(a+b)\*(a^2-4\*a\*b+b^2)\*ln(e^(1/2)+cot(d\*x+c)\*e^(1/2))-2^(1/2)\*(e\*cot(d\*x+c))^(1/2))/d/e^(9/2)\*2^(1/2)-1/4\*(a+b)\*(a^2-4\*a\*b+b^2)\*ln(e^(1/2)+cot(d\*x+c)\*e^(1/2)+2^(1/2)\*(e\*cot(d\*x+c))^(1/2))/d/e^(9/2)\*2^(1/2)-2\*b\*(3\*a^2-b^2)/d/e^4/(e\*cot(d\*x+c))^(1/2)

**Rubi [A]** time = 0.66, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3565, 3628, 3529, 3534, 1168, 1162, 617, 204, 1165, 628}

$$\frac{2a(a^2-3b^2)}{3de^3(e \cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4 \sqrt{e \cot(c+dx)}} + \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{9/2}} - \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(9/2), x]

[Out] ((a - b)\*(a^2 + 4\*a\*b + b^2)\*ArcTan[1 - (Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])/Sqrt[e]]/(Sqrt[2]\*d\*e^(9/2)) - ((a - b)\*(a^2 + 4\*a\*b + b^2)\*ArcTan[1 + (Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])/Sqrt[e]]/(Sqrt[2]\*d\*e^(9/2)) + (32\*a^2\*b)/(35\*d\*e^2\*(e\*Cot[c + d\*x])^(5/2)) - (2\*a\*(a^2 - 3\*b^2))/(3\*d\*e^3\*(e\*Cot[c + d\*x])^(3/2)) - (2\*b\*(3\*a^2 - b^2))/(d\*e^4\*Sqrt[e\*Cot[c + d\*x]]) + (2\*a^2\*(a + b\*Cot[c + d\*x]))/(7\*d\*e\*(e\*Cot[c + d\*x])^(7/2)) + ((a + b)\*(a^2 - 4\*a\*b + b^2)\*Log[Sqrt[e] + Sqrt[e]\*Cot[c + d\*x] - Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])/(2\*Sqrt[2]\*d\*e^(9/2)) - ((a + b)\*(a^2 - 4\*a\*b + b^2)\*Log[Sqrt[e] + Sqrt[e]\*Cot[c + d\*x] + Sqrt[2]\*Sqrt[e\*Cot[c + d\*x]])/(2\*Sqrt[2]\*d\*e^(9/2))

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 3529

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(
m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

### Rule 3628

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*tan[(e_) +
(f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{9/2}} dx &= \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} - \frac{2 \int \frac{-8a^2be^2 + \frac{7}{2}a(a^2 - 3b^2)e^2 \cot(c+dx) + \frac{1}{2}b(5a^2 - 7b^2)e^2 \cot^2(c+dx)}{(e \cot(c+dx))^{7/2}} dx}{7e^3} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} - \frac{2 \int \frac{\frac{7}{2}a(a^2 - 3b^2)e^3 + \frac{7}{2}b(3a^2 - b^2)e^3 \cot(c+dx)}{(e \cot(c+dx))^{5/2}} dx}{7e^5} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} - \frac{2a(a^2 - 3b^2)}{3de^3(e \cot(c + dx))^{3/2}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} - \frac{2 \int \frac{\frac{7}{2}b(3a^2 - b^2)e^3}{(e \cot(c+dx))^{3/2}} dx}{7e^5} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} - \frac{2a(a^2 - 3b^2)}{3de^3(e \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} - \frac{2a(a^2 - 3b^2)}{3de^3(e \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} - \frac{2a(a^2 - 3b^2)}{3de^3(e \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} - \frac{2a(a^2 - 3b^2)}{3de^3(e \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{32a^2b}{35de^2(e \cot(c + dx))^{5/2}} - \frac{2a(a^2 - 3b^2)}{3de^3(e \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{de^4 \sqrt{e \cot(c + dx)}} + \frac{2a^2(a + b \cot(c + dx))}{7de(e \cot(c + dx))^{7/2}} \\
&= \frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{9/2}} - \frac{(a - b)(a^2 + 4ab + b^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2} de^{9/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.68, size = 116, normalized size = 0.31

$$\frac{2 \tan^4(c + dx) \sqrt{e \cot(c + dx)} \left( 5a(a^2 - 3b^2) {}_2F_1\left(-\frac{7}{4}, 1; -\frac{3}{4}; -\cot^2(c + dx)\right) + b \left( 7(3a^2 - b^2) \cot(c + dx) {}_2F_1\left(-\frac{5}{4}, 1; -\frac{1}{4}; -\cot^2(c + dx)\right) \right) \right)}{35de^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^3/(e\*Cot[c + d\*x])^(9/2), x]

[Out] (2\*sqrt[e\*Cot[c + d\*x]]\*(5\*a\*(a^2 - 3\*b^2)\*Hypergeometric2F1[-7/4, 1, -3/4, -Cot[c + d\*x]^2] + b\*(b\*(15\*a + 7\*b\*Cot[c + d\*x]) + 7\*(3\*a^2 - b^2)\*Cot[c

+ d\*x]\*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d\*x]^2]))\*Tan[c + d\*x]^4)/(35\*d\*e^5)

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(9/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \cot(dx + c) + a)^3}{(e \cot(dx + c))^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(9/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^3/(e\*cot(d\*x + c))^(9/2), x)

**maple** [B] time = 0.45, size = 829, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(9/2),x)

[Out]  $\frac{1}{2}d^3a^3/e^5(e^2)^{1/4}2^{1/2}\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)-3/2d/e^5(e^2)^{1/4}2^{1/2}\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)*a*b^2-1/4d^3a^3/e^5(e^2)^{1/4}2^{1/2}\ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))+3/4d/e^5(e^2)^{1/4}2^{1/2}\ln((e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))*a*b^2-1/2d^3a^3/e^5(e^2)^{1/4}2^{1/2}\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)+3/2d/e^5(e^2)^{1/4}2^{1/2}\arctan(2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)*a*b^2-3/4d/e^42^{1/2}/(e^2)^{1/4}\ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))*a^2*b+1/4d/e^42^{1/2}/(e^2)^{1/4}\ln((e*\cot(d*x+c)-(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2})/(e*\cot(d*x+c)+(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2})*2^{1/2}+(e^2)^{1/2}))*b^3+3/2d/e^42^{1/2}/(e^2)^{1/4}\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1)$

$1) a^2 b - 1/2 d / e^4 2^{(1/2)} / (e^2)^{(1/4)} \arctan(-2^{(1/2)} / (e^2)^{(1/4)}) * (e \cot(d * x + c))^{(1/2)} + 1) * b^3 - 3/2 d / e^4 2^{(1/2)} / (e^2)^{(1/4)} \arctan(2^{(1/2)} / (e^2)^{(1/4)}) * (e \cot(d * x + c))^{(1/2)} + 1) * a^2 b + 1/2 d / e^4 2^{(1/2)} / (e^2)^{(1/4)} \arctan(2^{(1/2)} / (e^2)^{(1/4)}) * (e \cot(d * x + c))^{(1/2)} + 1) * b^3 + 2/7 d * a^3 / e / (e \cot(d * x + c))^{(7/2)} + 6/5 a^2 b / d / e^2 / (e \cot(d * x + c))^{(5/2)} - 2/3 a^3 / d / e^3 / (e \cot(d * x + c))^{(3/2)} + 2/d / e^3 a / (e \cot(d * x + c))^{(3/2)} * b^2 - 6/d / e^4 b / (e \cot(d * x + c))^{(1/2)} * a^2 + 2/d / e^4 b^3 / (e \cot(d * x + c))^{(1/2)}$

**maxima [A]** time = 0.77, size = 342, normalized size = 0.91

$$\frac{105 \left( \frac{2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} \right) + \frac{2 \sqrt{2} (a^3 + 3 a^2 b - 3 a b^2 - b^3) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} \right) + \sqrt{2} (a^3 - 3 a^2 b - 3 a b^2 + b^3)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^3/(e\*cot(d\*x+c))^(9/2),x, algorithm="maxima")

[Out]  $-1/420 * e * (105 * (2 * \sqrt{2}) * (a^3 + 3 * a^2 * b - 3 * a * b^2 - b^3) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e / \tan(d * x + c)}) / \sqrt{e}) / \sqrt{e} + 2 * \sqrt{2} * (a^3 + 3 * a^2 * b - 3 * a * b^2 - b^3) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e / \tan(d * x + c)}) / \sqrt{e}) / \sqrt{e} + \sqrt{2} * (a^3 - 3 * a^2 * b - 3 * a * b^2 + b^3) * \log(\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(d * x + c)}) + e + e / \tan(d * x + c)) / \sqrt{e} - \sqrt{2} * (a^3 - 3 * a^2 * b - 3 * a * b^2 + b^3) * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(d * x + c)}) + e + e / \tan(d * x + c)) / \sqrt{e}) / e^5 - 8 * (15 * a^3 * e^3 + 63 * a^2 * b * e^3 / \tan(d * x + c) - 35 * (a^3 - 3 * a * b^2) * e^3 / \tan(d * x + c)^2 - 105 * (3 * a^2 * b - b^3) * e^3 / \tan(d * x + c)^3) / (e^5 * (e / \tan(d * x + c))^{(7/2)}) / d$

**mupad [B]** time = 5.26, size = 1992, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^3/(e\*cot(c + d\*x))^(9/2),x)

[Out]  $\operatorname{atan}\left(\left(\left(e \cot(c + d * x)\right)^{(1/2)} * (16 * a^6 * d^3 * e^{14} - 16 * b^6 * d^3 * e^{14} + 240 * a^2 * b^4 * d^3 * e^{14} - 240 * a^4 * b^2 * d^3 * e^{14}) + (32 * a^3 * d^4 * e^{19} - 96 * a * b^2 * d^4 * e^{19})\right.\right.$

$$\begin{aligned}
& ) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * i + ((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) - (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} \\
& ) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * i) / (((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) + (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} - ((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) - (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} - 16 b^9 d^2 e^{10} + 48 a^8 b d^2 e^{10} + 96 a^4 b^5 d^2 e^{10} + 128 a^6 b^3 d^2 e^{10})) * (((a^5 b^6 + a^5 b^6 i - a^6 + b^6 - 15 a^2 b^4 - a^3 b^3 20i + 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * 2i + \operatorname{atan}((((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) + (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * i + ((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) - (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * i) / (((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) + (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} - ((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) - (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * i) / (((e \cot(c + dx))^{(1/2)}) * (16 a^6 d^3 e^{14} - 16 b^6 d^3 e^{14} + 240 a^2 b^4 d^3 e^{14} - 240 a^4 b^2 d^3 e^{14}) - (32 a^3 d^4 e^{19} - 96 a b^2 d^4 e^{19}) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)})) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} - 16 b^9 d^2 e^{10} + 48 a^8 b d^2 e^{10} + 96 a^4 b^5 d^2 e^{10} + 128 a^6 b^3 d^2 e^{10})) * (((a^5 b^6 + a^5 b^6 i + a^6 - b^6 + 15 a^2 b^4 - a^3 b^3 20i - 15 a^4 b^2) * i) / (4 d^2 e^9))^{(1/2)} * 2i + ((2 a^3 e) / 7 + (2 e \cot(c + dx))^2 * (3 a b^2 - a^3)) / 3 - 2 e \cot(c + dx)^3 * (3 a^2 b - b^3) + (6 a^2 b e \cot(c + dx)) / 5) / (d e^2 * (e \cot(c + dx))^{(7/2)})
\end{aligned}$$



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cot(c + dx))^3}{(e \cot(c + dx))^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*3/(e\*cot(d\*x+c))\*\*(9/2),x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*3/(e\*cot(c + d\*x))\*\*(9/2), x)

$$3.69 \quad \int \frac{(e \cot(c+dx))^{5/2}}{a+b \cot(c+dx)} dx$$

Optimal. Leaf size=325

$$\frac{e^{5/2}(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)} - \frac{e^{5/2}(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2}d(a^2+b^2)}$$

[Out]  $2*a^{(5/2)}*e^{(5/2)}*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/b^{(3/2)}/(a^2+b^2)/d-1/2*(a+b)*e^{(5/2)}*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/2*(a+b)*e^{(5/2)}*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}+1/4*(a-b)*e^{(5/2)}*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}-1/4*(a-b)*e^{(5/2)}*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}-2*e^2*(e*\cot(d*x+c))^{(1/2)}/b/d$

**Rubi [A]** time = 0.66, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3566, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{e^{5/2}(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)} - \frac{e^{5/2}(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)/(a + b\*Cot[c + d\*x]),x]

[Out]  $(2*a^{(5/2)}*e^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[a]*\text{Sqrt}[e])])/(b^{(3/2)}*(a^2 + b^2)*d) - ((a + b)*e^{(5/2)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)*d) + ((a + b)*e^{(5/2)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)*d) - (2*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(b*d) + ((a - b)*e^{(5/2)}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)*d) - ((a - b)*e^{(5/2)}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)*d)$

Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1168

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a,

$c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[-(a*c)]$

### Rule 3534

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}{\sqrt{(b_.)\tan[(e_.) + (f_.)x]}}], x\_Symbol] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[\frac{b*c + d*x^2}{b^2 + x^4}], x], \sqrt{b*\text{Tan}[e + f*x]}], x] /;$   $\text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0]$

### Rule 3566

$\text{Int}[\frac{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}]^{m_.*((c_.) + (d_.)\tan[(e_.) + (f_.)x])^{n_}}, x\_Symbol] \rightarrow \text{Simp}[(b^2*(a + b*\text{Tan}[e + f*x])^{m-2}*(c + d*\text{Tan}[e + f*x])^{n+1})/(d*f*(m+n-1)), x] + \text{Dist}[1/(d*(m+n-1)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{m-3}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a^3*d*(m+n-1) - b^2*(b*c*(m-2) + a*d*(1+n)) + b*d*(m+n-1)*(3*a^2 - b^2)*\text{Tan}[e + f*x] - b^2*(b*c*(m-2) - a*d*(3*m+2*n-4))*\text{Tan}[e + f*x]^2, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{IntegerQ}[2*m] \&\& \text{GtQ}[m, 2] \&\& (\text{GeQ}[n, -1] \mid \mid \text{IntegerQ}[m]) \&\& !( \text{IGtQ}[n, 2] \&\& ( !\text{IntegerQ}[m] \mid \mid (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0]) ) )$

### Rule 3634

$\text{Int}[\frac{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}{(c_.) + (d_.)\tan[(e_.) + (f_.)x]}]^{m_.*((A_.) + (C_.)\tan[(e_.) + (f_.)x])^2}, x\_Symbol] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m*(c + d*x)^n], x], \text{Tan}[e + f*x]], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& \text{EqQ}[A, C]$

### Rule 3653

$\text{Int}[\frac{((c_.) + (d_.)\tan[(e_.) + (f_.)x])^{n_.*((A_.) + (B_.)\tan[(e_.) + (f_.)x]) + (C_.)\tan[(e_.) + (f_.)x])^2}}{(a_.) + (b_.)\tan[(e_.) + (f_.)x]}], x\_Symbol] \rightarrow \text{Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*(1 + \text{Tan}[e + f*x]^2)]/(a + b*\text{Tan}[e + f*x]), x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0] \&\& !\text{LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{5/2}}{a + b \cot(c + dx)} dx &= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{bd} - \frac{2 \int \frac{\frac{ae^3}{2} + \frac{1}{2}be^3 \cot(c+dx) + \frac{1}{2}ae^3 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx}{b} \\
&= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{bd} - \frac{2 \int \frac{\frac{b^2e^3}{2} + \frac{1}{2}abe^3 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{b(a^2 + b^2)} - \frac{(a^3e^3) \int \frac{1 + \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx}{b(a^2 + b^2)} \\
&= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{bd} - \frac{4 \text{Subst} \left( \int \frac{-\frac{1}{2}b^2e^4 - \frac{1}{2}abe^3x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{b(a^2 + b^2)d} - \frac{(a^3e^3) \text{Subst} \left( \int \frac{1}{\sqrt{e \cot(c + dx)}} dx, x, \sqrt{e \cot(c + dx)} \right)}{b(a^2 + b^2)d} \\
&= -\frac{2e^2 \sqrt{e \cot(c + dx)}}{bd} + \frac{(2a^3e^2) \text{Subst} \left( \int \frac{1}{a + \frac{bx^2}{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{b(a^2 + b^2)d} - \frac{((a - b)e^3) \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{e} + \sqrt{e} \cot(c + dx)}{-e - \sqrt{2} \sqrt{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{2\sqrt{2} (a^2 + b^2)d} \\
&= \frac{2a^{5/2}e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{3/2} (a^2 + b^2) d} - \frac{2e^2 \sqrt{e \cot(c + dx)}}{bd} + \frac{((a - b)e^{5/2}) \text{Subst} \left( \int \frac{\sqrt{2} \sqrt{e} + \sqrt{e} \cot(c + dx)}{-e - \sqrt{2} \sqrt{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{2\sqrt{2} (a^2 + b^2)d} \\
&= \frac{2a^{5/2}e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{3/2} (a^2 + b^2) d} - \frac{2e^2 \sqrt{e \cot(c + dx)}}{bd} + \frac{(a - b)e^{5/2} \log(\sqrt{e} + \sqrt{e} \cot(c + dx))}{2\sqrt{2} (a^2 + b^2)d} \\
&= \frac{2a^{5/2}e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{3/2} (a^2 + b^2) d} - \frac{(a + b)e^{5/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a + b)e^{5/2} \tan^{-1} \left( \frac{\sqrt{2} \sqrt{e} + \sqrt{e} \cot(c + dx)}{-e - \sqrt{2} \sqrt{e}} \right)}{2\sqrt{2} (a^2 + b^2)d}
\end{aligned}$$

**Mathematica [C]** time = 0.87, size = 286, normalized size = 0.88

$$(e \cot(c + dx))^{5/2} \left( 8ab^{3/2} \cot^2(c + dx) {}_2F_1 \left( \frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx) \right) - 3 \left( -8a^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}} \right) + 8a^2 \sqrt{b} \sqrt{\cot(c + dx)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)/(a + b\*Cot[c + d\*x]),x]

[Out] ((e\*Cot[c + d\*x])^(5/2)\*(8\*a\*b^(3/2)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] - 3\*(2\*Sqrt[2]\*b^(5/2)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - 2\*Sqrt[2]\*b^(5/2)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - 8\*a^(5/2)\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]] + 8\*a^2\*Sqrt[b]\*Sqrt[Cot[c + d\*x]] + 8\*b^(5/2)\*Sqrt[Cot[c + d\*x]] + Sqrt[2]\*b^(5/2)\*Log[1 - S

```

qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*b^(5/2)*Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(12*b^(3/2)*(a^2 + b^2)*d*Cot[c + d*
x]^(5/2))

```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{5}{2}}}{b \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a), x)
```

**maple** [A] time = 0.67, size = 459, normalized size = 1.41

$$-\frac{2e^2\sqrt{e \cot(dx + c)}}{bd} + \frac{2e^3a^3 \arctan\left(\frac{\sqrt{e \cot(dx+c)} b}{\sqrt{aeb}}\right)}{db(a^2 + b^2)\sqrt{aeb}} + \frac{e^2b(e^2)^{\frac{1}{4}}\sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2 + \sqrt{e^2}}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}}\sqrt{e \cot(dx+c)}\sqrt{2 + \sqrt{e^2}}}\right)}{4d(a^2 + b^2)} + \frac{e^2b(e^2)^{\frac{1}{4}}}{4d(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x)
```

```
[Out] -2*e^2*(e*cot(d*x+c))^(1/2)/b/d+2/d*e^3/b*a^3/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))+1/4/d*e^2/(a^2+b^2)*b*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+1/2/d*e^2/(a^2+b^2)*b*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-1/2/d*e^2/(a^2+b^2)*b*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)+1/4/d*e^3/(a^2+b^2)*a/(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+1/2/d*e^3/(a^2+b^2)*a/(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
```

$(1/2)+1)-1/2/d*e^3/(a^2+b^2)*a/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)+1})$

**maxima [A]** time = 0.54, size = 259, normalized size = 0.80

$$\frac{8a^3e^2 \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^2b+b^3)\sqrt{abe}} + \frac{\frac{2\sqrt{2}(a+b) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a+b) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} - \frac{\sqrt{2}(a-b) \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}}}{a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+b\*cot(d\*x+c)),x, algorithm="maxima")

[Out]  $1/4*(8*a^3*e^2*\arctan(b*\sqrt{e/\tan(d*x+c)})/\sqrt{a*b*e})/((a^2*b+b^3)*\sqrt{a*b*e}) + (2*\sqrt{2}*(a+b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a+b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e})/\sqrt{e} - \sqrt{2}*(a-b)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}) + e + e/\tan(d*x+c))/\sqrt{e} + \sqrt{2}*(a-b)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}) + e + e/\tan(d*x+c))/\sqrt{e})*e^2/(a^2+b^2) - 8*e*\sqrt{e/\tan(d*x+c)}/b)*e/d$

**mupad [B]** time = 1.97, size = 5579, normalized size = 17.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c+d\*x))^(5/2)/(a+b\*cot(c+d\*x)),x)

[Out]  $(\operatorname{atan}(\frac{((32*(e*\cot(c+d*x))^{(1/2)}*(2*a^6*e^{20}-b^6*e^{20}))/b*d^4 + ((32*(12*a^6*b*d^2*e^{18}+a^2*b^5*d^2*e^{18}-15*a^4*b^3*d^2*e^{18}))/b*d^5 + ((32*(e*\cot(c+d*x))^{(1/2)}*(16*a^7*b*d^2*e^{15}-14*a*b^7*d^2*e^{15}+4*a^3*b^5*d^2*e^{15}+2*a^5*b^3*d^2*e^{15}))/b*d^4 - (((32*(4*a*b^8*d^4*e^{13}+8*a^3*b^6*d^4*e^{13}+4*a^5*b^4*d^4*e^{13}))/b*d^5 + (32*(e*\cot(c+d*x))^{(1/2)}*(-a^5*b^3*e^5)^{(1/2)}*(16*b^{10}*d^4*e^{10}+16*a^2*b^8*d^4*e^{10}-16*a^4*b^6*d^4*e^{10}-16*a^6*b^4*d^4*e^{10}))/b^4*d^5*(a^2+b^2)))*(-a^5*b^3*e^5)^{(1/2})/b^3*d*(a^2+b^2)))*(-a^5*b^3*e^5)^{(1/2})/b^3*d*(a^2+b^2)))*(-a^5*b^3*e^5)^{(1/2})/b^3*d*(a^2+b^2)))*(-a^5*b^3*e^5)^{(1/2)}*i)/b^3*d*(a^2+b^2))$







```

3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c + d*x))^(1/2)*
(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4*e^10 +
16*a^2*b^8*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*
(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x)
))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^
5*b^3*d^2*e^15))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(
1/2) - (32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(
b*d^5))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot
(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 -
a^2*d^2 + a*b*d^2*2i)))^(1/2) + (((((32*(4*a*b^8*d^4*e^13 + 8*a^3*b^6*d^4*
e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^(1/2)*(-(e^5*1i)
/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8
*d^4*e^10 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b*d^4))*(-(e^5*1i)
/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)*(
16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*
e^15))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (3
2*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5))*(-
(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x)
)^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4))*(-(e^5*1i)/(4*(b^2*d^2 - a^2*d^2 +
a*b*d^2*2i)))^(1/2) - (64*(a^5*e^23 - a^3*b^2*e^23))/(b*d^5)))*(-(e^5*1i)/
(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*2i

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c + dx))^{\frac{5}{2}}}{a + b \cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)/(a+b\*cot(d\*x+c)),x)

[Out] Integral((e\*cot(c + d\*x))\*\*(5/2)/(a + b\*cot(c + d\*x)), x)

$$3.70 \quad \int \frac{(e \cot(c+dx))^{3/2}}{a+b \cot(c+dx)} dx$$

**Optimal.** Leaf size=302

$$\frac{e^{3/2}(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)} + \frac{e^{3/2}(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2}d(a^2+b^2)}$$

[Out]  $-1/2*(a-b)*e^{(3/2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)}/d*2^{(1/2)}+1/2*(a-b)*e^{(3/2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)}/d*2^{(1/2)}-1/4*(a+b)*e^{(3/2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)}/d*2^{(1/2)}+1/4*(a+b)*e^{(3/2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)}/d*2^{(1/2)}-2*a^{(3/2)}*e^{(3/2)*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/(a^2+b^2)}/d/b^{(1/2)}$

**Rubi [A]** time = 0.38, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$ , Rules used = {3573, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{e^{3/2}(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)} + \frac{e^{3/2}(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*\text{Cot}[c+d*x])^{(3/2)}/(a+b*\text{Cot}[c+d*x]),x]$

[Out]  $(-2*a^{(3/2)}*e^{(3/2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/(\text{Sqrt}[a]*\text{Sqrt}[e])}]/(\text{Sqrt}[b]*(a^2+b^2)*d) - ((a-b)*e^{(3/2)*\text{ArcTan}[1-(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e]})/(\text{Sqrt}[2]*(a^2+b^2)*d) + ((a-b)*e^{(3/2)*\text{ArcTan}[1+(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])]/\text{Sqrt}[e]})/(\text{Sqrt}[2]*(a^2+b^2)*d) - ((a+b)*e^{(3/2)*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]-\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]]]})/(2*\text{Sqrt}[2]*(a^2+b^2)*d) + ((a+b)*e^{(3/2)*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]+\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]]]})/(2*\text{Sqrt}[2]*(a^2+b^2)*d)$

**Rule 63**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*

c)]

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3573

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(3/2)/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(c^2 + d^2), Int[Simp[a^2*c - b^2*c + 2*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] + Dist[(b*c - a*d)^2/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]^(n_))*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{3/2}}{a + b \cot(c + dx)} dx &= \frac{\int \frac{-ae^2 + be^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a^2 + b^2} + \frac{(a^2 e^2) \int \frac{1 + \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a + b \cot(c+dx))} dx}{a^2 + b^2} \\
&= \frac{2 \operatorname{Subst} \left( \int \frac{ae^3 - be^2 x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{(a^2 + b^2) d} + \frac{(a^2 e^2) \operatorname{Subst} \left( \int \frac{1}{\sqrt{-ex} (a - bx)} dx, x, -\cot(c + dx) \right)}{(a^2 + b^2) d} \\
&= -\frac{(2a^2 e) \operatorname{Subst} \left( \int \frac{1}{a + \frac{bx^2}{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{(a^2 + b^2) d} + \frac{((a - b)e^2) \operatorname{Subst} \left( \int \frac{e + x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{(a^2 + b^2) d} \\
&= -\frac{2a^{3/2} e^{3/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{b} (a^2 + b^2) d} - \frac{((a + b)e^{3/2}) \operatorname{Subst} \left( \int \frac{\sqrt{2} \sqrt{e} + 2x}{-e - \sqrt{2} \sqrt{e} x - x^2} dx, x, \sqrt{e \cot(c + dx)} \right)}{2\sqrt{2} (a^2 + b^2) d} \\
&= -\frac{2a^{3/2} e^{3/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{b} (a^2 + b^2) d} - \frac{(a + b)e^{3/2} \log \left( \sqrt{e} + \sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} \right)}{2\sqrt{2} (a^2 + b^2) d} \\
&= -\frac{2a^{3/2} e^{3/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{b} (a^2 + b^2) d} - \frac{(a - b)e^{3/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} (a^2 + b^2) d} + \frac{(a - b)e^{3/2} \tan^{-1} \left( \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} (a^2 + b^2) d}
\end{aligned}$$

**Mathematica [C]** time = 0.57, size = 249, normalized size = 0.82

$$\frac{(e \cot(c + dx))^{3/2} \left( 3a \left( 8\sqrt{a} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}} \right) \right) + \sqrt{2} \sqrt{b} \log \left( \cot(c + dx) - \sqrt{2} \sqrt{\cot(c + dx)} + 1 \right) - \sqrt{2} \sqrt{b} \log \left( \cot(c + dx) + \sqrt{2} \sqrt{\cot(c + dx)} + 1 \right) \right)}{(a^2 + b^2) d}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)/(a + b\*Cot[c + d\*x]),x]

[Out] -1/12\*((e\*Cot[c + d\*x])^(3/2)\*(8\*b^(3/2)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] + 3\*a\*(2\*Sqrt[2]\*Sqrt[b]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 2\*Sqrt[2]\*Sqrt[b]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 8\*Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]] + Sqrt[2]\*Sqrt[b]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Sqrt[b]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])))/(Sqrt[b]\*(a^2 + b^2)\*d\*Cot[c + d\*x]^(3/2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{b \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(3/2)/(b\*cot(d\*x + c) + a), x)

maple [A] time = 0.69, size = 429, normalized size = 1.42

$$\frac{2e^2 a^2 \arctan\left(\frac{\sqrt{e \cot(dx+c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2) \sqrt{aeb}} + \frac{ea(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d(a^2 + b^2)} + \frac{ea(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{2d(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x)

[Out] 
$$\begin{aligned} & -2/d*e^2*a^2/(a^2+b^2)/(a*e*b)^{(1/2)}*\arctan((e*cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)}) \\ & +1/4/d*e/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)} \\ & *(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot \\ & (d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))+1/2/d*e/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)} \\ & )*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/2/d*e/(a^2+b^2)*a*(e \\ & ^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/4/d \\ & *e^2/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d* \\ & x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)} \\ & )*2^{(1/2)}+(e^2)^{(1/2)}))-1/2/d*e^2/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan \\ & (2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2/d*e^2/(a^2+b^2)*b/(e^2)^{(1/4)} \\ & )*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1) \end{aligned}$$

**maxima [A]** time = 0.49, size = 236, normalized size = 0.78

$$\frac{8a^2e \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right) - \frac{2\sqrt{2}(a-b) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right) + 2\sqrt{2}(a-b) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right) + \frac{\sqrt{2}(a+b) \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{\sqrt{e}}}{\sqrt{abe}(a^2+b^2)} - \frac{\sqrt{2}(a+b) \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/4*(8*a^2*e*\arctan(b*\sqrt{e/\tan(d*x + c)})/\sqrt{a*b*e})/(\sqrt{a*b*e}*(a^2 + b^2)) - (2*\sqrt{2}*(a - b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a - b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + \sqrt{2}*(a + b)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e} - \sqrt{2}*(a + b)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e})*e/(a^2 + b^2))*e/d$$

**mupad [B]** time = 1.63, size = 5129, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(3/2)/(a + b\*cot(c + d\*x)),x)

[Out] 
$$\operatorname{atan}\left(\frac{\left(\frac{32(4a^2b^6d^4e^{12} + 8a^4b^4d^4e^{12} + 4a^6b^2d^4e^{12})}{d^5} - (32(e \cot(c + dx))^{1/2}((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2}(16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10})/d^4)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} + (32(e \cot(c + dx))^{1/2}(14ab^6d^2e^{13} - 4a^3b^4d^2e^{13} + 14a^5b^2d^2e^{13})/d^4)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} + (32(ab^5d^2e^{15} + 4a^5bd^2e^{15} - 15a^3b^3d^2e^{15})/d^5)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2}(b^5e^{16} + 2a^4b^4e^{16})/d^4)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} * 1i - \left(\frac{32(4a^2b^6d^4e^{12} + 8a^4b^4d^4e^{12} + 4a^6b^2d^4e^{12})}{d^5} + (32(e \cot(c + dx))^{1/2}((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2}(16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10})/d^4)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} + (32(e \cot(c + dx))^{1/2}(14ab^6d^2e^{13} - 4a^3b^4d^2e^{13} + 14a^5b^2d^2e^{13})/d^4)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} + (32(ab^5d^2e^{15} + 4a^5bd^2e^{15} - 15a^3b^3d^2e^{15})/d^5)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2}(b^5e^{16} + 2a^4b^4e^{16})/d^4)((e^3 1i)/(4(b^2d^2 - a^2d^2 + ab^2d^2 2i)))^{1/2} * 1i\right)}{4(b^2d^2 - a^2d^2 + ab^2d^2 2i)}\right)$$





$$\begin{aligned}
& 6 + 2*a^4*b*e^{16})/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)*1i)/((((((32*(4*a^2*b^6*d^4*e^{12} + 8*a^4*b^4*d^4*e^{12} + 4*a^6*b^2*d^4*e^{12}))/d^5 - (32*(e*cot(c + d*x))^{(1/2)}*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))))^{(1/2)}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10}))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} + (32*(e*cot(c + d*x))^{(1/2)}*(14*a*b^6*d^2*e^{13} - 4*a^3*b^4*d^2*e^{13} + 14*a^5*b^2*d^2*e^{13}))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} + (32*(a*b^5*d^2*e^{15} + 4*a^5*b*d^2*e^{15} - 15*a^3*b^3*d^2*e^{15}))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} - (32*(e*cot(c + d*x))^{(1/2)}*(b^5*e^{16} + 2*a^4*b*e^{16}))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} + (((((32*(4*a^2*b^6*d^4*e^{12} + 8*a^4*b^4*d^4*e^{12} + 4*a^6*b^2*d^4*e^{12}))/d^5 + (32*(e*cot(c + d*x))^{(1/2)}*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2))))^{(1/2)}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10}))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} - (32*(e*cot(c + d*x))^{(1/2)}*(14*a*b^6*d^2*e^{13} - 4*a^3*b^4*d^2*e^{13} + 14*a^5*b^2*d^2*e^{13}))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} + (32*(a*b^5*d^2*e^{15} + 4*a^5*b*d^2*e^{15} - 15*a^3*b^3*d^2*e^{15}))/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} + (32*(e*cot(c + d*x))^{(1/2)}*(b^5*e^{16} + 2*a^4*b*e^{16}))/d^4)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)} + (64*a^2*b^2*e^{18})/d^5)*(e^3/(4*(b^2*d^2*1i - a^2*d^2*1i + 2*a*b*d^2)))^{(1/2)}*2i - (atan(-((((((32*(a*b^5*d^2*e^{15} + 4*a^5*b*d^2*e^{15} - 15*a^3*b^3*d^2*e^{15}))/d^5 + (((((32*(4*a^2*b^6*d^4*e^{12} + 8*a^4*b^4*d^4*e^{12} + 4*a^6*b^2*d^4*e^{12}))/d^5 - (32*(e*cot(c + d*x))^{(1/2)}*(-a^3*b*e^3)^{(1/2)}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10}))/d^4*(b^3*d + a^2*b*d))))^{(1/2)}*(-a^3*b*e^3)^{(1/2)}))/(b^3*d + a^2*b*d) + (32*(e*cot(c + d*x))^{(1/2)}*(14*a*b^6*d^2*e^{13} - 4*a^3*b^4*d^2*e^{13} + 14*a^5*b^2*d^2*e^{13}))/d^4*(-a^3*b*e^3)^{(1/2)}))/(b^3*d + a^2*b*d) - (32*(e*cot(c + d*x))^{(1/2)}*(b^5*e^{16} + 2*a^4*b*e^{16}))/d^4*(-a^3*b*e^3)^{(1/2)}*1i)/(b^3*d + a^2*b*d) - (((((32*(a*b^5*d^2*e^{15} + 4*a^5*b*d^2*e^{15} - 15*a^3*b^3*d^2*e^{15}))/d^5 + (((((32*(4*a^2*b^6*d^4*e^{12} + 8*a^4*b^4*d^4*e^{12} + 4*a^6*b^2*d^4*e^{12}))/d^5 + (32*(e*cot(c + d*x))^{(1/2)}*(-a^3*b*e^3)^{(1/2)}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10}))/d^4*(b^3*d + a^2*b*d))))^{(1/2)}*(-a^3*b*e^3)^{(1/2)}))/(b^3*d + a^2*b*d) - (32*(e*cot(c + d*x))^{(1/2)}*(14*a*b^6*d^2*e^{13} - 4*a^3*b^4*d^2*e^{13} + 14*a^5*b^2*d^2*e^{13}))/d^4*(-a^3*b*e^3)^{(1/2)}))/(b^3*d + a^2*b*d) + (32*(e*cot(c + d*x))^{(1/2)}*(b^5*e^{16} + 2*a^4*b*e^{16}))/d^4*(-a^3*b*e^3)^{(1/2)}*1i)/(b^3*d + a^2*b*d)/((((((32*(a*b^5*d^2*e^{15} + 4*a^5*b*d^2*e^{15} - 15*a^3*b^3*d^2*e^{15}))/d^5 + (((((32*(4*a^2*b^6*d^4*e^{12} + 8*a^4*b^4*d^4*e^{12} + 4*a^6*b^2*d^4*e^{12}))/d^5 - (32*(e*cot(c + d*x))^{(1/2)}*(-a^3*b*e^3)^{(1/2)}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10}))/d^4*(b^3*d + a^2*b*d))))^{(1/2)}*(-a^3*b*e^3)^{(1/2)}))/(b^3*d + a^2*b*d) + (32*(e*cot(c + d*x))^{(1/2)}*(14*a*b^6*d^2*e^{13} - 4*a^3*b^4*d^2*e^{13} + 14*a^5*b^2*d^2*e^{13}))/d^4*(-a^3*b*e^3)^{(1/2)}))/(b^3*d + a^2*b*d) - (32*(e*cot(c + d*x))^{(1/2)}*(b^5*e^{16} +
\end{aligned}$$

```

2*a^4*b*e^16))/d^4)*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) + (((((32*(a*b^5*
d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5 + (((((32*(4*a^2*b^
6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c +
d*x))^(1/2)*(-a^3*b*e^3)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16
*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^4*(b^3*d + a^2*b*d)))*(-a^3*b*
e^3)^(1/2))/(b^3*d + a^2*b*d) - (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^
13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*(-a^3*b*e^3)^(1/2))/(b
^3*d + a^2*b*d))*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b*d) + (32*(e*cot(c + d*x
))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*(-a^3*b*e^3)^(1/2))/(b^3*d + a^2*b
*d) + (64*a^2*b^2*e^18)/d^5))*(-a^3*b*e^3)^(1/2)*2i)/(b^3*d + a^2*b*d)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c + dx))^{\frac{3}{2}}}{a + b \cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(3/2)/(a+b\*cot(d\*x+c)),x)

[Out] Integral((e\*cot(c + d\*x))\*\*(3/2)/(a + b\*cot(c + d\*x)), x)

$$3.71 \quad \int \frac{\sqrt{e \cot(c+dx)}}{a+b \cot(c+dx)} dx$$

Optimal. Leaf size=302

$$\frac{\sqrt{e}(a-b) \log\left(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{\sqrt{e}(a-b) \log\left(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)}\right)}{2\sqrt{2}d(a^2+b^2)}$$

[Out]  $1/2*(a+b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*e^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}-1/2*(a+b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})*e^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}-1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}*e^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}+1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}*e^{(1/2)}/(a^2+b^2)/d*2^{(1/2)}+2*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})*a^{(1/2)}*b^{(1/2)}*e^{(1/2)}/(a^2+b^2)/d$

**Rubi [A]** time = 0.38, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$ , Rules used = {3572, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{\sqrt{e}(a-b) \log\left(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e}\right)}{2\sqrt{2}d(a^2+b^2)} + \frac{\sqrt{e}(a-b) \log\left(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)}\right)}{2\sqrt{2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]/(a + b\*Cot[c + d\*x]),x]

[Out]  $(2*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[e]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(\text{Sqrt}[a]*\text{Sqrt}[e]))/((a^2 + b^2)*d) + ((a + b)*\text{Sqrt}[e]*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)*d) - ((a + b)*\text{Sqrt}[e]*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)*d) - ((a - b)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)*d) + ((a - b)*\text{Sqrt}[e]*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)*d)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*

c)]

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3572

```
Int[Sqrt[(a_) + (b_)*tan[(e_) + (f_)*(x_)]]/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/(c^2 + d^2), Int[Simp[a*c + b*d + (b*c - a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Dist[(d*(b*c - a*d))/(c^2 + d^2), Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e \cot(c+dx)}}{a+b \cot(c+dx)} dx &= \frac{\int \frac{be+ae \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a^2+b^2} - \frac{(abe) \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))} dx}{a^2+b^2} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{-be^2-axe^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} - \frac{(abe) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)} dx, x, -\cot(c+dx)\right)}{(a^2+b^2)d} \\
&= \frac{(2ab) \operatorname{Subst}\left(\int \frac{1}{a+\frac{bx^2}{e}} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} + \frac{((a-b)e) \operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} \\
&= \frac{2\sqrt{a}\sqrt{b}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)d} - \frac{((a-b)\sqrt{e}) \operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e+2x}}{-e-\sqrt{2}\sqrt{e}x-x^2} dx, x, \sqrt{e \cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)d} \\
&= \frac{2\sqrt{a}\sqrt{b}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)d} - \frac{(a-b)\sqrt{e} \log\left(\sqrt{e} + \sqrt{e} \cot(c+dx) - \sqrt{2}\sqrt{e \cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)d} \\
&= \frac{2\sqrt{a}\sqrt{b}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)d} + \frac{(a+b)\sqrt{e} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)d} - \frac{(a+b)\sqrt{e}}{\sqrt{2}(a^2+b^2)d}
\end{aligned}$$

**Mathematica** [C] time = 0.28, size = 226, normalized size = 0.75

$$\frac{\sqrt{e \cot(c+dx)} \left( 24\sqrt{a}\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right) - 8a \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right) + 3\sqrt{2}b \log\left(\cot(c+dx) + \sqrt{2}\sqrt{e \cot(c+dx)}\right) \right)}{(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]/(a + b\*Cot[c + d\*x]),x]

[Out] (Sqrt[e\*Cot[c + d\*x]]\*(6\*Sqrt[2]\*b\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - 6\*Sqrt[2]\*b\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 24\*Sqrt[a]\*Sqrt[b]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]] - 8\*a\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] + 3\*Sqrt[2]\*b\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - 3\*Sqrt[2]\*b\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/(12\*(a^2 + b^2)\*d\*Sqrt[Cot[c + d\*x]])

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out









$$\begin{aligned}
& \left( 4e^{11} + 12a^5b^3d^4e^{11} \right) / d^5 + (32(e \cot(c + dx))^{1/2} (-e / (4(b^2d^2i - a^2d^2i + 2ab^2d^2)))^{1/2} (16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10})) / d^4 (-e / (4(b^2d^2i - a^2d^2i + 2ab^2d^2)))^{1/2} - (32(e \cot(c + dx))^{1/2} (20a^3b^4d^2e^{11} - 14ab^6d^2e^{11} + 2a^5b^2d^2e^{11})) / d^4 (-e / (4(b^2d^2i - a^2d^2i + 2ab^2d^2)))^{1/2} + (32(e \cot(c + dx))^{1/2} (b^5e^{12} - 2a^2b^3e^{12})) / d^4 (-e / (4(b^2d^2i - a^2d^2i + 2ab^2d^2)))^{1/2} + (64ab^3e^{13}) / d^5 (-e / (4(b^2d^2i - a^2d^2i + 2ab^2d^2)))^{1/2} * 2i - \operatorname{atan}(\left( \left( \left( 32(13a^2b^4d^2e^{12} + a^4b^2d^2e^{12}) / d^5 + \left( \left( 32(12ab^7d^4e^{11} + 24a^3b^5d^4e^{11} + 12a^5b^3d^4e^{11}) / d^5 - (32(e \cot(c + dx))^{1/2} (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} (16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10}) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} + (32(e \cot(c + dx))^{1/2} (20a^3b^4d^2e^{11} - 14ab^6d^2e^{11} + 2a^5b^2d^2e^{11})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} \right) \right) (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2} (b^5e^{12} - 2a^2b^3e^{12})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} * 1i - \left( \left( \left( 32(13a^2b^4d^2e^{12} + a^4b^2d^2e^{12}) / d^5 + \left( \left( 32(12ab^7d^4e^{11} + 24a^3b^5d^4e^{11} + 12a^5b^3d^4e^{11}) / d^5 + (32(e \cot(c + dx))^{1/2} (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} (16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10}) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2} (20a^3b^4d^2e^{11} - 14ab^6d^2e^{11} + 2a^5b^2d^2e^{11})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} \right) \right) (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} + (32(e \cot(c + dx))^{1/2} (b^5e^{12} - 2a^2b^3e^{12})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} * 1i) / \left( \left( \left( 32(13a^2b^4d^2e^{12} + a^4b^2d^2e^{12}) / d^5 + \left( \left( 32(12ab^7d^4e^{11} + 24a^3b^5d^4e^{11} + 12a^5b^3d^4e^{11}) / d^5 - (32(e \cot(c + dx))^{1/2} (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} (16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10}) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2} (20a^3b^4d^2e^{11} - 14ab^6d^2e^{11} + 2a^5b^2d^2e^{11})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} \right) \right) (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} + (32(e \cot(c + dx))^{1/2} (20a^3b^4d^2e^{11} - 14ab^6d^2e^{11} + 2a^5b^2d^2e^{11})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2} (b^5e^{12} - 2a^2b^3e^{12})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} + \left( \left( \left( 32(13a^2b^4d^2e^{12} + a^4b^2d^2e^{12}) / d^5 + \left( \left( 32(12ab^7d^4e^{11} + 24a^3b^5d^4e^{11} + 12a^5b^3d^4e^{11}) / d^5 + (32(e \cot(c + dx))^{1/2} (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} (16b^9d^4e^{10} + 16a^2b^7d^4e^{10} - 16a^4b^5d^4e^{10} - 16a^6b^3d^4e^{10}) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} - (32(e \cot(c + dx))^{1/2} (20a^3b^4d^2e^{11} - 14ab^6d^2e^{11} + 2a^5b^2d^2e^{11})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} \right) \right) (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} + (32(e \cot(c + dx))^{1/2} (b^5e^{12} - 2a^2b^3e^{12})) / d^4 (-e1i) / (4(b^2d^2 - a^2d^2 + ab^2d^2 * 2i)))^{1/2} \right) \right)
\end{aligned}$$

$^{(1/2)} + (64*a*b^3*e^{13}/d^5))*(-(e*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i))^{(1/2)*2i}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(c + dx)}}{a + b \cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)/(a+b\*cot(d\*x+c)),x)

[Out] Integral(sqrt(e\*cot(c + d\*x))/(a + b\*cot(c + d\*x)), x)

$$3.72 \quad \int \frac{1}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx$$

**Optimal.** Leaf size=302

$$\frac{(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)} - \frac{(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)}$$

[Out]  $\frac{1}{2}*(a-b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}/e^{(1/2)}-1/2*(a-b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}/e^{(1/2)}+1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}/e^{(1/2)}-1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)/d*2^{(1/2)}/e^{(1/2)}-2*b^{(3/2)}*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d/a^{(1/2)}/e^{(1/2)}$

**Rubi [A]** time = 0.37, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$ , Rules used = {3574, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)} - \frac{(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])),x]

[Out]  $(-2*b^{(3/2)}*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)*d*Sqrt[e]) + ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) - ((a - b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) + ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d*Sqrt[e]) - ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d*Sqrt[e])$

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 1168

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*

c)]

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3574

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)/((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[1/(c^2 + d^2), Int[(a + b*Tan[e + f*x])^m*(c - d*Tan[e + f*x]), x], x] + Dist[d^2/(c^2 + d^2), Int[((a + b*Tan[e + f*x])^m*(1 + Tan[e + f*x]^2))/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)]^2), x_Symbol] :> Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))} dx &= \frac{\int \frac{a-b \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a^2+b^2} + \frac{b^2 \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))} dx}{a^2+b^2} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{-ae+bx^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-bx)} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} \\
&= -\frac{(a-b) \operatorname{Subst}\left(\int \frac{e+x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} - \frac{(a+b) \operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)}\right)}{(a^2+b^2)d} \\
&= -\frac{2b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{\sqrt{a} (a^2+b^2) d \sqrt{e}} - \frac{(a-b) \operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2} \sqrt{e} x+x^2} dx, x, \sqrt{e \cot(c+dx)}\right)}{2(a^2+b^2)d} \\
&= -\frac{2b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{\sqrt{a} (a^2+b^2) d \sqrt{e}} + \frac{(a+b) \log\left(\sqrt{e} + \sqrt{e} \cot(c+dx) - \sqrt{2}\right)}{2\sqrt{2} (a^2+b^2) d \sqrt{e}} \\
&= -\frac{2b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{\sqrt{a} (a^2+b^2) d \sqrt{e}} + \frac{(a-b) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2} (a^2+b^2) d \sqrt{e}} - \frac{(a+b) \log\left(\sqrt{e} + \sqrt{e} \cot(c+dx) - \sqrt{2}\right)}{2\sqrt{2} (a^2+b^2) d \sqrt{e}}
\end{aligned}$$

**Mathematica [C]** time = 0.25, size = 248, normalized size = 0.82

$$\sqrt{\cot(c+dx)} \left( -\frac{2b \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)}{3(a^2+b^2)} - \frac{a(2\sqrt{2} \log(\cot(c+dx) - \sqrt{2} \sqrt{\cot(c+dx)} + 1) - 2\sqrt{2} \log(\cot(c+dx) + \sqrt{2} \sqrt{\cot(c+dx)} + 1))}{8(a^2+b^2)} \right) / (d \sqrt{e \cot(c+dx)})$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])),x]

[Out] -((Sqrt[Cot[c + d\*x]]\*((2\*b^(3/2)\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]])/(Sqrt[a]\*(a^2 + b^2)) - (2\*b\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]/(3\*(a^2 + b^2)) - (a\*(4\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])] + 2\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - 2\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/(8\*(a^2 + b^2))))/(d\*Sqrt[e\*Cot[c + d\*x]])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cot(dx+c) + a) \sqrt{e \cot(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/((b\*cot(d\*x + c) + a)\*sqrt(e\*cot(d\*x + c))), x)

maple [A] time = 0.71, size = 423, normalized size = 1.40

$$\frac{2b^2 \arctan\left(\frac{\sqrt{e \cot(dx+c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2) \sqrt{aeb}} - \frac{a(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4de(a^2 + b^2)} - \frac{a(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{2de(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c)),x)

[Out] 
$$\begin{aligned} & -2/d*b^2/(a^2+b^2)/(a*e*b)^{(1/2)}*\arctan((e*cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)}) \\ & -1/4/d/e/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & -1/2/d/e/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)+1/2/d/e/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1) \\ & +1/4/d/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & +1/2/d/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/2/d/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1) \end{aligned}$$

**maxima** [A] time = 0.53, size = 239, normalized size = 0.79

$$e^{\left( \frac{8b^2 \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{\sqrt{abe}(a^2+b^2)e} + \frac{2\sqrt{2}(a-b)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a-b)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{\sqrt{2}(a+b)\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{(a^2+b^2)e} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/4*e*(8*b^2*\arctan(b*\sqrt{e/\tan(d*x+c)})/\sqrt{a*b*e})/(\sqrt{a*b*e}*(a^2+b^2)*e) + (2*\sqrt{2}*(a-b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(d*x+c)})/\sqrt{e}))/\sqrt{e} + 2*\sqrt{2}*(a-b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(d*x+c)})/\sqrt{e}))/\sqrt{e} + \sqrt{2}*(a+b)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}) + e + e/\tan(d*x+c))/\sqrt{e} - \sqrt{2}*(a+b)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}) + e + e/\tan(d*x+c))/\sqrt{e})/((a^2+b^2)*e))/d$

**mupad** [B] time = 1.77, size = 4871, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c+d\*x))^(1/2)\*(a+b\*cot(c+d\*x))),x)

[Out]  $\operatorname{atan}\left(\frac{\left(\frac{32*(5*a*b^5*e^9 + a^3*b^3*e^9)}{d^3} - \left(\frac{1}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}*\left(\frac{32*(16*b^8*d^2*e^{10} + 28*a^2*b^6*d^2*e^{10} + 8*a^4*b^4*d^2*e^{10} - 4*a^6*b^2*d^2*e^{10})}{d^3} - \left(\frac{16*(e*\cot(c+d*x))^{1/2}}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10})}{d^4}\right)/2 - \left(\frac{32*(e*\cot(c+d*x))^{1/2}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9)}{d^4}*\left(\frac{1}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}\right)/2}{\left(\frac{1}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}}\right)/2 + \frac{96*b^5*e^8*(e*\cot(c+d*x))^{1/2}}{d^4}*\left(\frac{1}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}*1i)/2 - \left(\frac{32*(5*a*b^5*e^9 + a^3*b^3*e^9)}{d^3} - \left(\frac{1}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}*\left(\frac{32*(16*b^8*d^2*e^{10} + 28*a^2*b^6*d^2*e^{10} + 8*a^4*b^4*d^2*e^{10} - 4*a^6*b^2*d^2*e^{10})}{d^3} + \left(\frac{16*(e*\cot(c+d*x))^{1/2}}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}*(16*b^9*d^4*e^{10} + 16*a^2*b^7*d^4*e^{10} - 16*a^4*b^5*d^4*e^{10} - 16*a^6*b^3*d^4*e^{10})}{d^4}\right)/2 + \left(\frac{32*(e*\cot(c+d*x))^{1/2}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9)}{d^4}*\left(\frac{1}{b^2*d^2*e^{1i} - a^2*d^2*e^{1i} + 2*a*b*d^2*e}\right)^{1/2}\right)/2$

$$\begin{aligned}
& 2*a^5*b^2*d^2*e^9)/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2 - (96*b^5*e^8*(e*\cot(c + d*x))^{(1/2)})/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)}*1i)/2)/((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)}*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (16*(e*\cot(c + d*x))^{(1/2)}*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)}*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4))/2 - (32*(e*\cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2 + (96*b^5*e^8*(e*\cot(c + d*x))^{(1/2)})/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2 + (((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)}*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (16*(e*\cot(c + d*x))^{(1/2)}*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)}*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4))/2 + (32*(e*\cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2 - (96*b^5*e^8*(e*\cot(c + d*x))^{(1/2)})/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)})/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^{(1/2)}*1i + \operatorname{atan}((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*\cot(c + d*x))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)} - (32*(e*\cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)} + (96*b^5*e^8*(e*\cot(c + d*x))^{(1/2)})/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*1i - (((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*\cot(c + d*x))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)} + (32*(e*\cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)} - (96*b^5*e^8*(e*\cot(c + d*x))^{(1/2)})/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*1i)/((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*\cot(c + d*x))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)} + (32*(e*\cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)} - (96*b^5*e^8*(e*\cot(c + d*x))^{(1/2)})/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i))))^{(1/2)}*1i)/
\end{aligned}$$

$$\begin{aligned}
& + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/ \\
& d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)}*(1i/(4*(b^2*d^2 \\
& *e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)} + (96*b^5*e^8*(e*cot(c + d*x))^{(1/2)} \\
& )/d^4)*(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)} + (((32*(5*a*b \\
& ^5*e^9 + a^3*b^3*e^9))/d^3 - (((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + \\
& 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^{(1/2)} \\
& *(1i/(4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)}*(16*b^9*d^4*e^10 + 1 \\
& 6*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*(1i/( \\
& 4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)} + (32*(e*cot(c + d*x))^{(1/ \\
& 2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1i/(4* \\
& (b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)}*(1i/(4*(b^2*d^2*e - a^2*d^2 \\
& *e + a*b*d^2*e*2i)))^{(1/2)} - (96*b^5*e^8*(e*cot(c + d*x))^{(1/2)})/d^4)*(1i/( \\
& 4*(b^2*d^2*e - a^2*d^2*e + a*b*d^2*e*2i)))^{(1/2)}))*(1i/(4*(b^2*d^2*e - a^2* \\
& d^2*e + a*b*d^2*e*2i)))^{(1/2)}*2i + (atan(((((((32*(5*a*b^5*e^9 + a^3*b^3*e^ \\
& 9))/d^3 - ((((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^ \\
& 10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*cot(c + d*x))^{(1/2)}*(-a*b^3*e)^{(1/2)} \\
& *(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3* \\
& d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2* \\
& d*e) - (32*(e*cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2 \\
& *a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e) \\
& ^{(1/2)})/(a^3*d*e + a*b^2*d*e) + (96*b^5*e^8*(e*cot(c + d*x))^{(1/2)})/d^4)*(- \\
& a*b^3*e)^{(1/2)}*1i)/(a^3*d*e + a*b^2*d*e) - ((((((32*(5*a*b^5*e^9 + a^3*b^3*e^ \\
& 9))/d^3 - ((((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^ \\
& 10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^{(1/2)}*(-a*b^3*e)^{(1/2)} \\
& )*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3 \\
& *d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2 \\
& *d*e) + (32*(e*cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + \\
& 2*a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e) \\
& ^{(1/2)})/(a^3*d*e + a*b^2*d*e) - (96*b^5*e^8*(e*cot(c + d*x))^{(1/2)})/d^4)*( \\
& -a*b^3*e)^{(1/2)}*1i)/(a^3*d*e + a*b^2*d*e)/(((((((32*(5*a*b^5*e^9 + a^3*b^3*e^ \\
& 9))/d^3 - ((((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^ \\
& 10 - 4*a^6*b^2*d^2*e^10))/d^3 - (32*(e*cot(c + d*x))^{(1/2)}*(-a*b^3*e)^{(1/ \\
& 2)}*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^ \\
& 3*d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^ \\
& 2*d*e) - (32*(e*cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + \\
& 2*a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2*d*e))*(-a*b^3* \\
& e)^{(1/2)})/(a^3*d*e + a*b^2*d*e) + (96*b^5*e^8*(e*cot(c + d*x))^{(1/2)})/d^4)* \\
& (-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2*d*e) + ((((((32*(5*a*b^5*e^9 + a^3*b^3*e^ \\
& 9))/d^3 - ((((((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^ \\
& 10 - 4*a^6*b^2*d^2*e^10))/d^3 + (32*(e*cot(c + d*x))^{(1/2)}*(-a*b^3*e)^{(1/2)} \\
& *(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3* \\
& d^4*e^10))/(d^4*(a^3*d*e + a*b^2*d*e)))*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2* \\
& d*e) + (32*(e*cot(c + d*x))^{(1/2)}*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2 \\
& *a^5*b^2*d^2*e^9))/d^4)*(-a*b^3*e)^{(1/2)})/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e) \\
& ^{(1/2)})/(a^3*d*e + a*b^2*d*e) - (96*b^5*e^8*(e*cot(c + d*x))^{(1/2)})/d^4)*(-
\end{aligned}$$

$a*b^3*e^{(1/2)}/(a^3*d*e + a*b^2*d*e))*(-a*b^3*e^{(1/2)*2i}/(a^3*d*e + a*b^2*d*e)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(1/2)/(a+b\*cot(d\*x+c)), x)

[Out] Integral(1/(sqrt(e\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))), x)

$$3.73 \quad \int \frac{1}{(e \cot(c+dx))^{3/2}(a+b \cot(c+dx))} dx$$

**Optimal.** Leaf size=325

$$\frac{(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2 + b^2)} - \frac{(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2 + b^2)}$$

[Out]  $2*b^{(5/2)}*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/a^{(3/2)}/(a^2+b^2)/d/e^{(3/2)}-1/2*(a+b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d/e^{(3/2)}*2^{(1/2)}+1/2*(a+b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d/e^{(3/2)}*2^{(1/2)}+1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)/d/e^{(3/2)}*2^{(1/2)}-1/4*(a-b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)/d/e^{(3/2)}*2^{(1/2)}+2/a/d/e/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.66, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3569, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{(a-b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2 + b^2)} - \frac{(a-b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])),x]

[Out]  $(2*b^{(5/2)}*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(a^{(3/2)}*(a^2 + b^2)*d*e^{(3/2)}) - ((a + b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^{(3/2)}) + ((a + b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^{(3/2)}) + 2/(a*d*e*Sqrt[e*Cot[c + d*x]]) + ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^{(3/2)}) - ((a - b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^{(3/2)})$

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1168

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x]] /; FreeQ[{a,

c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*c)]

### Rule 3534

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3569

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), x] + Dist[1/((m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3653

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2)/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d\*Tan[e + f\*x])^n\*Simp[b\*B + a\*(A - C) + (a\*B - b\*(A - C))\*Tan[e + f\*x], x], x], x] + Dist[(A\*b^2 - a\*b\*B + a^2\*C)/(a^2 + b^2), Int[((c + d\*Tan[e + f\*x])^n\*(1 + Tan[e + f\*x]^2))/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

### Rubi steps



$$\begin{aligned}
\int \frac{1}{(e \cot(c+dx))^{3/2}(a+b \cot(c+dx))} dx &= \frac{2}{ade\sqrt{e \cot(c+dx)}} + \frac{2 \int \frac{-\frac{be^2}{2} - \frac{1}{2}ae^2 \cot(c+dx) - \frac{1}{2}be^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))} dx}{ae^3} \\
&= \frac{2}{ade\sqrt{e \cot(c+dx)}} + \frac{2 \int \frac{-\frac{1}{2}abe^2 - \frac{1}{2}a^2e^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a(a^2+b^2)e^3} - \frac{b^3 \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{a(a^2+b^2)e^3} \\
&= \frac{2}{ade\sqrt{e \cot(c+dx)}} + \frac{4 \text{Subst} \left( \int \frac{\frac{1}{2}abe^3 + \frac{1}{2}a^2e^2x^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)} \right)}{a(a^2+b^2)de^3} \\
&= \frac{2}{ade\sqrt{e \cot(c+dx)}} + \frac{(2b^3) \text{Subst} \left( \int \frac{1}{\frac{a+bx^2}{e}} dx, x, \sqrt{e \cot(c+dx)} \right)}{a(a^2+b^2)de^2} \\
&= \frac{2b^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{3/2}(a^2+b^2)de^{3/2}} + \frac{2}{ade\sqrt{e \cot(c+dx)}} + \frac{(a-b) \text{Subst} \left( \int \frac{1}{\sqrt{e}} dx, x, \sqrt{e \cot(c+dx)} \right)}{a(a^2+b^2)de^{3/2}} \\
&= \frac{2b^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{3/2}(a^2+b^2)de^{3/2}} + \frac{2}{ade\sqrt{e \cot(c+dx)}} + \frac{(a-b) \log(\sqrt{e})}{a(a^2+b^2)de^{3/2}} \\
&= \frac{2b^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{3/2}(a^2+b^2)de^{3/2}} - \frac{(a+b) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2}(a^2+b^2)de^{3/2}} + \frac{(a-b) \log(\sqrt{e})}{a(a^2+b^2)de^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.45, size = 198, normalized size = 0.61

$$8b^2 {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; -\frac{b \cot(c+dx)}{a} \right) + a \left( 8a {}_2F_1 \left( -\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c+dx) \right) + \sqrt{2} b \sqrt{\cot(c+dx)} \left( -\log(\cot(c+dx) - \sqrt{e \cot(c+dx)}) - \log(\cot(c+dx) + \sqrt{e \cot(c+dx)}) \right) \right)$$

4ade

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])),x]

[Out] (8\*b^2\*Hypergeometric2F1[-1/2, 1, 1/2, -((b\*Cot[c + d\*x])/a)] + a\*(8\*a\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2] + Sqrt[2]\*b\*Sqrt[Cot[c + d\*x]]\*(-2\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] + Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])))/(4\*a\*(a^2 + b^2)\*d\*e\*Sqrt[e\*Cot[c + d\*x]])

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cot(dx+c) + a) (e \cot(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/((b\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(3/2)), x)

**maple** [A] time = 0.61, size = 459, normalized size = 1.41

$$\frac{2b^3 \arctan\left(\frac{\sqrt{e \cot(dx+c)} b}{\sqrt{aeb}}\right)}{dea(a^2 + b^2)\sqrt{aeb}} + \frac{b(e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d e^2 (a^2 + b^2)} + \frac{b(e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{2d e^2 (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c)),x)

[Out]  $\frac{2}{d} \frac{e}{a} \frac{b^3}{(a^2+b^2)} \frac{1}{(aeb)^{1/2}} \arctan\left(\frac{(e \cot(dx+c))^{1/2} b}{(aeb)^{1/2}}\right) + \frac{1}{4} \frac{d}{e} \frac{1}{(a^2+b^2)} \frac{b}{(e^2)^{1/4}} \frac{2^{1/2}}{(e \cot(dx+c))^{1/2}} \ln\left(\frac{(e \cot(dx+c))^{1/2} + (e^2)^{1/4} \sqrt{e \cot(dx+c)}}{(e \cot(dx+c))^{1/2} - (e^2)^{1/4} \sqrt{e \cot(dx+c)}}\right) + \frac{1}{2} \frac{d}{e} \frac{1}{(a^2+b^2)} \frac{b}{(e^2)^{1/4}} \frac{2^{1/2}}{(e \cot(dx+c))^{1/2}} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2}}\right) - \frac{1}{2} \frac{d}{e} \frac{1}{(a^2+b^2)} \frac{b}{(e^2)^{1/4}} \frac{2^{1/2}}{(e \cot(dx+c))^{1/2}} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2}}\right) + \frac{1}{4} \frac{d}{e} \frac{1}{(a^2+b^2)} \frac{a}{(e^2)^{1/4}} \frac{2^{1/2}}{(e \cot(dx+c))^{1/2}} \ln\left(\frac{(e \cot(dx+c))^{1/2} - (e^2)^{1/4} \sqrt{e \cot(dx+c)}}{(e \cot(dx+c))^{1/2} + (e^2)^{1/4} \sqrt{e \cot(dx+c)}}\right) + \frac{1}{2} \frac{d}{e} \frac{1}{(a^2+b^2)} \frac{a}{(e^2)^{1/4}} \frac{2^{1/2}}{(e \cot(dx+c))^{1/2}} \arctan\left(\frac{2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2}}\right) - \frac{1}{2} \frac{d}{e} \frac{1}{(a^2+b^2)} \frac{a}{(e^2)^{1/4}} \frac{2^{1/2}}{(e \cot(dx+c))^{1/2}} \arctan\left(\frac{-2^{1/2}}{(e^2)^{1/4} (e \cot(dx+c))^{1/2}}\right) + \frac{2}{a} \frac{d}{e} \frac{1}{(e \cot(dx+c))^{1/2}}$





$$\begin{aligned}
& 18 + 640a^{10}b^7d^8e^{18} - 256a^{12}b^5d^8e^{18} - 384a^{14}b^3d^8e^{18}) \\
& - (e \cot(c + dx))^{1/2} (512a^8b^8d^7e^{16} - 448a^{10}b^6d^7e^{16} + 128a^{12}b^4d^7e^{16} + 64a^{14}b^2d^7e^{16})) + 128a^7b^8d^6e^{15} - 32a^{11}b^4d^6e^{15} - 32a^{13}b^2d^6e^{15})) \cdot (-1/(4(b^2d^2e^3i - a^2d^2e^3i + 2ab^2d^2e^3)))^{1/2} - (\operatorname{atan}(\frac{((e \cot(c + dx))^{1/2} (64a^7b^7d^5e^{13} - 32a^9b^5d^5e^{13}) + ((-a^3b^5e^3)^{1/2} ((e \cot(c + dx))^{1/2} (512a^8b^8d^7e^{16} - 448a^{10}b^6d^7e^{16} + 128a^{12}b^4d^7e^{16} + 64a^{14}b^2d^7e^{16}) - ((-a^3b^5e^3)^{1/2} (256a^{12}b^5d^8e^{18} - 640a^{10}b^7d^8e^{18} - 512a^8b^9d^8e^{18} + 384a^{14}b^3d^8e^{18} + (e \cot(c + dx))^{1/2} (-a^3b^5e^3)^{1/2} (512a^9b^9d^9e^{19} + 512a^{11}b^7d^9e^{19} - 512a^{13}b^5d^9e^{19} - 512a^{15}b^3d^9e^{19}))/ (a^5d^3e^3 + a^3b^2d^3e^3)))/ (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} / (a^5d^3e^3 + a^3b^2d^3e^3) + 128a^7b^8d^6e^{15} - 32a^{11}b^4d^6e^{15} - 32a^{13}b^2d^6e^{15})) / (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} \cdot i) / (a^5d^3e^3 + a^3b^2d^3e^3) + (((e \cot(c + dx))^{1/2} (64a^7b^7d^5e^{13} - 32a^9b^5d^5e^{13}) + ((-a^3b^5e^3)^{1/2} ((e \cot(c + dx))^{1/2} (512a^8b^8d^7e^{16} - 448a^{10}b^6d^7e^{16} + 128a^{12}b^4d^7e^{16} + 64a^{14}b^2d^7e^{16}) - ((-a^3b^5e^3)^{1/2} (512a^8b^9d^8e^{18} + 640a^{10}b^7d^8e^{18} - 256a^{12}b^5d^8e^{18} - 384a^{14}b^3d^8e^{18} + ((e \cot(c + dx))^{1/2} (-a^3b^5e^3)^{1/2} (512a^9b^9d^9e^{19} + 512a^{11}b^7d^9e^{19} - 512a^{13}b^5d^9e^{19} - 512a^{15}b^3d^9e^{19}))/ (a^5d^3e^3 + a^3b^2d^3e^3)))/ (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} / (a^5d^3e^3 + a^3b^2d^3e^3) - 128a^7b^8d^6e^{15} + 32a^{11}b^4d^6e^{15} + 32a^{13}b^2d^6e^{15})) / (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} \cdot i) / (a^5d^3e^3 + a^3b^2d^3e^3) / (((e \cot(c + dx))^{1/2} (64a^7b^7d^5e^{13} - 32a^9b^5d^5e^{13}) + ((-a^3b^5e^3)^{1/2} ((e \cot(c + dx))^{1/2} (512a^8b^8d^7e^{16} - 448a^{10}b^6d^7e^{16} + 128a^{12}b^4d^7e^{16} + 64a^{14}b^2d^7e^{16}) - ((-a^3b^5e^3)^{1/2} (256a^{12}b^5d^8e^{18} - 640a^{10}b^7d^8e^{18} - 512a^8b^9d^8e^{18} + 384a^{14}b^3d^8e^{18} + ((e \cot(c + dx))^{1/2} (-a^3b^5e^3)^{1/2} (512a^9b^9d^9e^{19} + 512a^{11}b^7d^9e^{19} - 512a^{13}b^5d^9e^{19} - 512a^{15}b^3d^9e^{19}))/ (a^5d^3e^3 + a^3b^2d^3e^3)))/ (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} / (a^5d^3e^3 + a^3b^2d^3e^3) + 128a^7b^8d^6e^{15} - 32a^{11}b^4d^6e^{15} - 32a^{13}b^2d^6e^{15})) / (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} / (a^5d^3e^3 + a^3b^2d^3e^3) - (((e \cot(c + dx))^{1/2} (64a^7b^7d^5e^{13} - 32a^9b^5d^5e^{13}) + ((-a^3b^5e^3)^{1/2} ((e \cot(c + dx))^{1/2} (512a^8b^8d^7e^{16} - 448a^{10}b^6d^7e^{16} + 128a^{12}b^4d^7e^{16} + 64a^{14}b^2d^7e^{16}) - ((-a^3b^5e^3)^{1/2} (512a^8b^9d^8e^{18} + 640a^{10}b^7d^8e^{18} - 256a^{12}b^5d^8e^{18} - 384a^{14}b^3d^8e^{18} + ((e \cot(c + dx))^{1/2} (-a^3b^5e^3)^{1/2} (512a^9b^9d^9e^{19} + 512a^{11}b^7d^9e^{19} - 512a^{13}b^5d^9e^{19} - 512a^{15}b^3d^9e^{19}))/ (a^5d^3e^3 + a^3b^2d^3e^3)))/ (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} / (a^5d^3e^3 + a^3b^2d^3e^3) - 128a^7b^8d^6e^{15} + 32a^{11}b^4d^6e^{15} + 32a^{13}b^2d^6e^{15})) / (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} / (a^5d^3e^3 + a^3b^2d^3e^3)) \cdot (-a^3b^5e^3)^{1/2} \cdot 2i) / (a^5d^3e^3 + a^3b^2d^3e^3) + 2/(a \cdot d \cdot e \cdot (e \cot(c + dx))^{1/2})
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e \cot(c + dx))^{\frac{3}{2}} (a + b \cot(c + dx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(3/2)/(a+b\*cot(d\*x+c)),x)

[Out] Integral(1/((e\*cot(c + d\*x))\*\*(3/2)\*(a + b\*cot(c + d\*x))), x)

$$3.74 \quad \int \frac{1}{(e \cot(c+dx))^{5/2}(a+b \cot(c+dx))} dx$$

**Optimal.** Leaf size=351

$$\frac{(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d e^{5/2} (a^2 + b^2)} + \frac{(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d e^{5/2} (a^2 + b^2)}$$

[Out]  $-2*b^{(7/2)}*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/a^{(5/2)}/(a^2+b^2)/d/e^{(5/2)}+2/3/a/d/e/(e*\cot(d*x+c))^{(3/2)}-1/2*(a-b)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d/e^{(5/2)}*2^{(1/2)}+1/2*(a-b)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)/d/e^{(5/2)}*2^{(1/2)}-1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)/d/e^{(5/2)}*2^{(1/2)}+1/4*(a+b)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)/d/e^{(5/2)}*2^{(1/2)}-2*b/a^2/d/e^2/(e*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.96, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3569, 3649, 3654, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{(a+b) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d e^{5/2} (a^2 + b^2)} + \frac{(a+b) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d e^{5/2} (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(5/2)\*(a + b\*Cot[c + d\*x])),x]

[Out]  $(-2*b^{(7/2)}*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(a^{(5/2)}*(a^2 + b^2)*d*e^{(5/2)}) - ((a - b)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^{(5/2)}) + ((a - b)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)*d*e^{(5/2)}) + 2/(3*a*d*e*(e*Cot[c + d*x])^{(3/2)}) - (2*b)/(a^2*d*e^2*Sqrt[e*Cot[c + d*x]]) - ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^{(5/2)}) + ((a + b)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]])/(2*Sqrt[2]*(a^2 + b^2)*d*e^{(5/2)})$

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 1168

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[a\*c, 2]}, Dist[(d\*q + a\*e)/(2\*a\*c), Int[(q + c\*x^2)/(a + c\*x^4), x], x] + Dist[(d\*q - a\*e)/(2\*a\*c), Int[(q - c\*x^2)/(a + c\*x^4), x], x] /; FreeQ[{a,



c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && NegQ[-(a\*c)]

### Rule 3534

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/Sqrt[(b\_)\*tan[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[2/f, Subst[Int[(b\*c + d\*x^2)/(b^2 + x^4), x], x, Sqrt[b\*Tan[e + f\*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && NeQ[c^2 + d^2, 0]

### Rule 3569

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b^2\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), x] + Dist[1/(f\*(m + 1)\*(a^2 + b^2)\*(b\*c - a\*d)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2) - b\*(b\*c - a\*d)\*(m + 1)\*Tan[e + f\*x] - b^2\*d\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2\*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3634

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]

### Rule 3649

Int[((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (B\_)\*tan[(e\_) + (f\_)\*(x\_)]) + (C\_)\*tan[(e\_) + (f\_)\*(x\_)])^2, x\_Symbol] := Simp[((A\*b^2 - a\*(b\*B - a\*C))\*(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), x] + Dist[1/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 + b^2)), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*(c + d\*Tan[e + f\*x])^n\*Simp[A\*(a\*(b\*c - a\*d)\*(m + 1) - b^2\*d\*(m + n + 2)) + (b\*B - a\*C)\*(b\*c\*(m + 1) + a\*d\*(n + 1)) - (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B - b\*C)\*Tan[e + f\*x] - d\*(A\*b^2 - a\*(b\*B - a\*C))\*(m + n + 2)\*Tan[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

### Rule 3654

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dis
t[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*Ta
n[e + f*x], x], x] + Dist[(A*b^2 + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[
e + f*x])^n*(1 + Tan[e + f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a,
b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c + dx))^{5/2} (a + b \cot(c + dx))} dx &= \frac{2}{3ade(e \cot(c + dx))^{3/2}} + \frac{2 \int \frac{-\frac{3be^2}{2} - \frac{3}{2}ae^2 \cot(c+dx) - \frac{3}{2}be^2 \cot^2(c+dx)}{(e \cot(c+dx))^{3/2} (a+b \cot(c+dx))} dx}{3ae^3} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2b}{a^2de^2 \sqrt{e \cot(c + dx)}} + \frac{4 \int \frac{-\frac{3}{4}(a^2-b^2)e^4 + \frac{3}{4}b^2e^4}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx}{3a^2e^6} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2b}{a^2de^2 \sqrt{e \cot(c + dx)}} + \frac{4 \int \frac{-\frac{3}{4}a^3e^4 + \frac{3}{4}a^2be^4 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{3a^2(a^2 + b^2)e} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2b}{a^2de^2 \sqrt{e \cot(c + dx)}} + \frac{8 \operatorname{Subst} \left( \int \frac{\frac{3a^3e^5}{4} - \frac{3}{4}a^2be^4}{e^2 + x^4} dx \right)}{3a^2(a^2 + b^2)} \\
&= \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2b}{a^2de^2 \sqrt{e \cot(c + dx)}} - \frac{(2b^4) \operatorname{Subst} \left( \int \frac{1}{a + \frac{bx^2}{e}} dx \right)}{a^2(a^2 + b^2)} \\
&= -\frac{2b^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{5/2} (a^2 + b^2) de^{5/2}} + \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2b}{a^2de^2 \sqrt{e \cot(c + dx)}} \\
&= -\frac{2b^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{5/2} (a^2 + b^2) de^{5/2}} + \frac{2}{3ade(e \cot(c + dx))^{3/2}} - \frac{2b}{a^2de^2 \sqrt{e \cot(c + dx)}} \\
&= -\frac{2b^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{5/2} (a^2 + b^2) de^{5/2}} - \frac{(a - b) \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} (a^2 + b^2) de^{5/2}} + \dots
\end{aligned}$$

**Mathematica [C]** time = 0.28, size = 109, normalized size = 0.31

$$\frac{2 \left( b^2 {}_2F_1 \left( -\frac{3}{2}, 1; -\frac{1}{2}; -\frac{b \cot(c+dx)}{a} \right) + a \left( a {}_2F_1 \left( -\frac{3}{4}, 1; \frac{1}{4}; -\cot^2(c+dx) \right) - 3b \cot(c+dx) {}_2F_1 \left( -\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c+dx) \right) \right) \right)}{3ade (a^2 + b^2) (e \cot(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(5/2)\*(a + b\*Cot[c + d\*x])),x]

[Out] (2\*(b^2\*Hypergeometric2F1[-3/2, 1, -1/2, -((b\*Cot[c + d\*x])/a)] + a\*(a\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d\*x]^2] - 3\*b\*Cot[c + d\*x]\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d\*x]^2])))/(3\*a\*(a^2 + b^2)\*d\*e\*(e\*Cot[c + d\*x])^(3/2))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cot(dx + c) + a) (e \cot(dx + c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(5/2)/(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate(1/((b\*cot(d\*x + c) + a)\*(e\*cot(d\*x + c))^(5/2)), x)

**maple [A]** time = 0.63, size = 481, normalized size = 1.37

$$\frac{2b^4 \arctan\left(\frac{\sqrt{e \cot(dx+c)} b}{\sqrt{aeb}}\right)}{d e^2 a^2 (a^2 + b^2) \sqrt{aeb}} + \frac{a (e^2)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{e \cot(dx+c) + (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}{e \cot(dx+c) - (e^2)^{\frac{1}{4}} \sqrt{e \cot(dx+c)} \sqrt{2} + \sqrt{e^2}}\right)}{4d e^3 (a^2 + b^2)} + \frac{a (e^2)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{e \cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right)}{2d e^3 (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(d\*x+c))^(5/2)/(a+b\*cot(d\*x+c)),x)

[Out] 
$$-2/d/e^2/a^2*b^4/(a^2+b^2)/(a*e*b)^{(1/2)}*\arctan((e*\cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})+1/4/d/e^3/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))+1/2/d/e^3/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/2/d/e^3/(a^2+b^2)*a*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)-1/4/d/e^2/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))-1/2/d/e^2/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+1/2/d/e^2/(a^2+b^2)*b/(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)+2/3/a/d/e/(e*\cot(d*x+c))^{(3/2)}-2*b/a^2/d/e^2/(e*\cot(d*x+c))^{(1/2)}$$

**maxima** [A] time = 0.66, size = 280, normalized size = 0.80

$$e \left( \frac{24 b^4 \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^4+a^2b^2)\sqrt{abe}e^3} - \frac{3 \left( \frac{2 \sqrt{2}(a-b) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2}(a-b) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right) + \sqrt{2}(a+b) \log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e}{\tan(dx+c)}}\right)}{(a^2+b^2)e^3} \right)$$

12 d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-1/12*e*(24*b^4*\arctan(b*\sqrt{e/\tan(d*x+c)})/\sqrt{a*b*e})/((a^4+a^2*b^2)*\sqrt{a*b*e}*e^3)-3*(2*\sqrt{2}*(a-b)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e})/\sqrt{e}+2*\sqrt{2}*(a-b)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(d*x+c)}))/\sqrt{e})/\sqrt{e}+\sqrt{2}*(a+b)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e}-\sqrt{2}*(a+b)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x+c)}+e+e/\tan(d*x+c))/\sqrt{e})/((a^2+b^2)*e^3)-8*(a*e-3*b*e/\tan(d*x+c))/(a^2*e^3*(e/\tan(d*x+c))^{(3/2)})/d$$

**mupad** [B] time = 2.81, size = 6042, normalized size = 17.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c + d\*x))^(5/2)\*(a + b\*cot(c + d\*x))),x)

[Out]  $(2/(3*a*e) - (2*b*cot(c + d*x))/(a^2*e))/(d*(e*cot(c + d*x))^(3/2)) - atan($   
 $(((((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2$   
 $+ ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*((e*cot(c + d*x))^(1/2)$   
 $)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9$   
 $*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9$   
 $*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 256*a^18*b^8*d^8*e^26 + 192*a^20*b^6*$   
 $d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a^24*b^2*d^8*e^26))/2 - ((e*cot(c + d$   
 $*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d$   
 $^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2$   
 $*a*b*d^2*e^5))^(1/2))/2 + 192*a^15*b^9*d^6*e^21 - 16*a^19*b^5*d^6*e^21 - 16$   
 $*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5$   
 $))^(1/2)*1i + (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*$   
 $d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)$   
 $*(((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*((e*cot(c$   
 $+ d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*$   
 $(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 51$   
 $2*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^18*b^8*d^8*e^26 -$   
 $192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*b^2*d^8*e^26))/2 -$   
 $((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 1$   
 $28*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d$   
 $^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 - 192*a^15*b^9*d^6*e^21 + 16*a^19*b^5*$   
 $d^6*e^21 + 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i +$   
 $2*a*b*d^2*e^5))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 +$   
 $32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2$   
 $*e^5))^(1/2)*(((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)$   
 $*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2$   
 $*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*$   
 $d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^18*b^8$   
 $*d^8*e^26 - 192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*b^2*d^8$   
 $*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6$   
 $*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e$   
 $^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 - 192*a^15*b^9*d^6*e^21 +$   
 $16*a^19*b^5*d^6*e^21 + 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*$   
 $d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2) - (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*$   
 $d^5*e^18 + 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i$   
 $+ 2*a*b*d^2*e^5))^(1/2)*(((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2$   
 $*e^5))^(1/2)*((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i +$   
 $2*a*b*d^2*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512$   
 $*a^22*b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 2$   
 $56*a^18*b^8*d^8*e^26 + 192*a^20*b^6*d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a$   
 $^24*b^2*d^8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 44$   
 $8*a^19*b^6*d^7*e^23 - 128*a^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/$

$$\begin{aligned}
& ((b^2*d^2*e^5*i - a^2*d^2*e^5*i + 2*a*b*d^2*e^5))^{(1/2)}/2 + 192*a^{15}*b^9* \\
& d^6*e^{21} - 16*a^{19}*b^5*d^6*e^{21} - 16*a^{21}*b^3*d^6*e^{21}))/2)*(1/(b^2*d^2*e^5 \\
& *i - a^2*d^2*e^5*i + 2*a*b*d^2*e^5))^{(1/2)} + 64*a^{14}*b^8*d^4*e^{16}))*((1/(b \\
& ^2*d^2*e^5*i - a^2*d^2*e^5*i + 2*a*b*d^2*e^5))^{(1/2)}*i - \operatorname{atan}(((1/(4*(b \\
& ^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^{(1/2)}*(e*\cot(c + d*x))^{(1/2)}* \\
& (64*a^{14}*b^9*d^5*e^{18} + 32*a^{18}*b^5*d^5*e^{18}) + (1/(4*(b^2*d^2*e^5 - a^2*d \\
& ^2*e^5 + a*b*d^2*e^5*2i)))^{(1/2)}*((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d \\
& ^2*e^5*2i))))^{(1/2)}*((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)} \\
& ((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*(e*\cot(c \\
& + d*x))^{(1/2)}*(512*a^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5 \\
& *d^9*e^{28} - 512*a^{24}*b^3*d^9*e^{28}) - 512*a^{16}*b^{10}*d^8*e^{26} - 512*a^{18}*b^8* \\
& d^8*e^{26} + 384*a^{20}*b^6*d^8*e^{26} + 256*a^{22}*b^4*d^8*e^{26} - 128*a^{24}*b^2*d^8 \\
& *e^{26}) - (e*\cot(c + d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7* \\
& e^{23} - 128*a^{21}*b^4*d^7*e^{23} - 64*a^{23}*b^2*d^7*e^{23})) + 384*a^{15}*b^9*d^6*e^ \\
& 21 - 32*a^{19}*b^5*d^6*e^{21} - 32*a^{21}*b^3*d^6*e^{21}))*i + (1/(4*(b^2*d^2*e^5 \\
& - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^{(1/2)}*(e*\cot(c + d*x))^{(1/2)}*(64*a^{14}*b \\
& ^9*d^5*e^{18} + 32*a^{18}*b^5*d^5*e^{18}) + (1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a \\
& *b*d^2*e^5*2i)))^{(1/2)}*((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i) \\
& ))^{(1/2)}*((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*((1/( \\
& 4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*(e*\cot(c + d*x))^{(1 \\
& /2)}*(512*a^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5*d^9*e^{28} \\
& - 512*a^{24}*b^3*d^9*e^{28}) + 512*a^{16}*b^{10}*d^8*e^{26} + 512*a^{18}*b^8*d^8*e^{26} - \\
& 384*a^{20}*b^6*d^8*e^{26} - 256*a^{22}*b^4*d^8*e^{26} + 128*a^{24}*b^2*d^8*e^{26}) - ( \\
& e*\cot(c + d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128 \\
& *a^{21}*b^4*d^7*e^{23} - 64*a^{23}*b^2*d^7*e^{23})) - 384*a^{15}*b^9*d^6*e^{21} + 32*a^ \\
& 19*b^5*d^6*e^{21} + 32*a^{21}*b^3*d^6*e^{21}))*i)/(((1/(4*(b^2*d^2*e^5 - a^2*d^2 \\
& *e^5 + a*b*d^2*e^5*2i)))^{(1/2)}*(e*\cot(c + d*x))^{(1/2)}*(64*a^{14}*b^9*d^5*e^ \\
& 18 + 32*a^{18}*b^5*d^5*e^{18}) + (1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5 \\
& *2i)))^{(1/2)}*((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*( \\
& (1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*((1/(4*(b^2*d^ \\
& 2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*(e*\cot(c + d*x))^{(1/2)}*(512*a \\
& ^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5*d^9*e^{28} - 512*a^{24} \\
& *b^3*d^9*e^{28}) + 512*a^{16}*b^{10}*d^8*e^{26} + 512*a^{18}*b^8*d^8*e^{26} - 384*a^{20} \\
& *b^6*d^8*e^{26} - 256*a^{22}*b^4*d^8*e^{26} + 128*a^{24}*b^2*d^8*e^{26}) - (e*\cot(c + \\
& d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128*a^{21}*b^4* \\
& d^7*e^{23} - 64*a^{23}*b^2*d^7*e^{23})) - 384*a^{15}*b^9*d^6*e^{21} + 32*a^{19}*b^5*d^6 \\
& *e^{21} + 32*a^{21}*b^3*d^6*e^{21})) - (1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^ \\
& 2*e^5*2i)))^{(1/2)}*(e*\cot(c + d*x))^{(1/2)}*(64*a^{14}*b^9*d^5*e^{18} + 32*a^{18}*b \\
& ^5*d^5*e^{18}) + (1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^{(1/2)}* \\
& ((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*((1/(4*(b^2*d \\
& ^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*((1/(4*(b^2*d^2*e^5 - a^2*d \\
& ^2*e^5 + a*b*d^2*e^5*2i))))^{(1/2)}*(e*\cot(c + d*x))^{(1/2)}*(512*a^{18}*b^9*d^9*e \\
& ^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5*d^9*e^{28} - 512*a^{24}*b^3*d^9*e^{28} \\
& ) - 512*a^{16}*b^{10}*d^8*e^{26} - 512*a^{18}*b^8*d^8*e^{26} + 384*a^{20}*b^6*d^8*e^{26} \\
& + 256*a^{22}*b^4*d^8*e^{26} - 128*a^{24}*b^2*d^8*e^{26}) - (e*\cot(c + d*x))^{(1/2)}*(
\end{aligned}$$

$$\begin{aligned}
& 512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128*a^{21}*b^4*d^7*e^{23} - 64 \\
& *a^{23}*b^2*d^7*e^{23})) + 384*a^{15}*b^9*d^6*e^{21} - 32*a^{19}*b^5*d^6*e^{21} - 32*a^{21} \\
& *b^3*d^6*e^{21})) + 64*a^{14}*b^8*d^4*e^{16}))*((1/(4*(b^2*d^2*e^5 - a^2*d^2*e^5 + a*b*d^2*e^5*2i)))^{(1/2)}*2i - (\operatorname{atan}((((e*\cot(c + d*x))^{(1/2)}*(64*a^{14}*b \\
& ^9*d^5*e^{18} + 32*a^{18}*b^5*d^5*e^{18}) - ((-a^5*b^7*e^5)^{(1/2)}*(32*a^{19}*b^5*d^6 \\
& *e^{21} - 384*a^{15}*b^9*d^6*e^{21} + 32*a^{21}*b^3*d^6*e^{21} + (((e*\cot(c + d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128*a^{21}*b^4*d^7*e^{23} \\
& - 64*a^{23}*b^2*d^7*e^{23}) + ((-a^5*b^7*e^5)^{(1/2)}*(512*a^{16}*b^{10}*d^8*e^{26} \\
& + 512*a^{18}*b^8*d^8*e^{26} - 384*a^{20}*b^6*d^8*e^{26} - 256*a^{22}*b^4*d^8*e^{26} + 1 \\
& 28*a^{24}*b^2*d^8*e^{26} - ((e*\cot(c + d*x))^{(1/2)}*(-a^5*b^7*e^5)^{(1/2)}*(512*a^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5*d^9*e^{28} - 512*a^{24}* \\
& b^3*d^9*e^{28}))/ (a^5*d*e^5*(a^2 + b^2)))))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7 \\
& *e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)))))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e \\
& ^5)^{(1/2)}*1i)/ (a^5*d*e^5*(a^2 + b^2)) + (((e*\cot(c + d*x))^{(1/2)}*(64*a^{14}*b \\
& ^9*d^5*e^{18} + 32*a^{18}*b^5*d^5*e^{18}) - ((-a^5*b^7*e^5)^{(1/2)}*(384*a^{15}*b^9*d \\
& ^6*e^{21} - 32*a^{19}*b^5*d^6*e^{21} - 32*a^{21}*b^3*d^6*e^{21} + (((e*\cot(c + d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128*a^{21}*b^4*d^7*e^{23} \\
& - 64*a^{23}*b^2*d^7*e^{23}) - ((-a^5*b^7*e^5)^{(1/2)}*(512*a^{16}*b^{10}*d^8*e^{26} \\
& + 512*a^{18}*b^8*d^8*e^{26} - 384*a^{20}*b^6*d^8*e^{26} - 256*a^{22}*b^4*d^8*e^{26} + 1 \\
& 28*a^{24}*b^2*d^8*e^{26} + ((e*\cot(c + d*x))^{(1/2)}*(-a^5*b^7*e^5)^{(1/2)}*(512*a^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5*d^9*e^{28} - 512*a^{24}* \\
& b^3*d^9*e^{28}))/ (a^5*d*e^5*(a^2 + b^2)))))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7 \\
& *e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)))))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e \\
& ^5)^{(1/2)}*1i)/ (a^5*d*e^5*(a^2 + b^2)))/ (64*a^{14}*b^8*d^4*e^{16} - (((e*\cot(c + \\
& d*x))^{(1/2)}*(64*a^{14}*b^9*d^5*e^{18} + 32*a^{18}*b^5*d^5*e^{18}) - ((-a^5*b^7*e^5)^{(1/2)}*(32*a^{19}*b^5*d^6*e^{21} - 384*a^{15}*b^9*d^6*e^{21} + 32*a^{21}*b^3*d^6*e^{21} \\
& + (((e*\cot(c + d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128*a^{21}*b^4*d^7*e^{23} - 64*a^{23}*b^2*d^7*e^{23}) + ((-a^5*b^7*e^5)^{(1/2)}*(512*a^{16}*b^{10}*d^8*e^{26} + 512*a^{18}*b^8*d^8*e^{26} - 384*a^{20}*b^6*d^8*e^{26} - 25 \\
& 6*a^{22}*b^4*d^8*e^{26} + 128*a^{24}*b^2*d^8*e^{26} - ((e*\cot(c + d*x))^{(1/2)}*(-a^5 \\
& *b^7*e^5)^{(1/2)}*(512*a^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5 \\
& ^5*d^9*e^{28} - 512*a^{24}*b^3*d^9*e^{28}))/ (a^5*d*e^5*(a^2 + b^2)))))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)) + (((e*\cot(c + d*x))^{(1/2)}*(64*a^{14}*b^9*d^5*e^{18} + 32*a^{18}*b^5*d^5*e^{18}) - ((-a^5*b^7*e^5)^{(1/2)}*(384*a^{15}*b^9*d^6*e^{21} - 32*a^{19}*b^5*d^6*e^{21} - 32*a^{21}*b^3*d^6*e^{21} + (((e*\cot(c + d*x))^{(1/2)}*(512*a^{15}*b^{10}*d^7*e^{23} + 448*a^{19}*b^6*d^7*e^{23} - 128*a^{21}*b^4*d^7*e^{23} - 64*a^{23}*b^2*d^7*e^{23}) - ((-a^5*b^7*e^5)^{(1/2)}*(512*a^{16}*b^{10}*d^8*e^{26} + 512*a^{18}*b^8*d^8*e^{26} - 384*a^{20}*b^6*d^8*e^{26} - 256*a^{22}*b^4*d^8*e^{26} + 128*a^{24}*b^2*d^8*e^{26} + ((e*\cot(c + d*x))^{(1/2)}*(-a^5*b^7*e^5)^{(1/2)}*(512*a^{18}*b^9*d^9*e^{28} + 512*a^{20}*b^7*d^9*e^{28} - 512*a^{22}*b^5*d^9*e^{28} - 512*a^{24}*b^3*d^9*e^{28}))/ (a^5*d*e^5*(a^2 + b^2)))))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e^5)^{(1/2))/ (a^5*d*e^5*(a^2 + b^2)))*(-a^5*b^7*e^5)^{(1/2)}*2i)/ (a^5*d*e^5*(a^2 + b^2))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e \cot(c + dx))^{\frac{5}{2}} (a + b \cot(c + dx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(5/2)/(a+b\*cot(d\*x+c)),x)

[Out] Integral(1/((e\*cot(c + d\*x))\*\*(5/2)\*(a + b\*cot(c + d\*x))), x)



$$3.75 \quad \int \frac{(e \cot(c+dx))^{7/2}}{(a+b \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=437

$$\frac{e^{7/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2} - \frac{e^{7/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) + \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2}$$

[Out]  $a^{5/2} (3a^2 + 7b^2) e^{7/2} \arctan(b^{1/2} (e \cot(dx+c))^{1/2} / a^{1/2} / e^{1/2}) / b^{5/2} / (a^2 + b^2)^2 / d + a^2 e^2 (e \cot(dx+c))^{3/2} / b / (a^2 + b^2) / d / (a + b \cot(dx+c)) + 1/2 (a^2 - 2ab - b^2) e^{7/2} \arctan(1 - 2^{1/2} (e \cot(dx+c))^{1/2} / e^{1/2}) / (a^2 + b^2)^2 / d + 1/2 (a^2 - 2ab - b^2) e^{7/2} \arctan(1 + 2^{1/2} (e \cot(dx+c))^{1/2} / e^{1/2}) / (a^2 + b^2)^2 / d + 1/4 (a^2 + 2ab - b^2) e^{7/2} \ln(e^{1/2} + \cot(dx+c) e^{1/2} - 2^{1/2} (e \cot(dx+c))^{1/2}) / (a^2 + b^2)^2 / d + 1/4 (a^2 + 2ab - b^2) e^{7/2} \ln(e^{1/2} + \cot(dx+c) e^{1/2} + 2^{1/2} (e \cot(dx+c))^{1/2}) / (a^2 + b^2)^2 / d - (3a^2 + 2b^2) e^3 (e \cot(dx+c))^{1/2} / b^2 / (a^2 + b^2) / d$

**Rubi [A]** time = 1.11, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3565, 3647, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$-\frac{e^3 (3a^2 + 2b^2) \sqrt{e \cot(c + dx)}}{b^2 d (a^2 + b^2)} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{bd (a^2 + b^2) (a + b \cot(c + dx))} + \frac{e^{7/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(7/2)/(a + b\*Cot[c + d\*x])^2,x]

[Out]  $(a^{5/2} (3a^2 + 7b^2) e^{7/2} \text{ArcTan}[\text{Sqrt}[b] \text{Sqrt}[e \cot(c + dx)]] / (\text{Sqrt}[a] \text{Sqrt}[e])) / (b^{5/2} (a^2 + b^2)^2 d) + ((a^2 - 2ab - b^2) e^{7/2} \text{ArcTan}[1 - (\text{Sqrt}[2] \text{Sqrt}[e \cot(c + dx)]] / \text{Sqrt}[e])] / (\text{Sqrt}[2] (a^2 + b^2)^2 d) - ((a^2 - 2ab - b^2) e^{7/2} \text{ArcTan}[1 + (\text{Sqrt}[2] \text{Sqrt}[e \cot(c + dx)]] / \text{Sqrt}[e])] / (\text{Sqrt}[2] (a^2 + b^2)^2 d) - ((3a^2 + 2b^2) e^3 \text{Sqrt}[e \cot(c + dx)]) / (b^2 (a^2 + b^2) d) + (a^2 e^2 (e \cot(c + dx))^{3/2}) / (b (a^2 + b^2) d (a + b \cot(c + dx))) + ((a^2 + 2ab - b^2) e^{7/2} \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot(c + dx) - \text{Sqrt}[2] \text{Sqrt}[e \cot(c + dx)]]]) / (2 \text{Sqrt}[2] (a^2 + b^2)^2 d) - ((a^2 + 2ab - b^2) e^{7/2} \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cot(c + dx) + \text{Sqrt}[2] \text{Sqrt}[e \cot(c + dx)]]]) / (2 \text{Sqrt}[2] (a^2 + b^2)^2 d)$

**Rule 63**

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(
m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1
/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f
*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*
(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3647

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_)
+ (f_)*(x_)])^2), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

### Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{7/2}}{(a + b \cot(c + dx))^2} dx &= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} - \int \frac{\sqrt{e \cot(c+dx)} \left( -\frac{3}{2} a^2 e^3 + a b e^3 \cot(c+dx) - \frac{1}{2} (3a^2 + 2b^2) e^3 \cot^2(c+dx) \right)}{a + b \cot(c+dx)} \\
&= -\frac{(3a^2 + 2b^2) e^3 \sqrt{e \cot(c + dx)}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{2 \int \frac{-\frac{1}{4} a (3a^2 + 2b^2) e^3 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{b^2 (a^2 + b^2) d} \\
&= -\frac{(3a^2 + 2b^2) e^3 \sqrt{e \cot(c + dx)}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{2 \int \frac{\frac{1}{2} b^2 (a^2 - b^2) e^4 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{b^2 (a^2 + b^2) d} \\
&= -\frac{(3a^2 + 2b^2) e^3 \sqrt{e \cot(c + dx)}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{4 \text{Subst} \left( \int \frac{-\frac{1}{2} b^2 (a^2 - b^2) e^4 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}} dx \right)}{b^2 (a^2 + b^2) d} \\
&= -\frac{(3a^2 + 2b^2) e^3 \sqrt{e \cot(c + dx)}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{(a^3 (3a^2 + 7b^2) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right))}{b^2 (a^2 + b^2) d} \\
&= \frac{a^{5/2} (3a^2 + 7b^2) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{5/2} (a^2 + b^2)^2 d} - \frac{(3a^2 + 2b^2) e^3 \sqrt{e \cot(c + dx)}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} \\
&= \frac{a^{5/2} (3a^2 + 7b^2) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{5/2} (a^2 + b^2)^2 d} - \frac{(3a^2 + 2b^2) e^3 \sqrt{e \cot(c + dx)}}{b^2 (a^2 + b^2) d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{b (a^2 + b^2) d (a + b \cot(c + dx))} \\
&= \frac{a^{5/2} (3a^2 + 7b^2) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{5/2} (a^2 + b^2)^2 d} + \frac{(a^2 - 2ab - b^2) e^{7/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{2} (a^2 + b^2)^2 d}
\end{aligned}$$

**Mathematica** [C] time = 6.16, size = 445, normalized size = 1.02

$$(e \cot(c + dx))^{7/2} \left( \frac{2b^2 \cot^2(c+dx) {}_2F_1\left(2, \frac{9}{2}; \frac{11}{2}; -\frac{b \cot(c+dx)}{a}\right)}{9a^2(a^2+b^2)} + \frac{4ab \left(-7 \cot^2(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right) - 3 \cot^2(c+dx) + 7 \cot^2(c+dx)\right)}{21(a^2+b^2)^2} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(7/2)/(a + b\*Cot[c + d\*x])^2,x]

[Out] -(((e\*Cot[c + d\*x])^(7/2)\*((4\*a\*b\*Cot[c + d\*x]^(7/2))/(7\*(a^2 + b^2)^2) - (4\*a^2\*(3\*Cot[c + d\*x]^(5/2) - 5\*a\*(-3\*a\*(-(Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]])/b^(3/2)) + Sqrt[Cot[c + d\*x]]/b))/b + Cot[c + d\*x]^(3/2)/b)))/(15\*(a^2 + b^2)^2) + (4\*a\*b\*(7\*Cot[c + d\*x]^(3/2) - 3\*Cot[c + d\*x]^(7/2) - 7\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(21\*(a^2 + b^2)^2) + (2\*b^2\*Cot[c + d\*x]^(9/2)\*Hypergeometric2F1[2, 9/2, 11/2, -(b\*Cot[c + d\*x])/a]))/(9\*a^2\*(a^2 + b^2)) - ((a - b)\*(a + b)\*(40\*Sqrt[Cot[c + d\*x]] - 8\*Cot[c + d\*x]^(5/2) + (5\*(4\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]]) + 2\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - 2\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/2))/(20\*(a^2 + b^2)^2)))/(d\*Cot[c + d\*x]^(7/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{7/2}}{(b \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(7/2)/(b\*cot(d\*x + c) + a)^2, x)

maple [B] time = 0.83, size = 805, normalized size = 1.84

$$-\frac{2e^3\sqrt{e\cot(dx+c)}}{db^2} - \frac{e^4a^5\sqrt{e\cot(dx+c)}}{d(a^2+b^2)^2b^2(e\cot(dx+c)b+ae)} - \frac{e^4a^3\sqrt{e\cot(dx+c)}}{d(a^2+b^2)^2(e\cot(dx+c)b+ae)} + \frac{3e^4a^5\arctan\left(\frac{y}{x}\right)}{d(a^2+b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^2,x)

[Out] 
$$-2/d*e^3/b^2*(e*cot(d*x+c))^{(1/2)}-1/d*e^4*a^5/(a^2+b^2)^2/b^2*(e*cot(d*x+c))^{(1/2)}/(e*cot(d*x+c)*b+a*e)-1/d*e^4*a^3/(a^2+b^2)^2*(e*cot(d*x+c))^{(1/2)}/(e*cot(d*x+c)*b+a*e)+3/d*e^4*a^5/(a^2+b^2)^2/b^2/(a*e*b)^{(1/2)}*arctan((e*cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})+7/d*e^4*a^3/(a^2+b^2)^2/(a*e*b)^{(1/2)}*arctan((e*cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})+1/2/d*e^3/(a^2+b^2)^2*(e^2)^{(1/4)}*2^{(1/2)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a^2-1/2/d*e^3/(a^2+b^2)^2*(e^2)^{(1/4)}*2^{(1/2)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*b^2-1/4/d*e^3/(a^2+b^2)^2*(e^2)^{(1/4)}*2^{(1/2)}*ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))*a^2+1/4/d*e^3/(a^2+b^2)^2*(e^2)^{(1/4)}*2^{(1/2)}*ln((e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))*b^2-1/2/d*e^3/(a^2+b^2)^2*(e^2)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*a^2+1/2/d*e^3/(a^2+b^2)^2*(e^2)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)*b^2+1/2/d*e^4/(a^2+b^2)^2*a*b/(e^2)^{(1/4)}*2^{(1/2)}*ln((e*cot(d*x+c)-(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*cot(d*x+c)+(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))+1/d*e^4/(a^2+b^2)^2*a*b/(e^2)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)-1/d*e^4/(a^2+b^2)^2*a*b/(e^2)^{(1/4)}*2^{(1/2)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*cot(d*x+c))^{(1/2)}+1)$$

**maxima** [A] time = 0.46, size = 385, normalized size = 0.88

$$\left( \frac{4 a^3 e^3 \sqrt{\frac{e}{\tan(dx+c)}}}{(a^3 b^2 + a b^4) e + \frac{(a^2 b^3 + b^5) e}{\tan(dx+c)}} - \frac{4 (3 a^5 + 7 a^3 b^2) e^3 \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{a b e}}\right)}{(a^4 b^2 + 2 a^2 b^4 + b^6) \sqrt{a b e}} + \frac{2 \sqrt{2} (a^2 - 2 a b - b^2) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} + 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} + \frac{2 \sqrt{2} (a^2 - 2 a b - b^2) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{e} - 2 \sqrt{\frac{e}{\tan(dx+c)}}\right)}{2 \sqrt{e}}\right)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $-1/4*(4*a^3*e^3*\sqrt{e/\tan(dx+c)})/((a^3*b^2+a*b^4)*e+(a^2*b^3+b^5)*e/\tan(dx+c))-4*(3*a^5+7*a^3*b^2)*e^3*\arctan(b*\sqrt{e/\tan(dx+c)})/\sqrt{a*b*e}/((a^4*b^2+2*a^2*b^4+b^6)*\sqrt{a*b*e})+(2*\sqrt{2}*(a^2-2*a*b-b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(dx+c)}))/\sqrt{e})/\sqrt{e}+2*\sqrt{2}*(a^2-2*a*b-b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(dx+c)}))/\sqrt{e})/\sqrt{e}+\sqrt{2}*(a^2+2*a*b-b^2)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)})+e+e/\tan(dx+c))/\sqrt{e}-\sqrt{2}*(a^2+2*a*b-b^2)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)})+e+e/\tan(dx+c))/\sqrt{e}*e^3/(a^4+2*a^2*b^2+b^4)+8*e^2*\sqrt{e/\tan(dx+c)}/b^2)*e/d$

**mupad** [B] time = 3.97, size = 13244, normalized size = 30.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c+d\*x))^(7/2)/(a+b\*cot(c+d\*x))^2,x)

[Out]  $(\operatorname{atan}(\frac{((16*(e*\cot(c+d*x))^{1/2}*(9*a^{12}*e^{24}+2*b^{12}*e^{24}+4*a^2*b^{10}*e^{24}+2*a^4*b^8*e^{24}-49*a^6*b^6*e^{24}+7*a^8*b^4*e^{24}+33*a^{10}*b^2*e^{24}+4))/b^{11}*d^4+4*a^2*b^9*d^4+6*a^4*b^7*d^4+4*a^6*b^5*d^4+a^8*b^3*d^4)}{((16*(30*a^6*b^8*d^2*e^{21}-224*a^4*b^{10}*d^2*e^{21}-18*a^{14}*d^2*e^{21}+600*a^8*b^6*d^2*e^{21}+388*a^{10}*b^4*d^2*e^{21}+24*a^{12}*b^2*d^2*e^{21}))/b^{11}*d^5+4*a^2*b^9*d^5+6*a^4*b^7*d^5+4*a^6*b^5*d^5+a^8*b^3*d^5)}-\frac{((16*(e*\cot(c+d*x))^{1/2}*(72*a^{15}*b*d^2*e^{17}-60*a*b^{15}*d^2*e^{17}-52*a^3*b^{13}*d^2*e^{17}+72*a^5*b^{11}*d^2*e^{17}+448*a^7*b^9*d^2*e^{17}+1108*a^9*b^7*$





$$\begin{aligned}
& \left( 9d^2e^{17} + 1108a^9b^7d^2e^{17} + 1132a^{11}b^5d^2e^{17} + 480a^{13}b^3 \right. \\
& \left. *d^2e^{17} \right) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8 \\
& *b^3d^4) + \left( (16(8a^8b^{17}d^4e^{14} + 96a^3b^{15}d^4e^{14} + 360a^5b^{13} \right. \\
& \left. d^4e^{14} + 640a^7b^{11}d^4e^{14} + 600a^9b^9d^4e^{14} + 288a^{11}b^7d^4e^{14} \right. \\
& \left. + 56a^{13}b^5d^4e^{14}) \right) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4 \\
& *a^6b^5d^5 + a^8b^3d^5) - (8*(e*\cot(c + d*x))^{(1/2)}*(3a^2 + 7b^2)*(-a \\
& ^5b^5e^7)^{(1/2)}*(32b^{20}d^4e^{10} + 160a^2b^{18}d^4e^{10} + 288a^4b^{16} \\
& d^4e^{10} + 160a^6b^{14}d^4e^{10} - 160a^8b^{12}d^4e^{10} - 288a^{10}b^{10}d^4 \\
& e^{10} - 160a^{12}b^8d^4e^{10} - 32a^{14}b^6d^4e^{10})) / ((b^9d + 2a^2b^7 \\
& *d + a^4b^5d)*(b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + \\
& a^8b^3d^4)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7 \\
& *d + a^4b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7 \\
& *d + a^4b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7 \\
& *d + a^4b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7 \\
& *d + a^4b^5d)) + \left( (16*(e*\cot(c + d*x))^{(1/2)}*(9a^{12}e^{24} + 2b^{12} \right. \\
& \left. *e^{24} + 4a^2b^{10}e^{24} + 2a^4b^8e^{24} - 49a^6b^6e^{24} + 7a^8b^4e^{24} \right. \\
& \left. + 33a^{10}b^2e^{24}) \right) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5 \\
& *d^4 + a^8b^3d^4) - \left( (16*(30a^6b^8d^2e^{21} - 224a^4b^{10}d^2e^{21} - \right. \\
& \left. 18a^{14}d^2e^{21} + 600a^8b^6d^2e^{21} + 388a^{10}b^4d^2e^{21} + 24a^{12}b^2 \right. \\
& \left. d^2e^{21}) \right) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 + a^8b^3d^5) \\
& + \left( (16*(e*\cot(c + d*x))^{(1/2)}*(72a^{15}b^5d^2e^{17} - 60a^8b^{15} \right. \\
& \left. d^2e^{17} - 52a^3b^{13}d^2e^{17} + 72a^5b^{11}d^2e^{17} + 448a^7b^9d^2e^{17} \right. \\
& \left. + 1108a^9b^7d^2e^{17} + 1132a^{11}b^5d^2e^{17} + 480a^{13}b^3d^2e^{17} \right) \\
& \left. \right) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) \\
& - \left( (16*(8a^8b^{17}d^4e^{14} + 96a^3b^{15}d^4e^{14} + 360a^5b^{13}d^4e^{14} \right. \\
& \left. + 640a^7b^{11}d^4e^{14} + 600a^9b^9d^4e^{14} + 288a^{11}b^7d^4e^{14} + 56 \right. \\
& \left. *a^{13}b^5d^4e^{14}) \right) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 \\
& + a^8b^3d^5) + (8*(e*\cot(c + d*x))^{(1/2)}*(3a^2 + 7b^2)*(-a^5b^5e^7)^{(1/2)} \\
& *(32b^{20}d^4e^{10} + 160a^2b^{18}d^4e^{10} + 288a^4b^{16}d^4e^{10} \\
& + 160a^6b^{14}d^4e^{10} - 160a^8b^{12}d^4e^{10} - 288a^{10}b^{10}d^4e^{10} - \\
& 160a^{12}b^8d^4e^{10} - 32a^{14}b^6d^4e^{10})) / ((b^9d + 2a^2b^7d + a^4* \\
& b^5d)*(b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3 \\
& d^4)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7d + a^4* \\
& b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7d + a^4* \\
& b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7d + a^4* \\
& b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7d + a^4* \\
& b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} / (2*(b^9d + 2a^2b^7d + a^4* \\
& b^5d)) * (3a^2 + 7b^2) * (-a^5b^5e^7)^{(1/2)} * i / (b^9d + 2a^2b^7 \\
& *d + a^4b^5d) - \operatorname{atan}\left(\left(\left(\left(\left(16(8a^8b^{17}d^4e^{14} + 96a^3b^{15}d^4e^{14} + \right. \right. \right. \right. \\
& \left. \left. \left. \left. 360a^5b^{13}d^4e^{14} + 640a^7b^{11}d^4e^{14} + 600a^9b^9d^4e^{14} + 288 \right. \right. \right. \right. \\
& \left. \left. \left. \left. *a^{11}b^7d^4e^{14} + 56a^{13}b^5d^4e^{14}) \right) \right) \right) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + \right. \\
& \left. 4a^6b^5d^5 + a^8b^3d^5) - (16*(e*\cot(c + d*x))^{(1/2)}*(-e^7 / (4*(a^4d^2i + b^4d^2i + \right. \\
& \left. 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2*6i) \right) \right)^{(1/2)} * (32b^{20}d^4e^{10} + 160a^2b^{18}d^4e^{10} + 288a^4b^{16}d^4e^{10} \\
& + 160a^6b^{14}d^4e^{10} - 160a^8b^{12}d^4e^{10} - 288a^{10}b^{10}d^4e^{10} - \\
& 160a^{12}b^8d^4e^{10} - 32a^{14}b^6d^4e^{10})) / (b^{11}d^4 + 4a^2b^9d^4 +
\end{aligned}$$

$$\begin{aligned}
& (6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} + (16(e * \cot(c + dx))^{1/2} * (72a^{15}b^d^2e^{17} - 60a^b^{15}d^2e^{17} - 52a^3b^{13}d^2e^{17} + 72a^5b^{11}d^2e^{17} + 448a^7b^9d^2e^{17} + 1108a^9b^7d^2e^{17} + 1132a^{11}b^5d^2e^{17} + 480a^{13}b^3d^2e^{17})) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} - (16(30a^6b^8d^2e^{21} - 224a^4b^{10}d^2e^{21} - 18a^{14}d^2e^{21} + 600a^8b^6d^2e^{21} + 388a^{10}b^4d^2e^{21} + 24a^{12}b^2d^2e^{21})) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 + a^8b^3d^5) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} - (16(e * \cot(c + dx))^{1/2} * (9a^{12}e^{24} + 2b^{12}e^{24} + 4a^2b^{10}e^{24} + 2a^4b^8e^{24} - 49a^6b^6e^{24} + 7a^8b^4e^{24} + 33a^{10}b^2e^{24})) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} * i - ((((((16(8a^b^{17}d^4e^{14} + 96a^3b^{15}d^4e^{14} + 360a^5b^{13}d^4e^{14} + 640a^7b^{11}d^4e^{14} + 600a^9b^9d^4e^{14} + 288a^{11}b^7d^4e^{14} + 56a^{13}b^5d^4e^{14}))) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 + a^8b^3d^5) + (16(e * \cot(c + dx))^{1/2} * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} * (32b^{20}d^4e^{10} + 160a^2b^{18}d^4e^{10} + 288a^4b^{16}d^4e^{10} + 160a^6b^{14}d^4e^{10} - 160a^8b^{12}d^4e^{10} - 288a^{10}b^{10}d^4e^{10} - 160a^{12}b^8d^4e^{10} - 32a^{14}b^6d^4e^{10})) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} - (16(e * \cot(c + dx))^{1/2} * (72a^{15}b^d^2e^{17} - 60a^b^{15}d^2e^{17} - 52a^3b^{13}d^2e^{17} + 72a^5b^{11}d^2e^{17} + 448a^7b^9d^2e^{17} + 1108a^9b^7d^2e^{17} + 1132a^{11}b^5d^2e^{17} + 480a^{13}b^3d^2e^{17})) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} - (16(30a^6b^8d^2e^{21} - 224a^4b^{10}d^2e^{21} - 18a^{14}d^2e^{21} + 600a^8b^6d^2e^{21} + 388a^{10}b^4d^2e^{21} + 24a^{12}b^2d^2e^{21})) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 + a^8b^3d^5) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} + (16(e * \cot(c + dx))^{1/2} * (9a^{12}e^{24} + 2b^{12}e^{24} + 4a^2b^{10}e^{24} + 2a^4b^8e^{24} - 49a^6b^6e^{24} + 7a^8b^4e^{24} + 33a^{10}b^2e^{24})) / (b^{11}d^4 + 4a^2b^9d^4 + 6a^4b^7d^4 + 4a^6b^5d^4 + a^8b^3d^4) * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} * i) / ((32(7a^3b^7e^{28} + 3a^5b^5e^{28})) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 + a^8b^3d^5) + ((((((16(8a^b^{17}d^4e^{14} + 96a^3b^{15}d^4e^{14} + 360a^5b^{13}d^4e^{14} + 640a^7b^{11}d^4e^{14} + 600a^9b^9d^4e^{14} + 288a^{11}b^7d^4e^{14} + 56a^{13}b^5d^4e^{14}))) / (b^{11}d^5 + 4a^2b^9d^5 + 6a^4b^7d^5 + 4a^6b^5d^5 + a^8b^3d^5) - (16(e * \cot(c + dx))^{1/2} * (-e^{7/4} / (4(a^4d^2 * i + b^4d^2 * i + 4a^3b^3d^2 - 4a^3b^3d^2 - a^2b^2d^2 * 6i)))^{1/2} * (32b^{20}d^4e^{10} + 160a^2b^{18}d^4e^{10} + 288a^4b^{16}d^4e^{10} + 1
\end{aligned}$$

$$\begin{aligned}
& 60*a^6*b^{14}*d^4*e^{10} - 160*a^8*b^{12}*d^4*e^{10} - 288*a^{10}*b^{10}*d^4*e^{10} - 160 \\
& *a^{12}*b^8*d^4*e^{10} - 32*a^{14}*b^6*d^4*e^{10}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a \\
& ^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2*1 \\
& i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} + (16*(e*\cot(c + d* \\
& x))^{(1/2)}*(72*a^{15}*b*d^2*e^{17} - 60*a*b^{15}*d^2*e^{17} - 52*a^3*b^{13}*d^2*e^{17} + \\
& 72*a^5*b^{11}*d^2*e^{17} + 448*a^7*b^9*d^2*e^{17} + 1108*a^9*b^7*d^2*e^{17} + 1132 \\
& *a^{11}*b^5*d^2*e^{17} + 480*a^{13}*b^3*d^2*e^{17}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6* \\
& a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2* \\
& 1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} - (16*(30*a^6*b^8* \\
& d^2*e^{21} - 224*a^4*b^{10}*d^2*e^{21} - 18*a^{14}*d^2*e^{21} + 600*a^8*b^6*d^2*e^{21} \\
& + 388*a^{10}*b^4*d^2*e^{21} + 24*a^{12}*b^2*d^2*e^{21}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 \\
& + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-e^{7/(4*(a^4*d^2*1i + b^4* \\
& d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} - (16*(e*\cot(c \\
& + d*x))^{(1/2)}*(9*a^{12}*e^{24} + 2*b^{12}*e^{24} + 4*a^2*b^{10}*e^{24} + 2*a^4*b^8*e^{2 \\
& 4} - 49*a^6*b^6*e^{24} + 7*a^8*b^4*e^{24} + 33*a^{10}*b^2*e^{24}))/ (b^{11}*d^4 + 4*a^2 \\
& *b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^{7/(4*(a^4*d^2* \\
& 1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} + ((( \\
& ((16*(8*a*b^{17}*d^4*e^{14} + 96*a^3*b^{15}*d^4*e^{14} + 360*a^5*b^{13}*d^4*e^{14} + 64 \\
& 0*a^7*b^{11}*d^4*e^{14} + 600*a^9*b^9*d^4*e^{14} + 288*a^{11}*b^7*d^4*e^{14} + 56*a^{1 \\
& 3}*b^5*d^4*e^{14}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 \\
& + a^8*b^3*d^5) + (16*(e*\cot(c + d*x))^{(1/2)}*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2* \\
& 1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)}*(32*b^{20}*d^4*e^{10} \\
& + 160*a^2*b^{18}*d^4*e^{10} + 288*a^4*b^{16}*d^4*e^{10} + 160*a^6*b^{14}*d^4*e^{10} - 1 \\
& 60*a^8*b^{12}*d^4*e^{10} - 288*a^{10}*b^{10}*d^4*e^{10} - 160*a^{12}*b^8*d^4*e^{10} - 32* \\
& a^{14}*b^6*d^4*e^{10}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d \\
& ^4 + a^8*b^3*d^4))*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3* \\
& b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} - (16*(e*\cot(c + d*x))^{(1/2)}*(72*a^{15}*b*d^2 \\
& *e^{17} - 60*a*b^{15}*d^2*e^{17} - 52*a^3*b^{13}*d^2*e^{17} + 72*a^5*b^{11}*d^2*e^{17} + \\
& 448*a^7*b^9*d^2*e^{17} + 1108*a^9*b^7*d^2*e^{17} + 1132*a^{11}*b^5*d^2*e^{17} + 480 \\
& *a^{13}*b^3*d^2*e^{17}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5* \\
& d^4 + a^8*b^3*d^4))*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3 \\
& *b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} - (16*(30*a^6*b^8*d^2*e^{21} - 224*a^4*b^{10} \\
& d^2*e^{21} - 18*a^{14}*d^2*e^{21} + 600*a^8*b^6*d^2*e^{21} + 388*a^{10}*b^4*d^2*e^{21} \\
& + 24*a^{12}*b^2*d^2*e^{21}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6* \\
& b^5*d^5 + a^8*b^3*d^5))*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4 \\
& *a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)} + (16*(e*\cot(c + d*x))^{(1/2)}*(9*a^{12}*e \\
& ^{24} + 2*b^{12}*e^{24} + 4*a^2*b^{10}*e^{24} + 2*a^4*b^8*e^{24} - 49*a^6*b^6*e^{24} + 7* \\
& a^8*b^4*e^{24} + 33*a^{10}*b^2*e^{24}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 \\
& + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-e^{7/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^ \\
& 3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)}))*(-e^{7/(4*(a^4*d^2*1i + b^4* \\
& d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))})^{(1/2)}*2i - (2*e^3*(e \\
& *\cot(c + d*x))^{(1/2)})/(b^2*d) - \operatorname{atan}(((((((16*(8*a*b^{17}*d^4*e^{14} + 96*a^3*b \\
& ^{15}*d^4*e^{14} + 360*a^5*b^{13}*d^4*e^{14} + 640*a^7*b^{11}*d^4*e^{14} + 600*a^9*b^9* \\
& d^4*e^{14} + 288*a^{11}*b^7*d^4*e^{14} + 56*a^{13}*b^5*d^4*e^{14}))/ (b^{11}*d^5 + 4*a^2 \\
& *b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*(e*\cot(c + d*
\end{aligned}$$

$$\begin{aligned}
& x))^{(1/2)} * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * (32*b^20*d^4*e^{10} + 160*a^2*b^18*d^4*e^{10} + 288*a^4*b^16*d^4*e^{10} + 160*a^6*b^14*d^4*e^{10} - 160*a^8*b^12*d^4*e^{10} - 288*a^10*b^10*d^4*e^{10} - 160*a^12*b^8*d^4*e^{10} - 32*a^14*b^6*d^4*e^{10})) / (b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} + \\
& (16 * (e * \cot(c + d*x))^{(1/2)} * (72*a^15*b*d^2*e^{17} - 60*a*b^15*d^2*e^{17} - 52*a^3*b^13*d^2*e^{17} + 72*a^5*b^11*d^2*e^{17} + 448*a^7*b^9*d^2*e^{17} + 1108*a^9*b^7*d^2*e^{17} + 1132*a^11*b^5*d^2*e^{17} + 480*a^13*b^3*d^2*e^{17})) / (b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} - \\
& (16 * (30*a^6*b^8*d^2*e^{21} - 224*a^4*b^10*d^2*e^{21} - 18*a^14*d^2*e^{21} + 600*a^8*b^6*d^2*e^{21} + 388*a^10*b^4*d^2*e^{21} + 24*a^12*b^2*d^2*e^{21})) / (b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} - \\
& (16 * (e * \cot(c + d*x))^{(1/2)} * (9*a^12*e^{24} + 2*b^12*e^{24} + 4*a^2*b^10*e^{24} + 2*a^4*b^8*e^{24} - 49*a^6*b^6*e^{24} + 7*a^8*b^4*e^{24} + 33*a^10*b^2*e^{24})) / (b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * 1i - \\
& ((((((16 * (8*a*b^17*d^4*e^{14} + 96*a^3*b^15*d^4*e^{14} + 360*a^5*b^13*d^4*e^{14} + 640*a^7*b^11*d^4*e^{14} + 600*a^9*b^9*d^4*e^{14} + 288*a^11*b^7*d^4*e^{14} + 56*a^13*b^5*d^4*e^{14}))) / (b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (16 * (e * \cot(c + d*x))^{(1/2)} * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * \\
& (32*b^20*d^4*e^{10} + 160*a^2*b^18*d^4*e^{10} + 288*a^4*b^16*d^4*e^{10} + 160*a^6*b^14*d^4*e^{10} - 160*a^8*b^12*d^4*e^{10} - 288*a^10*b^10*d^4*e^{10} - 160*a^12*b^8*d^4*e^{10} - 32*a^14*b^6*d^4*e^{10})) / (b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} - \\
& (16 * (e * \cot(c + d*x))^{(1/2)} * (72*a^15*b*d^2*e^{17} - 60*a*b^15*d^2*e^{17} - 52*a^3*b^13*d^2*e^{17} + 72*a^5*b^11*d^2*e^{17} + 448*a^7*b^9*d^2*e^{17} + 1108*a^9*b^7*d^2*e^{17} + 1132*a^11*b^5*d^2*e^{17} + 480*a^13*b^3*d^2*e^{17})) / (b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} - \\
& (16 * (30*a^6*b^8*d^2*e^{21} - 224*a^4*b^10*d^2*e^{21} - 18*a^14*d^2*e^{21} + 600*a^8*b^6*d^2*e^{21} + 388*a^10*b^4*d^2*e^{21} + 24*a^12*b^2*d^2*e^{21})) / (b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} + \\
& (16 * (e * \cot(c + d*x))^{(1/2)} * (9*a^12*e^{24} + 2*b^12*e^{24} + 4*a^2*b^10*e^{24} + 2*a^4*b^8*e^{24} - 49*a^6*b^6*e^{24} + 7*a^8*b^4*e^{24} + 33*a^10*b^2*e^{24})) / (b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4)) * (- (e^{7*1i}) / (4 * (a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * 1i) / ((32 * (7*a^3*b^7*e^{28} + 3*a^5*b^5*e^{28})) / (b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + ((((((16 * (8*a*b^17*d^4*e^{14} + 96*a^3*b^15*d^4*e^{14} + 360*a^5*b^13*d^4*e^{14} + 640*a^7*b^11*d^4*e^{14} + 600*a^9*b^9*d^4
\end{aligned}$$

$$\begin{aligned}
& *e^{14} + 288*a^{11}*b^7*d^4*e^{14} + 56*a^{13}*b^5*d^4*e^{14}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) - (16*(e*\cot(c + d*x))^{(1/2)}*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)}*(32*b^20*d^4*e^{10} + 160*a^2*b^18*d^4*e^{10} + 288*a^4*b^16*d^4*e^{10} + 160*a^6*b^14*d^4*e^{10} - 160*a^8*b^12*d^4*e^{10} - 288*a^10*b^10*d^4*e^{10} - 160*a^12*b^8*d^4*e^{10} - 32*a^14*b^6*d^4*e^{10}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} + (16*(e*\cot(c + d*x))^{(1/2)}*(72*a^15*b*d^2*e^{17} - 60*a*b^15*d^2*e^{17} - 52*a^3*b^13*d^2*e^{17} + 72*a^5*b^11*d^2*e^{17} + 448*a^7*b^9*d^2*e^{17} + 1108*a^9*b^7*d^2*e^{17} + 1132*a^11*b^5*d^2*e^{17} + 480*a^13*b^3*d^2*e^{17}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} - (16*(30*a^6*b^8*d^2*e^{21} - 224*a^4*b^10*d^2*e^{21} - 18*a^14*d^2*e^{21} + 600*a^8*b^6*d^2*e^{21} + 388*a^10*b^4*d^2*e^{21} + 24*a^12*b^2*d^2*e^{21}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} - (16*(e*\cot(c + d*x))^{(1/2)}*(9*a^12*e^{24} + 2*b^12*e^{24} + 4*a^2*b^10*e^{24} + 2*a^4*b^8*e^{24} - 49*a^6*b^6*e^{24} + 7*a^8*b^4*e^{24} + 33*a^10*b^2*e^{24}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} + ((((((16*(8*a*b^17*d^4*e^{14} + 96*a^3*b^15*d^4*e^{14} + 360*a^5*b^13*d^4*e^{14} + 640*a^7*b^11*d^4*e^{14} + 600*a^9*b^9*d^4*e^{14} + 288*a^11*b^7*d^4*e^{14} + 56*a^13*b^5*d^4*e^{14}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5) + (16*(e*\cot(c + d*x))^{(1/2)}*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)}*(32*b^20*d^4*e^{10} + 160*a^2*b^18*d^4*e^{10} + 288*a^4*b^16*d^4*e^{10} + 160*a^6*b^14*d^4*e^{10} - 160*a^8*b^12*d^4*e^{10} - 288*a^10*b^10*d^4*e^{10} - 160*a^12*b^8*d^4*e^{10} - 32*a^14*b^6*d^4*e^{10}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} - (16*(e*\cot(c + d*x))^{(1/2)}*(72*a^15*b*d^2*e^{17} - 60*a*b^15*d^2*e^{17} - 52*a^3*b^13*d^2*e^{17} + 72*a^5*b^11*d^2*e^{17} + 448*a^7*b^9*d^2*e^{17} + 1108*a^9*b^7*d^2*e^{17} + 1132*a^11*b^5*d^2*e^{17} + 480*a^13*b^3*d^2*e^{17}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} - (16*(30*a^6*b^8*d^2*e^{21} - 224*a^4*b^10*d^2*e^{21} - 18*a^14*d^2*e^{21} + 600*a^8*b^6*d^2*e^{21} + 388*a^10*b^4*d^2*e^{21} + 24*a^12*b^2*d^2*e^{21}))/ (b^{11}*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} + (16*(e*\cot(c + d*x))^{(1/2)}*(9*a^12*e^{24} + 2*b^12*e^{24} + 4*a^2*b^10*e^{24} + 2*a^4*b^8*e^{24} - 49*a^6*b^6*e^{24} + 7*a^8*b^4*e^{24} + 33*a^10*b^2*e^{24}))/ (b^{11}*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)})))*(-(e^7*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)}*
\end{aligned}$$

$2i - (a^3 e^4 (e \cot(c + dx))^{1/2}) / ((a^2 + b^2) (a b^2 d e + b^3 d e \cot(c + dx)))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(7/2)/(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Timed out

$$3.76 \quad \int \frac{(e \cot(c+dx))^{5/2}}{(a+b \cot(c+dx))^2} dx$$

Optimal. Leaf size=393

$$\frac{e^{5/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2} - \frac{e^{5/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2}$$

[Out]  $-a^{3/2}*(a^2+5*b^2)*e^{5/2}*\arctan(b^{1/2}*(e*\cot(d*x+c))^{1/2}/a^{1/2}/e^{1/2})/b^{3/2}/(a^2+b^2)^2/d-1/2*(a^2+2*a*b-b^2)*e^{5/2}*\arctan(1-2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/(a^2+b^2)^2/d*2^{1/2}+1/2*(a^2+2*a*b-b^2)*e^{5/2}*\arctan(1+2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/(a^2+b^2)^2/d*2^{1/2}+1/4*(a^2-2*a*b-b^2)*e^{5/2}*ln(e^{1/2}+\cot(d*x+c))*e^{1/2}-2^{1/2}*(e*\cot(d*x+c))^{1/2}/(a^2+b^2)^2/d*2^{1/2}-1/4*(a^2-2*a*b-b^2)*e^{5/2}*ln(e^{1/2}+\cot(d*x+c))*e^{1/2}+2^{1/2}*(e*\cot(d*x+c))^{1/2}/(a^2+b^2)^2/d*2^{1/2}+a^2*e^{5/2}*(e*\cot(d*x+c))^{1/2}/b/(a^2+b^2)/d/(a+b*\cot(d*x+c))$

**Rubi [A]** time = 0.74, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3565, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{a^2 e^2 \sqrt{e \cot(c+dx)}}{bd (a^2 + b^2) (a + b \cot(c+dx))} + \frac{e^{5/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2} - \frac{e^{5/2} (a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)/(a + b\*Cot[c + d\*x])^2,x]

[Out]  $-((a^{3/2}*(a^2 + 5*b^2)*e^{5/2}*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(\text{Sqrt}[a]*\text{Sqrt}[e]))/(b^{3/2}*(a^2 + b^2)^2*d) - ((a^2 + 2*a*b - b^2)*e^{5/2}*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*e^{5/2}*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)^2*d) + (a^2*e^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(b*(a^2 + b^2)*d*(a + b*\text{Cot}[c + d*x])) + ((a^2 - 2*a*b - b^2)*e^{5/2}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^2*d) - ((a^2 - 2*a*b - b^2)*e^{5/2}*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^2*d)$

Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +



$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]]/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

### Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x\_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3565

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)^(m-2)*(a + b*Tan[e + f*x])^(
m-2)*(c + d*Tan[e + f*x])^(n+1))/(d*f*(n+1)*(c^2 + d^2)), x] - Dist[1
/(d*(n+1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m-3)*(c + d*Tan[e + f
*x])^(n+1)*Simp[a^2*d*(b*d*(m-2) - a*c*(n+1)) + b*(b*c - 2*a*d)*(b*c*
(m-2) + a*d*(n+1)) - d*(n+1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*
Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m+n-1) - b^2*(c^2*(m-2) - d^2*(n
+1)))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ[
n, -1] && IntegerQ[2*m]
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3653

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_)
+ (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{5/2}}{(a + b \cot(c + dx))^2} dx &= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{b (a^2 + b^2) d (a + b \cot(c + dx))} - \frac{\int \frac{-\frac{1}{2} a^2 e^3 + a b e^3 \cot(c + dx) - \frac{1}{2} (a^2 + 2b^2) e^3 \cot^2(c + dx)}{\sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} dx}{b (a^2 + b^2)} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{b (a^2 + b^2) d (a + b \cot(c + dx))} - \frac{\int \frac{2ab^2 e^3 + b(a^2 - b^2) e^3 \cot(c + dx)}{\sqrt{e \cot(c + dx)}} dx}{b (a^2 + b^2)^2} + \frac{(a^2 (a^2 + 5b^2) e^3)}{2} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{b (a^2 + b^2) d (a + b \cot(c + dx))} - \frac{2 \text{Subst} \left( \int \frac{-2ab^2 e^4 - b(a^2 - b^2) e^3 x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{b (a^2 + b^2)^2 d} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{b (a^2 + b^2) d (a + b \cot(c + dx))} - \frac{(a^2 (a^2 + 5b^2) e^2) \text{Subst} \left( \int \frac{1}{a + \frac{bx^2}{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{b (a^2 + b^2)^2 d} \\
&= -\frac{a^{3/2} (a^2 + 5b^2) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{3/2} (a^2 + b^2)^2 d} + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{((a^2 + 5b^2) e^3)}{2} \\
&= -\frac{a^{3/2} (a^2 + 5b^2) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{3/2} (a^2 + b^2)^2 d} + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{(a^2 + 5b^2) e^3}{2} \\
&= -\frac{a^{3/2} (a^2 + 5b^2) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right)}{b^{3/2} (a^2 + b^2)^2 d} - \frac{(a^2 + 2ab - b^2) e^{5/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c + dx)}}{\sqrt{a}} \right)}{\sqrt{2} (a^2 + b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** time = 2.80, size = 390, normalized size = 0.99

$$\frac{(e \cot(c + dx))^{5/2} \left( 12b^{7/2} (a^2 + b^2) \cot^{7/2}(c + dx) {}_2F_1 \left( 2, \frac{7}{2}; \frac{9}{2}; -\frac{b \cot(c + dx)}{a} \right) - 28a^2 b^{3/2} (a^2 - b^2) \cot^{3/2}(c + dx) {}_2F_1 \left( \dots \right) \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(5/2)/(a + b\*Cot[c + d\*x])^2,x]

[Out] -1/42\*((e\*Cot[c + d\*x])^(5/2))\*(-28\*a^2\*b^(3/2)\*(a^2 - b^2)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2] + 12\*b^(7/2)\*(a^2 + b^2)

\*Cot[c + d\*x]^(7/2)\*Hypergeometric2F1[2, 7/2, 9/2, -((b\*Cot[c + d\*x])/a)] - 7\*a^2\*(-6\*Sqrt[2]\*a\*b^(5/2)\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 6\*Sqrt[2]\*a\*b^(5/2)\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]]] + 24\*a^(7/2)\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]] - 24\*a^3\*Sqrt[b]\*Sqrt[Cot[c + d\*x]] - 24\*a\*b^(5/2)\*Sqrt[Cot[c + d\*x]] + 4\*a^2\*b^(3/2)\*Cot[c + d\*x]^(3/2) + 4\*b^(7/2)\*Cot[c + d\*x]^(3/2) - 3\*Sqrt[2]\*a\*b^(5/2)\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] + 3\*Sqrt[2]\*a\*b^(5/2)\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])))/(a^2\*b^(3/2)\*(a^2 + b^2)^2\*d\*Cot[c + d\*x]^(5/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(dx+c))^(5/2)/(a+b\*cot(dx+c))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{5}{2}}}{(b \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(dx+c))^(5/2)/(a+b\*cot(dx+c))^2,x, algorithm="giac")

[Out] integrate((e\*cot(dx + c))^(5/2)/(b\*cot(dx + c) + a)^2, x)

**maple** [B] time = 0.78, size = 784, normalized size = 1.99

$$\frac{e^3 a^4 \sqrt{e \cot(dx + c)}}{d(a^2 + b^2)^2 b(e \cot(dx + c)b + ae)} + \frac{e^3 a^2 b \sqrt{e \cot(dx + c)}}{d(a^2 + b^2)^2 (e \cot(dx + c)b + ae)} - \frac{e^3 a^4 \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2)^2 b \sqrt{aeb}} - \frac{5e^3 a^2 b \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2)^2 b \sqrt{aeb}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(dx+c))^(5/2)/(a+b\*cot(dx+c))^2,x)

[Out] 1/d\*e^3\*a^4/(a^2+b^2)^2/b\*(e\*cot(dx+c))^(1/2)/(e\*cot(dx+c)\*b+a\*e)+1/d\*e^3\*a^2/(a^2+b^2)^2\*b\*(e\*cot(dx+c))^(1/2)/(e\*cot(dx+c)\*b+a\*e)-1/d\*e^3\*a^4/(a^2+b^2)^2/b/(a\*e\*b)^(1/2)\*arctan((e\*cot(dx+c))^(1/2)\*b/(a\*e\*b)^(1/2))-5/d\*e^3\*a^2/(a^2+b^2)^2\*b/(a\*e\*b)^(1/2)\*arctan((e\*cot(dx+c))^(1/2)\*b/(a\*e\*b)^(1/2))+1/2/d\*e^2/(a^2+b^2)^2\*a\*b\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(dx+c)+(e^2)^(1/4)\*2^(1/2))

$$\begin{aligned} & \left( \frac{1}{4} \right) * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)} / (e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) + 1/d * e^2 / (a^2 + b^2)^2 * a * b * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) - 1/d * e^2 / (a^2 + b^2)^2 * a * b * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) + 1/4/d * e^3 / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * a^2 - 1/4/d * e^3 / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * b^2 + 1/2/d * e^3 / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a^2 - 1/2/d * e^3 / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * b^2 - 1/2/d * e^3 / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a^2 + 1/2/d * e^3 / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * b^2 \end{aligned}$$

**maxima [A]** time = 0.45, size = 359, normalized size = 0.91

$$\left( \frac{4a^2e^2\sqrt{\frac{e}{\tan(dx+c)}}}{(a^3b+ab^3)e+\frac{(a^2b^2+b^4)e}{\tan(dx+c)}} - \frac{4(a^4+5a^2b^2)e^2\arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^4b+2a^2b^3+b^5)\sqrt{abe}} + \frac{2\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2+2ab-b^2)\arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right) / 4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} * (4 * a^2 * e^2 * \sqrt{e / \tan(dx+c)}) / ((a^3 * b + a * b^3) * e + (a^2 * b^2 + b^4) * e / \tan(dx+c)) - 4 * (a^4 + 5 * a^2 * b^2) * e^2 * \arctan(b * \sqrt{e / \tan(dx+c)}) / \sqrt{a * b * e} / ((a^4 * b + 2 * a^2 * b^3 + b^5) * \sqrt{a * b * e}) + (2 * \sqrt{2} * (a^2 + 2 * a * b - b^2) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e / \tan(dx+c)})) / \sqrt{e}) / \sqrt{e} + 2 * \sqrt{2} * (a^2 + 2 * a * b - b^2) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e / \tan(dx+c)})) / \sqrt{e}) / \sqrt{e} - \sqrt{2} * (a^2 - 2 * a * b - b^2) * \log(\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx+c)}) + e + e / \tan(dx+c) / \sqrt{e} + \sqrt{2} * (a^2 - 2 * a * b - b^2) * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx+c)}) + e + e / \tan(dx+c) / \sqrt{e} * e^2 / (a^4 + 2 * a^2 * b^2 + b^4) * e / d$

**mupad [B]** time = 3.12, size = 12617, normalized size = 32.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e \cdot \cot(c + d \cdot x))^{5/2} / (a + b \cdot \cot(c + d \cdot x))^2, x)$

[Out]  $\text{atan}\left(\frac{\left(\frac{8(96a^2b^{14}d^4e^{13} + 480a^4b^{12}d^4e^{13} + 960a^6b^{10}d^4e^{13} + 960a^8b^8d^4e^{13} + 480a^{10}b^6d^4e^{13} + 96a^{12}b^4d^4e^{13})}{(b^9d^5 + a^8b^8d^5 + 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3d^5)} - (16(e \cdot \cot(c + d \cdot x))^{1/2} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} \cdot (32b^{18}d^4e^{10} + 160a^2b^{16}d^4e^{10} + 288a^4b^{14}d^4e^{10} + 160a^6b^{12}d^4e^{10} - 160a^8b^{10}d^4e^{10} - 288a^{10}b^8d^4e^{10} - 160a^{12}b^6d^4e^{10} - 32a^{14}b^4d^4e^{10})}{(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} + (16(e \cdot \cot(c + d \cdot x))^{1/2} \cdot (60a^3b^{13}d^2e^{15} + 8a^{13}b^8d^2e^{15} + 52a^3b^{11}d^2e^{15} + 128a^5b^9d^2e^{15} + 424a^7b^7d^2e^{15} + 380a^9b^5d^2e^{15} + 100a^{11}b^3d^2e^{15}))}{(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} + (8(4a^3b^{11}d^2e^{18} + 16a^{11}b^8d^2e^{18} - 304a^3b^9d^2e^{18} - 120a^5b^7d^2e^{18} + 320a^7b^5d^2e^{18} + 148a^9b^3d^2e^{18}))}{(b^9d^5 + a^8b^8d^5 + 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3d^5)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} + (16(e \cdot \cot(c + d \cdot x))^{1/2} \cdot (a^{10}e^{20} - 2b^{10}e^{20} - 4a^2b^8e^{20} - 27a^4b^6e^{20} + 15a^6b^4e^{20} + 9a^8b^2e^{20}))}{(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} \cdot 1i - \left(\frac{8(96a^2b^{14}d^4e^{13} + 480a^4b^{12}d^4e^{13} + 960a^6b^{10}d^4e^{13} + 960a^8b^8d^4e^{13} + 480a^{10}b^6d^4e^{13} + 96a^{12}b^4d^4e^{13})}{(b^9d^5 + a^8b^8d^5 + 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3d^5)} + (16(e \cdot \cot(c + d \cdot x))^{1/2} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} \cdot (32b^{18}d^4e^{10} + 160a^2b^{16}d^4e^{10} + 288a^4b^{14}d^4e^{10} + 160a^6b^{12}d^4e^{10} - 160a^8b^{10}d^4e^{10} - 288a^{10}b^8d^4e^{10} - 160a^{12}b^6d^4e^{10} - 32a^{14}b^4d^4e^{10}))}{(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} - (16(e \cdot \cot(c + d \cdot x))^{1/2} \cdot (60a^3b^{13}d^2e^{15} + 8a^{13}b^8d^2e^{15} + 52a^3b^{11}d^2e^{15} + 128a^5b^9d^2e^{15} + 424a^7b^7d^2e^{15} + 380a^9b^5d^2e^{15} + 100a^{11}b^3d^2e^{15}))}{(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} + (8(4a^3b^{11}d^2e^{18} + 16a^{11}b^8d^2e^{18} - 304a^3b^9d^2e^{18} - 120a^5b^7d^2e^{18} + 320a^7b^5d^2e^{18} + 148a^9b^3d^2e^{18}))}{(b^9d^5 + a^8b^8d^5 + 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3d^5)} \cdot ((e^5 \cdot 1i) / (4(a^4d^2 + b^4d^2 + a^3b^3d^2 \cdot 4i - a^3b^3d^2 \cdot 4i - 6a^2b^2d^2)))^{1/2} - (16(e \cdot \cot(c + d \cdot x))^{1/2} \cdot (a^{10}e^{20} - 2b^{10}e^{20} - 4a^2b^8e^{20} - 27a^4b^6e^{20} + 15a^6b^4e^{20} + 9a^8b^2e^{20}))}{(b^9d^4 +$

$$\begin{aligned}
& (a^8 b d^4 + 4 a^2 b^7 d^4 + 6 a^4 b^5 d^4 + 4 a^6 b^3 d^4) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} * 1i \\
& ) / ((16 * (a^8 e^{23} + 10 a^2 b^6 e^{23} + 27 a^4 b^4 e^{23} + 10 a^6 b^2 e^{23})) / (b^9 d^5 + a^8 b d^5 + 4 a^2 b^7 d^5 + 6 a^4 b^5 d^5 + 4 a^6 b^3 d^5) + (((((8 * (96 a^2 b^{14} d^4 e^{13} + 480 a^4 b^{12} d^4 e^{13} + 960 a^6 b^{10} d^4 e^{13} + 960 a^8 b^8 d^4 e^{13} + 480 a^{10} b^6 d^4 e^{13} + 96 a^{12} b^4 d^4 e^{13})) / (b^9 d^5 + a^8 b d^5 + 4 a^2 b^7 d^5 + 6 a^4 b^5 d^5 + 4 a^6 b^3 d^5) - (16 * (e * \cot(c + d * x))^{(1/2)} * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} * (32 b^{18} d^4 e^{10} + 160 a^2 b^{16} d^4 e^{10} + 288 a^4 b^{14} d^4 e^{10} + 160 a^6 b^{12} d^4 e^{10} - 160 a^8 b^{10} d^4 e^{10} - 288 a^{10} b^8 d^4 e^{10} - 160 a^{12} b^6 d^4 e^{10} - 32 a^{14} b^4 d^4 e^{10})) / (b^9 d^4 + a^8 b d^4 + 4 a^2 b^7 d^4 + 6 a^4 b^5 d^4 + 4 a^6 b^3 d^4)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (16 * (e * \cot(c + d * x))^{(1/2)} * (60 a b^{13} d^2 e^{15} + 8 a^{13} b d^2 e^{15} + 52 a^3 b^{11} d^2 e^{15} + 128 a^5 b^9 d^2 e^{15} + 424 a^7 b^7 d^2 e^{15} + 380 a^9 b^5 d^2 e^{15} + 100 a^{11} b^3 d^2 e^{15})) / (b^9 d^4 + a^8 b d^4 + 4 a^2 b^7 d^4 + 6 a^4 b^5 d^4 + 4 a^6 b^3 d^4)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (8 * (4 a b^{11} d^2 e^{18} + 16 a^{11} b d^2 e^{18} - 304 a^3 b^9 d^2 e^{18} - 120 a^5 b^7 d^2 e^{18} + 320 a^7 b^5 d^2 e^{18} + 148 a^9 b^3 d^2 e^{18})) / (b^9 d^5 + a^8 b d^5 + 4 a^2 b^7 d^5 + 6 a^4 b^5 d^5 + 4 a^6 b^3 d^5)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (16 * (e * \cot(c + d * x))^{(1/2)} * (a^{10} e^{20} - 2 b^{10} e^{20} - 4 a^2 b^8 e^{20} - 27 a^4 b^6 e^{20} + 15 a^6 b^4 e^{20} + 9 a^8 b^2 e^{20})) / (b^9 d^4 + a^8 b d^4 + 4 a^2 b^7 d^4 + 6 a^4 b^5 d^4 + 4 a^6 b^3 d^4)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (((((8 * (96 a^2 b^{14} d^4 e^{13} + 480 a^4 b^{12} d^4 e^{13} + 960 a^6 b^{10} d^4 e^{13} + 960 a^8 b^8 d^4 e^{13} + 480 a^{10} b^6 d^4 e^{13} + 96 a^{12} b^4 d^4 e^{13})) / (b^9 d^5 + a^8 b d^5 + 4 a^2 b^7 d^5 + 6 a^4 b^5 d^5 + 4 a^6 b^3 d^5) + (16 * (e * \cot(c + d * x))^{(1/2)} * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} * (32 b^{18} d^4 e^{10} + 160 a^2 b^{16} d^4 e^{10} + 288 a^4 b^{14} d^4 e^{10} + 160 a^6 b^{12} d^4 e^{10} - 160 a^8 b^{10} d^4 e^{10} - 288 a^{10} b^8 d^4 e^{10} - 160 a^{12} b^6 d^4 e^{10} - 32 a^{14} b^4 d^4 e^{10})) / (b^9 d^4 + a^8 b d^4 + 4 a^2 b^7 d^4 + 6 a^4 b^5 d^4 + 4 a^6 b^3 d^4)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} - (16 * (e * \cot(c + d * x))^{(1/2)} * (60 a b^{13} d^2 e^{15} + 8 a^{13} b d^2 e^{15} + 52 a^3 b^{11} d^2 e^{15} + 128 a^5 b^9 d^2 e^{15} + 424 a^7 b^7 d^2 e^{15} + 380 a^9 b^5 d^2 e^{15} + 100 a^{11} b^3 d^2 e^{15})) / (b^9 d^4 + a^8 b d^4 + 4 a^2 b^7 d^4 + 6 a^4 b^5 d^4 + 4 a^6 b^3 d^4)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (8 * (4 a b^{11} d^2 e^{18} + 16 a^{11} b d^2 e^{18} - 304 a^3 b^9 d^2 e^{18} - 120 a^5 b^7 d^2 e^{18} + 320 a^7 b^5 d^2 e^{18} + 148 a^9 b^3 d^2 e^{18})) / (b^9 d^5 + a^8 b d^5 + 4 a^2 b^7 d^5 + 6 a^4 b^5 d^5 + 4 a^6 b^3 d^5)) * ((e^{5*1i}) / (4 * (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} - (16 * (e * \cot(c + d * x))^{(1/2)} * (a^{10} e^{20} - 2 b^{10} e^{20} - 4 a^2 b^8 e^{20} - 27 a^4 b^6 e^{20} + 15 a^6 b^4 e^{20} + 9 a^8 b^2 e^{20})) / (b^9 d^4 + a^8 b d^4 + 4
\end{aligned}$$

$$\begin{aligned}
& *a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * ((e^5*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)}) * ((e^5*i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * 2i \\
& + \operatorname{atan}(\frac{(((((8*(96*a^2*b^14*d^4*e^{13} + 480*a^4*b^12*d^4*e^{13} + 960*a^6*b^10*d^4*e^{13} + 960*a^8*b^8*d^4*e^{13} + 480*a^{10}*b^6*d^4*e^{13} + 96*a^{12}*b^4*d^4*e^{13} + 0*d^4*e^{13} + 960*a^8*b^8*d^4*e^{13} + 480*a^{10}*b^6*d^4*e^{13} + 96*a^{12}*b^4*d^4*e^{13} + 960*a^6*b^10*d^4*e^{13} + 960*a^8*b^8*d^4*e^{13} + 480*a^{10}*b^6*d^4*e^{13} + 96*a^{12}*b^4*d^4*e^{13} + 0*d^4*e^{13}))/((b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*(e*\cot(c + d*x))^{(1/2)}*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^{(1/2)}*(32*b^18*d^4*e^{10} + 160*a^2*b^16*d^4*e^{10} + 288*a^4*b^14*d^4*e^{10} + 160*a^6*b^12*d^4*e^{10} - 160*a^8*b^10*d^4*e^{10} - 288*a^{10}*b^8*d^4*e^{10} - 160*a^{12}*b^6*d^4*e^{10} - 32*a^{14}*b^4*d^4*e^{10}))/((b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (16*(e*\cot(c + d*x))^{(1/2)}*(60*a*b^13*d^2*e^{15} + 8*a^{13}*b*d^2*e^{15} + 52*a^3*b^11*d^2*e^{15} + 128*a^5*b^9*d^2*e^{15} + 424*a^7*b^7*d^2*e^{15} + 380*a^9*b^5*d^2*e^{15} + 100*a^{11}*b^3*d^2*e^{15}))/((b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (8*(4*a*b^11*d^2*e^{18} + 16*a^{11}*b*d^2*e^{18} - 304*a^3*b^9*d^2*e^{18} - 120*a^5*b^7*d^2*e^{18} + 320*a^7*b^5*d^2*e^{18} + 148*a^9*b^3*d^2*e^{18}))/((b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (16*(e*\cot(c + d*x))^{(1/2)}*(a^{10}*e^{20} - 2*b^{10}*e^{20} - 4*a^2*b^8*e^{20} - 27*a^4*b^6*e^{20} + 15*a^6*b^4*e^{20} + 9*a^8*b^2*e^{20}))/((b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} * 1i - (((((8*(96*a^2*b^14*d^4*e^{13} + 480*a^4*b^12*d^4*e^{13} + 960*a^6*b^10*d^4*e^{13} + 960*a^8*b^8*d^4*e^{13} + 480*a^{10}*b^6*d^4*e^{13} + 96*a^{12}*b^4*d^4*e^{13}))/((b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (16*(e*\cot(c + d*x))^{(1/2)}*(e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)}*(32*b^18*d^4*e^{10} + 160*a^2*b^16*d^4*e^{10} + 288*a^4*b^14*d^4*e^{10} + 160*a^6*b^12*d^4*e^{10} - 160*a^8*b^10*d^4*e^{10} - 288*a^{10}*b^8*d^4*e^{10} - 160*a^{12}*b^6*d^4*e^{10} - 32*a^{14}*b^4*d^4*e^{10}))/((b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(e*\cot(c + d*x))^{(1/2)}*(60*a*b^13*d^2*e^{15} + 8*a^{13}*b*d^2*e^{15} + 52*a^3*b^11*d^2*e^{15} + 128*a^5*b^9*d^2*e^{15} + 424*a^7*b^7*d^2*e^{15} + 380*a^9*b^5*d^2*e^{15} + 100*a^{11}*b^3*d^2*e^{15}))/((b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (8*(4*a*b^11*d^2*e^{18} + 16*a^{11}*b*d^2*e^{18} - 304*a^3*b^9*d^2*e^{18} - 120*a^5*b^7*d^2*e^{18} + 320*a^7*b^5*d^2*e^{18} + 148*a^9*b^3*d^2*e^{18}))/((b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5)) * (e^5/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(e*\cot(c + d*x))^{(1/2)}*(a^{10}*e^{20} - 2*b^{10}*e^{20} - 4*a^2*b^8*e^{20} - 27*a^4*b^6*e^{20} + 15*a^6*b^4*e^{20} + 9*a^8*b^2*e^{20}))/((b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4)) * (e^5/(4*(a^
\end{aligned}$$



$$\begin{aligned}
& (4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)} \\
& *i)/((16*(a^8*e^23 + 10*a^2*b^6*e^23 + 27*a^4*b^4*e^23 + 10*a^6*b^2*e^23)) \\
& / (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (( \\
& ((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*d^4*e^13 \\
& + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*e^13)) / (b^9 \\
& *d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (16*(e \\
& *cot(c + d*x))^{(1/2)}*(e^5/(4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3 \\
& *b*d^2 - a^2*b^2*d^2*6i))^{(1/2)}*(32*b^18*d^4*e^10 + 160*a^2*b^16*d^4*e^10 \\
& + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^10*d^4*e^10 - 2 \\
& 88*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d^4*e^10)) / (b^9*d^4 \\
& + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*( \\
& a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/ \\
& 2)} + (16*(e*cot(c + d*x))^{(1/2)}*(60*a*b^13*d^2*e^15 + 8*a^13*b*d^2*e^15 + 5 \\
& 2*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7*d^2*e^15 + 380*a^9 \\
& *b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15)) / (b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 \\
& + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3 \\
& *d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)} + (8*(4*a*b^11*d^2*e^18 + 1 \\
& 6*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b^7*d^2*e^18 + 320*a^7*b \\
& ^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18)) / (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + \\
& 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*(e^5/(4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3 \\
& *d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)}*( \\
& a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^4*b^6*e^20 + 15*a^6*b^4*e^2 \\
& 0 + 9*a^8*b^2*e^20)) / (b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + \\
& 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d \\
& ^2 - a^2*b^2*d^2*6i))^{(1/2)} + (((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12* \\
& d^4*e^13 + 960*a^6*b^10*d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4* \\
& e^13 + 96*a^12*b^4*d^4*e^13)) / (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4* \\
& b^5*d^5 + 4*a^6*b^3*d^5) + (16*(e*cot(c + d*x))^{(1/2)}*(e^5/(4*(a^4*d^2*i + \\
& b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)}*(32*b^18* \\
& d^4*e^10 + 160*a^2*b^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4 \\
& *e^10 - 160*a^8*b^10*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^ \\
& 10 - 32*a^14*b^4*d^4*e^10)) / (b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^ \\
& 5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4* \\
& a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)}*(60*a*b^13 \\
& *d^2*e^15 + 8*a^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 \\
& + 424*a^7*b^7*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15)) / (b \\
& ^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/( \\
& 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{( \\
& 1/2)} + (8*(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - \\
& 120*a^5*b^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18)) / (b^9* \\
& d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*(e^5/(4*( \\
& a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/ \\
& 2)} - (16*(e*cot(c + d*x))^{(1/2)}*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - \\
& 27*a^4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20)) / (b^9*d^4 + a^8*b*d^4 \\
& + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*(e^5/(4*(a^4*d^2*i + b^4
\end{aligned}$$

$$\begin{aligned}
& *d^{2*1i} + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)})) * (e^5 / (4*(a^4*d^{2*1i} + b^4*d^{2*1i} + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))^{(1/2)} \\
& *2i + (\operatorname{atan}(\frac{((a^2 + 5*b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(a^{10}*e^{20} - 2*b^{10}*e^{20} - 4*a^2*b^8*e^{20} - 27*a^4*b^6*e^{20} + 15*a^6*b^4*e^{20} + 9*a^8*b^2*e^{20})))}{(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) + ((a^2 + 5*b^2)*((8*(4*a*b^{11}*d^2*e^{18} + 16*a^{11}*b*d^2*e^{18} - 304*a^3*b^9*d^2*e^{18} - 120*a^5*b^7*d^2*e^{18} + 320*a^7*b^5*d^2*e^{18} + 148*a^9*b^3*d^2*e^{18})))}{(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (((16*(e*\cot(c + d*x))^{(1/2)}*(60*a*b^{13}*d^2*e^{15} + 8*a^{13}*b*d^2*e^{15} + 52*a^3*b^{11}*d^2*e^{15} + 128*a^5*b^9*d^2*e^{15} + 424*a^7*b^7*d^2*e^{15} + 380*a^9*b^5*d^2*e^{15} + 100*a^{11}*b^3*d^2*e^{15}))/ (b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) + (((8*(96*a^2*b^{14}*d^4*e^{13} + 480*a^4*b^{12}*d^4*e^{13} + 960*a^6*b^{10}*d^4*e^{13} + 960*a^8*b^8*d^4*e^{13} + 480*a^{10}*b^6*d^4*e^{13} + 96*a^{12}*b^4*d^4*e^{13}))/ (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (8*(e*\cot(c + d*x))^{(1/2)}*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^{(1/2)}*(32*b^{18}*d^4*e^{10} + 160*a^2*b^{16}*d^4*e^{10} + 288*a^4*b^{14}*d^4*e^{10} + 160*a^6*b^{12}*d^4*e^{10} - 160*a^8*b^{10}*d^4*e^{10} - 288*a^{10}*b^8*d^4*e^{10} - 160*a^{12}*b^6*d^4*e^{10} - 32*a^{14}*b^4*d^4*e^{10}))/((b^7*d + 2*a^2*b^5*d + a^4*b^3*d)*(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))) * (a^2 + 5*b^2)*(-a^3*b^3*e^5)^{(1/2)}) / (2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d))) * (a^2 + 5*b^2)*(-a^3*b^3*e^5)^{(1/2)}) / (2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d))) * (-a^3*b^3*e^5)^{(1/2)} * i) / (2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)) + ((a^2 + 5*b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(a^{10}*e^{20} - 2*b^{10}*e^{20} - 4*a^2*b^8*e^{20} - 27*a^4*b^6*e^{20} + 15*a^6*b^4*e^{20} + 9*a^8*b^2*e^{20}))) / (b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) - ((a^2 + 5*b^2)*((8*(4*a*b^{11}*d^2*e^{18} + 16*a^{11}*b*d^2*e^{18} - 304*a^3*b^9*d^2*e^{18} - 120*a^5*b^7*d^2*e^{18} + 320*a^7*b^5*d^2*e^{18} + 148*a^9*b^3*d^2*e^{18}))) / (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) - (((16*(e*\cot(c + d*x))^{(1/2)}*(60*a*b^{13}*d^2*e^{15} + 8*a^{13}*b*d^2*e^{15} + 52*a^3*b^{11}*d^2*e^{15} + 128*a^5*b^9*d^2*e^{15} + 424*a^7*b^7*d^2*e^{15} + 380*a^9*b^5*d^2*e^{15} + 100*a^{11}*b^3*d^2*e^{15}))/ (b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4) - (((8*(96*a^2*b^{14}*d^4*e^{13} + 480*a^4*b^{12}*d^4*e^{13} + 960*a^6*b^{10}*d^4*e^{13} + 960*a^8*b^8*d^4*e^{13} + 480*a^{10}*b^6*d^4*e^{13} + 96*a^{12}*b^4*d^4*e^{13}))/ (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5) + (8*(e*\cot(c + d*x))^{(1/2)}*(a^2 + 5*b^2)*(-a^3*b^3*e^5)^{(1/2)}*(32*b^{18}*d^4*e^{10} + 160*a^2*b^{16}*d^4*e^{10} + 288*a^4*b^{14}*d^4*e^{10} + 160*a^6*b^{12}*d^4*e^{10} - 160*a^8*b^{10}*d^4*e^{10} - 288*a^{10}*b^8*d^4*e^{10} - 160*a^{12}*b^6*d^4*e^{10} - 32*a^{14}*b^4*d^4*e^{10}))/((b^7*d + 2*a^2*b^5*d + a^4*b^3*d)*(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))) * (a^2 + 5*b^2)*(-a^3*b^3*e^5)^{(1/2)}) / (2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d))) * (a^2 + 5*b^2)*(-a^3*b^3*e^5)^{(1/2)}) / (2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d))) * (-a^3*b^3*e^5)^{(1/2)} * i) / (2*(b^7*d + 2*a^2*b^5*d + a^4*b^3*d)) / ((16*(a^8*e^{23} + 10*a^2*b^6*e^{23} + 27*a^4*b^4*e^{23} + 10*a^6*b^2*e^{23}))/ (b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5
\end{aligned}$$

$$\begin{aligned}
&^5 + 4a^6b^3d^5) + ((a^2 + 5b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(a^{10}e^{20} \\
&- 2b^{10}e^{20} - 4a^2b^8e^{20} - 27a^4b^6e^{20} + 15a^6b^4e^{20} + 9a^8 \\
&*b^2e^{20}))/((b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3 \\
&*d^4) + ((a^2 + 5b^2)*((8*(4a*b^{11}d^2e^{18} + 16a^{11}b*d^2e^{18} - 304a \\
&^3b^9d^2e^{18} - 120a^5b^7d^2e^{18} + 320a^7b^5d^2e^{18} + 148a^9b^3 \\
&*d^2e^{18}))/((b^9d^5 + a^8b^8d^5 + 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3 \\
&*d^5) + (((16*(e*\cot(c + d*x))^{(1/2)}*(60a*b^{13}d^2e^{15} + 8a^{13}b*d^2e^{15} \\
&+ 52a^3b^{11}d^2e^{15} + 128a^5b^9d^2e^{15} + 424a^7b^7d^2e^{15} + 3 \\
&80a^9b^5d^2e^{15} + 100a^{11}b^3d^2e^{15}))/((b^9d^4 + a^8b^8d^4 + 4a^2b^7 \\
&d^4 + 6a^4b^5d^4 + 4a^6b^3d^4) + (((8*(96a^2b^{14}d^4e^{13} + 480 \\
&*a^4b^{12}d^4e^{13} + 960a^6b^{10}d^4e^{13} + 960a^8b^8d^4e^{13} + 480a^{10}b^6 \\
&d^4e^{13} + 96a^{12}b^4d^4e^{13}))/((b^9d^5 + a^8b^8d^5 + 4a^2b^7d^5 \\
&+ 6a^4b^5d^5 + 4a^6b^3d^5) - (8*(e*\cot(c + d*x))^{(1/2)}*(a^2 + 5b^2 \\
&)*(-a^3b^3e^5)^{(1/2)}*(32b^{18}d^4e^{10} + 160a^2b^{16}d^4e^{10} + 288a^4b^{14} \\
&d^4e^{10} + 160a^6b^{12}d^4e^{10} - 160a^8b^{10}d^4e^{10} - 288a^{10}b^8d^4e^{10} - \\
&160a^{12}b^6d^4e^{10} - 32a^{14}b^4d^4e^{10}))/((b^7d + 2a^2b^5d + a^4b^3d) \\
&*(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4))) \\
&*(a^2 + 5b^2)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) \\
&*(a^2 + 5b^2)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) \\
&*(a^2 + 5b^2)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) \\
&)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) - ((a^2 + 5b^2) \\
&)*((16*(e*\cot(c + d*x))^{(1/2)}*(a^{10}e^{20} - 2b^{10}e^{20} - 4a^2b^8e^{20} \\
&- 27a^4b^6e^{20} + 15a^6b^4e^{20} + 9a^8b^2e^{20}))/((b^9d^4 + a^8b^8d^4 \\
&+ 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4) - ((a^2 + 5b^2)*((8*(4a \\
&*b^{11}d^2e^{18} + 16a^{11}b*d^2e^{18} - 304a^3b^9d^2e^{18} - 120a^5b^7d^2 \\
&*e^{18} + 320a^7b^5d^2e^{18} + 148a^9b^3d^2e^{18}))/((b^9d^5 + a^8b^8d^5 \\
&+ 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3d^5) - (((16*(e*\cot(c + d*x))^{(1/2)} \\
&*(60a*b^{13}d^2e^{15} + 8a^{13}b*d^2e^{15} + 52a^3b^{11}d^2e^{15} + 128a^5 \\
&b^9d^2e^{15} + 424a^7b^7d^2e^{15} + 380a^9b^5d^2e^{15} + 100a^{11}b^3d^2e^{15} \\
&^3d^2e^{15}))/((b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3 \\
&*d^4) - (((8*(96a^2b^{14}d^4e^{13} + 480a^4b^{12}d^4e^{13} + 960a^6b^{10}d^4e^{13} \\
&+ 960a^8b^8d^4e^{13} + 480a^{10}b^6d^4e^{13} + 96a^{12}b^4d^4e^{13}))/((b^9d^5 + a^8b^8d^5 \\
&+ 4a^2b^7d^5 + 6a^4b^5d^5 + 4a^6b^3d^5) + (8*(e*\cot(c + d*x))^{(1/2)}*(a^2 + 5b^2) \\
&)*(-a^3b^3e^5)^{(1/2)}*(32b^{18}d^4e^{10} + 160a^2b^{16}d^4e^{10} + 288a^4b^{14} \\
&d^4e^{10} + 160a^6b^{12}d^4e^{10} - 160a^8b^{10}d^4e^{10} - 288a^{10}b^8d^4e^{10} - \\
&160a^{12}b^6d^4e^{10} - 32a^{14}b^4d^4e^{10}))/((b^7d + 2a^2b^5d + a^4b^3d) \\
&*(b^9d^4 + a^8b^8d^4 + 4a^2b^7d^4 + 6a^4b^5d^4 + 4a^6b^3d^4))) \\
&*(a^2 + 5b^2)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) \\
&*(a^2 + 5b^2)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) \\
&*(a^2 + 5b^2)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)) \\
&)*(-a^3b^3e^5)^{(1/2)})/(2*(b^7d + 2a^2b^5d + a^4b^3d)))*(-a^3b^3e^5)^{(1/2)} \\
&)/(2*(b^7d + 2a^2b^5d + a^4b^3d)))*(-a^3b^3e^5)^{(1/2)}*1i)/(b^7d + 2a^2b^5d + a^4b^3d) \\
&+ (a^2e^3*(e*\cot(c + d*x))^{(1/2)})/(b*(a*d*e + b*d*e*\cot(c + d*x))*(a^2 + b^2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)/(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Timed out

$$3.77 \quad \int \frac{(e \cot(c+dx))^{3/2}}{(a+b \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=387

$$\frac{e^{3/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2} + \frac{e^{3/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx))}{2\sqrt{2} d (a^2 + b^2)^2}$$

[Out]  $-1/2*(a^2-2*a*b-b^2)*e^{(3/2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+1/2*(a^2-2*a*b-b^2)*e^{(3/2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-1/4*(a^2+2*a*b-b^2)*e^{(3/2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}+1/4*(a^2+2*a*b-b^2)*e^{(3/2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^2/d*2^{(1/2)}-(a^2-3*b^2)*e^{(3/2)*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})*a^{(1/2)}/(a^2+b^2)^2/d/b^{(1/2)}-a*e*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)/d/(a+b*\cot(d*x+c))}$

**Rubi [A]** time = 0.68, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3567, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{e^{3/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx) - \sqrt{2} \sqrt{e \cot(c + dx)} + \sqrt{e})}{2\sqrt{2} d (a^2 + b^2)^2} + \frac{e^{3/2} (a^2 + 2ab - b^2) \log(\sqrt{e} \cot(c + dx))}{2\sqrt{2} d (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(3/2)/(a + b\*Cot[c + d\*x])^2,x]

[Out]  $-((\text{Sqrt}[a]*(a^2 - 3*b^2)*e^{(3/2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(\text{Sqrt}[a]*\text{Sqrt}[e])}))/(\text{Sqrt}[b]*(a^2 + b^2)^2*d) - ((a^2 - 2*a*b - b^2)*e^{(3/2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]]}/(\text{Sqrt}[2]*(a^2 + b^2)^2*d) + ((a^2 - 2*a*b - b^2)*e^{(3/2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]]}/(\text{Sqrt}[2]*(a^2 + b^2)^2*d) - (a*e*\text{Sqrt}[e*\text{Cot}[c + d*x]])/((a^2 + b^2)*d*(a + b*\text{Cot}[c + d*x])) - ((a^2 + 2*a*b - b^2)*e^{(3/2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]]}]/(2*\text{Sqrt}[2]*(a^2 + b^2)^2*d) + ((a^2 + 2*a*b - b^2)*e^{(3/2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]]}]/(2*\text{Sqrt}[2]*(a^2 + b^2)^2*d)$

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]\}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

### Rule 617

$\text{Int}[\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\{(d\_)+ (e\_)*(x\_)\}/\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}, x\_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3567

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/((m
+ 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(
n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2*
a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
+ b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2
*m]
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3653

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_)
+ (f_)*(x_)])], x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{3/2}}{(a + b \cot(c + dx))^2} dx &= -\frac{ae\sqrt{e \cot(c + dx)}}{(a^2 + b^2)d(a + b \cot(c + dx))} - \frac{\int \frac{\frac{ae^2}{2} - be^2 \cot(c+dx) - \frac{1}{2}ae^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))} dx}{a^2 + b^2} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{(a^2 + b^2)d(a + b \cot(c + dx))} - \frac{\int \frac{(a^2-b^2)e^2 - 2abe^2 \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{(a^2 + b^2)^2} + \frac{(a(a^2 - 3b^2)e) \int \frac{1}{\sqrt{e \cot(c + dx)}} dx}{2(a^2 + b^2)} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{(a^2 + b^2)d(a + b \cot(c + dx))} - \frac{2 \text{Subst} \left( \int \frac{-(a^2-b^2)e^3 + 2abe^2 x^2}{e^2 + x^4} dx, x, \sqrt{e \cot(c + dx)} \right)}{(a^2 + b^2)^2 d} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{(a^2 + b^2)d(a + b \cot(c + dx))} - \frac{(a(a^2 - 3b^2)e) \text{Subst} \left( \int \frac{1}{a + \frac{bx^2}{e}} dx, x, \sqrt{e \cot(c + dx)} \right)}{(a^2 + b^2)^2 d} \\
&= -\frac{\sqrt{a} (a^2 - 3b^2) e^{3/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{b} (a^2 + b^2)^2 d} - \frac{ae\sqrt{e \cot(c + dx)}}{(a^2 + b^2)d(a + b \cot(c + dx))} - \frac{((a^2 + b^2)e) \int \frac{1}{\sqrt{e \cot(c + dx)}} dx}{2(a^2 + b^2)} \\
&= -\frac{\sqrt{a} (a^2 - 3b^2) e^{3/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{b} (a^2 + b^2)^2 d} - \frac{ae\sqrt{e \cot(c + dx)}}{(a^2 + b^2)d(a + b \cot(c + dx))} - \frac{(a^2 + b^2)e \int \frac{1}{\sqrt{e \cot(c + dx)}} dx}{2(a^2 + b^2)} \\
&= -\frac{\sqrt{a} (a^2 - 3b^2) e^{3/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{b} (a^2 + b^2)^2 d} - \frac{(a^2 - 2ab - b^2) e^{3/2} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2} (a^2 + b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** time = 3.33, size = 322, normalized size = 0.83

$$\frac{(e \cot(c + dx))^{3/2} \left( \frac{24b^2(a^2+b^2) \cot^{\frac{5}{2}}(c+dx) {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; -\frac{b \cot(c+dx)}{a}\right)}{a^2} - 240a^2 \left( \sqrt{\cot(c + dx)} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}} \right) \right) + 80ab \cot(c + dx)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)/(a + b\*Cot[c + d\*x])^2, x]

[Out] -1/60\*((e\*Cot[c + d\*x])^(3/2)\*(-240\*a^2\*(-((Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]])/Sqrt[b])) + Sqrt[Cot[c + d\*x]]) + 80\*a\*b\*Cot[c + d\*x]



$$\begin{aligned} & \left( \frac{3}{2} \right) + 80*a*b*\cot[c + d*x]^{\frac{3}{2}}*(-1 + \text{Hypergeometric2F1}[3/4, 1, 7/4, -\cot[c + d*x]^2]) \\ & + (24*b^2*(a^2 + b^2)*\cot[c + d*x]^{\frac{5}{2}}*\text{Hypergeometric2F1}[2, 5/2, 7/2, -((b*\cot[c + d*x])/a)])/a^2 + 15*(a - b)*(a + b)*(2*\sqrt{2}*\text{ArcTan}[1 - \sqrt{2}*\sqrt{\cot[c + d*x]}] \\ & - 2*\sqrt{2}*\text{ArcTan}[1 + \sqrt{2}*\sqrt{\cot[c + d*x]}]) + 8*\sqrt{\cot[c + d*x]} + \sqrt{2}*\text{Log}[1 - \sqrt{2}*\sqrt{\cot[c + d*x]}] \\ & + \cot[c + d*x] - \sqrt{2}*\text{Log}[1 + \sqrt{2}*\sqrt{\cot[c + d*x]}] + \cot[c + d*x] \end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{(b \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(3/2)/(b\*cot(d\*x + c) + a)^2, x)

**maple** [B] time = 0.76, size = 768, normalized size = 1.98

$$\frac{e^2 a^3 \sqrt{e \cot(dx + c)}}{d(a^2 + b^2)^2 (e \cot(dx + c) b + ae)} - \frac{e^2 a \sqrt{e \cot(dx + c)} b^2}{d(a^2 + b^2)^2 (e \cot(dx + c) b + ae)} - \frac{e^2 a^3 \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2)^2 \sqrt{aeb}} + \frac{3e^2 a \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^2,x)

[Out] 
$$\begin{aligned} & -1/d*e^2*a^3/(a^2+b^2)^2*(e*\cot(d*x+c))^{1/2}/(e*\cot(d*x+c)*b+a*e) - 1/d*e^2*a^3/(a^2+b^2)^2 \\ & *(e*\cot(d*x+c))^{1/2}/(e*\cot(d*x+c)*b+a*e)*b^2 - 1/d*e^2*a^3/(a^2+b^2)^2/(a*e*b)^{1/2}*\arctan((e*\cot(d*x+c))^{1/2}*b/(a*e*b)^{1/2}) \\ & + 3/d*e^2*a/(a^2+b^2)^2/(a*e*b)^{1/2}*\arctan((e*\cot(d*x+c))^{1/2}*b/(a*e*b)^{1/2})*b^2 \\ & - 1/2/d*e/(a^2+b^2)^2*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}+1) \\ & *a^2 + 1/2/d*e/(a^2+b^2)^2*(e^2)^{1/4}*2^{1/2}*\arctan(-2^{1/2}/(e^2)^{1/4}*(e*\cot(d*x+c))^{1/2}-1) \\ & *a^2 \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)+1} * b^2 + 1/4 / d * e / (a^2+b^2)^2 * (e^2)^{(1/4)} \\ & * 2^{(1/2)} * \ln((e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / \\ & (e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * \\ & a^2 - 1/4 / d * e / (a^2+b^2)^2 * (e^2)^{(1/4)} * 2^{(1/2)} * \ln((e * \cot(dx+c) + (e^2)^{(1/4)} * (e \\ & * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(d \\ & x+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * b^2 + 1/2 / d * e / (a^2+b^2)^2 * (e^2)^{(1/4)} * 2^{(1/ \\ & 2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)+1}) * a^2 - 1/2 / d * e / (a^2+b^2) \\ & ^2 * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)+1}) * b \\ & ^2 - 1/2 / d * e^2 / (a^2+b^2)^2 * a * b / (e^2)^{(1/4)} * 2^{(1/2)} * \ln((e * \cot(dx+c) - (e^2)^{(1/ \\ & 4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * c \\ & ot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) - 1 / d * e^2 / (a^2+b^2)^2 * a * b / (e^2)^{(1/4)} * \\ & 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)+1}) + 1 / d * e^2 / (a^2+b^2) \\ & ^2 * a * b / (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2 \\ & )+1}) \end{aligned}$$

**maxima** [A] time = 0.59, size = 344, normalized size = 0.89

$$\left( \frac{4ae\sqrt{\frac{e}{\tan(dx+c)}}}{(a^3+ab^2)e+\frac{(a^2b+b^3)e}{\tan(dx+c)}} + \frac{4(a^3-3ab^2)e\arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{abe}} - \frac{\left(2\sqrt{2}(a^2-2ab-b^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2-2ab-b^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/4 * (4 * a * e * \sqrt{e / \tan(dx+c)}) / ((a^3 + a * b^2) * e + (a^2 * b + b^3) * e / \tan(dx \\ & + c)) + 4 * (a^3 - 3 * a * b^2) * e * \arctan(b * \sqrt{e / \tan(dx+c)} / \sqrt{a * b * e}) / ((a \\ & ^4 + 2 * a^2 * b^2 + b^4) * \sqrt{a * b * e}) - (2 * \sqrt{2}) * (a^2 - 2 * a * b - b^2) * \arctan( \\ & 1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e / \tan(dx+c)}) / \sqrt{e}) / \sqrt{e} + 2 \\ & * \sqrt{2} * (a^2 - 2 * a * b - b^2) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e / \tan(dx+c)}) / \sqrt{e}) / \sqrt{e} + \sqrt{2} * (a^2 + 2 * a * b - b^2) * \log(\sqrt{2} \\ & * \sqrt{e} * \sqrt{e / \tan(dx+c)} + e + e / \tan(dx+c)) / \sqrt{e} - \sqrt{2} * (a^2 \\ & + 2 * a * b - b^2) * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx+c)} + e + e / \tan(dx+c)) / \sqrt{e} * e / (a^4 + 2 * a^2 * b^2 + b^4) * e / d \end{aligned}$$

**mupad** [B] time = 3.37, size = 11953, normalized size = 30.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*\cot(c + d*x))^{3/2}/(a + b*\cot(c + d*x))^2, x)$

[Out]  $(\text{atan}(\frac{((a^2 - 3*b^2)*((16*(e*\cot(c + d*x))^{1/2}*(2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4*b^5*e^{16} - 7*a^6*b^3*e^{16}))/((a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(2*a^{10}*b*d^2*e^{15} - 78*a^2*b^9*d^2*e^{15} + 8*a^4*b^7*d^2*e^{15} + 60*a^6*b^5*d^2*e^{15} - 24*a^8*b^3*d^2*e^{15}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((16*(e*\cot(c + d*x))^{1/2}*(20*a^3*b^{10}*d^2*e^{13} - 60*a*b^{12}*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^{11}*b^2*d^2*e^{13}))/((a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(40*a*b^{14}*d^4*e^{12} + 192*a^3*b^{12}*d^4*e^{12} + 360*a^5*b^{10}*d^4*e^{12} + 320*a^7*b^8*d^4*e^{12} + 120*a^9*b^6*d^4*e^{12} - 8*a^{13}*b^2*d^4*e^{12}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*\cot(c + d*x))^{1/2}*(a^2 - 3*b^2)*(-a*b*e^3)^{1/2}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + 288*a^4*b^{13}*d^4*e^{10} + 160*a^6*b^{11}*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^{10}*b^7*d^4*e^{10} - 160*a^{12}*b^5*d^4*e^{10} - 32*a^{14}*b^3*d^4*e^{10}))/((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e^3)^{1/2}))/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^{1/2}))/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^{1/2}))/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^{1/2})*i)/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)) + ((a^2 - 3*b^2)*((16*(e*\cot(c + d*x))^{1/2}*(2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4*b^5*e^{16} - 7*a^6*b^3*e^{16}))/((a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(2*a^{10}*b*d^2*e^{15} - 78*a^2*b^9*d^2*e^{15} + 8*a^4*b^7*d^2*e^{15} + 60*a^6*b^5*d^2*e^{15} - 24*a^8*b^3*d^2*e^{15}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + ((a^2 - 3*b^2)*((16*(e*\cot(c + d*x))^{1/2}*(20*a^3*b^{10}*d^2*e^{13} - 60*a*b^{12}*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^{11}*b^2*d^2*e^{13}))/((a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - ((a^2 - 3*b^2)*((16*(40*a*b^{14}*d^4*e^{12} + 192*a^3*b^{12}*d^4*e^{12} + 360*a^5*b^{10}*d^4*e^{12} + 320*a^7*b^8*d^4*e^{12} + 120*a^9*b^6*d^4*e^{12} - 8*a^{13}*b^2*d^4*e^{12}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*\cot(c + d*x))^{1/2}*(a^2 - 3*b^2)*(-a*b*e^3)^{1/2}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + 288*a^4*b^{13}*d^4*e^{10} + 160*a^6*b^{11}*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^{10}*b^7*d^4*e^{10} - 160*a^{12}*b^5*d^4*e^{10} - 32*a^{14}*b^3*d^4*e^{10}))/((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e^3)^{1/2}))/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^{1/2}))/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^{1/2})*i)/((2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))/((32*(3*a*b^6*e^{18} - a^3*b^4*e^{18}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((16*(e*\cot(c + d*x))^{1/2}*(2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4$

$$\begin{aligned}
& *b^5e^{16} - 7a^6b^3e^{16}))/ (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 \\
& *d^4 + 4a^6b^2d^4) + ((a^2 - 3b^2)*((16*(2a^{10}b*d^2e^{15} - 78a^2b^9 \\
& *d^2e^{15} + 8a^4b^7d^2e^{15} + 60a^6b^5d^2e^{15} - 24a^8b^3d^2e^{15}) \\
& ))/(a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) - ((a \\
& ^2 - 3b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(20a^3b^{10}d^2e^{13} - 60a*b^{12}d \\
& ^2e^{13} + 168a^5b^8d^2e^{13} + 40a^7b^6d^2e^{13} - 44a^9b^4d^2e^{13} \\
& + 4a^{11}b^2d^2e^{13}))/ (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 \\
& + 4a^6b^2d^4) + ((a^2 - 3b^2)*((16*(40a*b^{14}d^4e^{12} + 192a^3b^{12}d \\
& ^4e^{12} + 360a^5b^{10}d^4e^{12} + 320a^7b^8d^4e^{12} + 120a^9b^6d^4e^{12} \\
& - 8a^{13}b^2d^4e^{12}))/ (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 \\
& + 4a^6b^2d^5) - (8*(e*\cot(c + d*x))^{(1/2)}*(a^2 - 3b^2)*(-a*b*e^3)^{(1 \\
& /2)}*(32b^{17}d^4e^{10} + 160a^2b^{15}d^4e^{10} + 288a^4b^{13}d^4e^{10} + 160 \\
& *a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10}b^7d^4e^{10} - 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10}))/ ((b^5*d + 2a^2*b^3*d + a^4*b*d)*(a \\
& ^8*d^4 + b^8*d^4 + 4a^2*b^6*d^4 + 6a^4*b^4*d^4 + 4a^6*b^2*d^4)))*(-a*b*e \\
& ^3)^{(1/2)})/(2*(b^5*d + 2a^2*b^3*d + a^4*b*d))*(-a*b*e^3)^{(1/2)})/(2*(b^5*d \\
& + 2a^2*b^3*d + a^4*b*d))*(-a*b*e^3)^{(1/2)})/(2*(b^5*d + 2a^2*b^3*d + a^4 \\
& *b*d))*(-a*b*e^3)^{(1/2)})/(2*(b^5*d + 2a^2*b^3*d + a^4*b*d)) + ((a^2 - 3b \\
& ^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(2b^9e^{16} + a^8*b*e^{16} - 5a^2*b^7e^{16} + \\
& 17a^4*b^5e^{16} - 7a^6*b^3e^{16}))/ (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6 \\
& a^4b^4d^4 + 4a^6b^2d^4) - ((a^2 - 3b^2)*((16*(2a^{10}b*d^2e^{15} - 78a \\
& ^2b^9d^2e^{15} + 8a^4b^7d^2e^{15} + 60a^6b^5d^2e^{15} - 24a^8b^3d^2 \\
& e^{15}))/ (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) \\
& ) + ((a^2 - 3b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(20a^3b^{10}d^2e^{13} - 60a \\
& *b^{12}d^2e^{13} + 168a^5b^8d^2e^{13} + 40a^7b^6d^2e^{13} - 44a^9b^4d^2 \\
& e^{13} + 4a^{11}b^2d^2e^{13}))/ (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 \\
& + 4a^6b^2d^4) - ((a^2 - 3b^2)*((16*(40a*b^{14}d^4e^{12} + 192a^3 \\
& *b^{12}d^4e^{12} + 360a^5b^{10}d^4e^{12} + 320a^7b^8d^4e^{12} + 120a^9b^6 \\
& *d^4e^{12} - 8a^{13}b^2d^4e^{12}))/ (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4 \\
& b^4d^5 + 4a^6b^2d^5) + (8*(e*\cot(c + d*x))^{(1/2)}*(a^2 - 3b^2)*(-a*b* \\
& e^3)^{(1/2)}*(32b^{17}d^4e^{10} + 160a^2b^{15}d^4e^{10} + 288a^4b^{13}d^4e^{10} \\
& + 160a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10}b^7d^4e^{10} - \\
& 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10}))/ ((b^5*d + 2a^2*b^3*d + a^4* \\
& b*d)*(a^8*d^4 + b^8*d^4 + 4a^2*b^6*d^4 + 6a^4*b^4*d^4 + 4a^6*b^2*d^4)))* \\
& (-a*b*e^3)^{(1/2)})/(2*(b^5*d + 2a^2*b^3*d + a^4*b*d))*(-a*b*e^3)^{(1/2)})/(2 \\
& *(b^5*d + 2a^2*b^3*d + a^4*b*d))*(-a*b*e^3)^{(1/2)})/(2*(b^5*d + 2a^2*b^3* \\
& d + a^4*b*d))*(-a*b*e^3)^{(1/2)})/(2*(b^5*d + 2a^2*b^3*d + a^4*b*d)))*((a^2 \\
& - 3b^2)*(-a*b*e^3)^{(1/2)}*i)/(b^5*d + 2a^2*b^3*d + a^4*b*d) - \operatorname{atan}(((( \\
& (16*(40a*b^{14}d^4e^{12} + 192a^3b^{12}d^4e^{12} + 360a^5b^{10}d^4e^{12} + 3 \\
& 20a^7b^8d^4e^{12} + 120a^9b^6d^4e^{12} - 8a^{13}b^2d^4e^{12}))/ (a^8d^5 \\
& + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) - (16*(e*\cot(c \\
& + d*x))^{(1/2)}*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4a*b^3*d^2 - 4a^3*b*d^2 \\
& - a^2*b^2*d^2*6i))))^{(1/2)}*(32b^{17}d^4e^{10} + 160a^2b^{15}d^4e^{10} + 288a \\
& ^4b^{13}d^4e^{10} + 160a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10} \\
& *b^7d^4e^{10} - 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10}))/ (a^8d^4 + b
\end{aligned}$$

$$\begin{aligned}
& \left( 8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) * \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} + \left( 16 * (e*\cot(c + d*x))^{1/2} * (20*a^3*b^10*d^2*e^{13} - 60*a*b^12*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^11*b^2*d^2*e^{13}) \right) / \left( a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} - \left( 16*(2*a^10*b*d^2*e^{15} - 78*a^2*b^9*d^2*e^{15} + 8*a^4*b^7*d^2*e^{15} + 60*a^6*b^5*d^2*e^{15} - 24*a^8*b^3*d^2*e^{15}) \right) / \left( a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} - \left( 16*(e*\cot(c + d*x))^{1/2} * (2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4*b^5*e^{16} - 7*a^6*b^3*e^{16}) \right) / \left( a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} * i - \left( \left( \left( \left( 16*(40*a*b^14*d^4*e^{12} + 192*a^3*b^12*d^4*e^{12} + 360*a^5*b^10*d^4*e^{12} + 320*a^7*b^8*d^4*e^{12} + 120*a^9*b^6*d^4*e^{12} - 8*a^13*b^2*d^4*e^{12}) \right) \right) / \left( a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5 \right) + \left( 16*(e*\cot(c + d*x))^{1/2} * \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} * (32*b^17*d^4*e^{10} + 160*a^2*b^15*d^4*e^{10} + 288*a^4*b^13*d^4*e^{10} + 160*a^6*b^11*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^10*b^7*d^4*e^{10} - 160*a^12*b^5*d^4*e^{10} - 32*a^14*b^3*d^4*e^{10}) \right) \right) / \left( a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} - \left( 16*(e*\cot(c + d*x))^{1/2} * (20*a^3*b^10*d^2*e^{13} - 60*a*b^12*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^11*b^2*d^2*e^{13}) \right) / \left( a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} - \left( 16*(2*a^10*b*d^2*e^{15} - 78*a^2*b^9*d^2*e^{15} + 8*a^4*b^7*d^2*e^{15} + 60*a^6*b^5*d^2*e^{15} - 24*a^8*b^3*d^2*e^{15}) \right) / \left( a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} + \left( 16*(e*\cot(c + d*x))^{1/2} * (2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4*b^5*e^{16} - 7*a^6*b^3*e^{16}) \right) / \left( a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} * i / \left( \left( 32*(3*a*b^6*e^{18} - a^3*b^4*e^{18}) \right) / \left( a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5 \right) + \left( \left( \left( \left( 16*(40*a*b^14*d^4*e^{12} + 192*a^3*b^12*d^4*e^{12} + 360*a^5*b^10*d^4*e^{12} + 320*a^7*b^8*d^4*e^{12} + 120*a^9*b^6*d^4*e^{12} - 8*a^13*b^2*d^4*e^{12}) \right) \right) / \left( a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5 \right) - \left( 16*(e*\cot(c + d*x))^{1/2} * \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} * (32*b^17*d^4*e^{10} + 160*a^2*b^15*d^4*e^{10} + 288*a^4*b^13*d^4*e^{10} + 160*a^6*b^11*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^10*b^7*d^4*e^{10} - 160*a^12*b^5*d^4*e^{10} - 32*a^14*b^3*d^4*e^{10}) \right) \right) / \left( a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \right) \\
& \left( -e^3 / \left( 4*(a^4*d^2*i + b^4*d^2*i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i) \right) \right)^{1/2} + \left( 16*(e*\cot(c + d*x))^{1/2} * (20*a^3*b^10*d^2*e^{13} - 60
\end{aligned}$$

$$\begin{aligned}
& *a*b^{12}*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^{11}*b^2*d^2*e^{13})/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(2*a^{10}*b*d^2*e^{15} - 78*a^2*b^9*d^2*e^{15} + 8*a^4*b^7*d^2*e^{15} + 60*a^6*b^5*d^2*e^{15} - 24*a^8*b^3*d^2*e^{15}))/ (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)}*(2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4*b^5*e^{16} - 7*a^6*b^3*e^{16}))/ (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + ((((((16*(40*a*b^{14}*d^4*e^{12} + 192*a^3*b^{12}*d^4*e^{12} + 360*a^5*b^{10}*d^4*e^{12} + 320*a^7*b^8*d^4*e^{12} + 120*a^9*b^6*d^4*e^{12} - 8*a^{13}*b^2*d^4*e^{12}))/ (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^{(1/2)}*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^{(1/2)}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + 288*a^4*b^{13}*d^4*e^{10} + 160*a^6*b^{11}*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^{10}*b^7*d^4*e^{10} - 160*a^{12}*b^5*d^4*e^{10} - 32*a^{14}*b^3*d^4*e^{10}))/ (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)}*(20*a^3*b^{10}*d^2*e^{13} - 60*a*b^{12}*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^{11}*b^2*d^2*e^{13}))/ (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(2*a^{10}*b*d^2*e^{15} - 78*a^2*b^9*d^2*e^{15} + 8*a^4*b^7*d^2*e^{15} + 60*a^6*b^5*d^2*e^{15} - 24*a^8*b^3*d^2*e^{15}))/ (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)}*(2*b^9*e^{16} + a^8*b*e^{16} - 5*a^2*b^7*e^{16} + 17*a^4*b^5*e^{16} - 7*a^6*b^3*e^{16}))/ (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)})))*(-e^3/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)}*2i - atan(((((((16*(40*a*b^{14}*d^4*e^{12} + 192*a^3*b^{12}*d^4*e^{12} + 360*a^5*b^{10}*d^4*e^{12} + 320*a^7*b^8*d^4*e^{12} + 120*a^9*b^6*d^4*e^{12} - 8*a^{13}*b^2*d^4*e^{12}))/ (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c + d*x))^{(1/2)}*(-e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + 288*a^4*b^{13}*d^4*e^{10} + 160*a^6*b^{11}*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^{10}*b^7*d^4*e^{10} - 160*a^{12}*b^5*d^4*e^{10} - 32*a^{14}*b^3*d^4*e^{10}))/ (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2))))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)}*(20*a^3*b^{10}*d^2*e^{13} - 60*a*b^{12}*d^2*e^{13} + 168*a^5*b^8*d^2*e^{13} + 40*a^7*b^6*d^2*e^{13} - 44*a^9*b^4*d^2*e^{13} + 4*a^{11}*b^2*d^2*e^{13}))/ (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(-e^3*1i)/(4*(a^4*d^2 + b^4*d^2 +
\end{aligned}$$

$$\begin{aligned}
& a^3 b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} - (16(2 a^{10} b^2 d^2 e^{15} - 78 a^2 b^9 d^2 e^{15} + 8 a^4 b^7 d^2 e^{15} + 60 a^6 b^5 d^2 e^{15} - 24 a^8 b^3 d^2 e^{15})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} - (16(e \cot(c + d x))^{1/2} * (2 b^9 e^{16} + a^8 b e^{16} - 5 a^2 b^7 e^{16} + 17 a^4 b^5 e^{16} - 7 a^6 b^3 e^{16})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} * 1i - ((((((16(40 a^2 b^14 d^4 e^{12} + 192 a^3 b^12 d^4 e^{12} + 360 a^5 b^10 d^4 e^{12} + 320 a^7 b^8 d^4 e^{12} + 120 a^9 b^6 d^4 e^{12} - 8 a^{13} b^2 d^4 e^{12})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) + (16(e \cot(c + d x))^{1/2} * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} * (32 b^{17} d^4 e^{10} + 160 a^2 b^{15} d^4 e^{10} + 288 a^4 b^{13} d^4 e^{10} + 160 a^6 b^{11} d^4 e^{10} - 160 a^8 b^9 d^4 e^{10} - 288 a^{10} b^7 d^4 e^{10} - 160 a^{12} b^5 d^4 e^{10} - 32 a^{14} b^3 d^4 e^{10})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} - (16(e \cot(c + d x))^{1/2} * (20 a^3 b^{10} d^2 e^{13} - 60 a b^{12} d^2 e^{13} + 168 a^5 b^8 d^2 e^{13} + 40 a^7 b^6 d^2 e^{13} - 44 a^9 b^4 d^2 e^{13} + 4 a^{11} b^2 d^2 e^{13})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} - (16(2 a^{10} b^2 d^2 e^{15} - 78 a^2 b^9 d^2 e^{15} + 8 a^4 b^7 d^2 e^{15} + 60 a^6 b^5 d^2 e^{15} - 24 a^8 b^3 d^2 e^{15})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} + (16(e \cot(c + d x))^{1/2} * (2 b^9 e^{16} + a^8 b e^{16} - 5 a^2 b^7 e^{16} + 17 a^4 b^5 e^{16} - 7 a^6 b^3 e^{16})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} * 1i / ((32(3 a^3 b^6 e^{18} - a^3 b^4 e^{18})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) + ((((((16(40 a^2 b^14 d^4 e^{12} + 192 a^3 b^12 d^4 e^{12} + 360 a^5 b^10 d^4 e^{12} + 320 a^7 b^8 d^4 e^{12} + 120 a^9 b^6 d^4 e^{12} - 8 a^{13} b^2 d^4 e^{12})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) - (16(e \cot(c + d x))^{1/2} * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} * (32 b^{17} d^4 e^{10} + 160 a^2 b^{15} d^4 e^{10} + 288 a^4 b^{13} d^4 e^{10} + 160 a^6 b^{11} d^4 e^{10} - 160 a^8 b^9 d^4 e^{10} - 288 a^{10} b^7 d^4 e^{10} - 160 a^{12} b^5 d^4 e^{10} - 32 a^{14} b^3 d^4 e^{10})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} + (16(e \cot(c + d x))^{1/2} * (20 a^3 b^{10} d^2 e^{13} - 60 a b^{12} d^2 e^{13} + 168 a^5 b^8 d^2 e^{13} + 40 a^7 b^6 d^2 e^{13} - 44 a^9 b^4 d^2 e^{13} + 4 a^{11} b^2 d^2 e^{13})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * (-e^{3i}) / (4(a^4 d^2 + b^4 d^2 + a b^3 d^2 e^{4i} - a^3 b^3 d^2 e^{4i} - 6 a^2 b^2 d^2 e^2))^{1/2} - (16(2 a^{10} b^2 d^2 e^{15} - 78 a^2 b^9 d^2 e^{15} + 8 a^4 b^7 d^2 e^{15} + 60 a^6 b^5 d^2 e^{15} - 24 a^8 b^3 d^2 e^{15})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5
\end{aligned}$$

$$\begin{aligned}
& 5 + 6a^4b^4d^5 + 4a^6b^2d^5)) * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} - (16(e \cot(c + dx))^{1/2} * (2b^9e^{16} + a^8be^{16} - 5a^2b^7e^{16} + 17a^4b^5e^{16} - 7a^6b^3e^{16})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4)) \\
& * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} + (((((16(40ab^{14}d^4e^{12} + 192a^3b^{12}d^4e^{12} + 360a^5b^{10}d^4e^{12} + 320a^7b^8d^4e^{12} + 120a^9b^6d^4e^{12} - 8a^{13}b^2d^4e^{12})) / (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) \\
& + (16(e \cot(c + dx))^{1/2} * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} * (32b^{17}d^4e^{10} + 160a^2b^{15}d^4e^{10} + 288a^4b^{13}d^4e^{10} + 160a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10}b^7d^4e^{10} - 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4) \\
& ) * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} - (16(e \cot(c + dx))^{1/2} * (20a^3b^{10}d^2e^{13} - 60ab^{12}d^2e^{13} + 168a^5b^8d^2e^{13} + 40a^7b^6d^2e^{13} - 44a^9b^4d^2e^{13} + 4a^{11}b^2d^2e^{13})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4) \\
& ) * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} - (16(2a^{10}bd^2e^{15} - 78a^2b^9d^2e^{15} + 8a^4b^7d^2e^{15} + 60a^6b^5d^2e^{15} - 24a^8b^3d^2e^{15})) / (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) \\
& ) * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} + (16(e \cot(c + dx))^{1/2} * (2b^9e^{16} + a^8be^{16} - 5a^2b^7e^{16} + 17a^4b^5e^{16} - 7a^6b^3e^{16})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4) \\
& ) * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} + (16(e \cot(c + dx))^{1/2} * (2b^9e^{16} + a^8be^{16} - 5a^2b^7e^{16} + 17a^4b^5e^{16} - 7a^6b^3e^{16})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4) \\
& ) * (-e^{3i}) / (4(a^4d^2 + b^4d^2 + ab^3d^2 - a^3bd^2 - 6a^2b^2d^2))^{1/2} * 2i - (ae^2(e \cot(c + dx))^{1/2}) / ((ad^2e + bd^2e \cot(c + dx)) * (a^2 + b^2))
\end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c + dx))^{\frac{3}{2}}}{(a + b \cot(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(3/2)/(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Integral((e\*cot(c + d\*x))\*\*(3/2)/(a + b\*cot(c + d\*x))\*\*2, x)



$$3.78 \quad \int \frac{\sqrt{e \cot(c+dx)}}{(a+b \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=386

$$\frac{b\sqrt{e \cot(c+dx)}}{d(a^2+b^2)(a+b \cot(c+dx))} - \frac{\sqrt{e}(a^2-2ab-b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^2} + \frac{\sqrt{e}(a^2-2ab-b^2)}{2\sqrt{2}d(a^2+b^2)^2}$$

[Out]  $\frac{1}{2} \cdot (a^2 + 2ab - b^2) \cdot \arctan\left(\frac{1 - 2^{1/2} \cdot (e \cot(dx+c))^{1/2} / e^{1/2}}{e^{1/2} / (a^2 + b^2)^{1/2}}\right) \cdot e^{1/2} / (a^2 + b^2)^{1/2} - \frac{1}{2} \cdot (a^2 + 2ab - b^2) \cdot \arctan\left(\frac{1 + 2^{1/2} \cdot (e \cot(dx+c))^{1/2} / e^{1/2}}{e^{1/2} / (a^2 + b^2)^{1/2}}\right) \cdot e^{1/2} / (a^2 + b^2)^{1/2} - \frac{1}{4} \cdot (a^2 - 2ab - b^2) \cdot \ln(e^{1/2} + \cot(dx+c) \cdot e^{1/2}) - 2^{1/2} \cdot (e \cot(dx+c))^{1/2} \cdot e^{1/2} / (a^2 + b^2)^{1/2} + \frac{1}{4} \cdot (a^2 - 2ab - b^2) \cdot \ln(e^{1/2} + \cot(dx+c) \cdot e^{1/2}) + 2^{1/2} \cdot (e \cot(dx+c))^{1/2} \cdot e^{1/2} / (a^2 + b^2)^{1/2} + (3a^2 - b^2) \cdot \arctan\left(\frac{b^{1/2} \cdot (e \cot(dx+c))^{1/2} / a^{1/2}}{e^{1/2} / (a^2 + b^2)^{1/2}}\right) \cdot b^{1/2} \cdot e^{1/2} / (a^2 + b^2)^{1/2} + b \cdot (e \cot(dx+c))^{1/2} / (a^2 + b^2)^{1/2} / d / (a + b \cot(dx+c))$

**Rubi [A]** time = 0.65, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3568, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{b\sqrt{e \cot(c+dx)}}{d(a^2+b^2)(a+b \cot(c+dx))} - \frac{\sqrt{e}(a^2-2ab-b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^2} + \frac{\sqrt{e}(a^2-2ab-b^2)}{2\sqrt{2}d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]/(a + b\*Cot[c + d\*x])^2,x]

[Out]  $\frac{(\text{Sqrt}[b] \cdot (3a^2 - b^2) \cdot \text{Sqrt}[e] \cdot \text{ArcTan}[\frac{\text{Sqrt}[b] \cdot \text{Sqrt}[e \cot(c+dx)]}{\text{Sqrt}[a] \cdot \text{Sqrt}[e]}]) / (\text{Sqrt}[a] \cdot (a^2 + b^2)^2 \cdot d) + ((a^2 + 2ab - b^2) \cdot \text{Sqrt}[e] \cdot \text{ArcTan}[1 - \frac{\text{Sqrt}[2] \cdot \text{Sqrt}[e \cot(c+dx)]}{\text{Sqrt}[e]}]) / (\text{Sqrt}[2] \cdot (a^2 + b^2)^2 \cdot d) - ((a^2 + 2ab - b^2) \cdot \text{Sqrt}[e] \cdot \text{ArcTan}[1 + \frac{\text{Sqrt}[2] \cdot \text{Sqrt}[e \cot(c+dx)]}{\text{Sqrt}[e]}]) / (\text{Sqrt}[2] \cdot (a^2 + b^2)^2 \cdot d) + (b \cdot \text{Sqrt}[e \cot(c+dx)]) / ((a^2 + b^2) \cdot d \cdot (a + b \cot(c+dx))) - ((a^2 - 2ab - b^2) \cdot \text{Sqrt}[e] \cdot \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cdot \cot(c+dx) - \text{Sqrt}[2] \cdot \text{Sqrt}[e \cot(c+dx)]) / (2 \cdot \text{Sqrt}[2] \cdot (a^2 + b^2)^2 \cdot d) + ((a^2 - 2ab - b^2) \cdot \text{Sqrt}[e] \cdot \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \cdot \cot(c+dx) + \text{Sqrt}[2] \cdot \text{Sqrt}[e \cot(c+dx)]) / (2 \cdot \text{Sqrt}[2] \cdot (a^2 + b^2)^2 \cdot d)}$

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c - (a\*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]\}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

### Rule 617

$\text{Int}[\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}^{-1}, x\_Symbol] \ :> \ \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\{(d\_)+ (e\_)*(x\_)\}/\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}, x\_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3568

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n)/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(a^2 + b^2
)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*
(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Inte
gerQ[2*m]
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3653

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_)
+ (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e \cot(c+dx)}}{(a+b \cot(c+dx))^2} dx &= \frac{b\sqrt{e \cot(c+dx)}}{(a^2+b^2)d(a+b \cot(c+dx))} - \frac{\int \frac{-\frac{be}{2} - ae \cot(c+dx) + \frac{1}{2}be \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))} dx}{a^2+b^2} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{(a^2+b^2)d(a+b \cot(c+dx))} - \frac{\int \frac{-2abe - (a^2-b^2)e \cot(c+dx)}{\sqrt{e \cot(c+dx)}} dx}{(a^2+b^2)^2} - \frac{(b(3a^2-b^2)e) \int \frac{1}{\sqrt{e \cot(c+dx)}} dx}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{(a^2+b^2)d(a+b \cot(c+dx))} - \frac{2 \text{Subst} \left( \int \frac{2abe^2 + (a^2-b^2)ex^2}{e^2+x^4} dx, x, \sqrt{e \cot(c+dx)} \right)}{(a^2+b^2)^2 d} - \frac{(b(3a^2-b^2)e) \int \frac{1}{\sqrt{e \cot(c+dx)}} dx}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{(a^2+b^2)d(a+b \cot(c+dx))} + \frac{(b(3a^2-b^2)) \text{Subst} \left( \int \frac{1}{\frac{bx^2}{a+e}} dx, x, \sqrt{e \cot(c+dx)} \right)}{(a^2+b^2)^2 d} \\
&= \frac{\sqrt{b}(3a^2-b^2)\sqrt{e} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a}(a^2+b^2)^2 d} + \frac{b\sqrt{e \cot(c+dx)}}{(a^2+b^2)d(a+b \cot(c+dx))} - \frac{(a^2-2ab-b^2)\sqrt{e} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{(a^2+b^2)^2 d} \\
&= \frac{\sqrt{b}(3a^2-b^2)\sqrt{e} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a}(a^2+b^2)^2 d} + \frac{b\sqrt{e \cot(c+dx)}}{(a^2+b^2)d(a+b \cot(c+dx))} - \frac{(a^2-2ab-b^2)\sqrt{e} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{(a^2+b^2)^2 d} \\
&= \frac{\sqrt{b}(3a^2-b^2)\sqrt{e} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a}(a^2+b^2)^2 d} + \frac{(a^2+2ab-b^2)\sqrt{e} \tan^{-1} \left( 1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}} \right)}{\sqrt{2}(a^2+b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** time = 6.10, size = 401, normalized size = 1.04

$$\sqrt{e \cot(c+dx)} \left( \frac{2(a-b)(a+b) \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)}{3(a^2+b^2)^2} + \frac{4ab\sqrt{\cot(c+dx)}}{(a^2+b^2)^2} - \frac{\sqrt{b} \left( \sqrt{a} \sqrt{b} \sqrt{\cot(c+dx)} - a \tan^{-1} \left( \frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}} \right) \right) - b \cot(c+dx)}{\sqrt{a}(a^2+b^2)(a+b \cot(c+dx))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]/(a + b\*Cot[c + d\*x])^2, x]

[Out] -((Sqrt[e\*Cot[c + d\*x]]\*((-4\*a^(3/2)\*Sqrt[b]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]])/(a^2 + b^2)^2 + (4\*a\*b\*Sqrt[Cot[c + d\*x]])/(a^2 + b^2)^2 - (Sqrt[b]\*(-a\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]]) + Sqrt[a]\*Sqrt[

b)\*Sqrt[Cot[c + d\*x]] - b\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]]\*Cot[c + d\*x])/(Sqrt[a]\*(a^2 + b^2)\*(a + b\*Cot[c + d\*x])) + (2\*(a - b)\*(a + b)\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2])/(3\*(a^2 + b^2)^2) - (a\*b\*(2\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]])] - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])] + 8\*Sqrt[Cot[c + d\*x]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]])/(2\*(a^2 + b^2)^2))/(d\*Sqrt[Cot[c + d\*x]])

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(dx + c)}}{(b \cot(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(e\*cot(d\*x + c))/(b\*cot(d\*x + c) + a)^2, x)

**maple** [B] time = 0.82, size = 749, normalized size = 1.94

$$\frac{eb\sqrt{e \cot(dx + c)} a^2}{d(a^2 + b^2)^2 (e \cot(dx + c) b + ae)} + \frac{e b^3 \sqrt{e \cot(dx + c)}}{d(a^2 + b^2)^2 (e \cot(dx + c) b + ae)} + \frac{3eb \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{aeb}}\right) a^2}{d(a^2 + b^2)^2 \sqrt{aeb}} - \frac{e b^3 \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{aeb}}\right)}{d(a^2 + b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c))^2,x)

[Out] 1/d\*e\*b/(a^2+b^2)^2\*(e\*cot(d\*x+c))^(1/2)/(e\*cot(d\*x+c)\*b+a\*e)\*a^2+1/d\*e\*b^3/(a^2+b^2)^2\*(e\*cot(d\*x+c))^(1/2)/(e\*cot(d\*x+c)\*b+a\*e)+3/d\*e\*b/(a^2+b^2)^2/(a\*e\*b)^(1/2)\*arctan((e\*cot(d\*x+c))^(1/2)\*b/(a\*e\*b)^(1/2))\*a^2-1/d\*e\*b^3/(a^2+b^2)^2/(a\*e\*b)^(1/2)\*arctan((e\*cot(d\*x+c))^(1/2)\*b/(a\*e\*b)^(1/2))-1/2/d/(a^2+b^2)^2\*a\*b\*(e^2)^(1/4)\*2^(1/2)\*ln((e\*cot(d\*x+c)+(e^2)^(1/4)\*(e\*cot(d\*x

$$\begin{aligned}
& +c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) - 1/d / (a^2 + b^2)^2 * a * b * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) + 1/d / (a^2 + b^2)^2 * a * b * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) + 1/2/d * e / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a^2 - 1/2/d * e / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * b^2 - 1/2/d * e / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a^2 + 1/2/d * e / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * b^2 - 1/4/d * e / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * a^2 + 1/4/d * e / (a^2 + b^2)^2 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * b^2
\end{aligned}$$

**maxima [A]** time = 0.67, size = 343, normalized size = 0.89

$$e \left( \frac{4b \sqrt{\frac{e}{\tan(dx+c)}}}{(a^3+ab^2)e + \frac{(a^2b+b^3)e}{\tan(dx+c)}} + \frac{4(3a^2b-b^3) \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{abe}} - \frac{2\sqrt{2}(a^2+2ab-b^2) \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2+2ab-b^2) \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*e\*(4\*b\*sqrt(e/tan(d\*x + c))/((a^3 + a\*b^2)\*e + (a^2\*b + b^3)\*e/tan(d\*x + c)) + 4\*(3\*a^2\*b - b^3)\*arctan(b\*sqrt(e/tan(d\*x + c))/sqrt(a\*b\*e))/((a^4 + 2\*a^2\*b^2 + b^4)\*sqrt(a\*b\*e)) - (2\*sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) + sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/(a^4 + 2\*a^2\*b^2 + b^4)/d

**mupad [B]** time = 3.08, size = 11731, normalized size = 30.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(1/2)/(a + b\*cot(c + d\*x))^2,x)

```
[Out] (b*e*(e*cot(c + d*x))^(1/2))/((a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2)) - a
tan(((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^11
+ 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11)))/(a
^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*
cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*
d^2 - a^2*b^2*d^2*6i))))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 2
88*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a
^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8*d^4
+ b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4*d^2*
1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^(1/2) + (16
*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5
*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*
e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))
*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6
i))))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2
*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*
a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*d^2*1
i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^(1/2) - (16*(e*cot(c + d*
x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))
/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4
*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^(
1/2)*1i - ((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4
*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11
)))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (1
6*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a
^3*b*d^2 - a^2*b^2*d^2*6i))))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^1
0 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 -
288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(a^8
*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4
*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^(1/2)
- (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 8
8*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2
*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*
d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*
d^2*6i))))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^
6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5
+ 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*
d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))))^(1/2) + (16*(e*cot(c
+ d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e
^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*
(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i
))))^(1/2)*1i)/(((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^1
5*d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4
*e^11)))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5)
- (16*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2
```

$$\begin{aligned}
& - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)}*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10)) \\
& / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4 \\
& *(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)}*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 \\
& - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6 \\
& *b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5 \\
& *b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + \\
& b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(e \\
& cot(c + d*x))^{(1/2)}*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4 \\
& ^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15 \\
& *d^4*e^11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) \\
& + (16*(e*cot(c + d*x))^{(1/2)}*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)}*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4 \\
& *e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10) \\
& ))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i))) \\
& ^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)}*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a^11*b^2*d^2*e^11) \\
& ))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (8*(52*a*b^10*d^2 \\
& *e^12 - 128*a^3*b^8*d^2*e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) \\
& *(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} + (16*(e \\
& cot(c + d*x))^{(1/2)}*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2 \\
& *d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)} - (16*(b^7*e^13 - 9*a^4*b^3*e^13))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 \\
& + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^{(1/2)}*2i - (atan((((3*a^2 - b^2)*((16*(e*cot(c + d*x))^{(1/2)} \\
& *(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - (((8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2 \\
& *e^12 - 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((16*(e*cot(c + \\
& d*x))^{(1/2)}*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^11 - 88*a^5*b^8*d^2*e^11
\end{aligned}$$



$$\begin{aligned}
& 11 + 40*a^7*b^6*d^2*e^{11} + 84*a^9*b^4*d^2*e^{11} + 4*a^{11}*b^2*d^2*e^{11})/(a^8 \\
& *d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + (((8*(320 \\
& *a^6*b^9*d^4*e^{11} - 96*a^2*b^{13}*d^4*e^{11} - 32*b^{15}*d^4*e^{11} + 480*a^8*b^7*d \\
& ^4*e^{11} + 288*a^{10}*b^5*d^4*e^{11} + 64*a^{12}*b^3*d^4*e^{11}))/((a^8*d^5 + b^8*d^5 \\
& + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^{(1/ \\
& 2)}*(3*a^2 - b^2)*(-a*b*e)^{(1/2)}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + \\
& 288*a^4*b^{13}*d^4*e^{10} + 160*a^6*b^{11}*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288 \\
& *a^{10}*b^7*d^4*e^{10} - 160*a^{12}*b^5*d^4*e^{10} - 32*a^{14}*b^3*d^4*e^{10}))/((a^5*d \\
& + 2*a^3*b^2*d + a*b^4*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 \\
& + 4*a^6*b^2*d^4)))*(3*a^2 - b^2)*(-a*b*e)^{(1/2)})/(2*(a^5*d + 2*a^3*b^2*d \\
& + a*b^4*d)))*(3*a^2 - b^2)*(-a*b*e)^{(1/2)})/(2*(a^5*d + 2*a^3*b^2*d + a*b^4* \\
& d)))*(3*a^2 - b^2)*(-a*b*e)^{(1/2)})/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d))*(-a \\
& *b*e)^{(1/2)}*i)/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)) + ((3*a^2 - b^2)*((16*( \\
& e*cot(c + d*x))^{(1/2)}*(3*b^9*e^{12} - 3*a^2*b^7*e^{12} + 17*a^4*b^5*e^{12} - 9*a^ \\
& 6*b^3*e^{12}))/((a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2 \\
& *d^4) + (((8*(52*a*b^{10}*d^2*e^{12} - 128*a^3*b^8*d^2*e^{12} - 24*a^5*b^6*d^2*e^ \\
& 12 + 160*a^7*b^4*d^2*e^{12} + 4*a^9*b^2*d^2*e^{12}))/((a^8*d^5 + b^8*d^5 + 4*a^2 \\
& *b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (((16*(e*cot(c + d*x))^{(1/2)}*(6 \\
& 8*a*b^{12}*d^2*e^{11} + 20*a^3*b^{10}*d^2*e^{11} - 88*a^5*b^8*d^2*e^{11} + 40*a^7*b^6 \\
& *d^2*e^{11} + 84*a^9*b^4*d^2*e^{11} + 4*a^{11}*b^2*d^2*e^{11}))/((a^8*d^4 + b^8*d^4 \\
& + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) - (((8*(320*a^6*b^9*d^4*e^ \\
& 11 - 96*a^2*b^{13}*d^4*e^{11} - 32*b^{15}*d^4*e^{11} + 480*a^8*b^7*d^4*e^{11} + 288*a \\
& ^{10}*b^5*d^4*e^{11} + 64*a^{12}*b^3*d^4*e^{11}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^ \\
& 5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (8*(e*cot(c + d*x))^{(1/2)}*(3*a^2 - b^2 \\
& )*(-a*b*e)^{(1/2)}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + 288*a^4*b^{13}*d \\
& ^4*e^{10} + 160*a^6*b^{11}*d^4*e^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^{10}*b^7*d^4*e \\
& ^{10} - 160*a^{12}*b^5*d^4*e^{10} - 32*a^{14}*b^3*d^4*e^{10}))/((a^5*d + 2*a^3*b^2*d \\
& + a*b^4*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d \\
& ^4)))*(3*a^2 - b^2)*(-a*b*e)^{(1/2)})/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3 \\
& *a^2 - b^2)*(-a*b*e)^{(1/2)})/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(3*a^2 - b \\
& ^2)*(-a*b*e)^{(1/2)})/(2*(a^5*d + 2*a^3*b^2*d + a*b^4*d)))*(-a*b*e)^{(1/2)}*i) \\
& /((16*(b^7*e^{13} - 9*a^4*b^3*e^{13}))/((a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 \\
& + 4*a^6*b^2*d^5) + ((3*a^2 - b^2)*((16*(e*cot(c + d*x))^{(1/2)}*(3*b^9*e^{12} - 3*a^2*b^7*e^{12} \\
& + 17*a^4*b^5*e^{12} - 9*a^6*b^3*e^{12}))/((a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 \\
& + 4*a^6*b^2*d^4) - (((8*(52*a*b^{10}*d^2*e^{12} - 128*a^3*b^8*d^2*e^{12} - 24*a \\
& ^5*b^6*d^2*e^{12} + 160*a^7*b^4*d^2*e^{12} + 4*a^9*b^2*d^2*e^{12}))/((a^8*d^5 + b^ \\
& 8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) + (((16*(e*cot(c + d \\
& *x))^{(1/2)}*(68*a*b^{12}*d^2*e^{11} + 20*a^3*b^{10}*d^2*e^{11} - 88*a^5*b^8*d^2*e^{11} \\
& + 40*a^7*b^6*d^2*e^{11} + 84*a^9*b^4*d^2*e^{11} + 4*a^{11}*b^2*d^2*e^{11}))/((a^8*d \\
& ^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + (((8*(320*a \\
& ^6*b^9*d^4*e^{11} - 96*a^2*b^{13}*d^4*e^{11} - 32*b^{15}*d^4*e^{11} + 480*a^8*b^7*d^4 \\
& *e^{11} + 288*a^{10}*b^5*d^4*e^{11} + 64*a^{12}*b^3*d^4*e^{11}))/((a^8*d^5 + b^8*d^5 + \\
& 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*x))^{(1/2)} \\
& *(3*a^2 - b^2)*(-a*b*e)^{(1/2)}*(32*b^{17}*d^4*e^{10} + 160*a^2*b^{15}*d^4*e^{10} + 2
\end{aligned}$$

$$\begin{aligned}
& (88a^4b^{13}d^4e^{10} + 160a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10}b^7d^4e^{10} - 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10}) / ((a^5d + 2a^3b^2d + ab^4d)(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4)) \\
& \times (3a^2 - b^2)(-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) \times (3a^2 - b^2)(-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) \\
& \times (3a^2 - b^2)(-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) \times (-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) - ((3a^2 - b^2) \times ((16(e \cot(c + dx))^{(1/2)} \times (3b^9e^{12} - 3a^2b^7e^{12} + 17a^4b^5e^{12} - 9a^6b^3e^{12})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4) \\
& + ((8(52ab^{10}d^2e^{12} - 128a^3b^8d^2e^{12} - 24a^5b^6d^2e^{12} + 160a^7b^4d^2e^{12} + 4a^9b^2d^2e^{12}))) / (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) - (((16(e \cot(c + dx))^{(1/2)} \times (68ab^{12}d^2e^{11} + 20a^3b^{10}d^2e^{11} - 88a^5b^8d^2e^{11} + 40a^7b^6d^2e^{11} + 84a^9b^4d^2e^{11} + 4a^{11}b^2d^2e^{11})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4) - ((8(320a^6b^9d^4e^{11} - 96a^2b^{13}d^4e^{11} - 32b^{15}d^4e^{11} + 480a^8b^7d^4e^{11} + 288a^{10}b^5d^4e^{11} + 64a^{12}b^3d^4e^{11})) / (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) + (8(e \cot(c + dx))^{(1/2)} \times (3a^2 - b^2)(-abe)^{(1/2)} \times (32b^{17}d^4e^{10} + 160a^2b^{15}d^4e^{10} + 288a^4b^{13}d^4e^{10} + 160a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10}b^7d^4e^{10} - 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10})) / ((a^5d + 2a^3b^2d + ab^4d)(a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4))) \times (3a^2 - b^2)(-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) \times (3a^2 - b^2)(-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) \times (3a^2 - b^2)(-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d)) \times (-abe)^{(1/2)} / (2(a^5d + 2a^3b^2d + ab^4d))) \times (3a^2 - b^2)(-abe)^{(1/2)} \times i) / (a^5d + 2a^3b^2d + ab^4d) - \operatorname{atan}((((((8(320a^6b^9d^4e^{11} - 96a^2b^{13}d^4e^{11} - 32b^{15}d^4e^{11} + 480a^8b^7d^4e^{11} + 288a^{10}b^5d^4e^{11} + 64a^{12}b^3d^4e^{11})) / (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5) - (16(e \cot(c + dx))^{(1/2)} \times ((e \cdot i) / (4(a^4d^2 + b^4d^2 + ab^3d^2 \cdot 4i - a^3b \cdot d^2 \cdot 4i - 6a^2b^2d^2)))^{(1/2)} \times (32b^{17}d^4e^{10} + 160a^2b^{15}d^4e^{10} + 288a^4b^{13}d^4e^{10} + 160a^6b^{11}d^4e^{10} - 160a^8b^9d^4e^{10} - 288a^{10}b^7d^4e^{10} - 160a^{12}b^5d^4e^{10} - 32a^{14}b^3d^4e^{10})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4)) \times ((e \cdot i) / (4(a^4d^2 + b^4d^2 + ab^3d^2 \cdot 4i - a^3b \cdot d^2 \cdot 4i - 6a^2b^2d^2)))^{(1/2)} + (16(e \cot(c + dx))^{(1/2)} \times (68ab^{12}d^2e^{11} + 20a^3b^{10}d^2e^{11} - 88a^5b^8d^2e^{11} + 40a^7b^6d^2e^{11} + 84a^9b^4d^2e^{11} + 4a^{11}b^2d^2e^{11})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4)) \times ((e \cdot i) / (4(a^4d^2 + b^4d^2 + ab^3d^2 \cdot 4i - a^3b \cdot d^2 \cdot 4i - 6a^2b^2d^2)))^{(1/2)} + (8(52ab^{10}d^2e^{12} - 128a^3b^8d^2e^{12} - 24a^5b^6d^2e^{12} + 160a^7b^4d^2e^{12} + 4a^9b^2d^2e^{12}))) / (a^8d^5 + b^8d^5 + 4a^2b^6d^5 + 6a^4b^4d^5 + 4a^6b^2d^5)) \times ((e \cdot i) / (4(a^4d^2 + b^4d^2 + ab^3d^2 \cdot 4i - a^3b \cdot d^2 \cdot 4i - 6a^2b^2d^2)))^{(1/2)} - (16(e \cot(c + dx))^{(1/2)} \times (3b^9e^{12} - 3a^2b^7e^{12} + 17a^4b^5e^{12} - 9a^6b^3e^{12})) / (a^8d^4 + b^8d^4 + 4a^2b^6d^4 + 6a^4b^4d^4 + 4a^6b^2d^4)
\end{aligned}$$

$$\begin{aligned}
& + 4*a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2 \\
& *4i - 6*a^2*b^2*d^2)))^{(1/2)} * 1i - (((((8*(320*a^6*b^9*d^4*e^{11} - 96*a^2*b^1 \\
& 3*d^4*e^{11} - 32*b^15*d^4*e^{11} + 480*a^8*b^7*d^4*e^{11} + 288*a^10*b^5*d^4*e^{1 \\
& 1 + 64*a^12*b^3*d^4*e^{11}))) / (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d \\
& ^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^{(1/2)} * ((e*1i) / (4*(a^4*d^2 + b^4* \\
& d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * (32*b^17*d^4*e^1 \\
& 0 + 160*a^2*b^15*d^4*e^{10} + 288*a^4*b^13*d^4*e^{10} + 160*a^6*b^11*d^4*e^{10} - \\
& 160*a^8*b^9*d^4*e^{10} - 288*a^10*b^7*d^4*e^{10} - 160*a^12*b^5*d^4*e^{10} - 32* \\
& a^14*b^3*d^4*e^{10}))) / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4* \\
& a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - \\
& 6*a^2*b^2*d^2)))^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)} * (68*a*b^12*d^2*e^{11} + \\
& 20*a^3*b^10*d^2*e^{11} - 88*a^5*b^8*d^2*e^{11} + 40*a^7*b^6*d^2*e^{11} + 84*a^9*b \\
& ^4*d^2*e^{11} + 4*a^11*b^2*d^2*e^{11}))) / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6* \\
& a^4*b^4*d^4 + 4*a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i \\
& - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} + (8*(52*a*b^10*d^2*e^{12} - 128*a^3* \\
& b^8*d^2*e^{12} - 24*a^5*b^6*d^2*e^{12} + 160*a^7*b^4*d^2*e^{12} + 4*a^9*b^2*d^2*e \\
& ^{12}))) / (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5)) * \\
& ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2 \\
& )))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)} * (3*b^9*e^{12} - 3*a^2*b^7*e^{12} + 17*a^ \\
& 4*b^5*e^{12} - 9*a^6*b^3*e^{12}))) / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^ \\
& 4*d^4 + 4*a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3* \\
& b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * 1i) / (((((8*(320*a^6*b^9*d^4*e^{11} - 96*a^ \\
& 2*b^13*d^4*e^{11} - 32*b^15*d^4*e^{11} + 480*a^8*b^7*d^4*e^{11} + 288*a^10*b^5*d^ \\
& 4*e^{11} + 64*a^12*b^3*d^4*e^{11}))) / (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4* \\
& b^4*d^5 + 4*a^6*b^2*d^5) - (16*(e*cot(c + d*x))^{(1/2)} * ((e*1i) / (4*(a^4*d^2 + \\
& b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} * (32*b^17*d^ \\
& 4*e^{10} + 160*a^2*b^15*d^4*e^{10} + 288*a^4*b^13*d^4*e^{10} + 160*a^6*b^11*d^4*e \\
& ^{10} - 160*a^8*b^9*d^4*e^{10} - 288*a^10*b^7*d^4*e^{10} - 160*a^12*b^5*d^4*e^{10} \\
& - 32*a^14*b^3*d^4*e^{10}))) / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 \\
& + 4*a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2 \\
& *4i - 6*a^2*b^2*d^2)))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)} * (68*a*b^12*d^2*e^ \\
& 11 + 20*a^3*b^10*d^2*e^{11} - 88*a^5*b^8*d^2*e^{11} + 40*a^7*b^6*d^2*e^{11} + 84* \\
& a^9*b^4*d^2*e^{11} + 4*a^11*b^2*d^2*e^{11}))) / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 \\
& + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^ \\
& 2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} + (8*(52*a*b^10*d^2*e^{12} - 128 \\
& *a^3*b^8*d^2*e^{12} - 24*a^5*b^6*d^2*e^{12} + 160*a^7*b^4*d^2*e^{12} + 4*a^9*b^2* \\
& d^2*e^{12}))) / (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d \\
& ^5)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^ \\
& 2*d^2)))^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)} * (3*b^9*e^{12} - 3*a^2*b^7*e^{12} + \\
& 17*a^4*b^5*e^{12} - 9*a^6*b^3*e^{12}))) / (a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a \\
& ^4*b^4*d^4 + 4*a^6*b^2*d^4)) * ((e*1i) / (4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - \\
& a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^{(1/2)} + (((((8*(320*a^6*b^9*d^4*e^{11} - 96* \\
& a^2*b^13*d^4*e^{11} - 32*b^15*d^4*e^{11} + 480*a^8*b^7*d^4*e^{11} + 288*a^10*b^5* \\
& d^4*e^{11} + 64*a^12*b^3*d^4*e^{11}))) / (a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^ \\
& 4*b^4*d^5 + 4*a^6*b^2*d^5) + (16*(e*cot(c + d*x))^{(1/2)} * ((e*1i) / (4*(a^4*d^2
\end{aligned}$$

$$\begin{aligned}
& + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2))^{(1/2)} * (32 b^{17} d^4 e^{10} + 160 a^2 b^{15} d^4 e^{10} + 288 a^4 b^{13} d^4 e^{10} + 160 a^6 b^{11} d^4 e^{10} \\
& * e^{10} - 160 a^8 b^9 d^4 e^{10} - 288 a^{10} b^7 d^4 e^{10} - 160 a^{12} b^5 d^4 e^{10} - 32 a^{14} b^3 d^4 e^{10}) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * ((e^{1i}) / (4 (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} - (16 (e \cot(c + d * x))^{(1/2)} * (68 a b^{12} d^2 e^{11} + 20 a^3 b^{10} d^2 e^{11} - 88 a^5 b^8 d^2 e^{11} + 40 a^7 b^6 d^2 e^{11} + 8 \\
& 4 a^9 b^4 d^2 e^{11} + 4 a^{11} b^2 d^2 e^{11})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * ((e^{1i}) / (4 (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (8 (52 a b^{10} d^2 e^{12} - 128 a^3 b^8 d^2 e^{12} - 24 a^5 b^6 d^2 e^{12} + 160 a^7 b^4 d^2 e^{12} + 4 a^9 b^2 d^2 e^{12})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) * ((e^{1i}) / (4 (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} + (16 (e \cot(c + d * x))^{(1/2)} * (3 b^9 e^{12} - 3 a^2 b^7 e^{12} + 17 a^4 b^5 e^{12} - 9 a^6 b^3 e^{12})) / (a^8 d^4 + b^8 d^4 + 4 a^2 b^6 d^4 + 6 a^4 b^4 d^4 + 4 a^6 b^2 d^4) * ((e^{1i}) / (4 (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} - (16 (b^7 e^{13} - 9 a^4 b^3 e^{13})) / (a^8 d^5 + b^8 d^5 + 4 a^2 b^6 d^5 + 6 a^4 b^4 d^5 + 4 a^6 b^2 d^5) * ((e^{1i}) / (4 (a^4 d^2 + b^4 d^2 + a b^3 d^2 * 4i - a^3 b d^2 * 4i - 6 a^2 b^2 d^2)))^{(1/2)} * 2i
\end{aligned}$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(c + dx)}}{(a + b \cot(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(1/2)/(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Integral(sqrt(e\*cot(c + d\*x))/(a + b\*cot(c + d\*x))\*\*2, x)

$$3.79 \quad \int \frac{1}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=394

$$\frac{b^2 \sqrt{e \cot(c+dx)}}{ade (a^2 + b^2) (a + b \cot(c+dx))} + \frac{(a^2 + 2ab - b^2) \log(\sqrt{e \cot(c+dx)} - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)^2} - \frac{(a^2 + 2ab}{$$

[Out]  $-b^{3/2}*(5*a^2+b^2)*\arctan(b^{1/2}*(e*\cot(d*x+c))^{1/2}/a^{1/2}/e^{1/2})/a^{3/2}/(a^2+b^2)^2/d/e^{1/2}+1/2*(a^2-2*a*b-b^2)*\arctan(1-2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/(a^2+b^2)^2/d*2^{1/2}/e^{1/2}-1/2*(a^2-2*a*b-b^2)*\arctan(1+2^{1/2}*(e*\cot(d*x+c))^{1/2}/e^{1/2})/(a^2+b^2)^2/d*2^{1/2}/e^{1/2}+1/4*(a^2+2*a*b-b^2)*\ln(e^{1/2}+\cot(d*x+c)*e^{1/2}-2^{1/2}*(e*\cot(d*x+c))^{1/2})/(a^2+b^2)^2/d*2^{1/2}/e^{1/2}-1/4*(a^2+2*a*b-b^2)*\ln(e^{1/2}+\cot(d*x+c)*e^{1/2}+2^{1/2}*(e*\cot(d*x+c))^{1/2})/(a^2+b^2)^2/d*2^{1/2}/e^{1/2}-b^2*(e*\cot(d*x+c))^{1/2}/a/(a^2+b^2)/d/e/(a+b*\cot(d*x+c))$

**Rubi [A]** time = 0.74, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$ , Rules used = {3569, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{b^2 \sqrt{e \cot(c+dx)}}{ade (a^2 + b^2) (a + b \cot(c+dx))} + \frac{(a^2 + 2ab - b^2) \log(\sqrt{e \cot(c+dx)} - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)^2} - \frac{(a^2 + 2ab}{$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(\text{Sqrt}[e*\text{Cot}[c + d*x]]*(a + b*\text{Cot}[c + d*x])^2), x]$

[Out]  $-((b^{3/2}*(5*a^2 + b^2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])]/(\text{Sqrt}[a]*\text{Sqrt}[e]))/(a^{3/2}*(a^2 + b^2)^2*d*\text{Sqrt}[e])) + ((a^2 - 2*a*b - b^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)^2*d*\text{Sqrt}[e]) - ((a^2 - 2*a*b - b^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)^2*d*\text{Sqrt}[e]) - (b^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(a*(a^2 + b^2)*d*e*(a + b*\text{Cot}[c + d*x])) + ((a^2 + 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^2*d*\text{Sqrt}[e]) - ((a^2 + 2*a*b - b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^2*d*\text{Sqrt}[e])$

**Rule 63**

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b +$

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] :> -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]\}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] :> \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

### Rule 617

$\text{Int}[\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}^{-1}, x\_Symbol] :> \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\{(d\_)+ (e\_)*(x\_)\}/\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}, x\_Symbol] :> \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] :> \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] :> \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

### Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

### Rule 3569

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

### Rule 3653

```
Int((((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_)
+ (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)])^2)/((a_) + (b_)*tan[(e_)
+ (f_)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))^2} dx &= -\frac{b^2 \sqrt{e \cot(c+dx)}}{a(a^2+b^2) de(a+b \cot(c+dx))} - \frac{\int \frac{-\frac{1}{2}(2a^2+b^2)e+abe \cot(c+dx)-\frac{1}{2}b^2e \cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx}{a(a^2+b^2)e} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{a(a^2+b^2) de(a+b \cot(c+dx))} + \frac{(b^2(5a^2+b^2)) \int \frac{1+\cot^2(c+dx)}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx}{2a(a^2+b^2)^2} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{a(a^2+b^2) de(a+b \cot(c+dx))} + \frac{(b^2(5a^2+b^2)) \text{Subst}\left(\int \frac{1}{\sqrt{-ex}(a-b \cot(c+dx))} dx\right)}{2a(a^2+b^2)} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{a(a^2+b^2) de(a+b \cot(c+dx))} - \frac{(a^2-2ab-b^2) \text{Subst}\left(\int \frac{e+x^2}{e^2+x^4} dx\right)}{(a^2+b^2)^2 d} \\
&= -\frac{b^{3/2}(5a^2+b^2) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{a^{3/2}(a^2+b^2)^2 d \sqrt{e}} - \frac{b^2 \sqrt{e \cot(c+dx)}}{a(a^2+b^2) de(a+b \cot(c+dx))} \\
&= -\frac{b^{3/2}(5a^2+b^2) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{a^{3/2}(a^2+b^2)^2 d \sqrt{e}} - \frac{b^2 \sqrt{e \cot(c+dx)}}{a(a^2+b^2) de(a+b \cot(c+dx))} \\
&= -\frac{b^{3/2}(5a^2+b^2) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{a^{3/2}(a^2+b^2)^2 d \sqrt{e}} + \frac{(a^2-2ab-b^2) \tan^{-1}\left(1-\frac{b}{a}\right)}{\sqrt{2}(a^2+b^2)^2 d \sqrt{e}}
\end{aligned}$$

**Mathematica [C]** time = 2.86, size = 300, normalized size = 0.76

$$\sqrt{\cot(c+dx)} \left( \frac{24b^2(a^2+b^2) \sqrt{\cot(c+dx)} \left( \frac{a}{a+b \cot(c+dx)} + \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} \sqrt{\cot(c+dx)}} \right)}{a^2} + 96\sqrt{a} b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}}\right) - 32ab \cot^2(c+dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^2),x]



```
[Out] -1/24*(Sqrt[Cot[c + d*x]]*(96*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c +
d*x]])/Sqrt[a]] + (24*b^2*(a^2 + b^2)*Sqrt[Cot[c + d*x]]*((Sqrt[a]*ArcTan[(
Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(Sqrt[b]*Sqrt[Cot[c + d*x]]) + a/(a +
b*Cot[c + d*x]))) / a^2 - 32*a*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1
, 7/4, -Cot[c + d*x]^2] - 6*Sqrt[2]*(a - b)*(a + b)*(2*ArcTan[1 - Sqrt[2]*S
qrt[Cot[c + d*x]]) - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]) + Log[1 - Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]])))/(a^2 + b^2)^2*d*Sqrt[e*Cot[c + d*x]])
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cot(dx + c) + a)^2 \sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

[Out] integrate(1/((b\*cot(d\*x + c) + a)^2\*sqrt(e\*cot(d\*x + c))), x)

**maple** [B] time = 0.85, size = 765, normalized size = 1.94

$$\frac{b^2 a \sqrt{e \cot(dx + c)}}{d(a^2 + b^2)^2 (e \cot(dx + c) b + a e)} - \frac{b^4 \sqrt{e \cot(dx + c)}}{d(a^2 + b^2)^2 a (e \cot(dx + c) b + a e)} - \frac{5b^2 a \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{a e b}}\right)}{d(a^2 + b^2)^2 \sqrt{a e b}} - \frac{b^4 \arctan\left(\frac{\sqrt{e \cot(dx + c)} b}{\sqrt{a e b}}\right)}{d(a^2 + b^2)^2 \sqrt{a e b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x)
```

```
[Out] -1/d*b^2/(a^2+b^2)^2*a*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)*b+a*e)-1/d*b^4/(a
^2+b^2)^2/a*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)*b+a*e)-5/d*b^2/(a^2+b^2)^2*a
/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))-1/d*b^4/(a^2+b
^2)^2/a/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))-1/2/d/e/(
a^2+b^2)^2*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
```

$$\begin{aligned} & /2)+1)*a^{2+1/2}/d/e/(a^2+b^2)^2*(e^2)^{(1/4)*2^{(1/2)}}*\arctan(2^{(1/2)}/(e^2)^{(1/4)} \\ & *(e*\cot(dx+c))^{(1/2)+1}*b^{2+1/2}/d/e/(a^2+b^2)^2*(e^2)^{(1/4)*2^{(1/2)}}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)} \\ & *(e*\cot(dx+c))^{(1/2)+1}*a^{2-1/2}/d/e/(a^2+b^2)^2*(e^2)^{(1/4)*2^{(1/2)}}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)} \\ & *(e*\cot(dx+c))^{(1/2)+1}*b^{2-1/4}/d/e/(a^2+b^2)^2*(e^2)^{(1/4)*2^{(1/2)}}*\ln((e*\cot(dx+c)+(e^2)^{(1/4)}*(e*\cot(dx+c)) \\ & *(e^2)^{(1/2)+1}+(e^2)^{(1/2)))/(e*\cot(dx+c)-(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)*2^{(1/2)}}+(e^2)^{(1/2))} \\ & *(a^{2+1/4}/d/e/(a^2+b^2)^2*(e^2)^{(1/4)*2^{(1/2)}}*\ln((e*\cot(dx+c)+(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)*2^{(1/2)}}+(e^2)^{(1/2))} \\ & /(e*\cot(dx+c)-(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)*2^{(1/2)}}+(e^2)^{(1/2))})*b^{2+1/2}/d/(a^2+b^2)^2*a*b/(e^2)^{(1/4)*2^{(1/2)}} \\ & *\ln((e*\cot(dx+c)-(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)*2^{(1/2)}}+(e^2)^{(1/2)))/(e*\cot(dx+c)+(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)*2^{(1/2)}}+(e^2)^{(1/2))} \\ & ))+1/d/(a^2+b^2)^2*a*b/(e^2)^{(1/4)*2^{(1/2)}}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)+1}-1/d/(a^2+b^2)^2*a*b/(e^2)^{(1/4)*2^{(1/2)}} \\ & *\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(dx+c))^{(1/2)+1}) \end{aligned}$$

**maxima [A]** time = 0.53, size = 360, normalized size = 0.91

$$\left( \frac{4b^2 \sqrt{\frac{e}{\tan(dx+c)}}}{(a^4+a^2b^2)e^2 + \frac{(a^3b+ab^3)e^2}{\tan(dx+c)}} + \frac{4(5a^2b^2+b^4) \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^5+2a^3b^2+ab^4)\sqrt{abe}} + \frac{2\sqrt{2}(a^2-2ab-b^2) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^2-2ab-b^2) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right) / 4d$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(dx+c))^(1/2)/(a+b\*cot(dx+c))^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/4*(4*b^2*\sqrt{e/\tan(dx+c)})/((a^4+a^2*b^2)*e^2+(a^3*b+a*b^3)*e^2/\tan(dx+c)) \\ & +4*(5*a^2*b^2+b^4)*\arctan(b*\sqrt{e/\tan(dx+c)})/\sqrt{a*b*e}/((a^5+2*a^3*b^2+a*b^4)*\sqrt{a*b*e}) \\ & +\frac{(2*\sqrt{2}*(a^2-2*a*b-b^2)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}+2*\sqrt{e/\tan(dx+c)})))/\sqrt{e}}{\sqrt{e}} \\ & +\frac{2*\sqrt{2}*(a^2-2*a*b-b^2)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e}-2*\sqrt{e/\tan(dx+c)})))/\sqrt{e}}{\sqrt{e}} \\ & +\frac{\sqrt{2}*(a^2+2*a*b-b^2)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)}+e+e/\tan(dx+c))/\sqrt{e}}{\sqrt{e}} \\ & -\frac{\sqrt{2}*(a^2+2*a*b-b^2)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(dx+c)}+e+e/\tan(dx+c))/\sqrt{e}}{\sqrt{e}} \end{aligned}$$

**mupad [B]** time = 8.16, size = 9400, normalized size = 23.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c+dx))^(1/2)\*(a+b\*cot(c+dx))^2),x)

```
[Out] (log(- (((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2)))/(a*d) -
256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*e*(
a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (64*b^2*e^9*(e*co
t(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2)))/(a*d^2*
(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a^6 +
b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(a*1i -
b)^4))^(1/2))/2 - (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6 + 7*a^2
*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^4))^(1/2
))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(a^4*d^2*e*1i
+ b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e))^(1/2))/
2 - log(- (((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2)))/(a*d
) + 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*
e*(a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 + (64*b^2*e^9*(e
*cot(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2)))/(a*d
^2*(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a^6
+ b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(a*1
i - b)^4))^(1/2))/2 + (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6 + 7*
a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^4))^(
1/2))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(4*(a^4*d^2
*e*1i + b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e)))^
(1/2) + atan((( -1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*
e*4i - a^3*b*d^2*e*4i)))^(1/2))*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9
+ 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^
5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*
(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)
))^(1/2))*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i
- a^3*b*d^2*e*4i)))^(1/2))*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10
+ 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 2
16*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d
^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*(-1
i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*
e*4i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d
^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*
e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10
)))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)
) + (16*(e*cot(c + d*x))^(1/2)*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 31
6*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^
4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 +
6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2
*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^(1/2) + (16*(e*cot(c + d*x))^(1
/2)*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4
+ a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i - (-1i/
(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*
4i)))^(1/2))*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e
^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4
```

$$\begin{aligned}
& *a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (16*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*(e*cot(c + d*x))^{(1/2)}*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) - (16*(e*cot(c + d*x))^{(1/2)}*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)} - (16*(e*cot(c + d*x))^{(1/2)}*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4))*1i)/((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*((-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*((16*(16*a*b^16*d^4*e^10 + 136*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - (16*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*(e*cot(c + d*x))^{(1/2)}*(32*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) + (16*(e*cot(c + d*x))^{(1/2)}*(8*a*b^14*d^2*e^9 + 36*a^3*b^12*d^2*e^9 + 316*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4*d^2*e^9 - 4*a^13*b^2*d^2*e^9))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)))*(-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)} + (16*(e*cot(c + d*x))^{(1/2)}*(b^11*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^6*b^5*e^8))/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i - a^3*b*d^2*e*4i)))^{(1/2)}*((16*(24*a^2*b^11*d^2*e^9 - 2*b^13*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + (-1i/(4*(a^4*d^2*e + b^4*d^2*e - 6*a^2*b^2*d^2*e + a*b^3*d^2*e*4i -
\end{aligned}$$

$$\begin{aligned}
& a^3 b d^2 e^{4i} \Big)^{(1/2)} * \left( \frac{-1i}{4(a^4 d^2 e + b^4 d^2 e - 6a^2 b^2 d^2 e + a b^3 d^2 e^{4i} - a^3 b d^2 e^{4i})} \right)^{(1/2)} * \left( \frac{16(16a^3 b^{16} d^4 e^{10} + 136a^3 b^{14} d^4 e^{10} + 432a^5 b^{12} d^4 e^{10} + 680a^7 b^{10} d^4 e^{10} + 560a^9 b^8 d^4 e^{10} + 216a^{11} b^6 d^4 e^{10} + 16a^{13} b^4 d^4 e^{10} - 8a^{15} b^2 d^4 e^{10})}{(a^{10} d^5 + a^2 b^8 d^5 + 4a^4 b^6 d^5 + 6a^6 b^4 d^5 + 4a^8 b^2 d^5) + (16(-1i/(4(a^4 d^2 e + b^4 d^2 e - 6a^2 b^2 d^2 e + a b^3 d^2 e^{4i} - a^3 b d^2 e^{4i})))^{(1/2)} * (\operatorname{ecot}(c + dx))^{(1/2)} * (32a^2 b^{17} d^4 e^{10} + 160a^4 b^{15} d^4 e^{10} + 288a^6 b^{13} d^4 e^{10} + 160a^8 b^{11} d^4 e^{10} - 160a^{10} b^9 d^4 e^{10} - 288a^{12} b^7 d^4 e^{10} - 160a^{14} b^5 d^4 e^{10} - 32a^{16} b^3 d^4 e^{10}))}{(a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4)} - (16(\operatorname{ecot}(c + dx))^{(1/2)} * (8a^3 b^{14} d^2 e^9 + 36a^3 b^{12} d^2 e^9 + 316a^5 b^{10} d^2 e^9 + 552a^7 b^8 d^2 e^9 + 256a^9 b^6 d^2 e^9 - 12a^{11} b^4 d^2 e^9 - 4a^{13} b^2 d^2 e^9))}{(a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4)} \right) * \left( \frac{-1i}{4(a^4 d^2 e + b^4 d^2 e - 6a^2 b^2 d^2 e + a b^3 d^2 e^{4i} - a^3 b d^2 e^{4i})} \right)^{(1/2)} - (16(\operatorname{ecot}(c + dx))^{(1/2)} * (b^{11} e^8 + 7a^2 b^9 e^8 + 11a^4 b^7 e^8 - 27a^6 b^5 e^8)) / (a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4) + (32(a^3 b^8 e^8 + 5a^3 b^6 e^8)) / (a^{10} d^5 + a^2 b^8 d^5 + 4a^4 b^6 d^5 + 6a^6 b^4 d^5 + 4a^8 b^2 d^5)) * \left( \frac{-1i}{4(a^4 d^2 e + b^4 d^2 e - 6a^2 b^2 d^2 e + a b^3 d^2 e^{4i} - a^3 b d^2 e^{4i})} \right)^{(1/2)} * 2i + (\operatorname{atan}(\left( \frac{(5a^2 + b^2) * ((16(\operatorname{ecot}(c + dx))^{(1/2)} * (b^{11} e^8 + 7a^2 b^9 e^8 + 11a^4 b^7 e^8 - 27a^6 b^5 e^8)) / (a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4) + ((5a^2 + b^2) * ((16(24a^2 b^{11} d^2 e^9 - 2b^{13} d^2 e^9 + 196a^4 b^9 d^2 e^9 + 120a^6 b^7 d^2 e^9 - 50a^8 b^5 d^2 e^9)) / (a^{10} d^5 + a^2 b^8 d^5 + 4a^4 b^6 d^5 + 6a^6 b^4 d^5 + 4a^8 b^2 d^5) + (5a^2 + b^2) * ((16(\operatorname{ecot}(c + dx))^{(1/2)} * (8a^3 b^{14} d^2 e^9 + 36a^3 b^{12} d^2 e^9 + 316a^5 b^{10} d^2 e^9 + 552a^7 b^8 d^2 e^9 + 256a^9 b^6 d^2 e^9 - 12a^{11} b^4 d^2 e^9 - 4a^{13} b^2 d^2 e^9)) / (a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4) + ((5a^2 + b^2) * ((16(16a^3 b^{16} d^4 e^{10} + 136a^3 b^{14} d^4 e^{10} + 432a^5 b^{12} d^4 e^{10} + 680a^7 b^{10} d^4 e^{10} + 560a^9 b^8 d^4 e^{10} + 216a^{11} b^6 d^4 e^{10} + 16a^{13} b^4 d^4 e^{10} - 8a^{15} b^2 d^4 e^{10})) / (a^{10} d^5 + a^2 b^8 d^5 + 4a^4 b^6 d^5 + 6a^6 b^4 d^5 + 4a^8 b^2 d^5) - (8(\operatorname{ecot}(c + dx))^{(1/2)} * (5a^2 + b^2) * (-a^3 b^3 e)^{(1/2)} * (32a^2 b^{17} d^4 e^{10} + 160a^4 b^{15} d^4 e^{10} + 288a^6 b^{13} d^4 e^{10} + 160a^8 b^{11} d^4 e^{10} - 160a^{10} b^9 d^4 e^{10} - 288a^{12} b^7 d^4 e^{10} - 160a^{14} b^5 d^4 e^{10} - 32a^{16} b^3 d^4 e^{10}))}{(a^7 d e + a^3 b^4 d e + 2a^5 b^2 d e)} * (a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4)) * (-a^3 b^3 e)^{(1/2)}) / (2(a^7 d e + a^3 b^4 d e + 2a^5 b^2 d e))) * (-a^3 b^3 e)^{(1/2)}) / (2(a^7 d e + a^3 b^4 d e + 2a^5 b^2 d e))) * (-a^3 b^3 e)^{(1/2)}) / (2(a^7 d e + a^3 b^4 d e + 2a^5 b^2 d e))) * (-a^3 b^3 e)^{(1/2)} * 1i) / (2(a^7 d e + a^3 b^4 d e + 2a^5 b^2 d e)) + ((5a^2 + b^2) * ((16(\operatorname{ecot}(c + dx))^{(1/2)} * (b^{11} e^8 + 7a^2 b^9 e^8 + 11a^4 b^7 e^8 - 27a^6 b^5 e^8)) / (a^{10} d^4 + a^2 b^8 d^4 + 4a^4 b^6 d^4 + 6a^6 b^4 d^4 + 4a^8 b^2 d^4) - ((5a^2 + b^2) * ((16(24a^2 b^{11} d^2 e^9 - 2b^{13} d^2 e^9 + 196a^4 b^9 d^2 e^9 + 120a^6 b^7 d^2 e^9 - 50a^8 b^5 d^2 e^9)) / (a^{10} d^5 + a^2
\end{aligned}$$

$$\begin{aligned}
& *b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - ((5*a^2 + b^2)* \\
& ((16*(e*\cot(c + d*x))^{(1/2)}*(8*a*b^{14}*d^2*e^9 + 36*a^3*b^{12}*d^2*e^9 + 316*a \\
& ^5*b^{10}*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^{11}*b^4*d \\
& ^2*e^9 - 4*a^{13}*b^2*d^2*e^9)))/(a^{10}*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a \\
& ^6*b^4*d^4 + 4*a^8*b^2*d^4) - ((5*a^2 + b^2)*((16*(16*a*b^{16}*d^4*e^{10} + 136 \\
& *a^3*b^{14}*d^4*e^{10} + 432*a^5*b^{12}*d^4*e^{10} + 680*a^7*b^{10}*d^4*e^{10} + 560*a^ \\
& 9*b^8*d^4*e^{10} + 216*a^{11}*b^6*d^4*e^{10} + 16*a^{13}*b^4*d^4*e^{10} - 8*a^{15}*b^2* \\
& d^4*e^{10}))/((a^{10}*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8* \\
& b^2*d^5) + (8*(e*\cot(c + d*x))^{(1/2)}*(5*a^2 + b^2)*(-a^3*b^3*e)^{(1/2)}*(32*a \\
& ^2*b^{17}*d^4*e^{10} + 160*a^4*b^{15}*d^4*e^{10} + 288*a^6*b^{13}*d^4*e^{10} + 160*a^8* \\
& b^{11}*d^4*e^{10} - 160*a^{10}*b^9*d^4*e^{10} - 288*a^{12}*b^7*d^4*e^{10} - 160*a^{14}*b^ \\
& 5*d^4*e^{10} - 32*a^{16}*b^3*d^4*e^{10}))/((a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e \\
& )*(a^{10}*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4)) \\
& )*(-a^3*b^3*e)^{(1/2)})/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^ \\
& 3*e)^{(1/2)})/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^{(1/2)} \\
& )/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3*e)^{(1/2)}*1i)/(2*(a \\
& ^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))/((32*(a*b^8*e^8 + 5*a^3*b^6*e^8))/ \\
& (a^{10}*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) + ( \\
& (5*a^2 + b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(b^{11}*e^8 + 7*a^2*b^9*e^8 + 11*a^ \\
& 4*b^7*e^8 - 27*a^6*b^5*e^8)))/(a^{10}*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^ \\
& 6*b^4*d^4 + 4*a^8*b^2*d^4) + ((5*a^2 + b^2)*((16*(24*a^2*b^{11}*d^2*e^9 - 2*b \\
& ^{13}*d^2*e^9 + 196*a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^ \\
& 9)))/(a^{10}*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5 \\
& ) + ((5*a^2 + b^2)*((16*(e*\cot(c + d*x))^{(1/2)}*(8*a*b^{14}*d^2*e^9 + 36*a^3*b \\
& ^{12}*d^2*e^9 + 316*a^5*b^{10}*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2* \\
& e^9 - 12*a^{11}*b^4*d^2*e^9 - 4*a^{13}*b^2*d^2*e^9)))/(a^{10}*d^4 + a^2*b^8*d^4 + \\
& 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4) + ((5*a^2 + b^2)*((16*(16*a* \\
& b^{16}*d^4*e^{10} + 136*a^3*b^{14}*d^4*e^{10} + 432*a^5*b^{12}*d^4*e^{10} + 680*a^7*b^{1 \\
& 0}*d^4*e^{10} + 560*a^9*b^8*d^4*e^{10} + 216*a^{11}*b^6*d^4*e^{10} + 16*a^{13}*b^4*d^4 \\
& *e^{10} - 8*a^{15}*b^2*d^4*e^{10}))/((a^{10}*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a \\
& ^6*b^4*d^5 + 4*a^8*b^2*d^5) - (8*(e*\cot(c + d*x))^{(1/2)}*(5*a^2 + b^2)*(-a^3 \\
& *b^3*e)^{(1/2)}*(32*a^2*b^{17}*d^4*e^{10} + 160*a^4*b^{15}*d^4*e^{10} + 288*a^6*b^{13} \\
& d^4*e^{10} + 160*a^8*b^{11}*d^4*e^{10} - 160*a^{10}*b^9*d^4*e^{10} - 288*a^{12}*b^7*d^4 \\
& *e^{10} - 160*a^{14}*b^5*d^4*e^{10} - 32*a^{16}*b^3*d^4*e^{10}))/((a^7*d*e + a^3*b^4* \\
& d*e + 2*a^5*b^2*d*e)*(a^{10}*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 \\
& + 4*a^8*b^2*d^4)))*(-a^3*b^3*e)^{(1/2)})/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5* \\
& b^2*d*e)))*(-a^3*b^3*e)^{(1/2)})/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)) \\
& )*(-a^3*b^3*e)^{(1/2)})/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(-a^3*b^3 \\
& *e)^{(1/2)})/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)) - ((5*a^2 + b^2)*((1 \\
& 6*(e*\cot(c + d*x))^{(1/2)}*(b^{11}*e^8 + 7*a^2*b^9*e^8 + 11*a^4*b^7*e^8 - 27*a^ \\
& 6*b^5*e^8)))/(a^{10}*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8 \\
& *b^2*d^4) - ((5*a^2 + b^2)*((16*(24*a^2*b^{11}*d^2*e^9 - 2*b^{13}*d^2*e^9 + 196 \\
& *a^4*b^9*d^2*e^9 + 120*a^6*b^7*d^2*e^9 - 50*a^8*b^5*d^2*e^9)))/(a^{10}*d^5 + a \\
& ^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^8*b^2*d^5) - ((5*a^2 + b^2 \\
& )*((16*(e*\cot(c + d*x))^{(1/2)}*(8*a*b^{14}*d^2*e^9 + 36*a^3*b^{12}*d^2*e^9 + 316
\end{aligned}$$

```

*a^5*b^10*d^2*e^9 + 552*a^7*b^8*d^2*e^9 + 256*a^9*b^6*d^2*e^9 - 12*a^11*b^4
*d^2*e^9 - 4*a^13*b^2*d^2*e^9)/(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6
*a^6*b^4*d^4 + 4*a^8*b^2*d^4) - ((5*a^2 + b^2)*((16*(16*a*b^16*d^4*e^10 + 1
36*a^3*b^14*d^4*e^10 + 432*a^5*b^12*d^4*e^10 + 680*a^7*b^10*d^4*e^10 + 560*
a^9*b^8*d^4*e^10 + 216*a^11*b^6*d^4*e^10 + 16*a^13*b^4*d^4*e^10 - 8*a^15*b^
2*d^4*e^10))/(a^10*d^5 + a^2*b^8*d^5 + 4*a^4*b^6*d^5 + 6*a^6*b^4*d^5 + 4*a^
8*b^2*d^5) + (8*(e*cot(c + d*x))^(1/2)*(5*a^2 + b^2)*(-a^3*b^3*e)^(1/2)*(32
*a^2*b^17*d^4*e^10 + 160*a^4*b^15*d^4*e^10 + 288*a^6*b^13*d^4*e^10 + 160*a^
8*b^11*d^4*e^10 - 160*a^10*b^9*d^4*e^10 - 288*a^12*b^7*d^4*e^10 - 160*a^14*
b^5*d^4*e^10 - 32*a^16*b^3*d^4*e^10))/((a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d
*e)*(a^10*d^4 + a^2*b^8*d^4 + 4*a^4*b^6*d^4 + 6*a^6*b^4*d^4 + 4*a^8*b^2*d^4
)))*(-a^3*b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e))*(-a^3*
b^3*e)^(1/2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e))*(-a^3*b^3*e)^(1/
2))/(2*(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e))*(-a^3*b^3*e)^(1/2))/(2*(a^
7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e)))*(5*a^2 + b^2)*(-a^3*b^3*e)^(1/2)*1i
)/(a^7*d*e + a^3*b^4*d*e + 2*a^5*b^2*d*e) - (b^2*(e*cot(c + d*x))^(1/2))/(a
*(a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(1/2)/(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Integral(1/(sqrt(e\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))\*\*2), x)

$$3.80 \quad \int \frac{1}{(e \cot(c+dx))^{3/2} (a+b \cot(c+dx))^2} dx$$

**Optimal.** Leaf size=437

$$\frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2 + b^2)^2} - \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2} de^{3/2} (a^2 + b^2)^2}$$

[Out]  $b^{(5/2)} * (7*a^2 + 3*b^2) * \arctan(b^{(1/2)} * (e * \cot(d*x+c))^{(1/2)} / a^{(1/2)} / e^{(1/2)}) / a^{(5/2)} / (a^2 + b^2)^2 / d / e^{(3/2)} - 1/2 * (a^2 + 2*a*b - b^2) * \arctan(1 - 2^{(1/2)} * (e * \cot(d*x+c))^{(1/2)} / e^{(1/2)}) / (a^2 + b^2)^2 / d / e^{(3/2)} * 2^{(1/2)} + 1/2 * (a^2 + 2*a*b - b^2) * \arctan(1 + 2^{(1/2)} * (e * \cot(d*x+c))^{(1/2)} / e^{(1/2)}) / (a^2 + b^2)^2 / d / e^{(3/2)} * 2^{(1/2)} + 1/4 * (a^2 - 2*a*b - b^2) * \ln(e^{(1/2)} + \cot(d*x+c) * e^{(1/2)}) - 2^{(1/2)} * (e * \cot(d*x+c))^{(1/2)} / (a^2 + b^2)^2 / d / e^{(3/2)} * 2^{(1/2)} - 1/4 * (a^2 - 2*a*b - b^2) * \ln(e^{(1/2)} + \cot(d*x+c) * e^{(1/2)}) + 2^{(1/2)} * (e * \cot(d*x+c))^{(1/2)} / (a^2 + b^2)^2 / d / e^{(3/2)} * 2^{(1/2)} + (2*a^2 + 3*b^2) / a^2 / (a^2 + b^2) / d / e / (e * \cot(d*x+c))^{(1/2)} - b^2 / a / (a^2 + b^2) / d / e / (a * b * \cot(d*x+c)) / (e * \cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 1.09, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3569, 3649, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2 + b^2)^2} - \frac{(a^2 - 2ab - b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)})}{2\sqrt{2} de^{3/2} (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])^2),x]

[Out]  $(b^{(5/2)} * (7*a^2 + 3*b^2) * \text{ArcTan}[(\text{Sqrt}[b] * \text{Sqrt}[e * \text{Cot}[c + d*x]])] / (\text{Sqrt}[a] * \text{Sqrt}[e])) / (a^{(5/2)} * (a^2 + b^2)^2 * d * e^{(3/2)}) - ((a^2 + 2*a*b - b^2) * \text{ArcTan}[1 - (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / \text{Sqrt}[e]]) / (\text{Sqrt}[2] * (a^2 + b^2)^2 * d * e^{(3/2)}) + ((a^2 + 2*a*b - b^2) * \text{ArcTan}[1 + (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / \text{Sqrt}[e]]) / (\text{Sqrt}[2] * (a^2 + b^2)^2 * d * e^{(3/2)}) + (2*a^2 + 3*b^2) / (a^2 * (a^2 + b^2) * d * e * \text{Sqrt}[e * \text{Cot}[c + d*x]]) - b^2 / (a * (a^2 + b^2) * d * e * \text{Sqrt}[e * \text{Cot}[c + d*x]]) * (a + b * \text{Cot}[c + d*x]) + ((a^2 - 2*a*b - b^2) * \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] * \text{Cot}[c + d*x] - \text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]])] / (2 * \text{Sqrt}[2] * (a^2 + b^2)^2 * d * e^{(3/2)}) - ((a^2 - 2*a*b - b^2) * \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] * \text{Cot}[c + d*x] + \text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]])] / (2 * \text{Sqrt}[2] * (a^2 + b^2)^2 * d * e^{(3/2)})$

Rule 63



```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

#### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

#### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 3534

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_
)]], x_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqr
t[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &&
NeQ[c^2 + d^2, 0]
```

Rule 3569

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3634

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_)
+ (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2), x_Symbol] :=
Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3649

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_)
+ (f_)*(x_)])^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

$b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{LtQ}[m, -1] \&\& !$   
 $(\text{ILtQ}[n, -1] \&\& ( !\text{IntegerQ}[m] || (\text{EqQ}[c, 0] \&\& \text{NeQ}[a, 0])))$

### Rule 3653

$\text{Int}[(((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_.) + (B_.)*\tan[(e_.) + (f_.)*(x_.)] + (C_.)*\tan[(e_.) + (f_.)*(x_.)]^2))/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x\_Symbol] \text{:> Dist}[1/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n * \text{Simp}[b*B + a*(A - C) + (a*B - b*(A - C))*\text{Tan}[e + f*x], x], x] + \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*(1 + \text{Tan}[e + f*x]^2))/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0] \&\& !\text{LeQ}[n, -1]$

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c+dx))^{3/2} (a+b \cot(c+dx))^2} dx &= -\frac{b^2}{a(a^2+b^2) de \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} - \int \frac{-\frac{1}{2}(2a^2+3b^2)e+ab}{(e \cot(c+dx))^{3/2} (a+b \cot(c+dx))^2} dx \\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} - \frac{b^2}{a(a^2+b^2) de \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} \\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} - \frac{b^2}{a(a^2+b^2) de \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} \\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} - \frac{b^2}{a(a^2+b^2) de \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} \\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} - \frac{b^2}{a(a^2+b^2) de \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} \\
&= \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} - \frac{b^2}{a(a^2+b^2) de \sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} \\
&= \frac{b^{5/2} (7a^2+3b^2) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{5/2} (a^2+b^2)^2 de^{3/2}} + \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} \\
&= \frac{b^{5/2} (7a^2+3b^2) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{5/2} (a^2+b^2)^2 de^{3/2}} + \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}} \\
&= \frac{b^{5/2} (7a^2+3b^2) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}} \right)}{a^{5/2} (a^2+b^2)^2 de^{3/2}} - \frac{(a^2+2ab-b^2) \tan^{-1} \left( 1 - \frac{b \cot(c+dx)}{a} \right)}{\sqrt{2} (a^2+b^2)^2 a}
\end{aligned}$$

**Mathematica [C]** time = 0.63, size = 244, normalized size = 0.56

$$\frac{8a^2b^2 {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\frac{b \cot(c+dx)}{a}\right) + 4b^2(a^2+b^2) {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; -\frac{b \cot(c+dx)}{a}\right) + a^2\left(4(a^2-b^2) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; -\cot^2(c+dx)\right) + \frac{2a^2+3b^2}{a^2(a^2+b^2) de \sqrt{e \cot(c+dx)}}\right)}{\sqrt{2} (a^2+b^2)^2 a}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])^2),x]



$$\begin{aligned} & x+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) + 1/d/e^2/(a^2+b^2)^2 * a * b * (e^2)^{(1/4)} * 2^{(1/2)} \\ & * \arctan(2^{(1/2)}/(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) - 1/d/e^2/(a^2+b^2)^2 * a \\ & * b * (e^2)^{(1/4)} * 2^{(1/2)} * \arctan(-2^{(1/2)}/(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) + \\ & 1/2/d/e/(a^2+b^2)^2 * 2^{(1/2)}/(e^2)^{(1/4)} * \arctan(2^{(1/2)}/(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) \\ & * a^{-2-1/2}/d/e/(a^2+b^2)^2 * 2^{(1/2)}/(e^2)^{(1/4)} * \arctan(2^{(1/2)}/(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) \\ & * b^{-2-1/2}/d/e/(a^2+b^2)^2 * 2^{(1/2)}/(e^2)^{(1/4)} * \arctan(-2^{(1/2)}/(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) \\ & * a^{-2+1/2}/d/e/(a^2+b^2)^2 * 2^{(1/2)}/(e^2)^{(1/4)} * \arctan(-2^{(1/2)}/(e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) \\ & * b^{-2+1/4}/d/e/(a^2+b^2)^2 * 2^{(1/2)}/(e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) \\ & * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) \\ & ) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) * b^{-2} + \\ & 2/a^2/d/e/(e * \cot(dx+c))^{(1/2)} \end{aligned}$$

**maxima** [A] time = 0.72, size = 402, normalized size = 0.92

$$e^{\left( \frac{4 \left( 2(a^3 + ab^2)e + \frac{(2a^2b + 3b^3)e}{\tan(dx+c)} \right)}{(a^5 + a^3b^2)e^3 \sqrt{\frac{e}{\tan(dx+c)}} + (a^4b + a^2b^3)e^2 \left( \frac{e}{\tan(dx+c)} \right)^{\frac{3}{2}}} \right)^{\frac{3}{2}}} + \frac{4(7a^2b^3 + 3b^5) \arctan\left( \frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}} \right)}{(a^6 + 2a^4b^2 + a^2b^4) \sqrt{abe} e^2} + \frac{2\sqrt{2}(a^2 + 2ab - b^2) \arctan\left( \frac{\sqrt{2}(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}} \right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*e\*(4\*(2\*(a^3 + a\*b^2)\*e + (2\*a^2\*b + 3\*b^3)\*e/tan(d\*x + c))/((a^5 + a^3\*b^2)\*e^3\*sqrt(e/tan(d\*x + c)) + (a^4\*b + a^2\*b^3)\*e^2\*(e/tan(d\*x + c))^(3/2)) + 4\*(7\*a^2\*b^3 + 3\*b^5)\*arctan(b\*sqrt(e/tan(d\*x + c))/sqrt(a\*b\*e))/((a^6 + 2\*a^4\*b^2 + a^2\*b^4)\*sqrt(a\*b\*e)\*e^2) + (2\*sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) + 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) + 2\*sqrt(2)\*(a^2 + 2\*a\*b - b^2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*sqrt(e) - 2\*sqrt(e/tan(d\*x + c)))/sqrt(e))/sqrt(e) - sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*log(sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e) + sqrt(2)\*(a^2 - 2\*a\*b - b^2)\*log(-sqrt(2)\*sqrt(e)\*sqrt(e/tan(d\*x + c)) + e + e/tan(d\*x + c))/sqrt(e))/((a^4 + 2\*a^2\*b^2 + b^4)\*e^2))/d

**mupad** [B] time = 4.34, size = 15251, normalized size = 34.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& (3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)}*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19) + 768*a^16*b^27*d^8*e^18 + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)} - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 + 26496*a^25*b^14*d^6*e^15 + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)}*1i)/(((e*cot(c + d*x))^{(1/2)}*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)}*(26496*a^25*b^14*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 - ((e*cot(c + d*x))^{(1/2)}*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 156160*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16 + 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)}*(768*a^16*b^27*d^8*e^18 - (e*cot(c + d*x))^{(1/2)}*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)}*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 84480*a^34*b^11*d^9*e^19 - 56320*a^36*b^9*d^9*e^19 - 22528*a^38*b^7*d^9*e^19 - 5120*a^40*b^5*d^9*e^19 - 512*a^42*b^3*d^9*e^19) + 8704*a^18*b^25*d^8*e^18 + 44288*a^20*b^23*d^8*e^18 + 133120*a^22*b^21*d^8*e^18 + 261120*a^24*b^19*d^8*e^18 + 347136*a^26*b^17*d^8*e^18 + 311808*a^28*b^15*d^8*e^18 + 178176*a^30*b^13*d^8*e^18 + 49920*a^32*b^11*d^8*e^18 - 7680*a^34*b^9*d^8*e^18 - 12032*a^36*b^7*d^8*e^18 - 4096*a^38*b^5*d^8*e^18 - 512*a^40*b^3*d^8*e^18))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)} + 33984*a^27*b^12*d^6*e^15 + 18624*a^29*b^10*d^6*e^15 + 5376*a^31*b^8*d^6*e^15 + 1152*a^33*b^6*d^6*e^15 + 288*a^35*b^4*d^6*e^15 + 32*a^37*b^2*d^6*e^15))*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3))^{(1/2)} - ((e*cot(c
\end{aligned}$$



$$\begin{aligned}
& c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16}*b^{21}*d^5*e^{13} + 4224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{22}*b^{15}*d^5*e^{13} - 2816 \\
& *a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136*a^{28}*b^9*d^5*e^{13} - 56 \\
& 0*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}) - (1i/(4*(a^4*d^2*e^3 + b^4*d^2 \\
& *e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^{(1/2)}*((e*cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13440*a^{17}*b^{24}*d^7*e^{16} + \\
& 69056*a^{19}*b^{22}*d^7*e^{16} + 202752*a^{21}*b^{20}*d^7*e^{16} + 372800*a^{23}*b^{18}*d^ \\
& 7*e^{16} + 443136*a^{25}*b^{16}*d^7*e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} + 156160*a^{2 \\
& 9}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} + 3200*a^{33}*b^8*d^7*e^{16} + 704*a \\
& ^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16}) - (1i/(4*( \\
& a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2 \\
& *d^2*e^3)))^{(1/2)}*((e*cot(c + d*x))^{(1/2)}*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 \\
& + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^{(1/2)}*(512*a^{ \\
& 18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 563 \\
& 20*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} \\
& - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9 \\
& *e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 5120*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9 \\
& *e^{19}) + 768*a^{16}*b^{27}*d^8*e^{18} + 8704*a^{18}*b^{25}*d^8*e^{18} + 44288*a^{20}*b^{23} \\
& *d^8*e^{18} + 133120*a^{22}*b^{21}*d^8*e^{18} + 261120*a^{24}*b^{19}*d^8*e^{18} + 347136* \\
& a^{26}*b^{17}*d^8*e^{18} + 311808*a^{28}*b^{15}*d^8*e^{18} + 178176*a^{30}*b^{13}*d^8*e^{18} \\
& + 49920*a^{32}*b^{11}*d^8*e^{18} - 7680*a^{34}*b^9*d^8*e^{18} - 12032*a^{36}*b^7*d^8*e^{ \\
& 18} - 4096*a^{38}*b^5*d^8*e^{18} - 512*a^{40}*b^3*d^8*e^{18}))* (1i/(4*(a^4*d^2*e^3 + \\
& b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^{( \\
& 1/2)} - 1152*a^{15}*b^{24}*d^6*e^{15} - 8448*a^{17}*b^{22}*d^6*e^{15} - 23776*a^{19}*b^{20} \\
& d^6*e^{15} - 29664*a^{21}*b^{18}*d^6*e^{15} - 6528*a^{23}*b^{16}*d^6*e^{15} + 26496*a^{25} \\
& b^{14}*d^6*e^{15} + 33984*a^{27}*b^{12}*d^6*e^{15} + 18624*a^{29}*b^{10}*d^6*e^{15} + 5376* \\
& a^{31}*b^8*d^6*e^{15} + 1152*a^{33}*b^6*d^6*e^{15} + 288*a^{35}*b^4*d^6*e^{15} + 32*a^3 \\
& 7*b^2*d^6*e^{15}))* (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3 \\
& *b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^{(1/2)} + 144*a^{14}*b^{21}*d^4*e^{12} + 1296* \\
& a^{16}*b^{19}*d^4*e^{12} + 4880*a^{18}*b^{17}*d^4*e^{12} + 10000*a^{20}*b^{15}*d^4*e^{12} + 1 \\
& 2080*a^{22}*b^{13}*d^4*e^{12} + 8624*a^{24}*b^{11}*d^4*e^{12} + 3376*a^{26}*b^9*d^4*e^{12} \\
& + 560*a^{28}*b^7*d^4*e^{12}))* (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3 \\
& *4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^{(1/2)}*2i - atan(((1/(a^4*d^2* \\
& e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e \\
& ^3*6i)))^{(1/2)}*((e*cot(c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16} \\
& b^{21}*d^5*e^{13} + 4224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{ \\
& 22}*b^{15}*d^5*e^{13} - 2816*a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136 \\
& *a^{28}*b^9*d^5*e^{13} - 560*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}))/2 + ((1 \\
& /(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2 \\
& *b^2*d^2*e^3*6i)))^{(1/2)}*(13248*a^{25}*b^{14}*d^6*e^{15} - 576*a^{15}*b^{24}*d^6*e^{15} \\
& - 4224*a^{17}*b^{22}*d^6*e^{15} - 11888*a^{19}*b^{20}*d^6*e^{15} - 14832*a^{21}*b^{18}*d^6* \\
& e^{15} - 3264*a^{23}*b^{16}*d^6*e^{15} - (((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4 \\
& *a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i)))^{(1/2)}*(384*a^{16}*b^2 \\
& 7*d^8*e^{18} - ((e*cot(c + d*x))^{(1/2)}*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + \\
& 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i)))^{(1/2)}*(512*a^{18}*b^
\end{aligned}$$

$$\begin{aligned}
& 27*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} \\
& - 22528*a^{38}*b^7*d^9*e^{19} - 5120*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19} \\
& ))/4 + 4352*a^{18}*b^{25}*d^8*e^{18} + 22144*a^{20}*b^{23}*d^8*e^{18} + 66560*a^{22}*b^{21}*d^8*e^{18} + 130560*a^{24}*b^{19}*d^8*e^{18} + 173568*a^{26}*b^{17}*d^8*e^{18} + 155904*a^{28}*b^{15}*d^8*e^{18} + 89088*a^{30}*b^{13}*d^8*e^{18} + 24960*a^{32}*b^{11}*d^8*e^{18} - 3840*a^{34}*b^9*d^8*e^{18} - 6016*a^{36}*b^7*d^8*e^{18} - 2048*a^{38}*b^5*d^8*e^{18} - 256*a^{40}*b^3*d^8*e^{18}))/2 + ((e*cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13440*a^{17}*b^{24}*d^7*e^{16} + 69056*a^{19}*b^{22}*d^7*e^{16} + 202752*a^{21}*b^{20}*d^7*e^{16} + 372800*a^{23}*b^{18}*d^7*e^{16} + 443136*a^{25}*b^{16}*d^7*e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} + 156160*a^{29}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} + 3200*a^{33}*b^8*d^7*e^{16} + 704*a^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16}))/2*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)})/2 + 16992*a^{27}*b^{12}*d^6*e^{15} + 9312*a^{29}*b^{10}*d^6*e^{15} + 2688*a^{31}*b^8*d^6*e^{15} + 576*a^{33}*b^6*d^6*e^{15} + 144*a^{35}*b^4*d^6*e^{15} + 16*a^{37}*b^2*d^6*e^{15}))/2)*1i + (1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(((e*cot(c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16}*b^{21}*d^5*e^{13} + 4224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{22}*b^{15}*d^5*e^{13} - 2816*a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136*a^{28}*b^9*d^5*e^{13} - 560*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}))/2 - ((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(13248*a^{25}*b^{14}*d^6*e^{15} - 576*a^{15}*b^{24}*d^6*e^{15} - 4224*a^{17}*b^{22}*d^6*e^{15} - 11888*a^{19}*b^{20}*d^6*e^{15} - 14832*a^{21}*b^{18}*d^6*e^{15} - 3264*a^{23}*b^{16}*d^6*e^{15} - (((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(((e*cot(c + d*x))^{(1/2)}*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(512*a^{18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 5120*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19}))/4 + 384*a^{16}*b^{27}*d^8*e^{18} + 4352*a^{18}*b^{25}*d^8*e^{18} + 22144*a^{20}*b^{23}*d^8*e^{18} + 66560*a^{22}*b^{21}*d^8*e^{18} + 130560*a^{24}*b^{19}*d^8*e^{18} + 173568*a^{26}*b^{17}*d^8*e^{18} + 155904*a^{28}*b^{15}*d^8*e^{18} + 89088*a^{30}*b^{13}*d^8*e^{18} + 24960*a^{32}*b^{11}*d^8*e^{18} - 3840*a^{34}*b^9*d^8*e^{18} - 6016*a^{36}*b^7*d^8*e^{18} - 2048*a^{38}*b^5*d^8*e^{18} - 256*a^{40}*b^3*d^8*e^{18}))/2 - ((e*cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13440*a^{17}*b^{24}*d^7*e^{16} + 69056*a^{19}*b^{22}*d^7*e^{16} + 202752*a^{21}*b^{20}*d^7*e^{16} + 372800*a^{23}*b^{18}*d^7*e^{16} + 443136*a^{25}*b^{16}*d^7*e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} + 156160*a^{29}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} + 3200*a^{33}*b^8*d^7*e^{16} + 704*a^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16}))/2*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)})/2 + 16992*a^{27}*b^{12}*d^6*e^{15} + 9312*a^{29}*b^{10}*d^6*e^{15} + 2688*a^{31}*b^8*d^6*e^{15} + 576*a^{33}*b^6*d^6*e^{15}
\end{aligned}$$

$$\begin{aligned}
& ^{15} + 144*a^{35}*b^4*d^6*e^{15} + 16*a^{37}*b^2*d^6*e^{15}))/2)*1i)/((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*((e*\cot(c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16}*b^{21}*d^5*e^{13} + 4224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{22}*b^{15}*d^5*e^{13} - 2816*a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136*a^{28}*b^9*d^5*e^{13} - 560*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}))/2 + ((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(13248*a^{25}*b^{14}*d^6*e^{15} - 576*a^{15}*b^{24}*d^6*e^{15} - 4224*a^{17}*b^{22}*d^6*e^{15} - 11888*a^{19}*b^{20}*d^6*e^{15} - 14832*a^{21}*b^{18}*d^6*e^{15} - 3264*a^{23}*b^{16}*d^6*e^{15} - (((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(384*a^{16}*b^{27}*d^8*e^{18} - ((e*\cot(c + d*x))^{(1/2)}*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(512*a^{18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 5120*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19}))/4 + 4352*a^{18}*b^{25}*d^8*e^{18} + 22144*a^{20}*b^{23}*d^8*e^{18} + 66560*a^{22}*b^{21}*d^8*e^{18} + 130560*a^{24}*b^{19}*d^8*e^{18} + 173568*a^{26}*b^{17}*d^8*e^{18} + 155904*a^{28}*b^{15}*d^8*e^{18} + 89088*a^{30}*b^{13}*d^8*e^{18} + 24960*a^{32}*b^{11}*d^8*e^{18} - 3840*a^{34}*b^9*d^8*e^{18} - 6016*a^{36}*b^7*d^8*e^{18} - 2048*a^{38}*b^5*d^8*e^{18} - 256*a^{40}*b^3*d^8*e^{18}))/2 + ((e*\cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13440*a^{17}*b^{24}*d^7*e^{16} + 69056*a^{19}*b^{22}*d^7*e^{16} + 202752*a^{21}*b^{20}*d^7*e^{16} + 372800*a^{23}*b^{18}*d^7*e^{16} + 443136*a^{25}*b^{16}*d^7*e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} + 156160*a^{29}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} + 3200*a^{33}*b^8*d^7*e^{16} + 704*a^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16}))/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)})/2 + 16992*a^{27}*b^{12}*d^6*e^{15} + 9312*a^{29}*b^{10}*d^6*e^{15} + 2688*a^{31}*b^8*d^6*e^{15} + 576*a^{33}*b^6*d^6*e^{15} + 144*a^{35}*b^4*d^6*e^{15} + 16*a^{37}*b^2*d^6*e^{15}))/2) - (1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*((e*\cot(c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16}*b^{21}*d^5*e^{13} + 4224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{22}*b^{15}*d^5*e^{13} - 2816*a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136*a^{28}*b^9*d^5*e^{13} - 560*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}))/2 - (((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(13248*a^{25}*b^{14}*d^6*e^{15} - 576*a^{15}*b^{24}*d^6*e^{15} - 4224*a^{17}*b^{22}*d^6*e^{15} - 11888*a^{19}*b^{20}*d^6*e^{15} - 14832*a^{21}*b^{18}*d^6*e^{15} - 3264*a^{23}*b^{16}*d^6*e^{15} - (((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*((e*\cot(c + d*x))^{(1/2)}*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*(512*a^{18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 51
\end{aligned}$$

$$\begin{aligned}
& 20*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19}))/4 + 384*a^{16}*b^{27}*d^8*e^{18} + \\
& 4352*a^{18}*b^{25}*d^8*e^{18} + 22144*a^{20}*b^{23}*d^8*e^{18} + 66560*a^{22}*b^{21}*d^8*e^{18} \\
& + 130560*a^{24}*b^{19}*d^8*e^{18} + 173568*a^{26}*b^{17}*d^8*e^{18} + 155904*a^{28}*b^{15}*d^8*e^{18} \\
& + 89088*a^{30}*b^{13}*d^8*e^{18} + 24960*a^{32}*b^{11}*d^8*e^{18} - 3840*a^{34}*b^9*d^8*e^{18} \\
& - 6016*a^{36}*b^7*d^8*e^{18} - 2048*a^{38}*b^5*d^8*e^{18} - 256*a^{40}*b^3*d^8*e^{18}))/2 - ((e*\cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13 \\
& 440*a^{17}*b^{24}*d^7*e^{16} + 69056*a^{19}*b^{22}*d^7*e^{16} + 202752*a^{21}*b^{20}*d^7*e^{16} \\
& + 372800*a^{23}*b^{18}*d^7*e^{16} + 443136*a^{25}*b^{16}*d^7*e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} \\
& + 156160*a^{29}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} + 3200*a^{33}*b^8*d^7*e^{16} \\
& + 704*a^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16}))/2)*(1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a*b^3*d^2*e^3 - 4 \\
& *a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)})/2 + 16992*a^{27}*b^{12}*d^6*e^{15} + \\
& 9312*a^{29}*b^{10}*d^6*e^{15} + 2688*a^{31}*b^8*d^6*e^{15} + 576*a^{33}*b^6*d^6*e^{15} + \\
& 144*a^{35}*b^4*d^6*e^{15} + 16*a^{37}*b^2*d^6*e^{15}))/2) + 144*a^{14}*b^{21}*d^4*e^{12} \\
& + 1296*a^{16}*b^{19}*d^4*e^{12} + 4880*a^{18}*b^{17}*d^4*e^{12} + 10000*a^{20}*b^{15}*d^4* \\
& e^{12} + 12080*a^{22}*b^{13}*d^4*e^{12} + 8624*a^{24}*b^{11}*d^4*e^{12} + 3376*a^{26}*b^9*d^4* \\
& e^{12} + 560*a^{28}*b^7*d^4*e^{12}))*((1/(a^4*d^2*e^3*1i + b^4*d^2*e^3*1i + 4*a \\
& *b^3*d^2*e^3 - 4*a^3*b*d^2*e^3 - a^2*b^2*d^2*e^3*6i))^{(1/2)}*1i - (\operatorname{atan}((((e*\cot(c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16}*b^{21}*d^5*e^{13} + 4 \\
& 224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{22}*b^{15}*d^5*e^{13} \\
& - 2816*a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136*a^{28}*b^9*d^5*e^{13} \\
& - 560*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}) + ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(26496*a^{25}*b^{14}*d^6*e^{15} - 8448*a^{17}*b^{22}*d^6*e^{15} - 23776*a^{19}*b^{20}*d^6*e^{15} - 29664*a^{21}*b^{18}*d^6*e^{15} - 6528*a^{23}*b^{16}*d^6*e^{15} - 1 \\
& 152*a^{15}*b^{24}*d^6*e^{15} + 33984*a^{27}*b^{12}*d^6*e^{15} + 18624*a^{29}*b^{10}*d^6*e^{15} \\
& 5 + 5376*a^{31}*b^8*d^6*e^{15} + 1152*a^{33}*b^6*d^6*e^{15} + 288*a^{35}*b^4*d^6*e^{15} \\
& + 32*a^{37}*b^2*d^6*e^{15} - ((7*a^2 + 3*b^2)*((e*\cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13440*a^{17}*b^{24}*d^7*e^{16} + 69056*a^{19}*b^{22}*d^7*e^{16} + 20 \\
& 2752*a^{21}*b^{20}*d^7*e^{16} + 372800*a^{23}*b^{18}*d^7*e^{16} + 443136*a^{25}*b^{16}*d^7* \\
& e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} + 156160*a^{29}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} \\
& + 3200*a^{33}*b^8*d^7*e^{16} + 704*a^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16} \\
& + ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(768*a^{16}*b^{27}*d^8*e^{18} + 8704*a^{18}*b^{25}*d^8*e^{18} + 44288*a^{20}*b^{23}*d^8*e^{18} \\
& + 133120*a^{22}*b^{21}*d^8*e^{18} + 261120*a^{24}*b^{19}*d^8*e^{18} + 347136*a^{26}*b^{17}*d^8*e^{18} \\
& + 311808*a^{28}*b^{15}*d^8*e^{18} + 178176*a^{30}*b^{13}*d^8*e^{18} + 49920*a^{32}*b^{11}*d^8*e^{18} - 7680*a^{34}*b^9*d^8*e^{18} - 12032*a^{36}*b^7*d^8*e^{18} - 409 \\
& 6*a^{38}*b^5*d^8*e^{18} - 512*a^{40}*b^3*d^8*e^{18} - ((e*\cot(c + d*x))^{(1/2)}*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(512*a^{18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 51 \\
& 20*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*((-a^5*b^5*e^3)^{(1/2)})/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*((7*a^2 + 3*b^2)*(-a^5*b^5
\end{aligned}$$



$$\begin{aligned}
& 18 + 311808*a^{28}*b^{15}*d^8*e^{18} + 178176*a^{30}*b^{13}*d^8*e^{18} + 49920*a^{32}*b^{11}*d^8*e^{18} - 7680*a^{34}*b^9*d^8*e^{18} - 12032*a^{36}*b^7*d^8*e^{18} - 4096*a^{38}*b^5*d^8*e^{18} - 512*a^{40}*b^3*d^8*e^{18} - ((e*\cot(c + d*x))^{(1/2)}*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(512*a^{18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 5120*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3))))/(2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*(-a^5*b^5*e^3)^{(1/2)}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)) - (((e*\cot(c + d*x))^{(1/2)}*(144*a^{14}*b^{23}*d^5*e^{13} + 1248*a^{16}*b^{21}*d^5*e^{13} + 4224*a^{18}*b^{19}*d^5*e^{13} + 6720*a^{20}*b^{17}*d^5*e^{13} + 3872*a^{22}*b^{15}*d^5*e^{13} - 2816*a^{24}*b^{13}*d^5*e^{13} - 5632*a^{26}*b^{11}*d^5*e^{13} - 3136*a^{28}*b^9*d^5*e^{13} - 560*a^{30}*b^7*d^5*e^{13} + 32*a^{32}*b^5*d^5*e^{13}) - ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(26496*a^{25}*b^{14}*d^6*e^{15} - 8448*a^{17}*b^{22}*d^6*e^{15} - 23776*a^{19}*b^{20}*d^6*e^{15} - 29664*a^{21}*b^{18}*d^6*e^{15} - 6528*a^{23}*b^{16}*d^6*e^{15} - 1152*a^{15}*b^{24}*d^6*e^{15} + 33984*a^{27}*b^{12}*d^6*e^{15} + 18624*a^{29}*b^{10}*d^6*e^{15} + 5376*a^{31}*b^8*d^6*e^{15} + 1152*a^{33}*b^6*d^6*e^{15} + 288*a^{35}*b^4*d^6*e^{15} + 32*a^{37}*b^2*d^6*e^{15} + ((7*a^2 + 3*b^2)*((e*\cot(c + d*x))^{(1/2)}*(1152*a^{15}*b^{26}*d^7*e^{16} + 13440*a^{17}*b^{24}*d^7*e^{16} + 69056*a^{19}*b^{22}*d^7*e^{16} + 202752*a^{21}*b^{20}*d^7*e^{16} + 372800*a^{23}*b^{18}*d^7*e^{16} + 443136*a^{25}*b^{16}*d^7*e^{16} + 337792*a^{27}*b^{14}*d^7*e^{16} + 156160*a^{29}*b^{12}*d^7*e^{16} + 37632*a^{31}*b^{10}*d^7*e^{16} + 3200*a^{33}*b^8*d^7*e^{16} + 704*a^{35}*b^6*d^7*e^{16} + 512*a^{37}*b^4*d^7*e^{16} + 64*a^{39}*b^2*d^7*e^{16}) - ((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(768*a^{16}*b^{27}*d^8*e^{18} + 8704*a^{18}*b^{25}*d^8*e^{18} + 44288*a^{20}*b^{23}*d^8*e^{18} + 133120*a^{22}*b^{21}*d^8*e^{18} + 261120*a^{24}*b^{19}*d^8*e^{18} + 347136*a^{26}*b^{17}*d^8*e^{18} + 311808*a^{28}*b^{15}*d^8*e^{18} + 178176*a^{30}*b^{13}*d^8*e^{18} + 49920*a^{32}*b^{11}*d^8*e^{18} - 7680*a^{34}*b^9*d^8*e^{18} - 12032*a^{36}*b^7*d^8*e^{18} - 4096*a^{38}*b^5*d^8*e^{18} - 512*a^{40}*b^3*d^8*e^{18} + ((e*\cot(c + d*x))^{(1/2)}*(7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*(512*a^{18}*b^{27}*d^9*e^{19} + 5120*a^{20}*b^{25}*d^9*e^{19} + 22528*a^{22}*b^{23}*d^9*e^{19} + 56320*a^{24}*b^{21}*d^9*e^{19} + 84480*a^{26}*b^{19}*d^9*e^{19} + 67584*a^{28}*b^{17}*d^9*e^{19} - 67584*a^{32}*b^{13}*d^9*e^{19} - 84480*a^{34}*b^{11}*d^9*e^{19} - 56320*a^{36}*b^9*d^9*e^{19} - 22528*a^{38}*b^7*d^9*e^{19} - 5120*a^{40}*b^5*d^9*e^{19} - 512*a^{42}*b^3*d^9*e^{19}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))))*(-a^5*b^5*e^3)^{(1/2)}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)))*((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}))/((2*(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)) + 144*a^{14}*b^{21}*d^4*e^{12} + 1296*a^{16}*b^{19}*d^4*e^{12} + 4880*a^{18}*b^{17}*d^4*e^{12} + 10000*a^{20}*b^{15}*d^4*e^{12} + 12080*a^{22}*b^{13}*d^4*e^{12} + 8624*a^{24}*b^{11}*d^4*e^{12} + 3376*a^{26}*b^9*d^4*e^{12} + 560*a^{28}*b^7*d^4*e^{12}))*((7*a^2 + 3*b^2)*(-a^5*b^5*e^3)^{(1/2)}*i)/(a^9*d*e^3 + a^5*b^4*d*e^3 + 2*a^7*b^2*d*e^3)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e \cot(c + dx))^{\frac{3}{2}} (a + b \cot(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(3/2)/(a+b\*cot(d\*x+c))\*\*2,x)

[Out] Integral(1/((e\*cot(c + d\*x))\*\*(3/2)\*(a + b\*cot(c + d\*x))\*\*2), x)

$$3.81 \quad \int \frac{(e \cot(c+dx))^{9/2}}{(a+b \cot(c+dx))^3} dx$$

Optimal. Leaf size=529

$$\frac{e^{9/2}(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3} + \frac{e^{9/2}(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

[Out]  $\frac{1}{4}a^{5/2}(15a^4+46a^2b^2+63b^4)e^{9/2}\arctan(b^{1/2}(e\cot(dx+c))^{1/2}/a^{1/2}/e^{1/2})/b^{7/2}/(a^2+b^2)^3/d+1/2a^2e^2(e\cot(dx+c))^{5/2}/b/(a^2+b^2)/d/(a+b\cot(dx+c))^{2+1/4}a^2(5a^2+13b^2)e^3(e\cot(dx+c))^{3/2}/b^2/(a^2+b^2)^2/d/(a+b\cot(dx+c))+1/2(a-b)(a^2+4a*b+b^2)e^{9/2}\arctan(1-2^{1/2}(e\cot(dx+c))^{1/2}/e^{1/2})/(a^2+b^2)^3/d*2^{1/2}-1/2(a-b)(a^2+4a*b+b^2)e^{9/2}\arctan(1+2^{1/2}(e\cot(dx+c))^{1/2}/e^{1/2})/(a^2+b^2)^3/d*2^{1/2}-1/4(a+b)(a^2-4a*b+b^2)e^{9/2}\ln(e^{1/2}+\cot(dx+c)*e^{1/2}-2^{1/2}(e\cot(dx+c))^{1/2})/(a^2+b^2)^3/d*2^{1/2}+1/4(a+b)(a^2-4a*b+b^2)e^{9/2}\ln(e^{1/2}+\cot(dx+c)*e^{1/2}+2^{1/2}(e\cot(dx+c))^{1/2})/(a^2+b^2)^3/d*2^{1/2}-1/4(15a^4+31a^2b^2+8b^4)e^4(e\cot(dx+c))^{1/2}/b^3/(a^2+b^2)^2/d$

**Rubi [A]** time = 1.63, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$ , Rules used = {3565, 3645, 3647, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{e^4(31a^2b^2+15a^4+8b^4)\sqrt{e\cot(c+dx)}}{4b^3d(a^2+b^2)^2} + \frac{a^2e^3(5a^2+13b^2)(e\cot(c+dx))^{3/2}}{4b^2d(a^2+b^2)^2(a+b\cot(c+dx))} + \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd(a^2+b^2)(a+b\cot(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(9/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out]  $(a^{5/2}(15a^4+46a^2b^2+63b^4)e^{9/2}\text{ArcTan}[\text{Sqrt}[b]\text{Sqrt}[e\text{Cot}[c+d*x]]]/(\text{Sqrt}[a]\text{Sqrt}[e]))/(4b^{7/2}(a^2+b^2)^3d) + ((a-b)(a^2+4a*b+b^2)e^{9/2}\text{ArcTan}[1-(\text{Sqrt}[2]\text{Sqrt}[e\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2](a^2+b^2)^3d) - ((a-b)(a^2+4a*b+b^2)e^{9/2}\text{ArcTan}[1+(\text{Sqrt}[2]\text{Sqrt}[e\text{Cot}[c+d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2](a^2+b^2)^3d) - ((15a^4+31a^2b^2+8b^4)e^4\text{Sqrt}[e\text{Cot}[c+d*x]]/(4b^3(a^2+b^2)^2d) + (a^2e^2(e\cot(c+dx))^{5/2})/(2b(a^2+b^2)d(a+b\cot(c+dx)))^2 + (a^2(5a^2+13b^2)e^3(e\cot(c+dx))^{3/2})/(4b^2(a^2+b^2)^2d(a+b\cot(c+dx))) - ((a+b)(a^2-4a*b+b^2)e^{9/2}\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]\text{Cot}[c+d*x] - \text{Sqrt}[2]\text{Sqrt}[e\text{Cot}[c+d*x]])]/(2\text{Sqrt}[2](a^2+b^2)^3d) - ((a+b)(a^2-4a*b+b^2)e^{9/2}\text{Log}[\text{Sqrt}[e] - \text{Sqrt}[e]\text{Cot}[c+d*x] + \text{Sqrt}[2]\text{Sqrt}[e\text{Cot}[c+d*x]])]/(2\text{Sqrt}[2](a^2+b^2)^3d)$



$b^2)^3d) + ((a + b)(a^2 - 4ab + b^2)e^{(9/2)}\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]\text{Cot}[c + dx] + \text{Sqrt}[2]\text{Sqrt}[e\text{Cot}[c + dx]])]/(2\text{Sqrt}[2](a^2 + b^2)^3d)$

### Rule 63

$\text{Int}[(a_. + (b_.)(x_))^{(m_)}((c_.) + (d_.)(x_))^{(n_)}, x\_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 204

$\text{Int}[(a_. + (b_.)(x_)^2)^{-1}, x\_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 205

$\text{Int}[(a_. + (b_.)(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

### Rule 617

$\text{Int}[(a_. + (b_.)(x_) + (c_.)(x_)^2)^{-1}, x\_Symbol] := \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[(d_. + (e_.)(x_))/((a_.) + (b_.)(x_) + (c_.)(x_)^2), x\_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[(d_. + (e_.)(x_)^2)/((a_.) + (c_.)(x_)^4), x\_Symbol] := \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[(d_. + (e_.)(x_)^2)/((a_.) + (c_.)(x_)^4), x\_Symbol] := \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x],$

$x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[(d + (e_*)*(x_)^2)/(a + (c_*)*(x_)^4), x\_Symbol] \text{ :> } \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

### Rule 3534

$\text{Int}[(c + (d_*)*\tan[(e_*) + (f_*)*(x_)])/ \text{Sqrt}[(b_*)*\tan[(e_*) + (f_*)*(x_)]], x\_Symbol] \text{ :> } \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\text{Tan}[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3565

$\text{Int}[(a + (b_*)*\tan[(e_*) + (f_*)*(x_)])^{(m_*)} * ((c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)])^{(n_*)}, x\_Symbol] \text{ :> } \text{Simp}[(b*c - a*d)^2 * (a + b*\text{Tan}[e + f*x])^{(m-2)} * (c + d*\text{Tan}[e + f*x])^{(n+1)} / (d*f*(n+1)*(c^2 + d^2)), x] - \text{Dist}[1 / (d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-3)} * (c + d*\text{Tan}[e + f*x])^{(n+1)} * \text{Simp}[a^2*d*(b*d*(m-2) - a*c*(n+1)) + b*(b*c - 2*a*d)*(b*c*(m-2) + a*d*(n+1)) - d*(n+1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*\text{Tan}[e + f*x] - b*(a*d*(2*b*c - a*d)*(m+n-1) - b^2*(c^2*(m-2) - d^2*(n+1)))*\text{Tan}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{GtQ}[m, 2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*m]$

### Rule 3634

$\text{Int}[(a + (b_*)*\tan[(e_*) + (f_*)*(x_)])^{(m_*)} * ((c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)])^{(n_*)} * (A + (C_*)*\tan[(e_*) + (f_*)*(x_)])^2, x\_Symbol] \text{ :> } \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m * (c + d*x)^n, x], x, \text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n, x\} \ \&\& \ \text{EqQ}[A, C]$

### Rule 3645

$\text{Int}[(a + (b_*)*\tan[(e_*) + (f_*)*(x_)])^{(m_*)} * ((c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)])^{(n_*)} * (A + (B_*)*\tan[(e_*) + (f_*)*(x_)]) + (C_*)*\tan[(e_*) + (f_*)*(x_)])^2, x\_Symbol] \text{ :> } \text{Simp}[(A*d^2 + c*(c*C - B*d)) * (a + b*\text{Tan}[e + f*x])^m * (c + d*\text{Tan}[e + f*x])^{(n+1)} / (d*f*(n+1)*(c^2 + d^2)), x] - \text{Dist}[1 / (d*(n+1)*(c^2 + d^2)), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m-1)} * (c + d*\text{Tan}[e$

```

+ f*x]]^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3647

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{9/2}}{(a + b \cot(c + dx))^3} dx &= \frac{a^2 e^2 (e \cot(c + dx))^{5/2}}{2b(a^2 + b^2)d(a + b \cot(c + dx))^2} - \frac{\int \frac{(e \cot(c + dx))^{3/2} \left(-\frac{5}{2}a^2 e^3 + 2abe^3 \cot(c + dx) - \frac{1}{2}(5a^2 + 4b^2)e^3 \cot^2(c + dx)\right)}{(a + b \cot(c + dx))^2} dx}{2b(a^2 + b^2)} \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{5/2}}{2b(a^2 + b^2)d(a + b \cot(c + dx))^2} + \frac{a^2(5a^2 + 13b^2)e^3(e \cot(c + dx))^{3/2}}{4b^2(a^2 + b^2)^2 d(a + b \cot(c + dx))} + \frac{\int \frac{\sqrt{e \cot(c + dx)}}{(a + b \cot(c + dx))^2} dx}{4b^2(a^2 + b^2)} \\
&= -\frac{(15a^4 + 31a^2b^2 + 8b^4)e^4 \sqrt{e \cot(c + dx)}}{4b^3(a^2 + b^2)^2 d} + \frac{a^2 e^2 (e \cot(c + dx))^{5/2}}{2b(a^2 + b^2)d(a + b \cot(c + dx))^2} + \frac{a^2}{4b^2} \\
&= -\frac{(15a^4 + 31a^2b^2 + 8b^4)e^4 \sqrt{e \cot(c + dx)}}{4b^3(a^2 + b^2)^2 d} + \frac{a^2 e^2 (e \cot(c + dx))^{5/2}}{2b(a^2 + b^2)d(a + b \cot(c + dx))^2} + \frac{a^2}{4b^2} \\
&= -\frac{(15a^4 + 31a^2b^2 + 8b^4)e^4 \sqrt{e \cot(c + dx)}}{4b^3(a^2 + b^2)^2 d} + \frac{a^2 e^2 (e \cot(c + dx))^{5/2}}{2b(a^2 + b^2)d(a + b \cot(c + dx))^2} + \frac{a^2}{4b^2} \\
&= -\frac{(15a^4 + 31a^2b^2 + 8b^4)e^4 \sqrt{e \cot(c + dx)}}{4b^3(a^2 + b^2)^2 d} + \frac{a^2 e^2 (e \cot(c + dx))^{5/2}}{2b(a^2 + b^2)d(a + b \cot(c + dx))^2} + \frac{a^2}{4b^2} \\
&= \frac{a^{5/2}(15a^4 + 46a^2b^2 + 63b^4)e^{9/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}}\right)}{4b^{7/2}(a^2 + b^2)^3 d} - \frac{(15a^4 + 31a^2b^2 + 8b^4)e^4 \sqrt{e \cot(c + dx)}}{4b^3(a^2 + b^2)^2 d} \\
&= \frac{a^{5/2}(15a^4 + 46a^2b^2 + 63b^4)e^{9/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}}\right)}{4b^{7/2}(a^2 + b^2)^3 d} - \frac{(15a^4 + 31a^2b^2 + 8b^4)e^4 \sqrt{e \cot(c + dx)}}{4b^3(a^2 + b^2)^2 d} \\
&= \frac{a^{5/2}(15a^4 + 46a^2b^2 + 63b^4)e^{9/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}}\right)}{4b^{7/2}(a^2 + b^2)^3 d} + \frac{(a - b)(a^2 + 4ab + b^2)e^{9/2}}{\sqrt{2}(a^2 + b^2)}
\end{aligned}$$

**Mathematica [C]** time = 6.26, size = 556, normalized size = 1.05

$$(e \cot(c + dx))^{9/2} \frac{4b^2 \cot^{\frac{11}{2}}(c+dx) {}_2F_1\left(2, \frac{11}{2}; \frac{13}{2}; -\frac{b \cot(c+dx)}{a}\right)}{11a(a^2+b^2)^2} - \frac{2a(a^2-3b^2) \left(-7 \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right) - 3 \cot^{\frac{7}{2}}(c+dx) + 7 \cot^{\frac{3}{2}}(c+dx)\right)}{21(a^2+b^2)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*Cot[c + d\*x])^(9/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out] -(((e\*Cot[c + d\*x])^(9/2)\*((2\*b\*(3\*a^2 - b^2)\*Cot[c + d\*x])^(9/2))/(9\*(a^2 + b^2)^3) - (2\*a\*(3\*a^2 - b^2)\*(15\*Cot[c + d\*x]^(7/2) - 7\*a\*((3\*Cot[c + d\*x]^(5/2))/b - (5\*a\*((-3\*a\*(-((Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]]))/b^(3/2)) + Sqrt[Cot[c + d\*x]]/b))/b + Cot[c + d\*x]^(3/2)/b))/b))/(105\*(a^2 + b^2)^3) - (2\*a\*(a^2 - 3\*b^2)\*(7\*Cot[c + d\*x]^(3/2) - 3\*Cot[c + d\*x]^(7/2) - 7\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(21\*(a^2 + b^2)^3) + (4\*b^2\*Cot[c + d\*x]^(11/2)\*Hypergeometric2F1[2, 11/2, 13/2, -((b\*Cot[c + d\*x])/a)))/(11\*a\*(a^2 + b^2)^2) + (2\*b^2\*Cot[c + d\*x]^(11/2)\*Hypergeometric2F1[3, 11/2, 13/2, -((b\*Cot[c + d\*x])/a)))/(11\*a^3\*(a^2 + b^2)) - (b\*(3\*a^2 - b^2)\*(360\*Sqrt[Cot[c + d\*x]] - 72\*Cot[c + d\*x]^(5/2) + 40\*Cot[c + d\*x]^(9/2) + 45\*(2\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])]) + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*S

qrt[Cot[c + d\*x]] + Cot[c + d\*x])))/(180\*(a^2 + b^2)^3)))/(d\*Cot[c + d\*x]^(9/2)))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(9/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{9}{2}}}{(b \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(9/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(9/2)/(b\*cot(d\*x + c) + a)^3, x)

**maple** [B] time = 0.85, size = 1254, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(9/2)/(a+b\*cot(d\*x+c))^3,x)

[Out] 
$$\begin{aligned} & -2/d*e^4/b^3*(e*cot(d*x+c))^(1/2)-9/4/d*e^5*a^7/b^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)-13/2/d*e^5*a^5/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)-17/4/d*e^5*a^3*b^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)-7/4/d*e^6*a^8/b^3/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)-11/2/d*e^6*a^6/b/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)-15/4/d*e^6*a^4*b/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)+15/4/d*e^5*a^7/b^3/(a^2+b^2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))+23/2/d*e^5*a^5/b/(a^2+b^2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))+63/4/d*e^5*a^3*b/(a^2+b^2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))-3/2/d*e^4/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^2*b+1/2/d*e^4/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*b^3+3/2/d*e^4/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^2*b-1/2/d*e^4/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2) \end{aligned}$$

$$\begin{aligned}
& +1) * b^3 - 3/4/d * e^4 / (a^2 + b^2)^3 * (e^2)^{(1/4)} * 2^{(1/2)} * \ln((e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * a^2 * b + 1/4/d * e^4 / (a^2 + b^2)^3 * (e^2)^{(1/4)} * 2^{(1/2)} * \ln((e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * b^3 - 1/2/d * e^5 / (a^2 + b^2)^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a^3 + 3/2/d * e^5 / (a^2 + b^2)^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a * b^2 - 1/4/d * e^5 / (a^2 + b^2)^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * a^3 + 3/4/d * e^5 / (a^2 + b^2)^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \ln((e * \cot(dx+c) - (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)}) / (e * \cot(dx+c) + (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} * 2^{(1/2)} + (e^2)^{(1/2)})) * a * b^2 + 1/2/d * e^5 / (a^2 + b^2)^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a^3 - 3/2/d * e^5 / (a^2 + b^2)^3 * 2^{(1/2)} / (e^2)^{(1/4)} * \arctan(-2^{(1/2)} / (e^2)^{(1/4)} * (e * \cot(dx+c))^{(1/2)} + 1) * a * b^2
\end{aligned}$$

**maxima [A]** time = 0.61, size = 537, normalized size = 1.02

$$\left( \frac{(15a^7 + 46a^5b^2 + 63a^3b^4)e^4 \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^6b^3 + 3a^4b^5 + 3a^2b^7 + b^9)\sqrt{abe}} - \frac{\left( \frac{2\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3) \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3) \arctan\left(\frac{\sqrt{2}}{\sqrt{e}}\right)}{\sqrt{e}} \right)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(9/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\begin{aligned}
& 1/4 * ((15 * a^7 + 46 * a^5 * b^2 + 63 * a^3 * b^4) * e^4 * \arctan(b * \sqrt{e / \tan(dx + c)}) / \sqrt{a * b * e}) / ((a^6 * b^3 + 3 * a^4 * b^5 + 3 * a^2 * b^7 + b^9) * \sqrt{a * b * e}) - (2 * \sqrt{2} * (a^3 + 3 * a^2 * b - 3 * a * b^2 - b^3) * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} + 2 * \sqrt{e / \tan(dx + c)})) / \sqrt{e}) / \sqrt{e} + 2 * \sqrt{2} * (a^3 + 3 * a^2 * b - 3 * a * b^2 - b^3) * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * \sqrt{e} - 2 * \sqrt{e / \tan(dx + c)})) / \sqrt{e}) / \sqrt{e} - \sqrt{2} * (a^3 - 3 * a^2 * b - 3 * a * b^2 + b^3) * \log(\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx + c)} + e + e / \tan(dx + c)) / \sqrt{e} + \sqrt{2} * (a^3 - 3 * a^2 * b - 3 * a * b^2 + b^3) * \log(-\sqrt{2} * \sqrt{e} * \sqrt{e / \tan(dx + c)} + e + e / \tan(dx + c)) / \sqrt{e} * e^4 / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) - 8 * e^3 * \sqrt{e / \tan(dx + c)} / b^3 - ((7 * a^6 + 15 * a^4 * b^2) * e^5 * \sqrt{e / \tan(dx + c)} + (9 * a^5 * b
\end{aligned}$

$$+ 17*a^3*b^3)*e^4*(e/\tan(d*x + c))^{(3/2)})/((a^6*b^3 + 2*a^4*b^5 + a^2*b^7)*e^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*e^2/\tan(d*x + c) + (a^4*b^5 + 2*a^2*b^7 + b^9)*e^2/\tan(d*x + c)^2))*e/d$$

**mupad [B]** time = 10.40, size = 20651, normalized size = 39.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*\cot(c + d*x))^{(9/2)}/(a + b*\cot(c + d*x))^{3}, x)$

[Out]  $\text{atan}(\frac{((128*a*b^{26}*d^4*e^{15} + 3648*a^3*b^{24}*d^4*e^{15} + 25536*a^5*b^{22}*d^4*e^{15} + 88320*a^7*b^{20}*d^4*e^{15} + 182784*a^9*b^{18}*d^4*e^{15} + 244608*a^{11}*b^{16}*d^4*e^{15} + 217728*a^{13}*b^{14}*d^4*e^{15} + 128256*a^{15}*b^{12}*d^4*e^{15} + 48000*a^{17}*b^{10}*d^4*e^{15} + 10304*a^{19}*b^8*d^4*e^{15} + 960*a^{21}*b^6*d^4*e^{15})/(b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5) + ((e*\cot(c + d*x))^{(1/2)}*(-(e^9*i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)}*(512*b^{30}*d^4*e^{10} + 4608*a^2*b^{28}*d^4*e^{10} + 17920*a^4*b^{26}*d^4*e^{10} + 38400*a^6*b^{24}*d^4*e^{10} + 46080*a^8*b^{22}*d^4*e^{10} + 21504*a^{10}*b^{20}*d^4*e^{10} - 21504*a^{12}*b^{18}*d^4*e^{10} - 46080*a^{14}*b^{16}*d^4*e^{10} - 38400*a^{16}*b^{14}*d^4*e^{10} - 17920*a^{18}*b^{12}*d^4*e^{10} - 4608*a^{20}*b^{10}*d^4*e^{10} - 512*a^{22}*b^8*d^4*e^{10})/(b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4))*(-(e^9*i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)}*(1800*a^{23}*b*d^2*e^{19} - 1472*a*b^{23}*d^2*e^{19} - 1024*a^3*b^{21}*d^2*e^{19} + 8448*a^5*b^{19}*d^2*e^{19} + 46088*a^7*b^{17}*d^2*e^{19} + 177344*a^9*b^{15}*d^2*e^{19} + 402912*a^{11}*b^{13}*d^2*e^{19} + 541632*a^{13}*b^{11}*d^2*e^{19} + 455472*a^{15}*b^9*d^2*e^{19} + 248064*a^{17}*b^7*d^2*e^{19} + 87008*a^{19}*b^5*d^2*e^{19} + 18240*a^{21}*b^3*d^2*e^{19})/(b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4))*(-(e^9*i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)} + (2250*a^{20}*b*d^2*e^{24} + 32*a^2*b^{19}*d^2*e^{24} + 12288*a^4*b^{17}*d^2*e^{24} - 10974*a^6*b^{15}*d^2*e^{24} - 105162*a^8*b^{13}*d^2*e^{24} - 150758*a^{10}*b^{11}*d^2*e^{24} - 85314*a^{12}*b^9*d^2*e^{24} - 3578*a^{14}*b^7*d^2*e^{24} + 22210*a^{16}*b^5*d^2*e^{24} + 11550*a^{18}*b^3*d^2*e^{24})/(b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5))*(-(e^9*i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)}*(32*b^{18}*e^{28} - 225*a^{18}*e^{28} + 128*a^2*b^{16}*e^{28} + 192*a^4*b^{14}*e^{28} - 3841*a^6*b^{12}*e^{28} + 18050*a^8*b^{10}*e^{28} + 26801*a^{10}*b^8*e^{28} + 16860*a^{12}*b^6*e^{28} + 4049*a^{14}*b^4*e^{28} - 30*a^{16}*b^2*e^{28} + 17*a^3*b^3)*e^4*(e/\tan(d*x + c))^{(3/2)}/((a^6*b^3 + 2*a^4*b^5 + a^2*b^7)*e^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*e^2/\tan(d*x + c) + (a^4*b^5 + 2*a^2*b^7 + b^9)*e^2/\tan(d*x + c)^2))*e/d$



$$\begin{aligned}
& \text{^28))}/(b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4)) * (-e^9 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4d^2 - a^3 * b^3d^2 * 20i + 15 * a^4 * b^2d^2)))^{(1/2)} * 1i - (((((128 * a * b^{26}d^4 * e^{15} + 3648 * a^3 * b^{24}d^4 * e^{15} + 25536 * a^5 * b^{22}d^4 * e^{15} + 88320 * a^7 * b^{20}d^4 * e^{15} + 182784 * a^9 * b^{18}d^4 * e^{15} + 244608 * a^{11} * b^{16}d^4 * e^{15} + 217728 * a^{13} * b^{14}d^4 * e^{15} + 128256 * a^{15} * b^{12}d^4 * e^{15} + 48000 * a^{17} * b^{10}d^4 * e^{15} + 10304 * a^{19} * b^8d^4 * e^{15} + 960 * a^{21} * b^6d^4 * e^{15}) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) - ((e * \cot(c + d * x))^{(1/2)} * (-e^9 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4d^2 - a^3 * b^3d^2 * 20i + 15 * a^4 * b^2d^2)))^{(1/2)} * (512 * b^{30}d^4 * e^{10} + 4608 * a^2 * b^{28}d^4 * e^{10} + 17920 * a^4 * b^{26}d^4 * e^{10} + 38400 * a^6 * b^{24}d^4 * e^{10} + 46080 * a^8 * b^{22}d^4 * e^{10} + 21504 * a^{10} * b^{20}d^4 * e^{10} - 21504 * a^{12} * b^{18}d^4 * e^{10} - 46080 * a^{14} * b^{16}d^4 * e^{10} - 38400 * a^{16} * b^{14}d^4 * e^{10} - 17920 * a^{18} * b^{12}d^4 * e^{10} - 4608 * a^{20} * b^{10}d^4 * e^{10} - 512 * a^{22} * b^8d^4 * e^{10})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4)) * (-e^9 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4d^2 - a^3 * b^3d^2 * 20i + 15 * a^4 * b^2d^2)))^{(1/2)} + ((e * \cot(c + d * x))^{(1/2)} * (1800 * a^{23} * b * d^2 * e^{19} - 1472 * a * b^{23}d^2 * e^{19} - 1024 * a^3 * b^{21}d^2 * e^{19} + 8448 * a^5 * b^{19}d^2 * e^{19} + 46088 * a^7 * b^{17}d^2 * e^{19} + 177344 * a^9 * b^{15}d^2 * e^{19} + 402912 * a^{11} * b^{13}d^2 * e^{19} + 541632 * a^{13} * b^{11}d^2 * e^{19} + 455472 * a^{15} * b^9d^2 * e^{19} + 248064 * a^{17} * b^7d^2 * e^{19} + 87008 * a^{19} * b^5d^2 * e^{19} + 18240 * a^{21} * b^3d^2 * e^{19})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4)) * (-e^9 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4d^2 - a^3 * b^3d^2 * 20i + 15 * a^4 * b^2d^2)))^{(1/2)} + (2250 * a^{20} * b * d^2 * e^24 + 32 * a^2 * b^{19}d^2 * e^24 + 12288 * a^4 * b^{17}d^2 * e^24 - 10974 * a^6 * b^{15}d^2 * e^24 - 105162 * a^8 * b^{13}d^2 * e^24 - 150758 * a^{10} * b^{11}d^2 * e^24 - 85314 * a^{12} * b^9d^2 * e^24 - 3578 * a^{14} * b^7d^2 * e^24 + 22210 * a^{16} * b^5d^2 * e^24 + 11550 * a^{18} * b^3d^2 * e^24) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5)) * (-e^9 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4d^2 - a^3 * b^3d^2 * 20i + 15 * a^4 * b^2d^2)))^{(1/2)} - ((e * \cot(c + d * x))^{(1/2)} * (32 * b^{18}e^{28} - 225 * a^{18}e^{28} + 128 * a^2 * b^{16}e^{28} + 192 * a^4 * b^{14}e^{28} - 3841 * a^6 * b^{12}e^{28} + 18050 * a^8 * b^{10}e^{28} + 26801 * a^{10} * b^8e^{28} + 16860 * a^{12} * b^6e^{28} + 4049 * a^{14} * b^4e^{28} - 30 * a^{16} * b^2e^{28})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4)) * (-e^9 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4d^2 - a^3 * b^3d^2 * 20i + 15 * a^4 * b^2d^2)))^{(1/2)} * 1i) / ((225 * a^{15} * e^{33} + 504 * a^3 * b^{12} * e^{33} + 872 * a^5 * b^{10} * e^{33} + 4457 * a^7 * b^8 * e^{33} + 5916 * a^9 * b^6 * e^{33} + 4006 * a^{11} * b^4 * e^{33} + 1380 * a^{13} * b^2 * e^{33}) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 +
\end{aligned}$$



$$\begin{aligned}
& (6i + a^5 b d^2 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 20i + 15 a^4 b^2 d^2))^{(1/2)} \cdot (512 b^30 d^4 e^{10} + 4608 a^2 b^28 d^4 e^{10} + 17920 a^4 b^26 d^4 e^{10} + \\
& 38400 a^6 b^24 d^4 e^{10} + 46080 a^8 b^22 d^4 e^{10} + 21504 a^{10} b^20 d^4 e^{10} - 21504 a^{12} b^{18} d^4 e^{10} - 46080 a^{14} b^{16} d^4 e^{10} - 38400 a^{16} b^{14} \\
& d^4 e^{10} - 17920 a^{18} b^{12} d^4 e^{10} - 4608 a^{20} b^{10} d^4 e^{10} - 512 a^{22} b^8 d^4 e^{10})) / (b^{21} d^4 + 8 a^2 b^{19} d^4 + 28 a^4 b^{17} d^4 + 56 a^6 b^{15} d^4 \\
& + 70 a^8 b^{13} d^4 + 56 a^{10} b^{11} d^4 + 28 a^{12} b^9 d^4 + 8 a^{14} b^7 d^4 + a^{16} b^5 d^4) \cdot (-e^9 1i) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 6i + a^5 b d^2 6i \\
& - 15 a^2 b^4 d^2 - a^3 b^3 d^2 20i + 15 a^4 b^2 d^2))^{(1/2)} + ((e \cot(c + d x))^{(1/2)} \cdot (1800 a^{23} b d^2 e^{19} - 1472 a b^{23} d^2 e^{19} - 1024 a^3 b^{21} \\
& d^2 e^{19} + 8448 a^5 b^{19} d^2 e^{19} + 46088 a^7 b^{17} d^2 e^{19} + 177344 a^9 b^{15} d^2 e^{19} + 402912 a^{11} b^{13} d^2 e^{19} + 541632 a^{13} b^{11} d^2 e^{19} + 4554 \\
& 72 a^{15} b^9 d^2 e^{19} + 248064 a^{17} b^7 d^2 e^{19} + 87008 a^{19} b^5 d^2 e^{19} + 18240 a^{21} b^3 d^2 e^{19})) / (b^{21} d^4 + 8 a^2 b^{19} d^4 + 28 a^4 b^{17} d^4 + 5 \\
& 6 a^6 b^{15} d^4 + 70 a^8 b^{13} d^4 + 56 a^{10} b^{11} d^4 + 28 a^{12} b^9 d^4 + 8 a^{14} b^7 d^4 + a^{16} b^5 d^4) \cdot (-e^9 1i) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 6i \\
& i + a^5 b d^2 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 20i + 15 a^4 b^2 d^2))^{(1/2)} + (2250 a^{20} b d^2 e^{24} + 32 a^2 b^{19} d^2 e^{24} + 12288 a^4 b^{17} d^2 e^{24} \\
& - 10974 a^6 b^{15} d^2 e^{24} - 105162 a^8 b^{13} d^2 e^{24} - 150758 a^{10} b^{11} d^2 e^{24} - 85314 a^{12} b^9 d^2 e^{24} - 3578 a^{14} b^7 d^2 e^{24} + 22210 a^{16} b^5 \\
& d^2 e^{24} + 11550 a^{18} b^3 d^2 e^{24}) / (b^{21} d^5 + 8 a^2 b^{19} d^5 + 28 a^4 b^{17} d^5 + 56 a^6 b^{15} d^5 + 70 a^8 b^{13} d^5 + 56 a^{10} b^{11} d^5 + 28 a^{12} b^9 \\
& d^5 + 8 a^{14} b^7 d^5 + a^{16} b^5 d^5) \cdot (-e^9 1i) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 6i + a^5 b d^2 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 20i + 15 a^4 b^2 d^2))^{(1/2)} \\
& - ((e \cot(c + d x))^{(1/2)} \cdot (32 b^{18} e^{28} - 225 a^{18} e^{28} + 128 a^2 b^{16} e^{28} + 192 a^4 b^{14} e^{28} - 3841 a^6 b^{12} e^{28} + 18050 a^8 b^{10} e^{28} \\
& + 26801 a^{10} b^8 e^{28} + 16860 a^{12} b^6 e^{28} + 4049 a^{14} b^4 e^{28} - 30 a^{16} b^2 e^{28})) / (b^{21} d^4 + 8 a^2 b^{19} d^4 + 28 a^4 b^{17} d^4 + 56 a^6 b^{15} d^4 \\
& + 70 a^8 b^{13} d^4 + 56 a^{10} b^{11} d^4 + 28 a^{12} b^9 d^4 + 8 a^{14} b^7 d^4 + a^{16} b^5 d^4) \cdot (-e^9 1i) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 6i + a^5 b d^2 6i \\
& i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 20i + 15 a^4 b^2 d^2))^{(1/2)} \cdot (-e^9 1i) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 6i + a^5 b d^2 6i - 15 a^2 b^4 d^2 - a^3 \\
& b^3 d^2 20i + 15 a^4 b^2 d^2))^{(1/2)} \cdot 2i - (((e \cot(c + d x))^{(1/2)} \cdot (7 a^6 e^6 + 15 a^4 b^2 e^6)) / (4 (a^4 + b^4 + 2 a^2 b^2))) + (b (e \cot(c + d x))^{(3/2)} \\
& \cdot (9 a^5 e^5 + 17 a^3 b^2 e^5)) / (4 (a^4 + b^4 + 2 a^2 b^2))) / (a^2 b^3 d e^2 + b^5 d e^2 \cot(c + d x)^2 + 2 a b^4 d e^2 \cot(c + d x)) + \operatorname{atan}((((((1 \\
& 28 a b^{26} d^4 e^{15} + 3648 a^3 b^{24} d^4 e^{15} + 25536 a^5 b^{22} d^4 e^{15} + 88320 a^7 b^{20} d^4 e^{15} + 182784 a^9 b^{18} d^4 e^{15} + 244608 a^{11} b^{16} d^4 e^{15} \\
& + 217728 a^{13} b^{14} d^4 e^{15} + 128256 a^{15} b^{12} d^4 e^{15} + 48000 a^{17} b^{10} d^4 e^{15} + 10304 a^{19} b^8 d^4 e^{15} + 960 a^{21} b^6 d^4 e^{15})) / (b^{21} d^5 + 8 a^2 b^{19} d^5 \\
& + 28 a^4 b^{17} d^5 + 56 a^6 b^{15} d^5 + 70 a^8 b^{13} d^5 + 56 a^{10} b^{11} d^5 + 28 a^{12} b^9 d^5 + 8 a^{14} b^7 d^5 + a^{16} b^5 d^5) + ((e \cot(c + d x))^{(1/2)} \cdot (-e^9 / (4 (b^6 d^2 1i \\
& - a^6 d^2 1i + 6 a b^5 d^2 + 6 a^5 b d^2 - a^2 b^4 d^2 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 15i)))^{(1/2)} \cdot (512 b^30 d^4 e^{10} + 4608 a^2 b^28 d^4 e^{10} + 17920 a^4 b^26 d^4 e^{10} + 38400 a^6 b^24 d^4
\end{aligned}$$

$$\begin{aligned}
& 4e^{10} + 46080a^8b^{22}d^4e^{10} + 21504a^{10}b^{20}d^4e^{10} - 21504a^{12}b^{18}d^4e^{10} - 46080a^{14}b^{16}d^4e^{10} - 38400a^{16}b^{14}d^4e^{10} - 17920a^{18}b^{12}d^4e^{10} - 4608a^{20}b^{10}d^4e^{10} - 512a^{22}b^8d^4e^{10}) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)} * (1800a^{23}b^d^2e^{19} - 1472a*b^{23}d^2e^{19} - 1024a^3b^{21}d^2e^{19} + 8448a^5b^{19}d^2e^{19} + 46088a^7b^{17}d^2e^{19} + 177344a^9b^{15}d^2e^{19} + 402912a^{11}b^{13}d^2e^{19} + 541632a^{13}b^{11}d^2e^{19} + 455472a^{15}b^9d^2e^{19} + 248064a^{17}b^7d^2e^{19} + 87008a^{19}b^5d^2e^{19} + 18240a^{21}b^3d^2e^{19})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((2250a^{20}b*d^2e^{24} + 32a^2b^{19}d^2e^{24} + 12288a^4b^{17}d^2e^{24} - 10974a^6b^{15}d^2e^{24} - 105162a^8b^{13}d^2e^{24} - 150758a^{10}b^{11}d^2e^{24} - 85314a^{12}b^9d^2e^{24} - 3578a^{14}b^7d^2e^{24} + 22210a^{16}b^5d^2e^{24} + 11550a^{18}b^3d^2e^{24}) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)} * (32*b^{18}e^{28} - 225*a^{18}e^{28} + 128*a^2*b^{16}e^{28} + 192*a^4*b^{14}e^{28} - 3841*a^6*b^{12}e^{28} + 18050*a^8*b^{10}e^{28} + 26801*a^{10}b^8e^{28} + 16860*a^{12}b^6e^{28} + 4049*a^{14}b^4e^{28} - 30*a^{16}b^2e^{28})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} * 1i - (((((128*a*b^{26}d^4e^{15} + 3648a^3b^{24}d^4e^{15} + 25536a^5b^{22}d^4e^{15} + 88320a^7b^{20}d^4e^{15} + 182784a^9b^{18}d^4e^{15} + 244608a^{11}b^{16}d^4e^{15} + 217728a^{13}b^{14}d^4e^{15} + 128256a^{15}b^{12}d^4e^{15} + 48000a^{17}b^{10}d^4e^{15} + 10304a^{19}b^8d^4e^{15} + 960a^{21}b^6d^4e^{15}) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) - ((e*\cot(c + d*x))^{(1/2)} * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} * (512*b^{30}d^4e^{10} + 4608a^2b^28d^4e^{10} + 17920a^4b^{26}d^4e^{10} + 38400a^6b^{24}d^4e^{10} + 46080a^8b^{22}d^4e^{10} + 21504a^{10}b^{20}d^4e^{10} - 21504a^{12}b^{18}d^4e^{10} - 46080a^{14}b^{16}d^4e^{10} - 38400a^{16}b^{14}d^4e^{10} - 17920a^{18}b^{12}d^4e^{10} - 4608a^{20}b^{10}d^4e^{10} - 512a^{22}b^8d^4e^{10})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)} * (1800a^{23}b^d^2e^{19}
\end{aligned}$$

$$\begin{aligned}
& - 1472*a*b^{23}*d^2*e^{19} - 1024*a^3*b^{21}*d^2*e^{19} + 8448*a^5*b^{19}*d^2*e^{19} + \\
& 46088*a^7*b^{17}*d^2*e^{19} + 177344*a^9*b^{15}*d^2*e^{19} + 402912*a^{11}*b^{13}*d^2*e^{19} + \\
& 541632*a^{13}*b^{11}*d^2*e^{19} + 455472*a^{15}*b^9*d^2*e^{19} + 248064*a^{17}*b^7*d^2*e^{19} + \\
& 87008*a^{19}*b^5*d^2*e^{19} + 18240*a^{21}*b^3*d^2*e^{19}))/ (b^{21}*d^4 + 8*a^2*b^{19}*d^4 + \\
& 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + \\
& 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + (2250*a^{20}*b*d^2*e^{24} + 32*a^2*b^{19}*d^2*e^{24} + 12288*a^4*b^{17}*d^2*e^{24} - 10974*a^6*b^{15}*d^2*e^{24} - 105162*a^8*b^{13}*d^2*e^{24} - 150758*a^{10}*b^{11}*d^2*e^{24} - 85314*a^{12}*b^9*d^2*e^{24} - 3578*a^{14}*b^7*d^2*e^{24} + 22210*a^{16}*b^5*d^2*e^{24} + 11550*a^{18}*b^3*d^2*e^{24}))/ (b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} - ((e*cot(c + d*x))^{(1/2)}*(32*b^{18}*e^{28} - 225*a^{18}*e^{28} + 128*a^2*b^{16}*e^{28} + 192*a^4*b^{14}*e^{28} - 3841*a^6*b^{12}*e^{28} + 18050*a^8*b^{10}*e^{28} + 26801*a^{10}*b^8*e^{28} + 16860*a^{12}*b^6*e^{28} + 4049*a^{14}*b^4*e^{28} - 30*a^{16}*b^2*e^{28}))/ (b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*1i)/((225*a^{15}*e^{33} + 504*a^3*b^{12}*e^{33} + 872*a^5*b^{10}*e^{33} + 4457*a^7*b^8*e^{33} + 5916*a^9*b^6*e^{33} + 4006*a^{11}*b^4*e^{33} + 1380*a^{13}*b^2*e^{33}))/ (b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5) + (((((128*a*b^{26}*d^4*e^{15} + 3648*a^3*b^{24}*d^4*e^{15} + 25536*a^5*b^{22}*d^4*e^{15} + 88320*a^7*b^{20}*d^4*e^{15} + 182784*a^9*b^{18}*d^4*e^{15} + 244608*a^{11}*b^{16}*d^4*e^{15} + 217728*a^{13}*b^{14}*d^4*e^{15} + 128256*a^{15}*b^{12}*d^4*e^{15} + 48000*a^{17}*b^{10}*d^4*e^{15} + 10304*a^{19}*b^8*d^4*e^{15} + 960*a^{21}*b^6*d^4*e^{15}))/ (b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5) + ((e*cot(c + d*x))^{(1/2)}*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*(512*b^{30}*d^4*e^{10} + 4608*a^2*b^{28}*d^4*e^{10} + 17920*a^4*b^{26}*d^4*e^{10} + 38400*a^6*b^{24}*d^4*e^{10} + 46080*a^8*b^{22}*d^4*e^{10} + 21504*a^{10}*b^{20}*d^4*e^{10} - 21504*a^{12}*b^{18}*d^4*e^{10} - 46080*a^{14}*b^{16}*d^4*e^{10} - 38400*a^{16}*b^{14}*d^4*e^{10} - 17920*a^{18}*b^{12}*d^4*e^{10} - 4608*a^{20}*b^{10}*d^4*e^{10} - 512*a^{22}*b^8*d^4*e^{10}))/ (b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} - ((e*cot(c + d*x))^{(1/2)}*(1800*a^{23}*b*d^2*e^{19} - 1472*a*b^{23}*d^2*e^{19} - 1024*a^3*b^{21}*d^2*e^{19} + 8448*a^5*b^{19}*d^2*e^{19} + 46088*a^7*b^{17}*d^2*e^{19} + 177344*a^9*b^{15}*d^2*e^{19} + 402912*a^{11}*b^{13}*d^2*e^{19} + 541632*a^{13}*b^{11}*d^2*e^{19} + 455472*a^{15}*b^9*d^2*e^{19} + 248064*a^{17}*b^7*d^2*e^{19} + 87008*a^{19}*b^5*
\end{aligned}$$

$$\begin{aligned}
& d^2e^{19} + 18240a^{21}b^3d^2e^{19}) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + (2250*a^{20}*b*d^2*e^{24} + 32*a^2*b^{19}*d^2*e^{24} + 12288*a^4*b^{17}*d^2*e^{24} - 10974*a^6*b^{15}*d^2*e^{24} - 105162*a^8*b^{13}*d^2*e^{24} - 150758*a^{10}*b^{11}*d^2*e^{24} - 85314*a^{12}*b^9*d^2*e^{24} - 3578*a^{14}*b^7*d^2*e^{24} + 22210*a^{16}*b^5*d^2*e^{24} + 11550*a^{18}*b^3*d^2*e^{24}) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((e*cot(c + d*x))^{(1/2)} * (32*b^{18}*e^{28} - 225*a^{18}*e^{28} + 128*a^2*b^{16}*e^{28} + 192*a^4*b^{14}*e^{28} - 3841*a^6*b^{12}*e^{28} + 18050*a^8*b^{10}*e^{28} + 26801*a^{10}*b^8*e^{28} + 16860*a^{12}*b^6*e^{28} + 4049*a^{14}*b^4*e^{28} - 30*a^{16}*b^2*e^{28})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((((((128*a*b^{26}d^4e^{15} + 3648*a^3*b^{24}d^4e^{15} + 25536*a^5*b^{22}d^4e^{15} + 88320*a^7*b^{20}d^4e^{15} + 182784*a^9*b^{18}d^4e^{15} + 244608*a^{11}b^{16}d^4e^{15} + 217728*a^{13}b^{14}d^4e^{15} + 128256*a^{15}b^{12}d^4e^{15} + 48000*a^{17}b^{10}d^4e^{15} + 10304*a^{19}b^8d^4e^{15} + 960*a^{21}b^6d^4e^{15}) / (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) - ((e*cot(c + d*x))^{(1/2)} * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} * (512*b^30*d^4e^{10} + 4608*a^2*b^{28}d^4e^{10} + 17920*a^4*b^{26}d^4e^{10} + 38400*a^6*b^{24}d^4e^{10} + 46080*a^8*b^{22}d^4e^{10} + 21504*a^{10}b^{20}d^4e^{10} - 21504*a^{12}b^{18}d^4e^{10} - 46080*a^{14}b^{16}d^4e^{10} - 38400*a^{16}b^{14}d^4e^{10} - 17920*a^{18}b^{12}d^4e^{10} - 4608*a^{20}b^{10}d^4e^{10} - 512*a^{22}b^8d^4e^{10})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((e*cot(c + d*x))^{(1/2)} * (1800*a^{23}b*d^2e^{19} - 1472*a*b^{23}d^2e^{19} - 1024*a^3b^{21}d^2e^{19} + 8448*a^5b^{19}d^2e^{19} + 46088*a^7b^{17}d^2e^{19} + 177344*a^9b^{15}d^2e^{19} + 402912*a^{11}b^{13}d^2e^{19} + 541632*a^{13}b^{11}d^2e^{19} + 455472*a^{15}b^9d^2e^{19} + 248064*a^{17}b^7d^2e^{19} + 87008*a^{19}b^5d^2e^{19} + 18240*a^{21}b^3d^2e^{19})) / (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) * (-e^9 / (4*(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + (2250*a^{20}*b*d^2*e^{24} + 32*a^2*b^{19}*d^2*e^{24} + 12288*a^4*b^{17}*d^2*e^{24} - 10974*a^6*b^{15}*d^2*e^{24} - 105162*a^8*b^{13}*d^2*e^{24} - 150758*a^{10}*b^{11}*d^2*e^{24} - 85314*a^{12}b^9d^2e^{24} - 3578*a^{14}b^7d^2e^{24} + 22210*a^{16}b^5d^2e^{24} + 1
\end{aligned}$$

$$\begin{aligned}
& 1550*a^{18}*b^3*d^2*e^{24})/(b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5)*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} \\
& - ((e*\cot(c + d*x))^{(1/2)}*(32*b^{18}*e^{28} - 225*a^{18}*e^{28} + 128*a^2*b^{16}*e^{28} + 192*a^4*b^{14}*e^{28} - 3841*a^6*b^{12}*e^{28} + 18050*a^8*b^{10}*e^{28} + 26801*a^{10}*b^8*e^{28} + 16860*a^{12}*b^6*e^{28} + 4049*a^{14}*b^4*e^{28} - 30*a^{16}*b^2*e^{28}))/ \\
& (b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4) \\
& )*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)})))*(-e^9/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*2i - (2*e^4*(e*\cot(c + d*x))^{(1/2)})/(b^3*d) + (a \\
& \tan((((e*\cot(c + d*x))^{(1/2)}*(32*b^{18}*e^{28} - 225*a^{18}*e^{28} + 128*a^2*b^{16}*e^{28} + 192*a^4*b^{14}*e^{28} - 3841*a^6*b^{12}*e^{28} + 18050*a^8*b^{10}*e^{28} + 26801*a^{10}*b^8*e^{28} + 16860*a^{12}*b^6*e^{28} + 4049*a^{14}*b^4*e^{28} - 30*a^{16}*b^2*e^{28}))/ \\
& (b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4) - (((2250*a^{20}*b*d^2*e^{24} + 32*a^2*b^{19}*d^2*e^{24} + 12288*a^4*b^{17}*d^2*e^{24} \\
& *e^{24} - 10974*a^6*b^{15}*d^2*e^{24} - 105162*a^8*b^{13}*d^2*e^{24} - 150758*a^{10}*b^{11}*d^2*e^{24} - 85314*a^{12}*b^9*d^2*e^{24} - 3578*a^{14}*b^7*d^2*e^{24} + 22210*a^{16}*b^5*d^2*e^{24} + 11550*a^{18}*b^3*d^2*e^{24}))/ \\
& (b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5) + (((e*\cot(c + d*x))^{(1/2)}*(1800 \\
& *a^{23}*b*d^2*e^{19} - 1472*a*b^{23}*d^2*e^{19} - 1024*a^3*b^{21}*d^2*e^{19} + 8448*a^5*b^{19}*d^2*e^{19} + 46088*a^7*b^{17}*d^2*e^{19} + 177344*a^9*b^{15}*d^2*e^{19} + 402912*a^{11}*b^{13}*d^2*e^{19} + 541632*a^{13}*b^{11}*d^2*e^{19} + 455472*a^{15}*b^9*d^2*e^{19} \\
& + 248064*a^{17}*b^7*d^2*e^{19} + 87008*a^{19}*b^5*d^2*e^{19} + 18240*a^{21}*b^3*d^2*e^{19}))/ \\
& (b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4) + (((128*a*b^{26}*d^4*e^{15} + 3648*a^3*b^{24}*d^4*e^{15} + 25536*a^5*b^{22}*d^4*e^{15} + 88320*a^7*b^{20}*d^4*e^{15} + 182784*a^9*b^{18}*d^4*e^{15} + 244608*a^{11}*b^{16}*d^4*e^{15} + 217728*a^{13}*b^{14}*d^4*e^{15} + 128256*a^{15}*b^{12}*d^4*e^{15} + 48000*a^{17}*b^{10}*d^4*e^{15} + 10304*a^{19}*b^8*d^4*e^{15} + 960*a^{21}*b^6*d^4*e^{15}))/ \\
& (b^{21}*d^5 + 8*a^2*b^{19}*d^5 + 28*a^4*b^{17}*d^5 + 56*a^6*b^{15}*d^5 + 70*a^8*b^{13}*d^5 + 56*a^{10}*b^{11}*d^5 + 28*a^{12}*b^9*d^5 + 8*a^{14}*b^7*d^5 + a^{16}*b^5*d^5) \\
& - ((e*\cot(c + d*x))^{(1/2)}*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^{(1/2)}*(512*b^{30}*d^4*e^{10} + 4608*a^2*b^{28}*d^4*e^{10} + 17920*a^4*b^{26}*d^4*e^{10} + 38400*a^6*b^{24}*d^4*e^{10} + 46080*a^8*b^{22}*d^4*e^{10} + 21504*a^{10}*b^{20}*d^4*e^{10} \\
& - 21504*a^{12}*b^{18}*d^4*e^{10} - 46080*a^{14}*b^{16}*d^4*e^{10} - 38400*a^{16}*b^{14}*d^4*e^{10} - 17920*a^{18}*b^{12}*d^4*e^{10} - 4608*a^{20}*b^{10}*d^4*e^{10} - 512*a^{22}*b^8*d^4*e^{10}))/ \\
& (8*(b^{13}*d + 3*a^2*b^{11}*d + 3*a^4*b^9*d + a^6*b^7*d)*(b^{21}*d^4 + 8*a^2*b^{19}*d^4 + 28*a^4*b^{17}*d^4 + 56*a^6*b^{15}*d^4 + 70*a^8*b^{13}*d^4 + 56*a^{10}*b^{11}*d^4 + 28*a^{12}*b^9*d^4 + 8*a^{14}*b^7*d^4 + a^{16}*b^5*d^4)))*(15*a^4 + 63*b^4 + 46*a^2*b^2)*(-a^5*b^7*e^9)^{(1/2)})/(8*(b^{13}*d + 3*a^2*b^{11}*d + 3
\end{aligned}$$

$$\begin{aligned}
& (a^4 b^9 d + a^6 b^7 d)) * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} * i / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) + (((e \cot(c + dx))^{(1/2)} * (32 b^{18} e^{28} - 225 a^{18} e^{28} + 128 a^2 b^{16} e^{28} + 192 a^4 b^{14} e^{28} - 3841 a^6 b^{12} e^{28} + 18050 a^8 b^{10} e^{28} + 26801 a^{10} b^8 e^{28} + 16860 a^{12} b^6 e^{28} + 4049 a^{14} b^4 e^{28} - 30 a^{16} b^2 e^{28})) / (b^{21} d^4 + 8 a^2 b^{19} d^4 + 28 a^4 b^{17} d^4 + 56 a^6 b^{15} d^4 + 70 a^8 b^{13} d^4 + 56 a^{10} b^{11} d^4 + 28 a^{12} b^9 d^4 + 8 a^{14} b^7 d^4 + a^{16} b^5 d^4) + (((2250 a^{20} b^2 d^2 e^{24} + 32 a^2 b^{19} d^2 e^{24} + 12288 a^4 b^{17} d^2 e^{24} - 10974 a^6 b^{15} d^2 e^{24} - 105162 a^8 b^{13} d^2 e^{24} - 150758 a^{10} b^{11} d^2 e^{24} - 85314 a^{12} b^9 d^2 e^{24} - 3578 a^{14} b^7 d^2 e^{24} + 22210 a^{16} b^5 d^2 e^{24} + 11550 a^{18} b^3 d^2 e^{24})) / (b^{21} d^5 + 8 a^2 b^{19} d^5 + 28 a^4 b^{17} d^5 + 56 a^6 b^{15} d^5 + 70 a^8 b^{13} d^5 + 56 a^{10} b^{11} d^5 + 28 a^{12} b^9 d^5 + 8 a^{14} b^7 d^5 + a^{16} b^5 d^5) - (((e \cot(c + dx))^{(1/2)} * (1800 a^{23} b^2 d^2 e^{19} - 1472 a^3 b^{23} d^2 e^{19} - 1024 a^3 b^{21} d^2 e^{19} + 8448 a^5 b^{19} d^2 e^{19} + 46088 a^7 b^{17} d^2 e^{19} + 177344 a^9 b^{15} d^2 e^{19} + 402912 a^{11} b^{13} d^2 e^{19} + 541632 a^{13} b^{11} d^2 e^{19} + 455472 a^{15} b^9 d^2 e^{19} + 248064 a^{17} b^7 d^2 e^{19} + 87008 a^{19} b^5 d^2 e^{19} + 18240 a^{21} b^3 d^2 e^{19})) / (b^{21} d^4 + 8 a^2 b^{19} d^4 + 28 a^4 b^{17} d^4 + 56 a^6 b^{15} d^4 + 70 a^8 b^{13} d^4 + 56 a^{10} b^{11} d^4 + 28 a^{12} b^9 d^4 + 8 a^{14} b^7 d^4 + a^{16} b^5 d^4) - (((128 a^4 b^{26} d^4 e^{15} + 3648 a^3 b^{24} d^4 e^{15} + 25536 a^5 b^{22} d^4 e^{15} + 88320 a^7 b^{20} d^4 e^{15} + 182784 a^9 b^{18} d^4 e^{15} + 244608 a^{11} b^{16} d^4 e^{15} + 217728 a^{13} b^{14} d^4 e^{15} + 128256 a^{15} b^{12} d^4 e^{15} + 48000 a^{17} b^{10} d^4 e^{15} + 10304 a^{19} b^8 d^4 e^{15} + 960 a^{21} b^6 d^4 e^{15})) / (b^{21} d^5 + 8 a^2 b^{19} d^5 + 28 a^4 b^{17} d^5 + 56 a^6 b^{15} d^5 + 70 a^8 b^{13} d^5 + 56 a^{10} b^{11} d^5 + 28 a^{12} b^9 d^5 + 8 a^{14} b^7 d^5 + a^{16} b^5 d^5) + ((e \cot(c + dx))^{(1/2)} * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} * (512 b^{30} d^4 e^{10} + 4608 a^2 b^{28} d^4 e^{10} + 17920 a^4 b^{26} d^4 e^{10} + 38400 a^6 b^{24} d^4 e^{10} + 46080 a^8 b^{22} d^4 e^{10} + 21504 a^{10} b^{20} d^4 e^{10} - 21504 a^{12} b^{18} d^4 e^{10} - 46080 a^{14} b^{16} d^4 e^{10} - 38400 a^{16} b^{14} d^4 e^{10} - 17920 a^{18} b^{12} d^4 e^{10} - 4608 a^{20} b^{10} d^4 e^{10} - 512 a^{22} b^8 d^4 e^{10})) / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) * (b^{21} d^4 + 8 a^2 b^{19} d^4 + 28 a^4 b^{17} d^4 + 56 a^6 b^{15} d^4 + 70 a^8 b^{13} d^4 + 56 a^{10} b^{11} d^4 + 28 a^{12} b^9 d^4 + 8 a^{14} b^7 d^4 + a^{16} b^5 d^4)) * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) * (15 a^4 + 63 b^4 + 46 a^2 b^2) * (-a^5 b^7 e^9)^{(1/2)} * i / (8 (b^{13} d + 3 a^2 b^{11} d + 3 a^4 b^9 d + a^6 b^7 d)) / ((225 a^{15} e^{33} + 504 a^3 b^{12} e^{33} + 872 a^5 b^{10} e^{33} + 4457 a^7 b^8 e^{33} + 5916 a^9 b^6 e^{33} + 4006 a^{11} b^4 e^{33} + 1380 a^{13} b^2 e^{33})) / (b^{21} d^5 + 8 a^2 b^{19} d^5 + 28 a^4 b^{17} d^5 + 56 a^6 b^{15} d^5 + 70 a^8 b^{13} d^5 + 56 a^{10} b^{11} d^5 + 28 a^{12} b^9 d^5 + 8 a^{14} b^7 d^5
\end{aligned}$$



$$\begin{aligned}
& + a^{16}b^5d^5) - (((e \cot(c + dx))^{1/2}) * (32b^{18}e^{28} - 225a^{18}e^{28} \\
& + 128a^2b^{16}e^{28} + 192a^4b^{14}e^{28} - 3841a^6b^{12}e^{28} + 18050a^8b^{10}e^{28} + 26801a^{10}b^8e^{28} + 16860a^{12}b^6e^{28} + 4049a^{14}b^4e^{28} - \\
& 30a^{16}b^2e^{28}))/ (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 \\
& + a^{16}b^5d^4) - (((2250a^{20}b^2d^2e^{24} + 32a^2b^{19}d^2e^{24} + 12288a^4b^{17}d^2e^{24} - 10974a^6b^{15}d^2e^{24} - 105162a^8b^{13}d^2e^{24} - \\
& 150758a^{10}b^{11}d^2e^{24} - 85314a^{12}b^9d^2e^{24} - 3578a^{14}b^7d^2e^{24} + 22210a^{16}b^5d^2e^{24} + 11550a^{18}b^3d^2e^{24}))/ (b^{21}d^5 + 8a^2b^{19}d^5 \\
& + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) + (((e \cot(c + dx))^{1/2}) * (1800a^{23}b^2d^2e^{19} - \\
& 1472a^2b^{23}d^2e^{19} - 1024a^3b^{21}d^2e^{19} + 8448a^5b^{19}d^2e^{19} + 46088a^7b^{17}d^2e^{19} + 177344a^9b^{15}d^2e^{19} + 402912a^{11}b^{13}d^2e^{19} + \\
& 541632a^{13}b^{11}d^2e^{19} + 455472a^{15}b^9d^2e^{19} + 248064a^{17}b^7d^2e^{19} + 87008a^{19}b^5d^2e^{19} + 18240a^{21}b^3d^2e^{19}))/ (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 \\
& + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) + (((128a^2b^{26}d^4e^{15} + 3648a^3b^{24}d^4e^{15} + 25536a^5b^{22}d^4e^{15} + 88320a^7b^{20}d^4e^{15} + 182784a^9b^{18}d^4e^{15} \\
& + 244608a^{11}b^{16}d^4e^{15} + 217728a^{13}b^{14}d^4e^{15} + 128256a^{15}b^{12}d^4e^{15} + 48000a^{17}b^{10}d^4e^{15} + 10304a^{19}b^8d^4e^{15} + 960a^{21}b^6d^4e^{15}))/ (b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 \\
& + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) - ((e \cot(c + dx))^{1/2}) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2} * (512b^{30}d^4e^{10} + 4608a^2b^{28}d^4e^{10} + 17920a^4b^{26}d^4e^{10} + 38400a^6b^{24}d^4e^{10} + 46080a^8b^{22}d^4e^{10} + 21504a^{10}b^{20}d^4e^{10} - 21504a^{12}b^{18}d^4e^{10} - 46080a^{14}b^{16}d^4e^{10} - 38400a^{16}b^{14}d^4e^{10} - 17920a^{18}b^{12}d^4e^{10} - 4608a^{20}b^{10}d^4e^{10} - 512a^{22}b^8d^4e^{10}))/ (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d) * (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2} / (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2} / (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2} / (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) + (((e \cot(c + dx))^{1/2}) * (32b^{18}e^{28} - 225a^{18}e^{28} + 128a^2b^{16}e^{28} + 192a^4b^{14}e^{28} - 3841a^6b^{12}e^{28} + 18050a^8b^{10}e^{28} + 26801a^{10}b^8e^{28} + 16860a^{12}b^6e^{28} + 4049a^{14}b^4e^{28} - 30a^{16}b^2e^{28}))/ (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) + (((2250a^{20}b^2d^2e^{24} + 32a^2b^{19}d^2e^{24} + 12288a^4b^{17}d^2e^{24} - 10974a^6b^{15}d^2e^{24} - 105162a^8b^{13}d^2e^{24} - 150758a^{10}b^{11}d^2e^{24} - 85314a^{12}b^9d^2e^{24} - 3578a^{14}b^7d^2e^{24} + 22210a^{16}b^5d^2e^{24} +
\end{aligned}$$

$$\frac{11550a^{18}b^3d^2e^{24}}{(b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) - (((e \cot(c + dx))^{1/2}) * (1800a^{23}b^2d^2e^{19} - 1472a^2b^{23}d^2e^{19} - 1024a^3b^{21}d^2e^{19} + 8448a^5b^{19}d^2e^{19} + 46088a^7b^{17}d^2e^{19} + 177344a^9b^{15}d^2e^{19} + 402912a^{11}b^{13}d^2e^{19} + 541632a^{13}b^{11}d^2e^{19} + 455472a^{15}b^9d^2e^{19} + 248064a^{17}b^7d^2e^{19} + 87008a^{19}b^5d^2e^{19} + 18240a^{21}b^3d^2e^{19}))}{(b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4) - (((128a^8b^{26}d^4e^{15} + 3648a^3b^{24}d^4e^{15} + 25536a^5b^{22}d^4e^{15} + 88320a^7b^{20}d^4e^{15} + 182784a^9b^{18}d^4e^{15} + 244608a^{11}b^{16}d^4e^{15} + 217728a^{13}b^{14}d^4e^{15} + 128256a^{15}b^{12}d^4e^{15} + 48000a^{17}b^{10}d^4e^{15} + 10304a^{19}b^8d^4e^{15} + 960a^{21}b^6d^4e^{15}))}{(b^{21}d^5 + 8a^2b^{19}d^5 + 28a^4b^{17}d^5 + 56a^6b^{15}d^5 + 70a^8b^{13}d^5 + 56a^{10}b^{11}d^5 + 28a^{12}b^9d^5 + 8a^{14}b^7d^5 + a^{16}b^5d^5) + ((e \cot(c + dx))^{1/2}) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2} * (512b^{30}d^4e^{10} + 4608a^2b^{28}d^4e^{10} + 17920a^4b^{26}d^4e^{10} + 38400a^6b^{24}d^4e^{10} + 46080a^8b^{22}d^4e^{10} + 21504a^{10}b^{20}d^4e^{10} - 21504a^{12}b^{18}d^4e^{10} - 46080a^{14}b^{16}d^4e^{10} - 38400a^{16}b^{14}d^4e^{10} - 17920a^{18}b^{12}d^4e^{10} - 4608a^{20}b^{10}d^4e^{10} - 512a^{22}b^8d^4e^{10}))}{(8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d) * (b^{21}d^4 + 8a^2b^{19}d^4 + 28a^4b^{17}d^4 + 56a^6b^{15}d^4 + 70a^8b^{13}d^4 + 56a^{10}b^{11}d^4 + 28a^{12}b^9d^4 + 8a^{14}b^7d^4 + a^{16}b^5d^4)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2}}{((8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2}) / (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2}) / (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2}) / (8*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d)) * (15a^4 + 63b^4 + 46a^2b^2) * (-a^5b^7e^9)^{1/2}) * i} / (4*(b^{13}d + 3a^2b^{11}d + 3a^4b^9d + a^6b^7d))$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(9/2)/(a+b\*cot(d\*x+c))\*\*3,x)

[Out] Timed out

$$3.82 \quad \int \frac{(e \cot(c+dx))^{7/2}}{(a+b \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=476

$$\frac{e^{7/2}(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3} - \frac{e^{7/2}(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

[Out]  $-1/4*a^{(3/2)}*(3*a^4+6*a^2*b^2+35*b^4)*e^{(7/2)}*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/b^{(5/2)}/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*\cot(d*x+c))^{(3/2)}/b/(a^2+b^2)/d/(a+b*\cot(d*x+c))^{(2+1/2)*(a+b)}*(a^2-4*a*b+b^2)*e^{(7/2)}*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)-1/2*(a+b)}*(a^2-4*a*b+b^2)*e^{(7/2)}*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)+1/4*(a-b)}*(a^2+4*a*b+b^2)*e^{(7/2)}*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)-1/4*(a-b)}*(a^2+4*a*b+b^2)*e^{(7/2)}*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)+1/4*a^2*(3*a^2+11*b^2)*e^3*(e*\cot(d*x+c))^{(1/2)}/b^2/(a^2+b^2)^2/d/(a+b*\cot(d*x+c))$

**Rubi [A]** time = 1.23, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3565, 3645, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{a^2 e^3 (3a^2 + 11b^2) \sqrt{e \cot(c+dx)}}{4b^2 d (a^2 + b^2)^2 (a + b \cot(c+dx))} + \frac{a^2 e^2 (e \cot(c+dx))^{3/2}}{2bd (a^2 + b^2) (a + b \cot(c+dx))^2} + \frac{e^{7/2}(a-b)(a^2+4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(7/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out]  $-(a^{(3/2)}*(3*a^4 + 6*a^2*b^2 + 35*b^4)*e^{(7/2)}*ArcTan[(Sqrt[b]*Sqrt[e*Cot[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(4*b^{(5/2)}*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^{(7/2)}*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*e^{(7/2)}*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 + b^2)^3*d) + (a^2*e^{(7/2)}*(e*\cot(c+d*x))^{(3/2)})/(2*b*(a^2 + b^2)*d*(a + b*\cot(c + d*x))^2) + (a^2*(3*a^2 + 11*b^2)*e^3*Sqrt[e*\cot(c + d*x)])/(4*b^2*(a^2 + b^2)^2*d*(a + b*\cot(c + d*x))) + ((a - b)*(a^2 + 4*a*b + b^2)*e^{(7/2)}*Log[Sqrt[e] + Sqrt[e]*\cot(c + d*x) - Sqrt[2]*Sqrt[e*\cot(c + d*x)]])/(2*Sqrt[2]*(a^2 + b^2)^3*d) - ((a - b)*(a^2 + 4*a*b + b^2)*e^{(7/2)}*Log[Sqrt[e] + Sqrt[e]*\cot(c + d*x) + Sqrt[2]*Sqrt[e*\cot(c + d*x)]])/(2*Sqrt[2]*(a^2 + b^2)^3*d)$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
```

$eQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 1168

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] := With[\{q = Rt[a*c, 2]\}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[\{a, c, d, e\}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& NeQ[c*d^2 - a*e^2, 0] \&\& NegQ[-(a*c)]$

### Rule 3534

$Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]]], x\_Symbol] := Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[\{b, c, d, e, f\}, x] \&\& NeQ[c^2 - d^2, 0] \&\& NeQ[c^2 + d^2, 0]$

### Rule 3565

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] := Simp[((b*c - a*d)^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[\{a, b, c, d, e, f\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 + b^2, 0] \&\& NeQ[c^2 + d^2, 0] \&\& GtQ[m, 2] \&\& LtQ[n, -1] \&\& IntegerQ[2*m]$

### Rule 3634

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)])^2, x\_Symbol] := Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; FreeQ[\{a, b, c, d, e, f, A, C, m, n\}, x] \&\& EqQ[A, C]$

### Rule 3645

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)]) + (C_)*tan[(e_) + (f_)*(x_)])^2, x\_Symbol] := Simp[((A*d^2 + c*(c*C - B*d))*(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 + d^2)), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*($

```

n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)])], x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{7/2}}{(a + b \cot(c + dx))^3} dx &= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} - \int \frac{\sqrt{e \cot(c + dx)} \left( -\frac{3}{2} a^2 e^3 + 2abe^3 \cot(c + dx) - \frac{1}{2} (3a^2 + 4b^2) e^3 \cot^2(c + dx) \right)}{(a + b \cot(c + dx))^2} \frac{1}{2b (a^2 + b^2)} \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} + \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \cot(c + dx)}}{4b^2 (a^2 + b^2)^2 d (a + b \cot(c + dx))} + \int \frac{\frac{1}{4} a^2 (3a^2 + 4b^2) e^3 \cot^2(c + dx)}{(a + b \cot(c + dx))^2} \frac{1}{2b (a^2 + b^2)} \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} + \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \cot(c + dx)}}{4b^2 (a^2 + b^2)^2 d (a + b \cot(c + dx))} + \int \frac{2ab^2 (a^2 + b^2) e^3 \cot(c + dx)}{(a + b \cot(c + dx))^2} \frac{1}{2b (a^2 + b^2)} \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} + \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \cot(c + dx)}}{4b^2 (a^2 + b^2)^2 d (a + b \cot(c + dx))} + \text{Subst} \left( \int \frac{2ab^2 (a^2 + b^2) e^3 \cot(c + dx)}{(a + b \cot(c + dx))^2} \frac{1}{2b (a^2 + b^2)} \right) \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} + \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \cot(c + dx)}}{4b^2 (a^2 + b^2)^2 d (a + b \cot(c + dx))} - \frac{(a^2 (3a^4 + 6a^2 b^2 + 35b^4) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right))}{4b^{5/2} (a^2 + b^2)^3 d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))} \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \cot(c + dx)}}{4b^2 (a^2 + b^2)^2 d (a + b \cot(c + dx))} - \frac{(a^2 (3a^4 + 6a^2 b^2 + 35b^4) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right))}{4b^{5/2} (a^2 + b^2)^3 d} + \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))} \\
&= \frac{a^2 e^2 (e \cot(c + dx))^{3/2}}{2b (a^2 + b^2) d (a + b \cot(c + dx))} + \frac{a^2 (3a^2 + 11b^2) e^3 \sqrt{e \cot(c + dx)}}{4b^2 (a^2 + b^2)^2 d (a + b \cot(c + dx))} - \frac{(a^2 (3a^4 + 6a^2 b^2 + 35b^4) e^{7/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right))}{4b^{5/2} (a^2 + b^2)^3 d} + \frac{(a + b) (a^2 - 4ab + b^2) e^{7/2}}{\sqrt{2} (a^2 + b^2)^{3/2}}
\end{aligned}$$

**Mathematica** [C] time = 6.19, size = 525, normalized size = 1.10

$$(e \cot(c + dx))^{7/2} \left( \frac{4b^2 \cot^2(c+dx) {}_2F_1\left(2, \frac{9}{2}; \frac{11}{2}; -\frac{b \cot(c+dx)}{a}\right)}{9a(a^2+b^2)^2} + \frac{2b(3a^2-b^2) \left(-7 \cot^2(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right) - 3 \cot^2(c+dx) + 7 \cot^2(c+dx)\right)}{21(a^2+b^2)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(7/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out] -(((e\*Cot[c + d\*x])^(7/2)\*((2\*b\*(3\*a^2 - b^2)\*Cot[c + d\*x])^(7/2))/(7\*(a^2 + b^2)^3) - (2\*a\*(3\*a^2 - b^2)\*(3\*Cot[c + d\*x])^(5/2) - 5\*a\*((-3\*a\*(-((Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]])/b^(3/2)) + Sqrt[Cot[c + d\*x]]/b))/b + Cot[c + d\*x]^(3/2)/b)))/(15\*(a^2 + b^2)^3) + (2\*b\*(3\*a^2 - b^2)\*(7\*Cot[c + d\*x]^(3/2) - 3\*Cot[c + d\*x]^(7/2) - 7\*Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(21\*(a^2 + b^2)^3) + (4\*b^2\*Cot[c + d\*x]^(9/2)\*Hypergeometric2F1[2, 9/2, 11/2, -(b\*Cot[c + d\*x])/a]))/(9\*a\*(a^2 + b^2)^2) + (2\*b^2\*Cot[c + d\*x]^(9/2)\*Hypergeometric2F1[3, 9/2, 11/2, -(b\*Cot[c + d\*x])/a]))/(9\*a^3\*(a^2 + b^2)) - (a\*(a^2 - 3\*b^2)\*(40\*Sqrt[Cot[c + d\*x]] - 8\*Cot[c + d\*x]^(5/2) + (5\*(4\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])] + 2\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - 2\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/2))/(20\*(a^2 + b^2)^3)))/(d\*Cot[c + d\*x]^(7/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out



**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{7}{2}}}{(b \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(7/2)/(b\*cot(d\*x + c) + a)^3, x)

**maple [B]** time = 0.88, size = 1232, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^3,x)

[Out] 
$$\begin{aligned} & 5/4/d*e^4*a^6/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2/b*(e*cot(d*x+c))^(3/2)+9/2 \\ & /d*e^4*a^4/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*b*(e*cot(d*x+c))^(3/2)+13/4/d \\ & *e^4*a^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*b^3*(e*cot(d*x+c))^(3/2)+3/4/d* \\ & e^5*a^7/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2/b^2*(e*cot(d*x+c))^(1/2)+7/2/d*e \\ & ^5*a^5/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)+11/4/d*e^5*a \\ & ^3/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*b^2*(e*cot(d*x+c))^(1/2)-3/4/d*e^4*a^ \\ & 6/(a^2+b^2)^3/b^2/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2) \\ & )-3/2/d*e^4*a^4/(a^2+b^2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a* \\ & e*b)^(1/2))-35/4/d*e^4*a^2/(a^2+b^2)^3*b^2/(a*e*b)^(1/2)*arctan((e*cot(d*x+ \\ & c))^(1/2)*b/(a*e*b)^(1/2))-1/4/d*e^3/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*ln((e* \\ & cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x \\ & +c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*a^3+3/4/d*e^3/(a \\ & ^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^( \\ & 1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^ \\ & (1/2)+(e^2)^(1/2)))*a*b^2-1/2/d*e^3/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan( \\ & 2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^3+3/2/d*e^3/(a^2+b^2)^3*(e^2) \\ & ^{(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a*b^2+1/2 \\ & /d*e^3/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d \\ & *x+c))^(1/2)+1)*a^3-3/2/d*e^3/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/ \\ & 2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a*b^2+3/4/d*e^4/(a^2+b^2)^3*2^(1/2)/ \\ & (e^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2) \\ & ^{(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)) \\ & )*a^2*b-1/4/d*e^4/(a^2+b^2)^3*2^(1/2)/(e^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1 \\ & /4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e* \\ & cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*b^3+3/2/d*e^4/(a^2+b^2)^3*2^(1/2)/( \\ & e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^2*b-1/2/d*e \end{aligned}$$

$$\frac{e^{4/(a^2+b^2)^{3/2}}(e^{1/4})^{\arctan(2^{1/2}/(e^{1/4})^{\cot(dx+c)})^{1/2+1}}b^{3-3/2}d^{-1}e^{-4/(a^2+b^2)^{3/2}}(e^{1/4})^{\arctan(-2^{1/2}/(e^{1/4})^{\cot(dx+c)})^{1/2+1}}a^2b^{1/2}d^{-1}e^{-4/(a^2+b^2)^{3/2}}(e^{1/4})^{\arctan(-2^{1/2}/(e^{1/4})^{\cot(dx+c)})^{1/2+1}}b^3}{(a^6b^2+3a^4b^4+3a^2b^6+b^8)\sqrt{abe}}$$

**maxima** [A] time = 0.75, size = 516, normalized size = 1.08

$$\left( \frac{(3a^6+6a^4b^2+35a^2b^4)e^3 \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^6b^2+3a^4b^4+3a^2b^6+b^8)\sqrt{abe}} + \frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3) \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3-3a^2b-3ab^2+b^3) \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{e}-2\sqrt{\frac{e}{\tan(dx+c)}})}{2\sqrt{e}}\right)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(7/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/4*((3*a^6 + 6*a^4*b^2 + 35*a^2*b^4)*e^3*\arctan(b*\sqrt{e/\tan(d*x + c)})/\sqrt{a*b*e})/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*\sqrt{a*b*e}) + (2*\sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + \sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e} - \sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e}) * e^3 / (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((3*a^5 + 11*a^3*b^2)*e^4*\sqrt{e/\tan(d*x + c)} + (5*a^4*b + 13*a^2*b^3)*e^3*(e/\tan(d*x + c))^(3/2))/((a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*e^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*e^2/\tan(d*x + c) + (a^4*b^4 + 2*a^2*b^6 + b^8)*e^2/\tan(d*x + c)^2)) * e/d \end{aligned}$$

**mupad** [B] time = 7.26, size = 20089, normalized size = 42.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(7/2)/(a + b\*cot(c + d\*x))^3,x)

[Out] 
$$\left( \frac{((e*\cot(c + d*x))^{1/2}*(3*a^5*e^5 + 11*a^3*b^2*e^5))/(4*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (e^4*(e*\cot(c + d*x))^{3/2}*(5*a^4 + 13*a^2*b^2))/(4*b*(a^4 + 2*a^2*b^2 + b^4))}{(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)} \right)$$



$$\begin{aligned}
& 4e^{14} + 14592a^{16}b^9d^4e^{14} + 2112a^{18}b^7d^4e^{14} + 64a^{20}b^5d^4e^{14}) / (b^{19}d^5 + 8a^2b^{17}d^5 + 28a^4b^{15}d^5 + 56a^6b^{13}d^5 + 70 \\
& a^8b^{11}d^5 + 56a^{10}b^9d^5 + 28a^{12}b^7d^5 + 8a^{14}b^5d^5 + a^{16}b^3d^5) - ((e \cot(c + dx))^{(1/2)} * ((e^7 * i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i \\
& - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2))))^{(1/2)} * (512 * b^{28} * d^4 * e^{10} + 4608 * a^2 * b^{26} * d^4 * e^{10} + 17920 * a^4 * b^{24} * d^4 * e^{10} \\
& + 38400 * a^6 * b^{22} * d^4 * e^{10} + 46080 * a^8 * b^{20} * d^4 * e^{10} + 21504 * a^{10} * b^{18} * d^4 * e^{10} - 21504 * a^{12} * b^{16} * d^4 * e^{10} - 46080 * a^{14} * b^{14} * d^4 * e^{10} - 38400 * a^{16} * b^{12} * d^4 * e^{10} \\
& - 17920 * a^{18} * b^{10} * d^4 * e^{10} - 4608 * a^{20} * b^8 * d^4 * e^{10} - 512 * a^{22} * b^6 * d^4 * e^{10})) / (b^{19} * d^4 + 8 * a^2 * b^{17} * d^4 + 28 * a^4 * b^{15} * d^4 + 56 * a^6 * b^{13} * d^4 + 70 * a^8 * b^{11} * d^4 + 56 * a^{10} * b^9 * d^4 + 28 * a^{12} * b^7 * d^4 + 8 * a^{14} * b^5 * d^4 + a^{16} * b^3 * d^4) * ((e^7 * i) / (4 * (b^6 * d^2 - a^6 * d^2 + a * b^5 * d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2))))^{(1/2)} + ((e \cot(c + dx))^{(1/2)} * (1472 * a * b^{21} * d^2 * e^{17} + 72 * a^{21} * b * d^2 * e^{17} + 1024 * a^3 * b^{19} * d^2 * e^{17} + 1352 * a^5 * b^{17} * d^2 * e^{17} + 28224 * a^7 * b^{15} * d^2 * e^{17} + 70240 * a^9 * b^{13} * d^2 * e^{17} + 72640 * a^{11} * b^{11} * d^2 * e^{17} + 39088 * a^{13} * b^9 * d^2 * e^{17} + 13248 * a^{15} * b^7 * d^2 * e^{17} + 3488 * a^{17} * b^5 * d^2 * e^{17} + 576 * a^{19} * b^3 * d^2 * e^{17})) / (b^{19} * d^4 + 8 * a^2 * b^{17} * d^4 + 28 * a^4 * b^{15} * d^4 + 56 * a^6 * b^{13} * d^4 + 70 * a^8 * b^{11} * d^4 + 56 * a^{10} * b^9 * d^4 + 28 * a^{12} * b^7 * d^4 + 8 * a^{14} * b^5 * d^4 + a^{16} * b^3 * d^4) * ((e^7 * i) / (4 * (b^6 * d^2 - a^6 * d^2 + a * b^5 * d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2))))^{(1/2)} * ((e^7 * i) / (4 * (b^6 * d^2 - a^6 * d^2 + a * b^5 * d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2))))^{(1/2)} - ((e \cot(c + dx))^{(1/2)} * (9 * a^{16} * e^{24} + 32 * b^{16} * e^{24} + 128 * a^2 * b^{14} * e^{24} + 1417 * a^4 * b^{12} * e^{24} - 6802 * a^6 * b^{10} * e^{24} - 1017 * a^8 * b^8 * e^{24} - 1020 * a^{10} * b^6 * e^{24} + 39 * a^{12} * b^4 * e^{24} - 18 * a^{14} * b^2 * e^{24})) / (b^{19} * d^4 + 8 * a^2 * b^{17} * d^4 + 28 * a^4 * b^{15} * d^4 + 56 * a^6 * b^{13} * d^4 + 70 * a^8 * b^{11} * d^4 + 56 * a^{10} * b^9 * d^4 + 28 * a^{12} * b^7 * d^4 + 8 * a^{14} * b^5 * d^4 + a^{16} * b^3 * d^4) * ((e^7 * i) / (4 * (b^6 * d^2 - a^6 * d^2 + a * b^5 * d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2))))^{(1/2)} * i) / ((9 * a^{12} * b * e^{28} + 280 * a^2 * b^{11} * e^{28} + 1553 * a^4 * b^9 * e^{28} + 492 * a^6 * b^7 * e^{28} + 270 * a^8 * b^5 * e^{28} + 36 * a^{10} * b^3 * e^{28})) / (b^{19} * d^5 + 8 * a^2 * b^{17} * d^5 + 28 * a^4 * b^{15} * d^5 + 56 * a^6 * b^{13} * d^5 + 70 * a^8 * b^{11} * d^5 + 56 * a^{10} * b^9 * d^5 + 28 * a^{12} * b^7 * d^5 + 8 * a^{14} * b^5 * d^5 + a^{16} * b^3 * d^5) + (((32 * a * b^{18} * d^2 * e^{21} - 18 * a^{19} * d^2 * e^{21} - 6528 * a^3 * b^{16} * d^2 * e^{21} + 275 * 8 * a^5 * b^{14} * d^2 * e^{21} + 26482 * a^7 * b^{12} * d^2 * e^{21} + 21582 * a^9 * b^{10} * d^2 * e^{21} + 7 * 594 * a^{11} * b^8 * d^2 * e^{21} + 3314 * a^{13} * b^6 * d^2 * e^{21} + 246 * a^{15} * b^4 * d^2 * e^{21} + 90 * a^{17} * b^2 * d^2 * e^{21})) / (b^{19} * d^5 + 8 * a^2 * b^{17} * d^5 + 28 * a^4 * b^{15} * d^5 + 56 * a^6 * b^{13} * d^5 + 70 * a^8 * b^{11} * d^5 + 56 * a^{10} * b^9 * d^5 + 28 * a^{12} * b^7 * d^5 + 8 * a^{14} * b^5 * d^5 + a^{16} * b^3 * d^5) + (((1600 * a^2 * b^{23} * d^4 * e^{14} + 12864 * a^4 * b^{21} * d^4 * e^{14} + 45312 * a^6 * b^{19} * d^4 * e^{14} + 91392 * a^8 * b^{17} * d^4 * e^{14} + 115584 * a^{10} * b^{15} * d^4 * e^{14} + 94080 * a^{12} * b^{13} * d^4 * e^{14} + 48384 * a^{14} * b^{11} * d^4 * e^{14} + 14592 * a^{16} * b^9 * d^4 * e^{14} + 2112 * a^{18} * b^7 * d^4 * e^{14} + 64 * a^{20} * b^5 * d^4 * e^{14})) / (b^{19} * d^5 + 8 * a^2 * b^{17} * d^5 + 28 * a^4 * b^{15} * d^5 + 56 * a^6 * b^{13} * d^5 + 70 * a^8 * b^{11} * d^5 + 56 * a^{10} * b^9 * d^5 + 28 * a^{12} * b^7 * d^5 + 8 * a^{14} * b^5 * d^5 + a^{16} * b^3 * d^5) + ((e \cot(c + dx))^{(1/2)} * ((e^7 * i) / (4 * (b^6 * d^2 - a^6 * d^2 + a * b^5 * d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2))))^{(1/2)} * (512 * b^{28} * d^4 * e^{10}
\end{aligned}$$

$$\begin{aligned}
& + 4608a^2b^{26}d^4e^{10} + 17920a^4b^{24}d^4e^{10} + 38400a^6b^{22}d^4e^{10} + 46080a^8b^{20}d^4e^{10} + 21504a^{10}b^{18}d^4e^{10} - 21504a^{12}b^{16}d^4e^{10} \\
& - 46080a^{14}b^{14}d^4e^{10} - 38400a^{16}b^{12}d^4e^{10} - 17920a^{18}b^{10}d^4e^{10} - 4608a^{20}b^8d^4e^{10} - 512a^{22}b^6d^4e^{10}) / (b^{19}d^4 \\
& + 8a^2b^{17}d^4 + 28a^4b^{15}d^4 + 56a^6b^{13}d^4 + 70a^8b^{11}d^4 + 56a^{10}b^9d^4 + 28a^{12}b^7d^4 + 8a^{14}b^5d^4 + a^{16}b^3d^4) * ((e^{7*1i}) \\
& / (4*(b^6d^2 - a^6d^2 + a*b^5d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)} * (1472*a*b^{21}d^2e^{17} \\
& + 72*a^{21}b*d^2e^{17} + 1024*a^3*b^{19}d^2e^{17} + 1352*a^5*b^{17}d^2e^{17} + 28224*a^7*b^{15}d^2e^{17} + 70240*a^9*b^{13}d^2e^{17} + 72640*a^{11}b^{11}d^2e^{17} \\
& + 39088*a^{13}b^9d^2e^{17} + 13248*a^{15}b^7d^2e^{17} + 3488*a^{17}b^5d^2e^{17} + 576*a^{19}b^3d^2e^{17})) / (b^{19}d^4 + 8a^2b^{17}d^4 + 28a^4b^{15}d^4 \\
& + 56a^6b^{13}d^4 + 70a^8b^{11}d^4 + 56a^{10}b^9d^4 + 28a^{12}b^7d^4 + 8a^{14}b^5d^4 + a^{16}b^3d^4) * ((e^{7*1i}) / (4*(b^6d^2 - a^6d^2 + a*b^5d^2*6i \\
& + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} * ((e^{7*1i}) / (4*(b^6d^2 - a^6d^2 + a*b^5d^2*6i \\
& - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)} * (9*a^{16}e^{24} + 32*b^{16}e^{24} + 128*a^2*b^{14}e^{24} + 1417*a^4*b^{12}e^{24} \\
& - 6802*a^6*b^{10}e^{24} - 1017*a^8*b^8e^{24} - 1020*a^{10}b^6e^{24} + 39*a^{12}b^4e^{24} - 18*a^{14}b^2e^{24})) / (b^{19}d^4 + 8a^2b^{17}d^4 + 28a^4b^{15}d^4 \\
& + 56a^6b^{13}d^4 + 70a^8b^{11}d^4 + 56a^{10}b^9d^4 + 28a^{12}b^7d^4 + 8a^{14}b^5d^4 + a^{16}b^3d^4) * ((e^{7*1i}) / (4*(b^6d^2 - a^6d^2 + a*b^5d^2*6i \\
& + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} + (((32*a*b^{18}d^2e^{21} - 18*a^{19}d^2e^{21} - 6528*a^3*b^{16}d^2e^{21} \\
& + 2758*a^5*b^{14}d^2e^{21} + 26482*a^7*b^{12}d^2e^{21} + 21582*a^9*b^{10}d^2e^{21} + 7594*a^{11}b^8d^2e^{21} + 3314*a^{13}b^6d^2e^{21} + 246*a^{15}b^4d^2e^{21} \\
& + 90*a^{17}b^2d^2e^{21}) / (b^{19}d^5 + 8a^2b^{17}d^5 + 28a^4b^{15}d^5 + 56a^6b^{13}d^5 + 70a^8b^{11}d^5 + 56a^{10}b^9d^5 + 28a^{12}b^7d^5 + 8a^{14}b^5d^5 \\
& + a^{16}b^3d^5) + (((1600*a^2*b^{23}d^4e^{14} + 12864*a^4*b^{21}d^4e^{14} + 45312*a^6*b^{19}d^4e^{14} + 91392*a^8*b^{17}d^4e^{14} + 115584*a^{10}b^{15}d^4e^{14} \\
& + 94080*a^{12}b^{13}d^4e^{14} + 48384*a^{14}b^{11}d^4e^{14} + 14592*a^{16}b^9d^4e^{14} + 2112*a^{18}b^7d^4e^{14} + 64*a^{20}b^5d^4e^{14}) / (b^{19}d^5 \\
& + 8a^2b^{17}d^5 + 28a^4b^{15}d^5 + 56a^6b^{13}d^5 + 70a^8b^{11}d^5 + 56a^{10}b^9d^5 + 28a^{12}b^7d^5 + 8a^{14}b^5d^5 + a^{16}b^3d^5) - ((e*\cot(c + d*x))^{(1/2)} * ((e^{7*1i}) / (4*(b^6d^2 - a^6d^2 + a*b^5d^2*6i \\
& + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} * (512*b^{28}d^4e^{10} + 4608*a^2*b^{26}d^4e^{10} + 17920*a^4*b^{24}d^4e^{10} + 38400*a^6*b^{22}d^4e^{10} \\
& + 46080*a^8*b^{20}d^4e^{10} + 21504*a^{10}b^{18}d^4e^{10} - 21504*a^{12}b^{16}d^4e^{10} - 46080*a^{14}b^{14}d^4e^{10} - 38400*a^{16}b^{12}d^4e^{10} - 17920*a^{18}b^{10}d^4e^{10} - 4608*a^{20}b^8d^4e^{10} - 512*a^{22}b^6d^4e^{10})) / (b^{19}d^4 \\
& + 8a^2b^{17}d^4 + 28a^4b^{15}d^4 + 56a^6b^{13}d^4 + 70a^8b^{11}d^4 + 56a^{10}b^9d^4 + 28a^{12}b^7d^4 + 8a^{14}b^5d^4 + a^{16}b^3d^4) * ((e^{7*1i}) / (4*(b^6d^2 - a^6d^2 + a*b^5d^2*6i \\
& + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)} * (1472*a*b^{21}d^2e^{17} + 72*a^{21}b*d^2e^{17} + 1024*a^3*b^{19}d^2e^{17} + 1352*a^5
\end{aligned}$$



$$\begin{aligned}
& a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i))^{(1/2)} + ((e \cot(c + d \cdot x))^{(1/2)} \cdot (9 a^{16} e^{24} + 32 b^{16} e^{24} + 128 a^2 b^{14} e^{24} + 1417 a^4 b^{12} e^{24} - 6802 a^6 b^{10} e^{24} - 1017 a^8 b^8 e^{24} - 1020 a^{10} b^6 e^{24} + 39 a^{12} b^4 e^{24} - 18 a^{14} b^2 e^{24})) / (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4)) \cdot (e^7 / (4 \cdot (b^6 d^2 \cdot 1i - a^6 d^2 \cdot 1i + 6 a \cdot b^5 d^2 + 6 a^5 b \cdot d^2 - a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i)))^{(1/2)} \cdot 1i - (((32 a \cdot b^{18} d^2 e^{21} - 18 a^{19} d^2 e^{21} - 6528 a^3 b^{16} d^2 e^{21} + 2758 a^5 b^{14} d^2 e^{21} + 26482 a^7 b^{12} d^2 e^{21} + 21582 a^9 b^{10} d^2 e^{21} + 7594 a^{11} b^8 d^2 e^{21} + 3314 a^{13} b^6 d^2 e^{21} + 246 a^{15} b^4 d^2 e^{21} + 90 a^{17} b^2 d^2 e^{21})) / (b^{19} d^5 + 8 a^2 b^{17} d^5 + 28 a^4 b^{15} d^5 + 56 a^6 b^{13} d^5 + 70 a^8 b^{11} d^5 + 56 a^{10} b^9 d^5 + 28 a^{12} b^7 d^5 + 8 a^{14} b^5 d^5 + a^{16} b^3 d^5) + (((1600 a^2 b^{23} d^4 e^{14} + 12864 a^4 b^{21} d^4 e^{14} + 45312 a^6 b^{19} d^4 e^{14} + 91392 a^8 b^{17} d^4 e^{14} + 115584 a^{10} b^{15} d^4 e^{14} + 94080 a^{12} b^{13} d^4 e^{14} + 48384 a^{14} b^{11} d^4 e^{14} + 14592 a^{16} b^9 d^4 e^{14} + 2112 a^{18} b^7 d^4 e^{14} + 64 a^{20} b^5 d^4 e^{14})) / (b^{19} d^5 + 8 a^2 b^{17} d^5 + 28 a^4 b^{15} d^5 + 56 a^6 b^{13} d^5 + 70 a^8 b^{11} d^5 + 56 a^{10} b^9 d^5 + 28 a^{12} b^7 d^5 + 8 a^{14} b^5 d^5 + a^{16} b^3 d^5) - ((e \cot(c + d \cdot x))^{(1/2)} \cdot (e^7 / (4 \cdot (b^6 d^2 \cdot 1i - a^6 d^2 \cdot 1i + 6 a \cdot b^5 d^2 + 6 a^5 b \cdot d^2 - a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i)))^{(1/2)} \cdot (512 b^{28} d^4 e^{10} + 4608 a^2 b^{26} d^4 e^{10} + 17920 a^4 b^{24} d^4 e^{10} + 38400 a^6 b^{22} d^4 e^{10} + 46080 a^8 b^{20} d^4 e^{10} + 21504 a^{10} b^{18} d^4 e^{10} - 21504 a^{12} b^{16} d^4 e^{10} - 46080 a^{14} b^{14} d^4 e^{10} - 38400 a^{16} b^{12} d^4 e^{10} - 17920 a^{18} b^{10} d^4 e^{10} - 4608 a^{20} b^8 d^4 e^{10} - 512 a^{22} b^6 d^4 e^{10})) / (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4)) \cdot (e^7 / (4 \cdot (b^6 d^2 \cdot 1i - a^6 d^2 \cdot 1i + 6 a \cdot b^5 d^2 + 6 a^5 b \cdot d^2 - a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i)))^{(1/2)} + ((e \cot(c + d \cdot x))^{(1/2)} \cdot (1472 a \cdot b^{21} d^2 e^{17} + 72 a^{21} b \cdot d^2 e^{17} + 1024 a^3 b^{19} d^2 e^{17} + 1352 a^5 b^{17} d^2 e^{17} + 28224 a^7 b^{15} d^2 e^{17} + 70240 a^9 b^{13} d^2 e^{17} + 72640 a^{11} b^{11} d^2 e^{17} + 39088 a^{13} b^9 d^2 e^{17} + 13248 a^{15} b^7 d^2 e^{17} + 3488 a^{17} b^5 d^2 e^{17} + 576 a^{19} b^3 d^2 e^{17})) / (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4)) \cdot (e^7 / (4 \cdot (b^6 d^2 \cdot 1i - a^6 d^2 \cdot 1i + 6 a \cdot b^5 d^2 + 6 a^5 b \cdot d^2 - a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i)))^{(1/2)} \cdot (e^7 / (4 \cdot (b^6 d^2 \cdot 1i - a^6 d^2 \cdot 1i + 6 a \cdot b^5 d^2 + 6 a^5 b \cdot d^2 - a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i)))^{(1/2)} - ((e \cot(c + d \cdot x))^{(1/2)} \cdot (9 a^{16} e^{24} + 32 b^{16} e^{24} + 128 a^2 b^{14} e^{24} + 1417 a^4 b^{12} e^{24} - 6802 a^6 b^{10} e^{24} - 1017 a^8 b^8 e^{24} - 1020 a^{10} b^6 e^{24} + 39 a^{12} b^4 e^{24} - 18 a^{14} b^2 e^{24})) / (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4)) \cdot (e^7 / (4 \cdot (b^6 d^2 \cdot 1i - a^6 d^2 \cdot 1i + 6 a \cdot b^5 d^2 + 6 a^5 b \cdot d^2 - a^2 b^4 d^2 \cdot 15i - 20 a^3 b^3 d^2 + a^4 b^2 d^2 \cdot 15i)))^{(1/2)} \cdot 1i) / ((9 a^{12} b e^{28} + 280 a^2 b^{11} e^{28} + 1553 a^4 b^9 e^{28} + 492 a^6 b^7 e^{28} + 270 a^8 b^5 e^{28} + 36 a^{10} b^3 e^{28})) / (b^{19} d^5 + 8 a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^{17}*d^5 + 28*a^4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + 70*a^8*b^{11}*d^5 + 56*a^{10} \\
& *b^9*d^5 + 28*a^{12}*b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^5) + (((32*a*b^{18}* \\
& d^2*e^{21} - 18*a^{19}*d^2*e^{21} - 6528*a^3*b^{16}*d^2*e^{21} + 2758*a^5*b^{14}*d^2*e^{21} \\
& + 26482*a^7*b^{12}*d^2*e^{21} + 21582*a^9*b^{10}*d^2*e^{21} + 7594*a^{11}*b^8*d^2* \\
& e^{21} + 3314*a^{13}*b^6*d^2*e^{21} + 246*a^{15}*b^4*d^2*e^{21} + 90*a^{17}*b^2*d^2*e^{21} \\
& 1)/(b^{19}*d^5 + 8*a^2*b^{17}*d^5 + 28*a^4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + 70*a^8* \\
& b^{11}*d^5 + 56*a^{10}*b^9*d^5 + 28*a^{12}*b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^ \\
& 5) + (((1600*a^2*b^{23}*d^4*e^{14} + 12864*a^4*b^{21}*d^4*e^{14} + 45312*a^6*b^{19}*d \\
& ^4*e^{14} + 91392*a^8*b^{17}*d^4*e^{14} + 115584*a^{10}*b^{15}*d^4*e^{14} + 94080*a^{12}* \\
& b^{13}*d^4*e^{14} + 48384*a^{14}*b^{11}*d^4*e^{14} + 14592*a^{16}*b^9*d^4*e^{14} + 2112*a \\
& ^{18}*b^7*d^4*e^{14} + 64*a^{20}*b^5*d^4*e^{14}))/ (b^{19}*d^5 + 8*a^2*b^{17}*d^5 + 28*a^ \\
& 4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + 70*a^8*b^{11}*d^5 + 56*a^{10}*b^9*d^5 + 28*a^{12}* \\
& b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^5) + ((e*cot(c + d*x))^{(1/2)}*(e^{7/(4* \\
& (b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20 \\
& *a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)}*(512*b^{28}*d^4*e^{10} + 4608*a^2*b^{26}* \\
& d^4*e^{10} + 17920*a^4*b^{24}*d^4*e^{10} + 38400*a^6*b^{22}*d^4*e^{10} + 46080*a^8*b^{20}* \\
& d^4*e^{10} + 21504*a^{10}*b^{18}*d^4*e^{10} - 21504*a^{12}*b^{16}*d^4*e^{10} - 46080*a \\
& ^{14}*b^{14}*d^4*e^{10} - 38400*a^{16}*b^{12}*d^4*e^{10} - 17920*a^{18}*b^{10}*d^4*e^{10} - 4 \\
& 608*a^{20}*b^8*d^4*e^{10} - 512*a^{22}*b^6*d^4*e^{10}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 \\
& + 28*a^4*b^{15}*d^4 + 56*a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 2 \\
& 8*a^{12}*b^7*d^4 + 8*a^{14}*b^5*d^4 + a^{16}*b^3*d^4))* (e^{7/(4*(b^6*d^2*1i - a^6* \\
& d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4 \\
& *b^2*d^2*15i))))^{(1/2)} - ((e*cot(c + d*x))^{(1/2)}*(1472*a*b^{21}*d^2*e^{17} + 72* \\
& a^{21}*b*d^2*e^{17} + 1024*a^3*b^{19}*d^2*e^{17} + 1352*a^5*b^{17}*d^2*e^{17} + 28224*a \\
& ^7*b^{15}*d^2*e^{17} + 70240*a^9*b^{13}*d^2*e^{17} + 72640*a^{11}*b^{11}*d^2*e^{17} + 390 \\
& 88*a^{13}*b^9*d^2*e^{17} + 13248*a^{15}*b^7*d^2*e^{17} + 3488*a^{17}*b^5*d^2*e^{17} + 5 \\
& 76*a^{19}*b^3*d^2*e^{17}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d^4 + 56*a^ \\
& 6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 + 8*a^{14}*b \\
& ^5*d^4 + a^{16}*b^3*d^4))* (e^{7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6* \\
& a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)}*(e \\
& ^{7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15 \\
& i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} + ((e*cot(c + d*x))^{(1/2)}*(9* \\
& a^{16}*e^{24} + 32*b^{16}*e^{24} + 128*a^2*b^{14}*e^{24} + 1417*a^4*b^{12}*e^{24} - 6802*a^ \\
& 6*b^{10}*e^{24} - 1017*a^8*b^8*e^{24} - 1020*a^{10}*b^6*e^{24} + 39*a^{12}*b^4*e^{24} - 1 \\
& 8*a^{14}*b^2*e^{24}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d^4 + 56*a^6*b^1 \\
& 3*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 + 8*a^{14}*b^5*d^ \\
& 4 + a^{16}*b^3*d^4))* (e^{7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b \\
& *d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} + (((32* \\
& a*b^{18}*d^2*e^{21} - 18*a^{19}*d^2*e^{21} - 6528*a^3*b^{16}*d^2*e^{21} + 2758*a^5*b^{14} \\
& *d^2*e^{21} + 26482*a^7*b^{12}*d^2*e^{21} + 21582*a^9*b^{10}*d^2*e^{21} + 7594*a^{11}*b \\
& ^8*d^2*e^{21} + 3314*a^{13}*b^6*d^2*e^{21} + 246*a^{15}*b^4*d^2*e^{21} + 90*a^{17}*b^2* \\
& d^2*e^{21}))/ (b^{19}*d^5 + 8*a^2*b^{17}*d^5 + 28*a^4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + \\
& 70*a^8*b^{11}*d^5 + 56*a^{10}*b^9*d^5 + 28*a^{12}*b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16} \\
& *b^3*d^5) + (((1600*a^2*b^{23}*d^4*e^{14} + 12864*a^4*b^{21}*d^4*e^{14} + 45312*a^6 \\
& *b^{19}*d^4*e^{14} + 91392*a^8*b^{17}*d^4*e^{14} + 115584*a^{10}*b^{15}*d^4*e^{14} + 9408
\end{aligned}$$



$$\begin{aligned}
& 0*a^{12}*b^{13}*d^4*e^{14} + 48384*a^{14}*b^{11}*d^4*e^{14} + 14592*a^{16}*b^9*d^4*e^{14} + \\
& 2112*a^{18}*b^7*d^4*e^{14} + 64*a^{20}*b^5*d^4*e^{14})/(b^{19}*d^5 + 8*a^2*b^{17}*d^5 \\
& + 28*a^4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + 70*a^8*b^{11}*d^5 + 56*a^{10}*b^9*d^5 + 2 \\
& 8*a^{12}*b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^5) - ((e*\cot(c + d*x))^{(1/2)}*( \\
& e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*1 \\
& 5i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)}*(512*b^{28}*d^4*e^{10} + 4608*a^ \\
& 2*b^{26}*d^4*e^{10} + 17920*a^4*b^{24}*d^4*e^{10} + 38400*a^6*b^{22}*d^4*e^{10} + 46080 \\
& *a^8*b^{20}*d^4*e^{10} + 21504*a^{10}*b^{18}*d^4*e^{10} - 21504*a^{12}*b^{16}*d^4*e^{10} - \\
& 46080*a^{14}*b^{14}*d^4*e^{10} - 38400*a^{16}*b^{12}*d^4*e^{10} - 17920*a^{18}*b^{10}*d^4*e \\
& ^{10} - 4608*a^{20}*b^8*d^4*e^{10} - 512*a^{22}*b^6*d^4*e^{10}))/ (b^{19}*d^4 + 8*a^2*b^ \\
& 17*d^4 + 28*a^4*b^{15}*d^4 + 56*a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9* \\
& d^4 + 28*a^{12}*b^7*d^4 + 8*a^{14}*b^5*d^4 + a^{16}*b^3*d^4))*(e^7/(4*(b^6*d^2*1i \\
& - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^ \\
& 2 + a^4*b^2*d^2*15i))))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)}*(1472*a*b^{21}*d^2*e^1 \\
& 7 + 72*a^{21}*b*d^2*e^{17} + 1024*a^3*b^{19}*d^2*e^{17} + 1352*a^5*b^{17}*d^2*e^{17} + \\
& 28224*a^7*b^{15}*d^2*e^{17} + 70240*a^9*b^{13}*d^2*e^{17} + 72640*a^{11}*b^{11}*d^2*e^1 \\
& 7 + 39088*a^{13}*b^9*d^2*e^{17} + 13248*a^{15}*b^7*d^2*e^{17} + 3488*a^{17}*b^5*d^2*e \\
& ^{17} + 576*a^{19}*b^3*d^2*e^{17}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d^4 \\
& + 56*a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 + 8 \\
& *a^{14}*b^5*d^4 + a^{16}*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d \\
& ^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1 \\
& /2)}*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4 \\
& *d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} - ((e*\cot(c + d*x))^{(1 \\
& /2)}*(9*a^{16}*e^{24} + 32*b^{16}*e^{24} + 128*a^2*b^{14}*e^{24} + 1417*a^4*b^{12}*e^{24} - \\
& 6802*a^6*b^{10}*e^{24} - 1017*a^8*b^8*e^{24} - 1020*a^{10}*b^6*e^{24} + 39*a^{12}*b^4*e \\
& ^{24} - 18*a^{14}*b^2*e^{24}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d^4 + 56* \\
& a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 + 8*a^{14} \\
& *b^5*d^4 + a^{16}*b^3*d^4))*(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + \\
& 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)})) \\
& *(e^7/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2 \\
& *15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)}*2i - (\operatorname{atan}((((e*\cot(c + \\
& d*x))^{(1/2)}*(9*a^{16}*e^{24} + 32*b^{16}*e^{24} + 128*a^2*b^{14}*e^{24} + 1417*a^4*b^{12} \\
& *e^{24} - 6802*a^6*b^{10}*e^{24} - 1017*a^8*b^8*e^{24} - 1020*a^{10}*b^6*e^{24} + 39*a^{12} \\
& *b^4*e^{24} - 18*a^{14}*b^2*e^{24}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d \\
& ^4 + 56*a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 \\
& + 8*a^{14}*b^5*d^4 + a^{16}*b^3*d^4) - (((32*a*b^{18}*d^2*e^{21} - 18*a^{19}*d^2*e^{21} \\
& - 6528*a^3*b^{16}*d^2*e^{21} + 2758*a^5*b^{14}*d^2*e^{21} + 26482*a^7*b^{12}*d^2*e^2 \\
& 1 + 21582*a^9*b^{10}*d^2*e^{21} + 7594*a^{11}*b^8*d^2*e^{21} + 3314*a^{13}*b^6*d^2*e^ \\
& 21 + 246*a^{15}*b^4*d^2*e^{21} + 90*a^{17}*b^2*d^2*e^{21}))/ (b^{19}*d^5 + 8*a^2*b^{17}*d \\
& ^5 + 28*a^4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + 70*a^8*b^{11}*d^5 + 56*a^{10}*b^9*d^5 \\
& + 28*a^{12}*b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^5) + (((e*\cot(c + d*x))^{(1 \\
& /2)}*(1472*a*b^{21}*d^2*e^{17} + 72*a^{21}*b*d^2*e^{17} + 1024*a^3*b^{19}*d^2*e^{17} + 1 \\
& 352*a^5*b^{17}*d^2*e^{17} + 28224*a^7*b^{15}*d^2*e^{17} + 70240*a^9*b^{13}*d^2*e^{17} + \\
& 72640*a^{11}*b^{11}*d^2*e^{17} + 39088*a^{13}*b^9*d^2*e^{17} + 13248*a^{15}*b^7*d^2*e^ \\
& 17 + 3488*a^{17}*b^5*d^2*e^{17} + 576*a^{19}*b^3*d^2*e^{17}))/ (b^{19}*d^4 + 8*a^2*b^1
\end{aligned}$$



$$\begin{aligned}
& d^4 e^{10} - 21504 a^{12} b^{16} d^4 e^{10} - 46080 a^{14} b^{14} d^4 e^{10} - 38400 a^{16} b^{12} d^4 e^{10} - 17920 a^{18} b^{10} d^4 e^{10} - 4608 a^{20} b^8 d^4 e^{10} - 512 a^{22} b^6 d^4 e^{10} \\
& \left. \right) / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d) (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4)) \\
& \left. \right) * (3 a^4 + 35 b^4 + 6 a^2 b^2) * (-a^3 b^5 e^7)^{(1/2)} / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d)) \\
& \left. \right) * (3 a^4 + 35 b^4 + 6 a^2 b^2) * (-a^3 b^5 e^7)^{(1/2)} / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d)) \\
& \left. \right) * (3 a^4 + 35 b^4 + 6 a^2 b^2) * (-a^3 b^5 e^7)^{(1/2)} * i / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d)) \\
& \left. \right) / ((9 a^{12} b^8 e^{28} + 280 a^{10} b^{10} e^{28} + 1553 a^8 b^{12} e^{28} + 492 a^6 b^{14} e^{28} + 270 a^4 b^{16} e^{28} + 36 a^2 b^{18} e^{28} + b^{19} d^5 + 8 a^2 b^{17} d^5 + 28 a^4 b^{15} d^5 + 56 a^6 b^{13} d^5 + 70 a^8 b^{11} d^5 + 56 a^{10} b^9 d^5 + 28 a^{12} b^7 d^5 + 8 a^{14} b^5 d^5 + a^{16} b^3 d^5) \\
& - (((e \cot(c + dx))^{(1/2)} * (9 a^{16} e^{24} + 32 b^{16} e^{24} + 128 a^2 b^{14} e^{24} + 1417 a^4 b^{12} e^{24} - 6802 a^6 b^{10} e^{24} - 1017 a^8 b^8 e^{24} - 1020 a^{10} b^6 e^{24} + 39 a^{12} b^4 e^{24} - 18 a^{14} b^2 e^{24})) / (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4) \\
& - (((32 a^2 b^{18} d^2 e^{21} - 18 a^{19} d^2 e^{21} - 6528 a^3 b^{16} d^2 e^{21} + 2758 a^5 b^{14} d^2 e^{21} + 26482 a^7 b^{12} d^2 e^{21} + 21582 a^9 b^{10} d^2 e^{21} + 7594 a^{11} b^8 d^2 e^{21} + 3314 a^{13} b^6 d^2 e^{21} + 246 a^{15} b^4 d^2 e^{21} + 90 a^{17} b^2 d^2 e^{21})) / (b^{19} d^5 + 8 a^2 b^{17} d^5 + 28 a^4 b^{15} d^5 + 56 a^6 b^{13} d^5 + 70 a^8 b^{11} d^5 + 56 a^{10} b^9 d^5 + 28 a^{12} b^7 d^5 + 8 a^{14} b^5 d^5 + a^{16} b^3 d^5) \\
& ) + (((e \cot(c + dx))^{(1/2)} * (1472 a^2 b^{21} d^2 e^{17} + 72 a^{21} b^2 d^2 e^{17} + 1024 a^3 b^{19} d^2 e^{17} + 1352 a^5 b^{17} d^2 e^{17} + 28224 a^7 b^{15} d^2 e^{17} + 70240 a^9 b^{13} d^2 e^{17} + 72640 a^{11} b^{11} d^2 e^{17} + 39088 a^{13} b^9 d^2 e^{17} + 13248 a^{15} b^7 d^2 e^{17} + 3488 a^{17} b^5 d^2 e^{17} + 576 a^{19} b^3 d^2 e^{17})) / (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4) \\
& + (((1600 a^2 b^{23} d^4 e^{14} + 12864 a^4 b^{21} d^4 e^{14} + 45312 a^6 b^{19} d^4 e^{14} + 91392 a^8 b^{17} d^4 e^{14} + 115584 a^{10} b^{15} d^4 e^{14} + 94080 a^{12} b^{13} d^4 e^{14} + 48384 a^{14} b^{11} d^4 e^{14} + 14592 a^{16} b^9 d^4 e^{14} + 2112 a^{18} b^7 d^4 e^{14} + 64 a^{20} b^5 d^4 e^{14})) / (b^{19} d^5 + 8 a^2 b^{17} d^5 + 28 a^4 b^{15} d^5 + 56 a^6 b^{13} d^5 + 70 a^8 b^{11} d^5 + 56 a^{10} b^9 d^5 + 28 a^{12} b^7 d^5 + 8 a^{14} b^5 d^5 + a^{16} b^3 d^5) \\
& - ((e \cot(c + dx))^{(1/2)} * (3 a^4 + 35 b^4 + 6 a^2 b^2) * (-a^3 b^5 e^7)^{(1/2)} * (512 b^{28} d^4 e^{10} + 4608 a^2 b^{26} d^4 e^{10} + 17920 a^4 b^{24} d^4 e^{10} + 38400 a^6 b^{22} d^4 e^{10} + 46080 a^8 b^{20} d^4 e^{10} + 21504 a^{10} b^{18} d^4 e^{10} - 21504 a^{12} b^{16} d^4 e^{10} - 46080 a^{14} b^{14} d^4 e^{10} - 38400 a^{16} b^{12} d^4 e^{10} - 17920 a^{18} b^{10} d^4 e^{10} - 4608 a^{20} b^8 d^4 e^{10} - 512 a^{22} b^6 d^4 e^{10})) / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d) (b^{19} d^4 + 8 a^2 b^{17} d^4 + 28 a^4 b^{15} d^4 + 56 a^6 b^{13} d^4 + 70 a^8 b^{11} d^4 + 56 a^{10} b^9 d^4 + 28 a^{12} b^7 d^4 + 8 a^{14} b^5 d^4 + a^{16} b^3 d^4)) \\
& \left. \right) * (3 a^4 + 35 b^4 + 6 a^2 b^2) * (-a^3 b^5 e^7)^{(1/2)} / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d)) * (3 a^4 + 35 b^4 + 6 a^2 b^2) * (-a^3 b^5 e^7)^{(1/2)} / (8 (b^{11} d + 3 a^2 b^9 d + 3 a^4 b^7 d + a^6 b^5 d))
\end{aligned}$$

$$\begin{aligned}
& 4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^{(1/2)}/(8*(b^{11}*d + 3*a^2*b^9*d + 3*a^4*b^7*d \\
& + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^{(1/2)}/(8*(b^{11} \\
& *d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)* \\
& (-a^3*b^5*e^7)^{(1/2)}/(8*(b^{11}*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)) \\
& + (((e*\cot(c + d*x))^{(1/2)}*(9*a^{16}*e^{24} + 32*b^{16}*e^{24} + 128*a^2*b^{14}*e^{24} \\
& + 1417*a^4*b^{12}*e^{24} - 6802*a^6*b^{10}*e^{24} - 1017*a^8*b^8*e^{24} - 1020*a^{10}* \\
& b^6*e^{24} + 39*a^{12}*b^4*e^{24} - 18*a^{14}*b^2*e^{24}))/ (b^{19}*d^4 + 8*a^2*b^{17}*d^4 \\
& + 28*a^4*b^{15}*d^4 + 56*a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + \\
& 28*a^{12}*b^7*d^4 + 8*a^{14}*b^5*d^4 + a^{16}*b^3*d^4) + (((32*a*b^{18}*d^2*e^{21} - \\
& 18*a^{19}*d^2*e^{21} - 6528*a^3*b^{16}*d^2*e^{21} + 2758*a^5*b^{14}*d^2*e^{21} + 26482* \\
& a^7*b^{12}*d^2*e^{21} + 21582*a^9*b^{10}*d^2*e^{21} + 7594*a^{11}*b^8*d^2*e^{21} + 3314 \\
& *a^{13}*b^6*d^2*e^{21} + 246*a^{15}*b^4*d^2*e^{21} + 90*a^{17}*b^2*d^2*e^{21}))/ (b^{19}*d^ \\
& 5 + 8*a^2*b^{17}*d^5 + 28*a^4*b^{15}*d^5 + 56*a^6*b^{13}*d^5 + 70*a^8*b^{11}*d^5 + \\
& 56*a^{10}*b^9*d^5 + 28*a^{12}*b^7*d^5 + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^5) - (((e* \\
& \cot(c + d*x))^{(1/2)}*(1472*a*b^{21}*d^2*e^{17} + 72*a^{21}*b*d^2*e^{17} + 1024*a^3*b \\
& ^{19}*d^2*e^{17} + 1352*a^5*b^{17}*d^2*e^{17} + 28224*a^7*b^{15}*d^2*e^{17} + 70240*a^9 \\
& *b^{13}*d^2*e^{17} + 72640*a^{11}*b^{11}*d^2*e^{17} + 39088*a^{13}*b^9*d^2*e^{17} + 13248 \\
& *a^{15}*b^7*d^2*e^{17} + 3488*a^{17}*b^5*d^2*e^{17} + 576*a^{19}*b^3*d^2*e^{17}))/ (b^{19} \\
& *d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d^4 + 56*a^6*b^{13}*d^4 + 70*a^8*b^{11}*d^4 \\
& + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 + 8*a^{14}*b^5*d^4 + a^{16}*b^3*d^4) - ((( \\
& 1600*a^2*b^{23}*d^4*e^{14} + 12864*a^4*b^{21}*d^4*e^{14} + 45312*a^6*b^{19}*d^4*e^{14} \\
& + 91392*a^8*b^{17}*d^4*e^{14} + 115584*a^{10}*b^{15}*d^4*e^{14} + 94080*a^{12}*b^{13}*d^4 \\
& *e^{14} + 48384*a^{14}*b^{11}*d^4*e^{14} + 14592*a^{16}*b^9*d^4*e^{14} + 2112*a^{18}*b^7* \\
& d^4*e^{14} + 64*a^{20}*b^5*d^4*e^{14}))/ (b^{19}*d^5 + 8*a^2*b^{17}*d^5 + 28*a^4*b^{15}*d^ \\
& ^5 + 56*a^6*b^{13}*d^5 + 70*a^8*b^{11}*d^5 + 56*a^{10}*b^9*d^5 + 28*a^{12}*b^7*d^5 \\
& + 8*a^{14}*b^5*d^5 + a^{16}*b^3*d^5) + ((e*\cot(c + d*x))^{(1/2)}*(3*a^4 + 35*b^4 \\
& + 6*a^2*b^2)*(-a^3*b^5*e^7)^{(1/2)}*(512*b^{28}*d^4*e^{10} + 4608*a^2*b^{26}*d^4*e^ \\
& ^{10} + 17920*a^4*b^{24}*d^4*e^{10} + 38400*a^6*b^{22}*d^4*e^{10} + 46080*a^8*b^{20}*d^4 \\
& *e^{10} + 21504*a^{10}*b^{18}*d^4*e^{10} - 21504*a^{12}*b^{16}*d^4*e^{10} - 46080*a^{14}*b^ \\
& ^{14}*d^4*e^{10} - 38400*a^{16}*b^{12}*d^4*e^{10} - 17920*a^{18}*b^{10}*d^4*e^{10} - 4608*a^ \\
& ^{20}*b^8*d^4*e^{10} - 512*a^{22}*b^6*d^4*e^{10}))/ (8*(b^{11}*d + 3*a^2*b^9*d + 3*a^4* \\
& b^7*d + a^6*b^5*d)*(b^{19}*d^4 + 8*a^2*b^{17}*d^4 + 28*a^4*b^{15}*d^4 + 56*a^6*b^ \\
& ^{13}*d^4 + 70*a^8*b^{11}*d^4 + 56*a^{10}*b^9*d^4 + 28*a^{12}*b^7*d^4 + 8*a^{14}*b^5*d \\
& ^4 + a^{16}*b^3*d^4)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^{(1/2)}/(8* \\
& (b^{11}*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2* \\
& b^2)*(-a^3*b^5*e^7)^{(1/2)}/(8*(b^{11}*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5 \\
& *d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^{(1/2)}/(8*(b^{11}*d + 3*a^2 \\
& *b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + 35*b^4 + 6*a^2*b^2)*(-a^3*b^5* \\
& e^7)^{(1/2)}/(8*(b^{11}*d + 3*a^2*b^9*d + 3*a^4*b^7*d + a^6*b^5*d)))*(3*a^4 + \\
& 35*b^4 + 6*a^2*b^2)*(-a^3*b^5*e^7)^{(1/2)}*1i)/(4*(b^{11}*d + 3*a^2*b^9*d + 3* \\
& a^4*b^7*d + a^6*b^5*d))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Timed out
```

$$3.83 \quad \int \frac{(e \cot(c+dx))^{5/2}}{(a+b \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=470

$$\frac{e^{5/2}(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3} - \frac{e^{5/2}(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

[Out]  $-1/2*(a-b)*(a^2+4*a*b+b^2)*e^{(5/2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^{3/d*2^{(1/2)}+1/2}*(a-b)*(a^2+4*a*b+b^2)*e^{(5/2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^{3/d*2^{(1/2)}+1/4}*(a+b)*(a^2-4*a*b+b^2)*e^{(5/2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^{3/d*2^{(1/2)}-1/4}*(a+b)*(a^2-4*a*b+b^2)*e^{(5/2)*\ln(e^{(1/2)}+\cot(d*x+c)*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^{3/d*2^{(1/2)}-1/4}*(a^4+18*a^2*b^2-15*b^4)*e^{(5/2)*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})}*a^{(1/2)}/b^{(3/2)}/(a^2+b^2)^{3/d+1/2}*a^2*e^2*(e*\cot(d*x+c))^{(1/2)}/b/(a^2+b^2)/d/(a+b*\cot(d*x+c))^{2-1/4}*a*(a^2+9*b^2)*e^2*(e*\cot(d*x+c))^{(1/2)}/b/(a^2+b^2)^2/d/(a+b*\cot(d*x+c))$

**Rubi [A]** time = 1.29, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3565, 3649, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$-\frac{ae^2(a^2+9b^2)\sqrt{e \cot(c+dx)}}{4bd(a^2+b^2)^2(a+b \cot(c+dx))} + \frac{a^2e^2\sqrt{e \cot(c+dx)}}{2bd(a^2+b^2)(a+b \cot(c+dx))^2} + \frac{e^{5/2}(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(5/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out]  $-(\text{Sqrt}[a]*(a^4+18*a^2*b^2-15*b^4)*e^{(5/2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(\text{Sqrt}[a]*\text{Sqrt}[e])]}]/(4*b^{(3/2)}*(a^2+b^2)^3*d) - ((a-b)*(a^2+4*a*b+b^2)*e^{(5/2)*\text{ArcTan}[1-(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e])]}]/(\text{Sqrt}[2]*(a^2+b^2)^3*d) + ((a-b)*(a^2+4*a*b+b^2)*e^{(5/2)*\text{ArcTan}[1+(\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]])/\text{Sqrt}[e])]}]/(\text{Sqrt}[2]*(a^2+b^2)^3*d) + (a^2*e^{2*\text{Sqrt}[e*\text{Cot}[c+d*x]])}/(2*b*(a^2+b^2)*d*(a+b*\text{Cot}[c+d*x])^2) - (a*(a^2+9*b^2)*e^2*\text{Sqrt}[e*\text{Cot}[c+d*x]])/(4*b*(a^2+b^2)^2*d*(a+b*\text{Cot}[c+d*x])) + ((a+b)*(a^2-4*a*b+b^2)*e^{(5/2)*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]-\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]]]}/(2*\text{Sqrt}[2]*(a^2+b^2)^3*d) - ((a+b)*(a^2-4*a*b+b^2)*e^{(5/2)*\text{Log}[\text{Sqrt}[e]+\text{Sqrt}[e]*\text{Cot}[c+d*x]+\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c+d*x]]]}/(2*\text{Sqrt}[2]*(a^2+b^2)^3*d)$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$\text{eQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[(d + (e \cdot x^2)/(a + (c \cdot x^4)), x_{\text{Symbol}}] \rightarrow \text{With}\{q = \text{Rt}[a \cdot c, 2]\}, \text{Dist}[(d \cdot q + a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q + c \cdot x^2)/(a + c \cdot x^4), x], x] + \text{Dist}[(d \cdot q - a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q - c \cdot x^2)/(a + c \cdot x^4), x], x] \ /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \ \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[-(a \cdot c)]$

### Rule 3534

$\text{Int}[(c + (d \cdot \tan(e + f \cdot x))/\text{Sqrt}[b \cdot \tan(e + f \cdot x)]), x_{\text{Symbol}}] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b \cdot c + d \cdot x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \cdot \tan(e + f \cdot x)]]], x] \ /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3565

$\text{Int}[(a + (b \cdot \tan(e + f \cdot x)))^m \cdot (c + (d \cdot \tan(e + f \cdot x)))^n, x_{\text{Symbol}}] \rightarrow \text{Simp}[(b \cdot c - a \cdot d)^2 \cdot (a + b \cdot \tan(e + f \cdot x))^{m-2} \cdot (c + d \cdot \tan(e + f \cdot x))^{n+1} / (d \cdot f \cdot (n+1) \cdot (c^2 + d^2)), x] - \text{Dist}[1 / (d \cdot (n+1) \cdot (c^2 + d^2)), \text{Int}[(a + b \cdot \tan(e + f \cdot x))^{m-3} \cdot (c + d \cdot \tan(e + f \cdot x))^{n+1} \cdot \text{Simp}[a^2 \cdot d \cdot (b \cdot d \cdot (m-2) - a \cdot c \cdot (n+1)) + b \cdot (b \cdot c - 2 \cdot a \cdot d) \cdot (b \cdot c \cdot (m-2) + a \cdot d \cdot (n+1)) - d \cdot (n+1) \cdot (3 \cdot a^2 \cdot b \cdot c - b^3 \cdot c - a^3 \cdot d + 3 \cdot a \cdot b^2 \cdot d) \cdot \tan(e + f \cdot x) - b \cdot (a \cdot d \cdot (2 \cdot b \cdot c - a \cdot d) \cdot (m+n-1) - b^2 \cdot (c^2 \cdot (m-2) - d^2 \cdot (n+1))) \cdot \tan(e + f \cdot x)^2, x], x], x] \ /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{GtQ}[m, 2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2 \cdot m]$

### Rule 3634

$\text{Int}[(a + (b \cdot \tan(e + f \cdot x)))^m \cdot (c + (d \cdot \tan(e + f \cdot x)))^n \cdot (A + (C \cdot \tan(e + f \cdot x))^2), x_{\text{Symbol}}] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x, \tan(e + f \cdot x)], x] \ /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n, x\} \ \&\& \ \text{EqQ}[A, C]$

### Rule 3649

$\text{Int}[(a + (b \cdot \tan(e + f \cdot x)))^m \cdot (c + (d \cdot \tan(e + f \cdot x)))^n \cdot (A + (B \cdot \tan(e + f \cdot x)) + (C \cdot \tan(e + f \cdot x))^2), x_{\text{Symbol}}] \rightarrow \text{Simp}[(A \cdot b^2 - a \cdot (b \cdot B - a \cdot C)) \cdot (a + b \cdot \tan(e + f \cdot x))^{m+1} \cdot (c + d \cdot \tan(e + f \cdot x))^{n+1} / (f \cdot (m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), \text{Int}[(a + b \cdot \tan(e + f \cdot x))^{m+1} \cdot (c + d \cdot \tan(e + f \cdot x))^n \cdot \text{Simp}[A \cdot (a \cdot (b \cdot c - a \cdot d) \cdot (m+1) - b^2 \cdot d \cdot ($



```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{5/2}}{(a + b \cot(c + dx))^3} dx &= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} - \frac{\int \frac{-\frac{1}{2} a^2 e^3 + 2 a b e^3 \cot(c + dx) - \frac{1}{2} (a^2 + 4 b^2) e^3 \cot^2(c + dx)}{\sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2} dx}{2b (a^2 + b^2)} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} - \frac{a (a^2 + 9 b^2) e^2 \sqrt{e \cot(c + dx)}}{4b (a^2 + b^2)^2 d (a + b \cot(c + dx))} + \frac{\int \frac{\frac{1}{4} a^2 (a^2 - 7 b^2)}{\dots}}{\dots} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} - \frac{a (a^2 + 9 b^2) e^2 \sqrt{e \cot(c + dx)}}{4b (a^2 + b^2)^2 d (a + b \cot(c + dx))} + \frac{\int \frac{-2 a b^2 (3 a^2 - \dots)}{\dots}}{\dots} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} - \frac{a (a^2 + 9 b^2) e^2 \sqrt{e \cot(c + dx)}}{4b (a^2 + b^2)^2 d (a + b \cot(c + dx))} + \frac{\text{Subst} \left( \int \frac{2 a \dots}{\dots} \right)}{\dots} \\
&= \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))^2} - \frac{a (a^2 + 9 b^2) e^2 \sqrt{e \cot(c + dx)}}{4b (a^2 + b^2)^2 d (a + b \cot(c + dx))} - \frac{(a (a^4 + 18 a^2 b^2 - 15 b^4) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right) + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))}}{4 b^{3/2} (a^2 + b^2)^3 d} \\
&= - \frac{\sqrt{a} (a^4 + 18 a^2 b^2 - 15 b^4) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right) + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))}}{4 b^{3/2} (a^2 + b^2)^3 d} + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))} \\
&= - \frac{\sqrt{a} (a^4 + 18 a^2 b^2 - 15 b^4) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right) + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))}}{4 b^{3/2} (a^2 + b^2)^3 d} - \frac{(a - b) (a^2 + 4 a b + b^2) e^{5/2} \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right) + \frac{a^2 e^2 \sqrt{e \cot(c + dx)}}{2b (a^2 + b^2) d (a + b \cot(c + dx))}}{4 b^{3/2} (a^2 + b^2)^3 d} - \frac{\sqrt{2} (a^2 + b^2)}{\dots}
\end{aligned}$$

**Mathematica** [C] time = 6.20, size = 488, normalized size = 1.04

$$(e \cot(c + dx))^{5/2} \left( \frac{4 b^2 \cot^2(c + dx) {}_2F_1 \left( 2, \frac{7}{2}; \frac{9}{2}; -\frac{b \cot(c + dx)}{a} \right)}{7 a (a^2 + b^2)^2} + \frac{2 a (a^2 - 3 b^2) \left( \cot^2(c + dx) - \cot^2(c + dx) {}_2F_1 \left( \frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c + dx) \right) \right)}{3 (a^2 + b^2)^3} + \frac{2 b (3 a^2 - b^2)}{5 (a^2 + b^2)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

```
[Out] -(((e*Cot[c + d*x])^(5/2)*((2*b*(3*a^2 - b^2)*Cot[c + d*x])^(5/2))/(5*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*(-3*a*(-((Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/b^(3/2)) + Sqrt[Cot[c + d*x]]/b) + Cot[c + d*x]^(3/2)))/(3*(a^2 + b^2)^3) + (2*a*(a^2 - 3*b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*b^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])/a)))/(7*a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*Cot[c + d*x])/a)))/(7*a^3*(a^2 + b^2)) + (b*(3*a^2 - b^2)*(40*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + (5*(4*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]) - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]])] + 2*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 2*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/2))/(20*(a^2 + b^2)^3))/(d*Cot[c + d*x]^(5/2))
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{5}{2}}}{(b \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^3, x)
```

**maple** [B] time = 1.00, size = 1229, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x)
```

[Out]  $-1/4/d*e^3*a^5/(a^2+b^2)^3/(e*\cot(d*x+c)*b+a*e)^2*(e*\cot(d*x+c))^{(3/2)}-5/2/d*e^3*a^3/(a^2+b^2)^3/(e*\cot(d*x+c)*b+a*e)^2*(e*\cot(d*x+c))^{(3/2)}*b^2-9/4/d*e^3*a/(a^2+b^2)^3/(e*\cot(d*x+c)*b+a*e)^2*(e*\cot(d*x+c))^{(3/2)}*b^4+1/4/d*e^4*a^6/(a^2+b^2)^3/(e*\cot(d*x+c)*b+a*e)^2/b*(e*\cot(d*x+c))^{(1/2)}-3/2/d*e^4*a^4/(a^2+b^2)^3/(e*\cot(d*x+c)*b+a*e)^2*b*(e*\cot(d*x+c))^{(1/2)}-7/4/d*e^4*a^2/(a^2+b^2)^3/(e*\cot(d*x+c)*b+a*e)^2*b^3*(e*\cot(d*x+c))^{(1/2)}-1/4/d*e^3*a^5/(a^2+b^2)^3/b/(a*e*b)^{(1/2)}*arctan((e*\cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})-9/2/d*e^3*a^3/(a^2+b^2)^3*b/(a*e*b)^{(1/2)}*arctan((e*\cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})+15/4/d*e^3*a/(a^2+b^2)^3*b^3/(a*e*b)^{(1/2)}*arctan((e*\cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})+3/2/d*e^2/(a^2+b^2)^3*(e^2)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^2*b-1/2/d*e^2/(a^2+b^2)^3*(e^2)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*b^3-3/2/d*e^2/(a^2+b^2)^3*(e^2)^{(1/4)}*2^{(1/2)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^2*b+1/2/d*e^2/(a^2+b^2)^3*(e^2)^{(1/4)}*2^{(1/2)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*b^3+3/4/d*e^2/(a^2+b^2)^3*(e^2)^{(1/4)}*2^{(1/2)}*ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))*a^2*b-1/4/d*e^2/(a^2+b^2)^3*(e^2)^{(1/4)}*2^{(1/2)}*ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))*b^3+1/2/d*e^3/(a^2+b^2)^3*2^{(1/2)}/(e^2)^{(1/4)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^3-3/2/d*e^3/(a^2+b^2)^3*2^{(1/2)}/(e^2)^{(1/4)}*arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a*b^2-1/2/d*e^3/(a^2+b^2)^3*2^{(1/2)}/(e^2)^{(1/4)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^3+3/2/d*e^3/(a^2+b^2)^3*2^{(1/2)}/(e^2)^{(1/4)}*arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a*b^2+1/4/d*e^3/(a^2+b^2)^3*2^{(1/2)}/(e^2)^{(1/4)}*ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))*a^3-3/4/d*e^3/(a^2+b^2)^3*2^{(1/2)}/(e^2)^{(1/4)}*ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))*a*b^2$

**maxima** [A] time = 0.56, size = 505, normalized size = 1.07

$$\left( \frac{(a^5+18a^3b^2-15ab^4)e^2 \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^6b+3a^4b^3+3a^2b^5+b^7)\sqrt{abe}} - \frac{\left( 2\sqrt{2}(a^3+3a^2b-3ab^2-b^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}+2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right) \right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3+3a^2b-3ab^2-b^3) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}\right)}{\sqrt{e}}\right)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(5/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-1/4*((a^5 + 18*a^3*b^2 - 15*a*b^4)*e^2*\arctan(b*\sqrt{e/\tan(d*x + c)})/\sqrt{a*b*e})/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*\sqrt{a*b*e}) - (2*\sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} - \sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e} + \sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e})*e^2/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((a^4 - 7*a^2*b^2)*e^3*\sqrt{e/\tan(d*x + c)} - (a^3*b + 9*a*b^3)*e^2*(e/\tan(d*x + c))^(3/2))/((a^6*b + 2*a^4*b^3 + a^2*b^5)*e^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*e^2/\tan(d*x + c) + (a^4*b^3 + 2*a^2*b^5 + b^7)*e^2/\tan(d*x + c)^2))*e/d$$

**mupad [B]** time = 6.51, size = 19256, normalized size = 40.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(5/2)/(a + b\*cot(c + d\*x))^3,x)

[Out] 
$$\operatorname{atan}\left(\frac{\left(\left(\left(10*a^{16}*b*d^2*e^{18} - 2398*a^2*b^{15}*d^2*e^{18} + 5238*a^4*b^{13}*d^2*e^{18} + 7386*a^6*b^{11}*d^2*e^{18} - 8322*a^8*b^9*d^2*e^{18} - 5498*a^{10}*b^7*d^2*e^{18} + 2946*a^{12}*b^5*d^2*e^{18} + 382*a^{14}*b^3*d^2*e^{18}\right)\right)\right)}{(b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) - \left(\left(832*a*b^{22}*d^4*e^{13} + 5952*a^3*b^{20}*d^4*e^{13} + 17664*a^5*b^{18}*d^4*e^{13} + 26880*a^7*b^{16}*d^4*e^{13} + 18816*a^9*b^{14}*d^4*e^{13} - 2688*a^{11}*b^{12}*d^4*e^{13} - 16128*a^{13}*b^{10}*d^4*e^{13} - 13056*a^{15}*b^8*d^4*e^{13} - 4800*a^{17}*b^6*d^4*e^{13} - 704*a^{19}*b^4*d^4*e^{13}\right)\right)}{(b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) + \left(\left(e*\cot(c + d*x)\right)^{1/2}\right)*\left(-\left(e^5*1i\right)\right)/\left(4*\left(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2\right)\right)\right)^{1/2} * \left(512*b^{26}*d^4*e^{10} + 4608*a^2*b^{24}*d^4*e^{10} + 17920*a^4*b^{22}*d^4*e^{10} + 38400*a^6*b^{20}*d^4*e^{10} + 46080*a^8*b^{18}*d^4*e^{10} + 21504*a^{10}*b^{16}*d^4*e^{10} - 21504*a^{12}*b^{14}*d^4*e^{10} - 46080*a^{14}*b^{12}*d^4*e^{10} - 38400*a^{16}*b^{10}*d^4*e^{10} - 17920*a^{18}*b^8*d^4*e^{10} - 4608*a^{20}*b^6*d^4*e^{10} - 512*a^{22}*b^4*d^4*e^{10}\right) / \left(b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4\right) * \left(-\left(e^5*1i\right)\right) / \left(4*\left(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2\right)\right)\right)^{1/2} - \left(\left(e*\cot(c + d*x)\right)^{1/2}\right)*\left(8*a^{19}*b*d^2*e^{15} - 1472*a*b^{19}*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + \dots\right)$$

$$\begin{aligned}
& 11328*a^5*b^15*d^2*e^15 + 10208*a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 \\
& - 5328*a^11*b^9*d^2*e^15 + 4032*a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 \\
& + 384*a^17*b^3*d^2*e^15) / (b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4 \\
& *b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^ \\
& 5*d^4 + 8*a^14*b^3*d^4) * (- (e^5*1i) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + \\
& a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2)) * \\
& (- (e^5*1i) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4 \\
& *d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2) * \\
& (a^14*e^20 - 32*b^14*e^20 + 97*a^2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^ \\
& 6*b^8*e^20 - 2300*a^8*b^6*e^20 + 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20)) / (b^1 \\
& 7*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 7 \\
& 0*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) * (- (e^5 \\
& *1i) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - \\
& a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) * 1i - (((10*a^16*b*d^2*e^18 - 239 \\
& 8*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e^18 + 7386*a^6*b^11*d^2*e^18 - 832 \\
& 2*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e^18 + 2946*a^12*b^5*d^2*e^18 + 382* \\
& a^14*b^3*d^2*e^18) / (b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^ \\
& 5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + \\
& 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a \\
& ^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688 \\
& *a^11*b^12*d^4*e^13 - 16128*a^13*b^10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - \\
& 4800*a^17*b^6*d^4*e^13 - 704*a^19*b^4*d^4*e^13) / (b^17*d^5 + a^16*b*d^5 + 8* \\
& a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10 \\
& *b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - ((e*cot(c + d*x))^(1/2) * (- (e \\
& ^5*1i) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 \\
& - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) * (512*b^26*d^4*e^10 + 4608*a^2* \\
& b^24*d^4*e^10 + 17920*a^4*b^22*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a \\
& ^8*b^18*d^4*e^10 + 21504*a^10*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46 \\
& 080*a^14*b^12*d^4*e^10 - 38400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 \\
& - 4608*a^20*b^6*d^4*e^10 - 512*a^22*b^4*d^4*e^10)) / (b^17*d^4 + a^16*b*d^4 \\
& + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56* \\
& a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) * (- (e^5*1i) / (4*(b^6*d^2 - \\
& a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + \\
& 15*a^4*b^2*d^2)))^(1/2) + ((e*cot(c + d*x))^(1/2) * (8*a^19*b*d^2*e^15 - 1472 \\
& *a*b^19*d^2*e^15 + 776*a^3*b^17*d^2*e^15 + 11328*a^5*b^15*d^2*e^15 + 10208* \\
& a^7*b^13*d^2*e^15 - 5056*a^9*b^11*d^2*e^15 - 5328*a^11*b^9*d^2*e^15 + 4032* \\
& a^13*b^7*d^2*e^15 + 3552*a^15*b^5*d^2*e^15 + 384*a^17*b^3*d^2*e^15)) / (b^17* \\
& d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4*b^13*d^4 + 56*a^6*b^11*d^4 + 70* \\
& a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b^5*d^4 + 8*a^14*b^3*d^4) * (- (e^5*1 \\
& i) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a \\
& ^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) * (- (e^5*1i) / (4*(b^6*d^2 - a^6*d^2 \\
& + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b \\
& ^2*d^2)))^(1/2) - ((e*cot(c + d*x))^(1/2) * (a^14*e^20 - 32*b^14*e^20 + 97*a^ \\
& 2*b^12*e^20 - 2082*a^4*b^10*e^20 + 3631*a^6*b^8*e^20 - 2300*a^8*b^6*e^20 + \\
& 79*a^10*b^4*e^20 + 30*a^12*b^2*e^20)) / (b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d
\end{aligned}$$

$$\begin{aligned}
&^4 + 28a^4b^{13}d^4 + 56a^6b^{11}d^4 + 70a^8b^9d^4 + 56a^{10}b^7d^4 + \\
&28a^{12}b^5d^4 + 8a^{14}b^3d^4)) * (-e^5 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2 \\
&)))^{(1/2)} * 1i) / (((10 * a^{16} * b * d^2 * e^{18} - 2398 * a^2 * b^{15} * d^2 * e^{18} + 5238 * a^4 * b^{13} * d^2 * e^{18} + 7386 * a^6 * b^{11} * d^2 * e^{18} - 8322 * a^8 * b^9 * d^2 * e^{18} - 5498 * a^{10} * b^7 * d^2 * e^{18} + 2946 * a^{12} * b^5 * d^2 * e^{18} + 382 * a^{14} * b^3 * d^2 * e^{18}) / (b^{17} * d^5 + a^{16} * b * d^5 + 8 * a^2 * b^{15} * d^5 + 28 * a^4 * b^{13} * d^5 + 56 * a^6 * b^{11} * d^5 + 70 * a^8 * b^9 * d^5 + 56 * a^{10} * b^7 * d^5 + 28 * a^{12} * b^5 * d^5 + 8 * a^{14} * b^3 * d^5) - (((832 * a * b^{22} * d^4 * e^{13} + 5952 * a^3 * b^{20} * d^4 * e^{13} + 17664 * a^5 * b^{18} * d^4 * e^{13} + 26880 * a^7 * b^{16} * d^4 * e^{13} + 18816 * a^9 * b^{14} * d^4 * e^{13} - 2688 * a^{11} * b^{12} * d^4 * e^{13} - 16128 * a^{13} * b^{10} * d^4 * e^{13} - 13056 * a^{15} * b^8 * d^4 * e^{13} - 4800 * a^{17} * b^6 * d^4 * e^{13} - 704 * a^{19} * b^4 * d^4 * e^{13}) / (b^{17} * d^5 + a^{16} * b * d^5 + 8 * a^2 * b^{15} * d^5 + 28 * a^4 * b^{13} * d^5 + 56 * a^6 * b^{11} * d^5 + 70 * a^8 * b^9 * d^5 + 56 * a^{10} * b^7 * d^5 + 28 * a^{12} * b^5 * d^5 + 8 * a^{14} * b^3 * d^5) + ((e * \cot(c + d * x))^{(1/2)} * (-e^5 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2)))^{(1/2)} * (512 * b^{26} * d^4 * e^{10} + 4608 * a^2 * b^{24} * d^4 * e^{10} + 17920 * a^4 * b^{22} * d^4 * e^{10} + 38400 * a^6 * b^{20} * d^4 * e^{10} + 46080 * a^8 * b^{18} * d^4 * e^{10} + 21504 * a^{10} * b^{16} * d^4 * e^{10} - 21504 * a^{12} * b^{14} * d^4 * e^{10} - 46080 * a^{14} * b^{12} * d^4 * e^{10} - 38400 * a^{16} * b^{10} * d^4 * e^{10} - 17920 * a^{18} * b^8 * d^4 * e^{10} - 4608 * a^{20} * b^6 * d^4 * e^{10} - 512 * a^{22} * b^4 * d^4 * e^{10}) / (b^{17} * d^4 + a^{16} * b * d^4 + 8 * a^2 * b^{15} * d^4 + 28 * a^4 * b^{13} * d^4 + 56 * a^6 * b^{11} * d^4 + 70 * a^8 * b^9 * d^4 + 56 * a^{10} * b^7 * d^4 + 28 * a^{12} * b^5 * d^4 + 8 * a^{14} * b^3 * d^4)) * (-e^5 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2)))^{(1/2)} - ((e * \cot(c + d * x))^{(1/2)} * (8 * a^{19} * b * d^2 * e^{15} - 1472 * a * b^{19} * d^2 * e^{15} + 776 * a^3 * b^{17} * d^2 * e^{15} + 11328 * a^5 * b^{15} * d^2 * e^{15} + 10208 * a^7 * b^{13} * d^2 * e^{15} - 5056 * a^9 * b^{11} * d^2 * e^{15} - 5328 * a^{11} * b^9 * d^2 * e^{15} + 4032 * a^{13} * b^7 * d^2 * e^{15} + 3552 * a^{15} * b^5 * d^2 * e^{15} + 384 * a^{17} * b^3 * d^2 * e^{15})) / (b^{17} * d^4 + a^{16} * b * d^4 + 8 * a^2 * b^{15} * d^4 + 28 * a^4 * b^{13} * d^4 + 56 * a^6 * b^{11} * d^4 + 70 * a^8 * b^9 * d^4 + 56 * a^{10} * b^7 * d^4 + 28 * a^{12} * b^5 * d^4 + 8 * a^{14} * b^3 * d^4)) * (-e^5 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2)))^{(1/2)} + ((e * \cot(c + d * x))^{(1/2)} * (a^{14} * e^{20} - 32 * b^{14} * e^{20} + 97 * a^2 * b^{12} * e^{20} - 2082 * a^4 * b^{10} * e^{20} + 3631 * a^6 * b^8 * e^{20} - 2300 * a^8 * b^6 * e^{20} + 79 * a^{10} * b^4 * e^{20} + 30 * a^{12} * b^2 * e^{20})) / (b^{17} * d^4 + a^{16} * b * d^4 + 8 * a^2 * b^{15} * d^4 + 28 * a^4 * b^{13} * d^4 + 56 * a^6 * b^{11} * d^4 + 70 * a^8 * b^9 * d^4 + 56 * a^{10} * b^7 * d^4 + 28 * a^{12} * b^5 * d^4 + 8 * a^{14} * b^3 * d^4)) * (-e^5 * 1i) / (4 * (b^6d^2 - a^6d^2 + a * b^5d^2 * 6i + a^5 * b * d^2 * 6i - 15 * a^2 * b^4 * d^2 - a^3 * b^3 * d^2 * 20i + 15 * a^4 * b^2 * d^2)))^{(1/2)} + (((10 * a^{16} * b * d^2 * e^{18} - 2398 * a^2 * b^{15} * d^2 * e^{18} + 5238 * a^4 * b^{13} * d^2 * e^{18} + 7386 * a^6 * b^{11} * d^2 * e^{18} - 8322 * a^8 * b^9 * d^2 * e^{18} - 5498 * a^{10} * b^7 * d^2 * e^{18} + 2946 * a^{12} * b^5 * d^2 * e^{18} + 382 * a^{14} * b^3 * d^2 * e^{18}) / (b^{17} * d^5 + a^{16} * b * d^5 + 8 * a^2 * b^{15} * d^5 + 28 * a^4 * b^{13} * d^5 + 56 * a^6 * b^{11} * d^5 + 70 * a^8 * b^9 * d^5 + 56 * a^{10} * b^7 * d^5 + 28 * a^{12} * b^5 * d^5 + 8 * a^{14} * b^3 * d^5) - (((832 * a * b^{22} * d^4 * e^{13} + 5952 * a^3 * b^{20} * d^4 * e^{13} + 17664 * a^5 * b^{18} * d^4 * e^{13} + 26880 * a^7 * b^{16} * d^4 * e^{13} + 18816 * a^9 * b^{14} * d^4 * e^{13} - 2688 * a^{11} * b^{12} * d^4 * e^{13} - 16128 * a^{13} * b^{10} * d^4 * e^{13} - 13056 * a^{15} * b^8 * d^4 * e
\end{aligned}$$

$$\begin{aligned}
& ^{13} - 4800*a^{17}*b^6*d^4*e^{13} - 704*a^{19}*b^4*d^4*e^{13})/(b^{17}*d^5 + a^{16}*b*d^5 \\
& + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 5 \\
& 6*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) - ((e*\cot(c + d*x))^{(1/2)} \\
& )*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 \\
& ^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)}*(512*b^{26}*d^4*e^{10} + 460 \\
& 8*a^2*b^{24}*d^4*e^{10} + 17920*a^4*b^{22}*d^4*e^{10} + 38400*a^6*b^{20}*d^4*e^{10} + 4 \\
& 6080*a^8*b^{18}*d^4*e^{10} + 21504*a^{10}*b^{16}*d^4*e^{10} - 21504*a^{12}*b^{14}*d^4*e^{10} \\
& 0 - 46080*a^{14}*b^{12}*d^4*e^{10} - 38400*a^{16}*b^{10}*d^4*e^{10} - 17920*a^{18}*b^8*d^4 \\
& 4*e^{10} - 4608*a^{20}*b^6*d^4*e^{10} - 512*a^{22}*b^4*d^4*e^{10}))/((b^{17}*d^4 + a^{16}* \\
& b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 \\
& + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4))*(-(e^5*1i)/(4*(b^6* \\
& d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2* \\
& 20i + 15*a^4*b^2*d^2)))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)}*(8*a^{19}*b*d^2*e^{15} \\
& - 1472*a*b^{19}*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + 11328*a^5*b^{15}*d^2*e^{15} + \\
& 10208*a^7*b^{13}*d^2*e^{15} - 5056*a^9*b^{11}*d^2*e^{15} - 5328*a^{11}*b^9*d^2*e^{15} + \\
& 4032*a^{13}*b^7*d^2*e^{15} + 3552*a^{15}*b^5*d^2*e^{15} + 384*a^{17}*b^3*d^2*e^{15}))/ \\
& (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 \\
& + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4))*(-( \\
& (e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 \\
& ^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)}))*(-(e^5*1i)/(4*(b^6*d^2 - a^ \\
& 6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15 \\
& *a^4*b^2*d^2)))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)}*(a^{14}*e^{20} - 32*b^{14}*e^{20} + \\
& 97*a^2*b^{12}*e^{20} - 2082*a^4*b^{10}*e^{20} + 3631*a^6*b^8*e^{20} - 2300*a^8*b^6*e \\
& ^{20} + 79*a^{10}*b^4*e^{20} + 30*a^{12}*b^2*e^{20}))/((b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2* \\
& b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7 \\
& *d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 \\
& + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2 \\
& ^2*d^2)))^{(1/2)} + (a^{11}*e^{23} - 120*a*b^{10}*e^{23} + 249*a^3*b^8*e^{23} - 388*a^5 \\
& *b^6*e^{23} + 302*a^7*b^4*e^{23} + 36*a^9*b^2*e^{23}))/((b^{17}*d^5 + a^{16}*b*d^5 + 8* \\
& a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10} \\
& *b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5)))*(-(e^5*1i)/(4*(b^6*d^2 - a^6 \\
& *d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15* \\
& a^4*b^2*d^2)))^{(1/2)}*2i - ((e^3*(e*\cot(c + d*x))^{(3/2)}*(9*a*b^2 + a^3))/(4* \\
& (a^4 + b^4 + 2*a^2*b^2)) - ((e*\cot(c + d*x))^{(1/2)}*(a^4*e^4 - 7*a^2*b^2*e^4 \\
& ))/(4*b*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2 \\
& *a*b*d*e^2*cot(c + d*x)) + atan((((10*a^{16}*b*d^2*e^{18} - 2398*a^2*b^{15}*d^2* \\
& e^{18} + 5238*a^4*b^{13}*d^2*e^{18} + 7386*a^6*b^{11}*d^2*e^{18} - 8322*a^8*b^9*d^2*e \\
& ^{18} - 5498*a^{10}*b^7*d^2*e^{18} + 2946*a^{12}*b^5*d^2*e^{18} + 382*a^{14}*b^3*d^2*e^ \\
& ^{18}))/((832*a*b^{22}*d^4*e^{13} + 5952*a^3*b^{20}*d^4*e^{13} + 17664*a^5*b^{18}*d^4*e^{13} \\
& 3 + 26880*a^7*b^{16}*d^4*e^{13} + 18816*a^9*b^{14}*d^4*e^{13} - 2688*a^{11}*b^{12}*d^4* \\
& e^{13} - 16128*a^{13}*b^{10}*d^4*e^{13} - 13056*a^{15}*b^8*d^4*e^{13} - 4800*a^{17}*b^6*d^4 \\
& ^4*e^{13} - 704*a^{19}*b^4*d^4*e^{13}))/((b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^{15}*d^5 + \\
& 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a
\end{aligned}$$



$$\begin{aligned}
& \sqrt{12b^5d^5 + 8a^{14}b^3d^5} + ((e \cot(c + dx))^{1/2} (-e^5 / (4(b^6d^2 * i - a^6d^2 * i + 6ab^5d^2 + 6a^5bd^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i))))^{1/2} (512b^{26}d^4e^{10} + 4608a^2b^{24}d^4e^{10} + 17920a^4b^{22}d^4e^{10} + 38400a^6b^{20}d^4e^{10} + 46080a^8b^{18}d^4e^{10} + 21504a^{10}b^{16}d^4e^{10} - 21504a^{12}b^{14}d^4e^{10} - 46080a^{14}b^{12}d^4e^{10} - 38400a^{16}b^{10}d^4e^{10} - 17920a^{18}b^8d^4e^{10} - 4608a^{20}b^6d^4e^{10} - 512a^{22}b^4d^4e^{10}) / (b^{17}d^4 + a^{16}bd^4 + 8a^2b^{15}d^4 + 28a^4b^{13}d^4 + 56a^6b^{11}d^4 + 70a^8b^9d^4 + 56a^{10}b^7d^4 + 28a^{12}b^5d^4 + 8a^{14}b^3d^4)) * (-e^5 / (4(b^6d^2 * i - a^6d^2 * i + 6ab^5d^2 + 6a^5bd^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i)))^{1/2} - ((e \cot(c + dx))^{1/2} (8a^{19}bd^2e^{15} - 1472a^ab^{19}d^2e^{15} + 776a^3b^{17}d^2e^{15} + 11328a^5b^{15}d^2e^{15} + 10208a^7b^{13}d^2e^{15} - 5056a^9b^{11}d^2e^{15} - 5328a^{11}b^9d^2e^{15} + 4032a^{13}b^7d^2e^{15} + 3552a^{15}b^5d^2e^{15} + 384a^{17}b^3d^2e^{15})) / (b^{17}d^4 + a^{16}bd^4 + 8a^2b^{15}d^4 + 28a^4b^{13}d^4 + 56a^6b^{11}d^4 + 70a^8b^9d^4 + 56a^{10}b^7d^4 + 28a^{12}b^5d^4 + 8a^{14}b^3d^4)) * (-e^5 / (4(b^6d^2 * i - a^6d^2 * i + 6ab^5d^2 + 6a^5bd^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i)))^{1/2} + ((e \cot(c + dx))^{1/2} (a^{14}e^{20} - 32b^{14}e^{20} + 97a^2b^{12}e^{20} - 2082a^4b^{10}e^{20} + 3631a^6b^8e^{20} - 2300a^8b^6e^{20} + 79a^{10}b^4e^{20} - 20 + 30a^{12}b^2e^{20})) / (b^{17}d^4 + a^{16}bd^4 + 8a^2b^{15}d^4 + 28a^4b^{13}d^4 + 56a^6b^{11}d^4 + 70a^8b^9d^4 + 56a^{10}b^7d^4 + 28a^{12}b^5d^4 + 8a^{14}b^3d^4)) * (-e^5 / (4(b^6d^2 * i - a^6d^2 * i + 6ab^5d^2 + 6a^5bd^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i)))^{1/2} * i - (((10a^{16}bd^2e^{18} - 2398a^2b^{15}d^2e^{18} + 5238a^4b^{13}d^2e^{18} + 7386a^6b^{11}d^2e^{18} - 8322a^8b^9d^2e^{18} - 5498a^{10}b^7d^2e^{18} + 2946a^{12}b^5d^2e^{18} + 382a^{14}b^3d^2e^{18})) / (b^{17}d^5 + a^{16}bd^5 + 8a^2b^{15}d^5 + 28a^4b^{13}d^5 + 56a^6b^{11}d^5 + 70a^8b^9d^5 + 56a^{10}b^7d^5 + 28a^{12}b^5d^5 + 8a^{14}b^3d^5) - (((832ab^{22}d^4e^{13} + 5952a^3b^{20}d^4e^{13} + 17664a^5b^{18}d^4e^{13} + 26880a^7b^{16}d^4e^{13} + 18816a^9b^{14}d^4e^{13} - 2688a^{11}b^{12}d^4e^{13} - 16128a^{13}b^{10}d^4e^{13} - 13056a^{15}b^8d^4e^{13} - 4800a^{17}b^6d^4e^{13} - 704a^{19}b^4d^4e^{13})) / (b^{17}d^5 + a^{16}bd^5 + 8a^2b^{15}d^5 + 28a^4b^{13}d^5 + 56a^6b^{11}d^5 + 70a^8b^9d^5 + 56a^{10}b^7d^5 + 28a^{12}b^5d^5 + 8a^{14}b^3d^5) - ((e \cot(c + dx))^{1/2} (-e^5 / (4(b^6d^2 * i - a^6d^2 * i + 6ab^5d^2 + 6a^5bd^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i))))^{1/2} (512b^{26}d^4e^{10} + 4608a^2b^{24}d^4e^{10} + 17920a^4b^{22}d^4e^{10} + 38400a^6b^{20}d^4e^{10} + 46080a^8b^{18}d^4e^{10} + 21504a^{10}b^{16}d^4e^{10} - 21504a^{12}b^{14}d^4e^{10} - 46080a^{14}b^{12}d^4e^{10} - 38400a^{16}b^{10}d^4e^{10} - 17920a^{18}b^8d^4e^{10} - 4608a^{20}b^6d^4e^{10} - 512a^{22}b^4d^4e^{10}) / (b^{17}d^4 + a^{16}bd^4 + 8a^2b^{15}d^4 + 28a^4b^{13}d^4 + 56a^6b^{11}d^4 + 70a^8b^9d^4 + 56a^{10}b^7d^4 + 28a^{12}b^5d^4 + 8a^{14}b^3d^4)) * (-e^5 / (4(b^6d^2 * i - a^6d^2 * i + 6ab^5d^2 + 6a^5bd^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i)))^{1/2} + ((e \cot(c + dx))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& )*(8*a^{19}*b*d^2*e^{15} - 1472*a*b^{19}*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + 11328 \\
& *a^5*b^{15}*d^2*e^{15} + 10208*a^7*b^{13}*d^2*e^{15} - 5056*a^9*b^{11}*d^2*e^{15} - 532 \\
& 8*a^{11}*b^9*d^2*e^{15} + 4032*a^{13}*b^7*d^2*e^{15} + 3552*a^{15}*b^5*d^2*e^{15} + 384 \\
& *a^{17}*b^3*d^2*e^{15}))/((b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13} \\
& d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 \\
& + 8*a^{14}*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5* \\
& b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*(-e^5/ \\
& (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - \\
& 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)}*(a^{14}* \\
& e^{20} - 32*b^{14}*e^{20} + 97*a^2*b^{12}*e^{20} - 2082*a^4*b^{10}*e^{20} + 3631*a^6*b^8* \\
& e^{20} - 2300*a^8*b^6*e^{20} + 79*a^{10}*b^4*e^{20} + 30*a^{12}*b^2*e^{20}))/((b^{17}*d^4 \\
& + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8* \\
& b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4))*(-e^5/(4*(b^ \\
& 6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^ \\
& 3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*1i)/((((10*a^{16}*b*d^2*e^{18} - 2398*a^2* \\
& b^{15}*d^2*e^{18} + 5238*a^4*b^{13}*d^2*e^{18} + 7386*a^6*b^{11}*d^2*e^{18} - 8322*a^8* \\
& b^9*d^2*e^{18} - 5498*a^{10}*b^7*d^2*e^{18} + 2946*a^{12}*b^5*d^2*e^{18} + 382*a^{14}*b \\
& ^3*d^2*e^{18}))/((b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56 \\
& *a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14} \\
& *b^3*d^5) - (((832*a*b^{22}*d^4*e^{13} + 5952*a^3*b^{20}*d^4*e^{13} + 17664*a^5*b^{1 \\
& 8*d^4*e^{13} + 26880*a^7*b^{16}*d^4*e^{13} + 18816*a^9*b^{14}*d^4*e^{13} - 2688*a^{11}* \\
& b^{12}*d^4*e^{13} - 16128*a^{13}*b^{10}*d^4*e^{13} - 13056*a^{15}*b^8*d^4*e^{13} - 4800*a \\
& ^{17}*b^6*d^4*e^{13} - 704*a^{19}*b^4*d^4*e^{13}))/((b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^ \\
& 15*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d \\
& ^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) + ((e*\cot(c + d*x))^{(1/2)}*(-e^5/(4*( \\
& b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20* \\
& a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*(512*b^{26}*d^4*e^{10} + 4608*a^2*b^{24}*d \\
& ^4*e^{10} + 17920*a^4*b^{22}*d^4*e^{10} + 38400*a^6*b^{20}*d^4*e^{10} + 46080*a^8*b^{1 \\
& 8*d^4*e^{10} + 21504*a^{10}*b^{16}*d^4*e^{10} - 21504*a^{12}*b^{14}*d^4*e^{10} - 46080*a^ \\
& 14*b^{12}*d^4*e^{10} - 38400*a^{16}*b^{10}*d^4*e^{10} - 17920*a^{18}*b^8*d^4*e^{10} - 460 \\
& 8*a^{20}*b^6*d^4*e^{10} - 512*a^{22}*b^4*d^4*e^{10}))/((b^{17}*d^4 + a^{16}*b*d^4 + 8*a^ \\
& 2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b \\
& ^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4))*(-e^5/(4*(b^6*d^2*1i - a^6*d^2* \\
& 1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2 \\
& *d^2*15i)))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)}*(8*a^{19}*b*d^2*e^{15} - 1472*a*b^1 \\
& 9*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + 11328*a^5*b^{15}*d^2*e^{15} + 10208*a^7*b^ \\
& 13*d^2*e^{15} - 5056*a^9*b^{11}*d^2*e^{15} - 5328*a^{11}*b^9*d^2*e^{15} + 4032*a^{13}*b \\
& ^7*d^2*e^{15} + 3552*a^{15}*b^5*d^2*e^{15} + 384*a^{17}*b^3*d^2*e^{15}))/((b^{17}*d^4 + \\
& a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^ \\
& 9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4))*(-e^5/(4*(b^6* \\
& d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3* \\
& b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)}*(-e^5/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a \\
& *b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i \\
& )))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)}*(a^{14}*e^{20} - 32*b^{14}*e^{20} + 97*a^2*b^{12} \\
& *e^{20} - 2082*a^4*b^{10}*e^{20} + 3631*a^6*b^8*e^{20} - 2300*a^8*b^6*e^{20} + 79*a^1
\end{aligned}$$

$$\begin{aligned}
& (0*b^4*e^{20} + 30*a^{12}*b^2*e^{20}))/ (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 2 \\
& 8*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4) * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 \\
& ^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + (((10*a^{16}*b*d^2*e^{18} - 2398*a^2*b^{15}*d^2*e^{18} + 5238*a^4*b^{13}*d^2*e^{18} \\
& + 7386*a^6*b^{11}*d^2*e^{18} - 8322*a^8*b^9*d^2*e^{18} - 5498*a^{10}*b^7*d^2*e^{18} + 2946*a^{12}*b^5*d^2*e^{18} + 382*a^{14}*b^3*d^2*e^{18}) / (b^{17}*d^5 + a^{16}*b*d^5 \\
& + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) - (((832*a*b^{22}*d^4*e^{13} + \\
& 5952*a^3*b^{20}*d^4*e^{13} + 17664*a^5*b^{18}*d^4*e^{13} + 26880*a^7*b^{16}*d^4*e^{13} + 18816*a^9*b^{14}*d^4*e^{13} - 2688*a^{11}*b^{12}*d^4*e^{13} - 16128*a^{13}*b^{10}*d^4* \\
& e^{13} - 13056*a^{15}*b^8*d^4*e^{13} - 4800*a^{17}*b^6*d^4*e^{13} - 704*a^{19}*b^4*d^4* \\
& e^{13}) / (b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) \\
& - ((e*\cot(c + d*x))^{(1/2)} * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} * (512*b^{26}*d^4*e^{10} + 4608*a^2*b^{24}*d^4*e^{10} + 17920*a^4*b^{22}*d^4*e^{10} + 38400*a^6*b^{20}*d^4*e^{10} + 46080*a^8*b^{18}*d^4*e^{10} + 21504*a^{10}*b^{16}*d^4*e^{10} - 21504*a^{12}*b^{14}*d^4*e^{10} - 46080*a^{14}*b^{12}*d^4*e^{10} - 38400*a^{16}*b^{10}*d^4*e^{10} - 17920*a^{18}*b^8*d^4*e^{10} - 4608*a^{20}*b^6*d^4*e^{10} - 512*a^{22}*b^4*d^4*e^{10})) / (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4) * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)} * (8*a^{19}*b*d^2*e^{15} - 1472*a*b^{19}*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + 11328*a^5*b^{15}*d^2*e^{15} + 10208*a^7*b^{13}*d^2*e^{15} - 5056*a^9*b^{11}*d^2*e^{15} - 5328*a^{11}*b^9*d^2*e^{15} + 4032*a^{13}*b^7*d^2*e^{15} + 3552*a^{15}*b^5*d^2*e^{15} + 384*a^{17}*b^3*d^2*e^{15})) / (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4) * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)} * (a^{14}*e^{20} - 32*b^{14}*e^{20} + 97*a^2*b^{12}*e^{20} - 2082*a^4*b^{10}*e^{20} + 3631*a^6*b^8*e^{20} - 2300*a^8*b^6*e^{20} + 79*a^{10}*b^4*e^{20} + 30*a^{12}*b^2*e^{20})) / (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d^4) * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} + (a^{11}*e^{23} - 120*a*b^{10}*e^{23} + 249*a^3*b^8*e^{23} - 388*a^5*b^6*e^{23} + 302*a^7*b^4*e^{23} + 36*a^9*b^2*e^{23}) / (b^{17}*d^5 + a^{16}*b*d^5 + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5)) * (-e^5 / (4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^{(1/2)} * 2i + (\operatorname{atan}((((e*\cot(c + d*x))^{(1/2)} * (a^{14}*e^{20} - 32*b^{14}*e^{20} + 97*a^2*b^{12}*e^{20} - 2082*a^4*b^{10}*e^{20}
\end{aligned}$$

$$\begin{aligned}
& 0 + 3631*a^6*b^8*e^{20} - 2300*a^8*b^6*e^{20} + 79*a^{10}*b^4*e^{20} + 30*a^{12}*b^2* \\
& e^{20}))/ (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b \\
& ^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d \\
& ^4) - (((10*a^{16}*b*d^2*e^{18} - 2398*a^2*b^{15}*d^2*e^{18} + 5238*a^4*b^{13}*d^2*e^{18} \\
& + 7386*a^6*b^{11}*d^2*e^{18} - 8322*a^8*b^9*d^2*e^{18} - 5498*a^{10}*b^7*d^2*e^{18} \\
& + 2946*a^{12}*b^5*d^2*e^{18} + 382*a^{14}*b^3*d^2*e^{18}))/ (b^{17}*d^5 + a^{16}*b*d^5 \\
& + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56* \\
& a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) - (((e*\cot(c + d*x))^{(1/2)} \\
& )*(8*a^{19}*b*d^2*e^{15} - 1472*a*b^{19}*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + 11328 \\
& *a^5*b^{15}*d^2*e^{15} + 10208*a^7*b^{13}*d^2*e^{15} - 5056*a^9*b^{11}*d^2*e^{15} - 532 \\
& 8*a^{11}*b^9*d^2*e^{15} + 4032*a^{13}*b^7*d^2*e^{15} + 3552*a^{15}*b^5*d^2*e^{15} + 384 \\
& *a^{17}*b^3*d^2*e^{15}))/ (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}* \\
& d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 \\
& + 8*a^{14}*b^3*d^4) + (((832*a*b^{22}*d^4*e^{13} + 5952*a^3*b^{20}*d^4*e^{13} + 17664 \\
& *a^5*b^{18}*d^4*e^{13} + 26880*a^7*b^{16}*d^4*e^{13} + 18816*a^9*b^{14}*d^4*e^{13} - 26 \\
& 88*a^{11}*b^{12}*d^4*e^{13} - 16128*a^{13}*b^{10}*d^4*e^{13} - 13056*a^{15}*b^8*d^4*e^{13} \\
& - 4800*a^{17}*b^6*d^4*e^{13} - 704*a^{19}*b^4*d^4*e^{13}))/ (b^{17}*d^5 + a^{16}*b*d^5 + \\
& 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56*a^ \\
& 10*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) - ((e*\cot(c + d*x))^{(1/2)}*(a \\
& ^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^{(1/2)}*(512*b^{26}*d^4*e^{10} + 4608*a^2* \\
& b^{24}*d^4*e^{10} + 17920*a^4*b^{22}*d^4*e^{10} + 38400*a^6*b^{20}*d^4*e^{10} + 46080*a \\
& ^8*b^{18}*d^4*e^{10} + 21504*a^{10}*b^{16}*d^4*e^{10} - 21504*a^{12}*b^{14}*d^4*e^{10} - 46 \\
& 080*a^{14}*b^{12}*d^4*e^{10} - 38400*a^{16}*b^{10}*d^4*e^{10} - 17920*a^{18}*b^8*d^4*e^{10} \\
& - 4608*a^{20}*b^6*d^4*e^{10} - 512*a^{22}*b^4*d^4*e^{10}))/ (8*(b^9*d + 3*a^2*b^7*d \\
& + 3*a^4*b^5*d + a^6*b^3*d)*(b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^ \\
& 4*b^{13}*d^4 + 56*a^6*b^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b \\
& ^5*d^4 + 8*a^{14}*b^3*d^4)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^{(1/2)}))/ \\
& (8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2 \\
& *b^2)*(-a*b^3*e^5)^{(1/2)}))/ (8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d \\
& ))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^{(1/2)}))/ (8*(b^9*d + 3*a^2*b^7*d \\
& + 3*a^4*b^5*d + a^6*b^3*d)))*(a^4 - 15*b^4 + 18*a^2*b^2)*(-a*b^3*e^5)^{(1/2)} \\
& )*1i))/ (8*(b^9*d + 3*a^2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d)) + (((e*\cot(c + d \\
& *x))^{(1/2)}*(a^{14}*e^{20} - 32*b^{14}*e^{20} + 97*a^2*b^{12}*e^{20} - 2082*a^4*b^{10}*e^{20} \\
& 0 + 3631*a^6*b^8*e^{20} - 2300*a^8*b^6*e^{20} + 79*a^{10}*b^4*e^{20} + 30*a^{12}*b^2* \\
& e^{20}))/ (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*d^4 + 56*a^6*b \\
& ^{11}*d^4 + 70*a^8*b^9*d^4 + 56*a^{10}*b^7*d^4 + 28*a^{12}*b^5*d^4 + 8*a^{14}*b^3*d \\
& ^4) + (((10*a^{16}*b*d^2*e^{18} - 2398*a^2*b^{15}*d^2*e^{18} + 5238*a^4*b^{13}*d^2*e^{18} \\
& + 7386*a^6*b^{11}*d^2*e^{18} - 8322*a^8*b^9*d^2*e^{18} - 5498*a^{10}*b^7*d^2*e^{18} \\
& + 2946*a^{12}*b^5*d^2*e^{18} + 382*a^{14}*b^3*d^2*e^{18}))/ (b^{17}*d^5 + a^{16}*b*d^5 \\
& + 8*a^2*b^{15}*d^5 + 28*a^4*b^{13}*d^5 + 56*a^6*b^{11}*d^5 + 70*a^8*b^9*d^5 + 56* \\
& a^{10}*b^7*d^5 + 28*a^{12}*b^5*d^5 + 8*a^{14}*b^3*d^5) + (((e*\cot(c + d*x))^{(1/2)} \\
& )*(8*a^{19}*b*d^2*e^{15} - 1472*a*b^{19}*d^2*e^{15} + 776*a^3*b^{17}*d^2*e^{15} + 11328 \\
& *a^5*b^{15}*d^2*e^{15} + 10208*a^7*b^{13}*d^2*e^{15} - 5056*a^9*b^{11}*d^2*e^{15} - 532 \\
& 8*a^{11}*b^9*d^2*e^{15} + 4032*a^{13}*b^7*d^2*e^{15} + 3552*a^{15}*b^5*d^2*e^{15} + 384 \\
& *a^{17}*b^3*d^2*e^{15}))/ (b^{17}*d^4 + a^{16}*b*d^4 + 8*a^2*b^{15}*d^4 + 28*a^4*b^{13}*
\end{aligned}$$





$2*b^7*d + 3*a^4*b^5*d + a^6*b^3*d))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c + dx))^{\frac{5}{2}}}{(a + b \cot(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))\*\*(5/2)/(a+b\*cot(d\*x+c))\*\*3, x)

[Out] Integral((e\*cot(c + d\*x))\*\*(5/2)/(a + b\*cot(c + d\*x))\*\*3, x)

$$3.84 \quad \int \frac{(e \cot(c+dx))^{3/2}}{(a+b \cot(c+dx))^3} dx$$

Optimal. Leaf size=461

$$\frac{e^{3/2}(a-b)(a^2+4ab+b^2)\log(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3} + \frac{e^{3/2}(a-b)(a^2+4ab+b^2)\log(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}-\sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

[Out]  $-1/2*(a+b)*(a^2-4*a*b+b^2)*e^{(3/2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/2*(a+b)*(a^2-4*a*b+b^2)*e^{(3/2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(a-b)*(a^2+4*a*b+b^2)*e^{(3/2)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}+1/4*(a-b)*(a^2+4*a*b+b^2)*e^{(3/2)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}-1/4*(3*a^4-26*a^2*b^2+3*b^4)*e^{(3/2)*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d/a^{(1/2)}/b^{(1/2)}-1/2*a*e*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)/d/(a+b*\cot(d*x+c))^{(1/2)}-1/4*(3*a^2-5*b^2)*e*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)^2/d/(a+b*\cot(d*x+c))}$

**Rubi [A]** time = 1.23, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3567, 3649, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{e^{3/2}(a-b)(a^2+4ab+b^2)\log(\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}+\sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3} + \frac{e^{3/2}(a-b)(a^2+4ab+b^2)\log(\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}-\sqrt{e})}{2\sqrt{2}d(a^2+b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(e\*Cot[c + d\*x])^(3/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out]  $-((3*a^4 - 26*a^2*b^2 + 3*b^4)*e^{(3/2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[a]*\text{Sqrt}[e])]})/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*(a^2 + b^2)^3*d) - ((a + b)*(a^2 - 4*a*b + b^2)*e^{(3/2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e])]})/(\text{Sqrt}[2]*(a^2 + b^2)^3*d) + ((a + b)*(a^2 - 4*a*b + b^2)*e^{(3/2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e])]})/(\text{Sqrt}[2]*(a^2 + b^2)^3*d) - (a*e*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*\text{Cot}[c + d*x])^2) - ((3*a^2 - 5*b^2)*e*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(4*(a^2 + b^2)^2*d*(a + b*\text{Cot}[c + d*x])) - ((a - b)*(a^2 + 4*a*b + b^2)*e^{(3/2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]]]})/(2*\text{Sqrt}[2]*(a^2 + b^2)^3*d) + ((a - b)*(a^2 + 4*a*b + b^2)*e^{(3/2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]]]})/(2*\text{Sqrt}[2]*(a^2 + b^2)^3*d)$



Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$\text{eQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[(d + (e \cdot x^2)/(a + (c \cdot x^4))) / x_{\text{Symbol}}] \rightarrow \text{With}\{q = \text{Rt}[a \cdot c, 2]\}, \text{Dist}[(d \cdot q + a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q + c \cdot x^2)/(a + c \cdot x^4), x], x] + \text{Dist}[(d \cdot q - a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q - c \cdot x^2)/(a + c \cdot x^4), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \ \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[-(a \cdot c)]$

### Rule 3534

$\text{Int}[(c + (d \cdot \tan[e + (f \cdot x)]) / \text{Sqrt}[b \cdot \tan[e + (f \cdot x)]]], x_{\text{Symbol}}] \rightarrow \text{Dist}[2/f, \text{Subst}[\text{Int}[(b \cdot c + d \cdot x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b \cdot \tan[e + f \cdot x]]], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3567

$\text{Int}[(a + (b \cdot \tan[e + (f \cdot x)])^m) \cdot (c + (d \cdot \tan[e + (f \cdot x)]))^n], x_{\text{Symbol}}] \rightarrow \text{Simp}[(b \cdot c - a \cdot d) \cdot (a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^{n-1} / (f \cdot (m+1) \cdot (a^2 + b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (a^2 + b^2)), \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^{n-2} \cdot \text{Simp}[a \cdot c^2 \cdot (m+1) + a \cdot d^2 \cdot (n-1) + b \cdot c \cdot d \cdot (m-n+2) - (b \cdot c^2 - 2 \cdot a \cdot c \cdot d - b \cdot d^2) \cdot (m+1) \cdot \tan[e + f \cdot x] - d \cdot (b \cdot c - a \cdot d) \cdot (m+n) \cdot \tan[e + f \cdot x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[1, n, 2] \ \&\& \ \text{IntegerQ}[2 \cdot m]$

### Rule 3634

$\text{Int}[(a + (b \cdot \tan[e + (f \cdot x)])^m) \cdot (c + (d \cdot \tan[e + (f \cdot x)]))^n \cdot (A + (C \cdot \tan[e + (f \cdot x)]))^2], x_{\text{Symbol}}] \rightarrow \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x, \tan[e + f \cdot x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n, x\} \ \&\& \ \text{EqQ}[A, C]$

### Rule 3649

$\text{Int}[(a + (b \cdot \tan[e + (f \cdot x)])^m) \cdot (c + (d \cdot \tan[e + (f \cdot x)]))^n \cdot (A + (B \cdot \tan[e + (f \cdot x)] + C \cdot \tan[e + (f \cdot x)]))^2], x_{\text{Symbol}}] \rightarrow \text{Simp}[(A \cdot b^2 - a \cdot (b \cdot B - a \cdot C)) \cdot (a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^{n+1} / (f \cdot (m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 + b^2)), \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot (c + d \cdot \tan[e + f \cdot x])^n \cdot \text{Simp}[A \cdot (a \cdot (b \cdot c - a \cdot d) \cdot (m+1) - b^2 \cdot d \cdot (m+n+2)) + (b \cdot B - a \cdot C) \cdot (b \cdot c \cdot (m+1) + a \cdot d \cdot (n+1)) - (m+1) \cdot (b \cdot c - a \cdot d)]$

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e \cot(c + dx))^{3/2}}{(a + b \cot(c + dx))^3} dx &= -\frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} - \frac{\int \frac{\frac{ae^2}{2} - 2be^2 \cot(c+dx) - \frac{3}{2}ae^2 \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))^2} dx}{2(a^2 + b^2)} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} - \frac{(3a^2 - 5b^2)e\sqrt{e \cot(c + dx)}}{4(a^2 + b^2)^2 d(a + b \cot(c + dx))} + \frac{\int \frac{-\frac{1}{4}a(5a^2 - 3b^2)}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} dx}{2(a^2 + b^2)} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} - \frac{(3a^2 - 5b^2)e\sqrt{e \cot(c + dx)}}{4(a^2 + b^2)^2 d(a + b \cot(c + dx))} + \frac{\int \frac{-2a^2(a^2 - 3b^2)}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} dx}{2(a^2 + b^2)} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} - \frac{(3a^2 - 5b^2)e\sqrt{e \cot(c + dx)}}{4(a^2 + b^2)^2 d(a + b \cot(c + dx))} + \frac{\text{Subst}\left(\int \frac{2a^2}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} dx\right)}{2(a^2 + b^2)} \\
&= -\frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} - \frac{(3a^2 - 5b^2)e\sqrt{e \cot(c + dx)}}{4(a^2 + b^2)^2 d(a + b \cot(c + dx))} - \frac{((3a^4 - 26a^2b^2 + 3b^4)e^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right))}{4\sqrt{a} \sqrt{b} (a^2 + b^2)^3 d} - \frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} \\
&= -\frac{(3a^4 - 26a^2b^2 + 3b^4)e^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{4\sqrt{a} \sqrt{b} (a^2 + b^2)^3 d} - \frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} \\
&= -\frac{(3a^4 - 26a^2b^2 + 3b^4)e^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{4\sqrt{a} \sqrt{b} (a^2 + b^2)^3 d} - \frac{ae\sqrt{e \cot(c + dx)}}{2(a^2 + b^2)d(a + b \cot(c + dx))^2} \\
&= -\frac{(3a^4 - 26a^2b^2 + 3b^4)e^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{4\sqrt{a} \sqrt{b} (a^2 + b^2)^3 d} - \frac{(a + b)(a^2 - 4ab + b^2)e^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{\sqrt{2} (a^2 + b^2)^3}
\end{aligned}$$

**Mathematica [C]** time = 6.16, size = 518, normalized size = 1.12

$$(e \cot(c + dx))^{3/2} \left( \frac{4b^2 \cot^{\frac{5}{2}}(c+dx) {}_2F_1\left(2, \frac{5}{2}; \frac{7}{2}; -\frac{b \cot(c+dx)}{a}\right)}{5a(a^2+b^2)^2} - \frac{2b(3a^2-b^2)\left(\cot^{\frac{3}{2}}(c+dx) - \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)\right)}{3(a^2+b^2)^3} + \frac{2b(3a^2-b^2)}{3(a^2+b^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*Cot[c + d\*x])^(3/2)/(a + b\*Cot[c + d\*x])^3,x]

[Out] -(((e\*Cot[c + d\*x])^(3/2)\*((-2\*a\*(3\*a^2 - b^2)\*(-(Sqrt[a]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]])/Sqrt[b]) + Sqrt[Cot[c + d\*x]])))/(a^2 + b^2)^3 + (2\*b\*(3\*a^2 - b^2)\*Cot[c + d\*x]^(3/2))/(3\*(a^2 + b^2)^3) - ((-3\*Sqrt[b]\*ArcTan[(Sqrt[b]\*Sqrt[Cot[c + d\*x]])/Sqrt[a]]\*Sqrt[Cot[c + d\*x]])/Sqrt[a] + (2\*b^2\*Cot[c + d\*x]^2)/(a + b\*Cot[c + d\*x])^2 + (3\*b\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x]))/(4\*b\*(a^2 + b^2)\*Sqrt[Cot[c + d\*x]]) - (2\*b\*(3\*a^2 - b^2)\*(Cot[c + d\*x]^(3/2) - Cot[c + d\*x]^(3/2)\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d\*x]^2]))/(3\*(a^2 + b^2)^3) + (4\*b^2\*Cot[c + d\*x]^(5/2)\*Hypergeometric2F1[2, 5/2, 7/2, -(b\*Cot[c + d\*x])/a]))/(5\*a\*(a^2 + b^2)^2) + (a\*(a^2 - 3\*b^2)\*(2\*(Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]]) - Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]])] + 8\*Sqrt[Cot[c + d\*x]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Cot[c + d\*x]] + Cot[c + d\*x]]))/(4\*(a^2 + b^2)^3)))/(d\*Cot[c + d\*x]^(3/2))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(dx + c))^{\frac{3}{2}}}{(b \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((e\*cot(d\*x + c))^(3/2)/(b\*cot(d\*x + c) + a)^3, x)

**maple** [B] time = 0.87, size = 1212, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^3,x)

```
[Out] -3/4/d*e^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)*b*a^4+1/
2/d*e^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)*a^2*b^3+5/4
/d*e^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)*b^5-5/4/d*e^
3/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)*a^5-1/2/d*e^3/(a^
2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)*a^3*b^2+3/4/d*e^3/(a^2
+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)*a*b^4-3/4/d*e^2/(a^2+b^
2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))*a^4+13/2/d*
e^2/(a^2+b^2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))*
a^2*b^2-3/4/d*e^2/(a^2+b^2)^3/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(
a*e*b)^(1/2))*b^4-1/2/d*e/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^3+3/2/d*e/(a^2+b^2)^3*(e^2)^(1/4)*2^(1
/2)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a*b^2+1/4/d*e/(a^2+
b^2)^3*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
))*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2)))*a^3-3/4/d*e/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*ln((e*cot(d*x+
c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*a*b^2+1/2/d*e/(a^2+b^2)^
3*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^
3-3/2/d*e/(a^2+b^2)^3*(e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)*a*b^2+3/2/d*e^2/(a^2+b^2)^3*2^(1/2)/(e^2)^(1/4)*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*a^2*b-1/2/d*e^2/(a^2+b^2)^3*2^(1
/2)/(e^2)^(1/4)*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*b^3-3/2
/d*e^2/(a^2+b^2)^3*2^(1/2)/(e^2)^(1/4)*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)*a^2*b+1/2/d*e^2/(a^2+b^2)^3*2^(1/2)/(e^2)^(1/4)*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)*b^3-3/4/d*e^2/(a^2+b^2)^3*2^(1/2)/(e
^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*
a^2*b+1/4/d*e^2/(a^2+b^2)^3*2^(1/2)/(e^2)^(1/4)*ln((e*cot(d*x+c)-(e^2)^(1/4
))*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))*b^3
```

**maxima** [A] time = 0.78, size = 492, normalized size = 1.07

$$\left( \frac{(3a^4 - 26a^2b^2 + 3b^4)e \arctan\left(\frac{b\sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)\sqrt{abe}} - \frac{2\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} + \frac{2\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e} + 2\sqrt{\frac{e}{\tan(dx+c)}}\right)}{2\sqrt{e}}\right)}{\sqrt{e}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 
$$-1/4*((3*a^4 - 26*a^2*b^2 + 3*b^4)*e*\arctan(b*\sqrt{e/\tan(d*x + c)})/\sqrt{a*b*e})/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*\sqrt{a*b*e}) - (2*\sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\arctan(1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} + 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + 2*\sqrt{2}*(a^3 - 3*a^2*b - 3*a*b^2 + b^3)*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*\sqrt{e} - 2*\sqrt{e/\tan(d*x + c)}))/\sqrt{e})/\sqrt{e} + \sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\log(\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e} - \sqrt{2}*(a^3 + 3*a^2*b - 3*a*b^2 - b^3)*\log(-\sqrt{2}*\sqrt{e}*\sqrt{e/\tan(d*x + c)} + e + e/\tan(d*x + c))/\sqrt{e})*e/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((5*a^3 - 3*a*b^2)*e^2*\sqrt{e/\tan(d*x + c)} + (3*a^2*b - 5*b^3)*e*(e/\tan(d*x + c))^(3/2))/((a^6 + 2*a^4*b^2 + a^2*b^4)*e^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*e^2/\tan(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*e^2/\tan(d*x + c)^2))*e/d$$

**mupad [B]** time = 6.21, size = 19000, normalized size = 41.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*cot(c + d\*x))^(3/2)/(a + b\*cot(c + d\*x))^3,x)

[Out] 
$$\operatorname{atan}\left(\frac{((518*a*b^{15}*d^2*e^{15} - 18*a^{15}*b*d^2*e^{15} - 4494*a^3*b^{13}*d^2*e^{15} + 3022*a^5*b^{11}*d^2*e^{15} + 17194*a^7*b^9*d^2*e^{15} + 5298*a^9*b^7*d^2*e^{15} - 3338*a^{11}*b^5*d^2*e^{15} + 506*a^{13}*b^3*d^2*e^{15})/(a^{16}*d^5 + b^{16}*d^5 + 8*a^2*b^{14}*d^5 + 28*a^4*b^{12}*d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + 56*a^{10}*b^6*d^5 + 28*a^{12}*b^4*d^5 + 8*a^{14}*b^2*d^5) + (((4224*a^4*b^{18}*d^4*e^{12} - 320*a^2*b^{20}*d^4*e^{12} - 192*b^{22}*d^4*e^{12} + 22272*a^6*b^{16}*d^4*e^{12} + 51072*a^8*b^{14}*d^4*e^{12} + 67200*a^{10}*b^{12}*d^4*e^{12} + 53760*a^{12}*b^{10}*d^4*e^{12} + 25344*a^{14}*b^8*d^4*e^{12} + 5952*a^{16}*b^6*d^4*e^{12} + 192*a^{18}*b^4*d^4*e^{12} - 128*a^{20}*b^2*d^4*e^{12})/(a^{16}*d^5 + b^{16}*d^5 + 8*a^2*b^{14}*d^5 + 28*a^4*b^{12}*d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + 56*a^{10}*b^6*d^5 + 28*a^{12}*b^4*d^5 + 8*a^{14}*b^2*d^5) + ((e*\cot(c + d*x))^{1/2}*((e^3*i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{1/2}*(512*b^{25}*d^4*e^{10} + 4608*a^2*b^{23}*d^4*e^{10} + 17920*a^4*b^{21}*d^4*e^{10} + 38400*a^6*b^{19}*d^4*e^{10} + 46080*a^8*b^{17}*d^4*e^{10} + 21504*a^{10}*b^{15}*d^4*e^{10} - 21504*a^{12}*b^{13}*d^4*e^{10} - 46080*a^{14}*b^{11}*d^4*e^{10} - 38400*a^{16}*b^9*d^4*e^{10} - 17920*a^{18}*b^7*d^4*e^{10} - 4608*a^{20}*b^5*d^4*e^{10} - 512*a^{22}*b^3*d^4*e^{10})/(a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + 28*a^4*b^{12}*d^4 + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28*a^{12}*b^4*d^4 + 8*a^{14}*b^2*d^4)}*((e^3*i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{1/2} - ((e*\cot(c + d*x))^{1/2}*(1544*a*b^{18}*d^2*e^{13} + 64*a^3*b^{16}*d^2*e^{13} - 7456*a^5*b^$$

$$\begin{aligned}
& 14*d^2*e^{13} - 576*a^7*b^{12}*d^2*e^{13} + 19504*a^9*b^{10}*d^2*e^{13} + 18880*a^{11}* \\
& b^8*d^2*e^{13} + 3808*a^{13}*b^6*d^2*e^{13} - 960*a^{15}*b^4*d^2*e^{13} + 8*a^{17}*b^2* \\
& d^2*e^{13}))/((a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + 28*a^4*b^{12}*d^4 + 56*a^6* \\
& b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28*a^{12}*b^4*d^4 + 8*a^{14}*b^2* \\
& *d^4))*((e^3*i)/((4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2* \\
& b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)})*((e^3*i)/((4*(b^6*d^2 \\
& ^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*2 \\
& 0i + 15*a^4*b^2*d^2))))^{(1/2)} + ((e*\cot(c + d*x))^{(1/2)}*(41*b^{13}*e^{16} + 9*a^{12}* \\
& b*e^{16} - 82*a^2*b^{11}*e^{16} + 1831*a^4*b^9*e^{16} - 4348*a^6*b^7*e^{16} + 1671 \\
& *a^8*b^5*e^{16} - 210*a^{10}*b^3*e^{16}))/((a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + \\
& 28*a^4*b^{12}*d^4 + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28* \\
& a^{12}*b^4*d^4 + 8*a^{14}*b^2*d^4))*((e^3*i)/((4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2 \\
& *6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{( \\
& 1/2)}*i - (((518*a*b^{15}*d^2*e^{15} - 18*a^{15}*b*d^2*e^{15} - 4494*a^3*b^{13}*d^2*e \\
& ^{15} + 3022*a^5*b^{11}*d^2*e^{15} + 17194*a^7*b^9*d^2*e^{15} + 5298*a^9*b^7*d^2*e^{15} \\
& - 3338*a^{11}*b^5*d^2*e^{15} + 506*a^{13}*b^3*d^2*e^{15}))/((a^{16}*d^5 + b^{16}*d^5 + \\
& 8*a^2*b^{14}*d^5 + 28*a^4*b^{12}*d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + 56*a^{10}* \\
& b^6*d^5 + 28*a^{12}*b^4*d^5 + 8*a^{14}*b^2*d^5) + (((4224*a^4*b^{18}*d^4*e^{12} \\
& - 320*a^2*b^{20}*d^4*e^{12} - 192*b^{22}*d^4*e^{12} + 22272*a^6*b^{16}*d^4*e^{12} + 51 \\
& 072*a^8*b^{14}*d^4*e^{12} + 67200*a^{10}*b^{12}*d^4*e^{12} + 53760*a^{12}*b^{10}*d^4*e^{12} \\
& + 25344*a^{14}*b^8*d^4*e^{12} + 5952*a^{16}*b^6*d^4*e^{12} + 192*a^{18}*b^4*d^4*e^{12} \\
& - 128*a^{20}*b^2*d^4*e^{12}))/((a^{16}*d^5 + b^{16}*d^5 + 8*a^2*b^{14}*d^5 + 28*a^4*b^{12}* \\
& d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + 56*a^{10}*b^6*d^5 + 28*a^{12}*b^4*d^5 \\
& ^5 + 8*a^{14}*b^2*d^5) - ((e*\cot(c + d*x))^{(1/2)})*((e^3*i)/((4*(b^6*d^2 - a^6* \\
& d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4* \\
& b^2*d^2))))^{(1/2)}*(512*b^{25}*d^4*e^{10} + 4608*a^2*b^{23}*d^4*e^{10} + 17920*a^4* \\
& b^{21}*d^4*e^{10} + 38400*a^6*b^{19}*d^4*e^{10} + 46080*a^8*b^{17}*d^4*e^{10} + 21504* \\
& a^{10}*b^{15}*d^4*e^{10} - 21504*a^{12}*b^{13}*d^4*e^{10} - 46080*a^{14}*b^{11}*d^4*e^{10} - \\
& 38400*a^{16}*b^9*d^4*e^{10} - 17920*a^{18}*b^7*d^4*e^{10} - 4608*a^{20}*b^5*d^4*e^{10} \\
& - 512*a^{22}*b^3*d^4*e^{10}))/((a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + 28*a^4*b^{12}* \\
& d^4 + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28*a^{12}*b^4*d^4 \\
& ^4 + 8*a^{14}*b^2*d^4))*((e^3*i)/((4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5* \\
& b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)} + ((e \\
& *\cot(c + d*x))^{(1/2)}*(1544*a*b^{18}*d^2*e^{13} + 64*a^3*b^{16}*d^2*e^{13} - 7456*a^5* \\
& b^{14}*d^2*e^{13} - 576*a^7*b^{12}*d^2*e^{13} + 19504*a^9*b^{10}*d^2*e^{13} + 18880*a^{11}* \\
& b^8*d^2*e^{13} + 3808*a^{13}*b^6*d^2*e^{13} - 960*a^{15}*b^4*d^2*e^{13} + 8*a^{17}* \\
& b^2*d^2*e^{13}))/((a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + 28*a^4*b^{12}*d^4 + 56 \\
& *a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28*a^{12}*b^4*d^4 + 8*a^{14} \\
& *b^2*d^4))*((e^3*i)/((4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2* \\
& b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))))^{(1/2)})*((e^3*i)/((4*(b^6*d^2 \\
& ^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4* \\
& b^2*d^2))))^{(1/2)} - ((e*\cot(c + d*x))^{(1/2)}*(41*b^{13}*e^{16} + \\
& 9*a^{12}*b*e^{16} - 82*a^2*b^{11}*e^{16} + 1831*a^4*b^9*e^{16} - 4348*a^6*b^7*e^{16} + \\
& 1671*a^8*b^5*e^{16} - 210*a^{10}*b^3*e^{16}))/((a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}* \\
& ^4 + 28*a^4*b^{12}*d^4 + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 +
\end{aligned}$$



$$\begin{aligned}
& (28a^{12}b^4d^4 + 8a^{14}b^2d^4)) * ((e^{3*1i}) / (4*(b^6d^2 - a^6d^2 + a*b^5 \\
& *d^2*6i + a^5*b*d^2*6i - 15a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15a^4*b^2*d^2) \\
& ))^{(1/2)*1i} / (((518*a*b^{15}*d^2*e^{15} - 18*a^{15}*b*d^2*e^{15} - 4494*a^3*b^{13}*d \\
& ^2*e^{15} + 3022*a^5*b^{11}*d^2*e^{15} + 17194*a^7*b^9*d^2*e^{15} + 5298*a^9*b^7*d \\
& ^2*e^{15} - 3338*a^{11}*b^5*d^2*e^{15} + 506*a^{13}*b^3*d^2*e^{15}) / (a^{16}*d^5 + b^{16}*d \\
& ^5 + 8*a^2*b^{14}*d^5 + 28*a^4*b^{12}*d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + \\
& 56*a^{10}*b^6*d^5 + 28*a^{12}*b^4*d^5 + 8*a^{14}*b^2*d^5) + (((4224*a^4*b^{18}*d^4* \\
& e^{12} - 320*a^2*b^{20}*d^4*e^{12} - 192*b^{22}*d^4*e^{12} + 22272*a^6*b^{16}*d^4*e^{12} \\
& + 51072*a^8*b^{14}*d^4*e^{12} + 67200*a^{10}*b^{12}*d^4*e^{12} + 53760*a^{12}*b^{10}*d^4* \\
& e^{12} + 25344*a^{14}*b^8*d^4*e^{12} + 5952*a^{16}*b^6*d^4*e^{12} + 192*a^{18}*b^4*d^4* \\
& e^{12} - 128*a^{20}*b^2*d^4*e^{12}) / (a^{16}*d^5 + b^{16}*d^5 + 8*a^2*b^{14}*d^5 + 28*a^ \\
& 4*b^{12}*d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + 56*a^{10}*b^6*d^5 + 28*a^{12}*b \\
& ^4*d^5 + 8*a^{14}*b^2*d^5) + ((e*cot(c + d*x))^{(1/2)} * ((e^{3*1i}) / (4*(b^6*d^2 - \\
& a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + \\
& 15*a^4*b^2*d^2)))^{(1/2)} * (512*b^{25}*d^4*e^{10} + 4608*a^2*b^{23}*d^4*e^{10} + 17920 \\
& *a^4*b^{21}*d^4*e^{10} + 38400*a^6*b^{19}*d^4*e^{10} + 46080*a^8*b^{17}*d^4*e^{10} + 21 \\
& 504*a^{10}*b^{15}*d^4*e^{10} - 21504*a^{12}*b^{13}*d^4*e^{10} - 46080*a^{14}*b^{11}*d^4*e^{10} \\
& - 38400*a^{16}*b^9*d^4*e^{10} - 17920*a^{18}*b^7*d^4*e^{10} - 4608*a^{20}*b^5*d^4*e \\
& ^{10} - 512*a^{22}*b^3*d^4*e^{10}) / (a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + 28*a^ \\
& 4*b^{12}*d^4 + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28*a^{12}*b \\
& ^4*d^4 + 8*a^{14}*b^2*d^4)) * ((e^{3*1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + \\
& a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} - \\
& ((e*cot(c + d*x))^{(1/2)} * (1544*a*b^{18}*d^2*e^{13} + 64*a^3*b^{16}*d^2*e^{13} - 745 \\
& 6*a^5*b^{14}*d^2*e^{13} - 576*a^7*b^{12}*d^2*e^{13} + 19504*a^9*b^{10}*d^2*e^{13} + 188 \\
& 80*a^{11}*b^8*d^2*e^{13} + 3808*a^{13}*b^6*d^2*e^{13} - 960*a^{15}*b^4*d^2*e^{13} + 8*a \\
& ^{17}*b^2*d^2*e^{13}) / (a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^{14}*d^4 + 28*a^4*b^{12}*d^4 \\
& + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d^4 + 28*a^{12}*b^4*d^4 + 8* \\
& a^{14}*b^2*d^4)) * ((e^{3*1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6 \\
& i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} * ((e^{3*1i}) / ( \\
& 4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b \\
& ^3*d^2*20i + 15*a^4*b^2*d^2)))^{(1/2)} + ((e*cot(c + d*x))^{(1/2)} * (41*b^{13}*e^{1 \\
& 6} + 9*a^{12}*b*e^{16} - 82*a^2*b^{11}*e^{16} + 1831*a^4*b^9*e^{16} - 4348*a^6*b^7*e^{1 \\
& 6} + 1671*a^8*b^5*e^{16} - 210*a^{10}*b^3*e^{16})) / (a^{16}*d^4 + b^{16}*d^4 + 8*a^2*b^ \\
& ^{14}*d^4 + 28*a^4*b^{12}*d^4 + 56*a^6*b^{10}*d^4 + 70*a^8*b^8*d^4 + 56*a^{10}*b^6*d \\
& ^4 + 28*a^{12}*b^4*d^4 + 8*a^{14}*b^2*d^4)) * ((e^{3*1i}) / (4*(b^6*d^2 - a^6*d^2 + a \\
& *b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2* \\
& d^2)))^{(1/2)} + (((518*a*b^{15}*d^2*e^{15} - 18*a^{15}*b*d^2*e^{15} - 4494*a^3*b^{13}* \\
& d^2*e^{15} + 3022*a^5*b^{11}*d^2*e^{15} + 17194*a^7*b^9*d^2*e^{15} + 5298*a^9*b^7*d \\
& ^2*e^{15} - 3338*a^{11}*b^5*d^2*e^{15} + 506*a^{13}*b^3*d^2*e^{15}) / (a^{16}*d^5 + b^{16}* \\
& d^5 + 8*a^2*b^{14}*d^5 + 28*a^4*b^{12}*d^5 + 56*a^6*b^{10}*d^5 + 70*a^8*b^8*d^5 + \\
& 56*a^{10}*b^6*d^5 + 28*a^{12}*b^4*d^5 + 8*a^{14}*b^2*d^5) + (((4224*a^4*b^{18}*d^4 \\
& *e^{12} - 320*a^2*b^{20}*d^4*e^{12} - 192*b^{22}*d^4*e^{12} + 22272*a^6*b^{16}*d^4*e^{12} \\
& + 51072*a^8*b^{14}*d^4*e^{12} + 67200*a^{10}*b^{12}*d^4*e^{12} + 53760*a^{12}*b^{10}*d^4 \\
& *e^{12} + 25344*a^{14}*b^8*d^4*e^{12} + 5952*a^{16}*b^6*d^4*e^{12} + 192*a^{18}*b^4*d^4 \\
& *e^{12} - 128*a^{20}*b^2*d^4*e^{12}) / (a^{16}*d^5 + b^{16}*d^5 + 8*a^2*b^{14}*d^5 + 28*a
\end{aligned}$$

$$\begin{aligned}
& ^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5) - ((e^{\cot(c+dx)})^{1/2} * ((e^{3i}) / (4(b^6d^2 - a^6d^2 + a^5b^2d^2 * 6i + a^5b^2d^2 * 6i - 15a^2b^4d^2 - a^3b^3d^2 * 20i + 15a^4b^2d^2))))^{1/2} * (512b^{25}d^4e^{10} + 4608a^2b^{23}d^4e^{10} + 17920a^4b^{21}d^4e^{10} + 38400a^6b^{19}d^4e^{10} + 46080a^8b^{17}d^4e^{10} + 21504a^{10}b^{15}d^4e^{10} - 21504a^{12}b^{13}d^4e^{10} - 46080a^{14}b^{11}d^4e^{10} - 38400a^{16}b^9d^4e^{10} - 17920a^{18}b^7d^4e^{10} - 4608a^{20}b^5d^4e^{10} - 512a^{22}b^3d^4e^{10})) / (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4)) * ((e^{3i}) / (4(b^6d^2 - a^6d^2 + a^5b^2d^2 * 6i + a^5b^2d^2 * 6i - 15a^2b^4d^2 - a^3b^3d^2 * 20i + 15a^4b^2d^2))))^{1/2} \\
& + ((e^{\cot(c+dx)})^{1/2} * (1544a^3b^{18}d^2e^{13} + 64a^3b^{16}d^2e^{13} - 7456a^5b^{14}d^2e^{13} - 576a^7b^{12}d^2e^{13} + 19504a^9b^{10}d^2e^{13} + 18880a^{11}b^8d^2e^{13} + 3808a^{13}b^6d^2e^{13} - 960a^{15}b^4d^2e^{13} + 8a^{17}b^2d^2e^{13})) / (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4)) * ((e^{3i}) / (4(b^6d^2 - a^6d^2 + a^5b^2d^2 * 6i + a^5b^2d^2 * 6i - 15a^2b^4d^2 - a^3b^3d^2 * 20i + 15a^4b^2d^2))))^{1/2} * ((e^{3i}) / (4(b^6d^2 - a^6d^2 + a^5b^2d^2 * 6i + a^5b^2d^2 * 6i - 15a^2b^4d^2 - a^3b^3d^2 * 20i + 15a^4b^2d^2))))^{1/2} - ((e^{\cot(c+dx)})^{1/2} * (41b^{13}e^{16} + 9a^{12}b^5e^{16} - 82a^2b^{11}e^{16} + 1831a^4b^9e^{16} - 4348a^6b^7e^{16} + 1671a^8b^5e^{16} - 210a^{10}b^3e^{16})) / (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4)) * ((e^{3i}) / (4(b^6d^2 - a^6d^2 + a^5b^2d^2 * 6i + a^5b^2d^2 * 6i - 15a^2b^4d^2 - a^3b^3d^2 * 20i + 15a^4b^2d^2))))^{1/2} + (28a^2b^8e^{18} - 15b^{10}e^{18} + 878a^4b^6e^{18} - 180a^6b^4e^{18} + 9a^8b^2e^{18}) / (a^{16}d^5 + b^{16}d^5 + 8a^2b^{14}d^5 + 28a^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5)) * ((e^{3i}) / (4(b^6d^2 - a^6d^2 + a^5b^2d^2 * 6i + a^5b^2d^2 * 6i - 15a^2b^4d^2 - a^3b^3d^2 * 20i + 15a^4b^2d^2))))^{1/2} * 2i + \operatorname{atan}(\frac{(518a^5b^{15}d^2e^{15} - 18a^{15}b^5d^2e^{15} - 4494a^3b^{13}d^2e^{15} + 3022a^5b^{11}d^2e^{15} + 17194a^7b^9d^2e^{15} + 5298a^9b^7d^2e^{15} - 3338a^{11}b^5d^2e^{15} + 506a^{13}b^3d^2e^{15})}{(a^{16}d^5 + b^{16}d^5 + 8a^2b^{14}d^5 + 28a^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5)} + \frac{((4224a^4b^{18}d^4e^{12} - 320a^2b^{20}d^4e^{12} - 192b^{22}d^4e^{12} + 22272a^6b^{16}d^4e^{12} + 51072a^8b^{14}d^4e^{12} + 67200a^{10}b^{12}d^4e^{12} + 53760a^{12}b^{10}d^4e^{12} + 25344a^{14}b^8d^4e^{12} + 5952a^{16}b^6d^4e^{12} + 192a^{18}b^4d^4e^{12} - 128a^{20}b^2d^4e^{12})}{(a^{16}d^5 + b^{16}d^5 + 8a^2b^{14}d^5 + 28a^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5)} + ((e^{\cot(c+dx)})^{1/2} * (e^3 / (4(b^6d^2 * 1i - a^6d^2 * 2 * 1i + 6a^5b^5d^2 + 6a^5b^5d^2 - a^2b^4d^2 * 15i - 20a^3b^3d^2 + a^4b^2d^2 * 15i))))^{1/2} * (512b^{25}d^4e^{10} + 4608a^2b^{23}d^4e^{10} + 17920a^4b^{21}d^4e^{10} + 38400a^6b^{19}d^4e^{10} + 46080a^8b^{17}d^4e^{10} + 21504a^{10}b^{15}d^4e^{10} - 21504a^{12}b^{13}d^4e^{10} - 46080a^{14}b^{11}d^4e^{10} -
\end{aligned}$$





$$\begin{aligned}
& ^{16}d^5 + 8a^2b^{14}d^5 + 28a^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5) + (((4224a^4b^{18} \\
& d^4e^{12} - 320a^2b^{20}d^4e^{12} - 192b^{22}d^4e^{12} + 22272a^6b^{16}d^4e^{12} + 51072a^8b^{14}d^4e^{12} + 67200a^{10}b^{12}d^4e^{12} + 53760a^{12}b^{10} \\
& d^4e^{12} + 25344a^{14}b^8d^4e^{12} + 5952a^{16}b^6d^4e^{12} + 192a^{18}b^4d^4e^{12} - 128a^{20}b^2d^4e^{12}))/ (a^{16}d^5 + b^{16}d^5 + 8a^2b^{14}d^5 + \\
& 28a^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5) - ((e \cot(c + dx))^{(1/2)}(e^3/(4(b^6d^2*1i \\
& - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 \\
& + a^4*b^2*d^2*15i))))^{(1/2)}(512b^{25}d^4e^{10} + 4608a^2b^{23}d^4e^{10} + \\
& 17920a^4b^{21}d^4e^{10} + 38400a^6b^{19}d^4e^{10} + 46080a^8b^{17}d^4e^{10} \\
& + 21504a^{10}b^{15}d^4e^{10} - 21504a^{12}b^{13}d^4e^{10} - 46080a^{14}b^{11}d^4e^{10} - 38400a^{16}b^9d^4e^{10} - 17920a^{18}b^7d^4e^{10} - 4608a^{20}b^5d^4e^{10} \\
& - 512a^{22}b^3d^4e^{10}))/ (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4) \\
& ) * (e^3/(4(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} + ((e \cot(c + dx))^{(1/2)}(1544a*b^{18}d^2e^{13} + 64a^3b^{16}d^2e^{13} \\
& - 7456a^5b^{14}d^2e^{13} - 576a^7b^{12}d^2e^{13} + 19504a^9b^{10}d^2e^{13} \\
& + 18880a^{11}b^8d^2e^{13} + 3808a^{13}b^6d^2e^{13} - 960a^{15}b^4d^2e^{13} \\
& + 8a^{17}b^2d^2e^{13}))/ (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4) \\
& ) * (e^3/(4(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} * (e^3/(4(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} - ((e \cot(c + dx))^{(1/2)}(41*b^13e^{16} + 9a^{12}b^*e^{16} - 82a^2b^{11}e^{16} + 1831a^4b^9e^{16} - 4348a^6b^7e^{16} + 1671a^8b^5e^{16} - 210a^{10}b^3e^{16}))/ (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4) \\
& ) * (e^3/(4(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} + (28a^2b^8e^{18} - 15b^{10}e^{18} + 878a^4b^6e^{18} - 180a^6b^4e^{18} + 9a^8b^2e^{18}))/ (a^{16}d^5 + b^{16}d^5 + 8a^2b^{14}d^5 + 28a^4b^{12}d^5 + 56a^6b^{10}d^5 + 70a^8b^8d^5 + 56a^{10}b^6d^5 + 28a^{12}b^4d^5 + 8a^{14}b^2d^5) \\
& ) * (e^3/(4(b^6d^2*1i - a^6d^2*1i + 6a*b^5d^2 + 6a^5*b*d^2 - a^2*b^4*d^2*15i - 20a^3*b^3*d^2 + a^4*b^2*d^2*15i))))^{(1/2)} * 2i - (((e \cot(c + dx))^{(1/2)}(5a^3e^3 - 3a*b^2e^3))/ (4(a^4 + b^4 + 2a^2b^2)) + (b*e^2*(e \cot(c + dx))^{(3/2)}(3a^2 - 5b^2))/ (4(a^4 + b^4 + 2a^2b^2)))/ (a^2*d*e^2 + b^2*d*e^2*cot(c + dx)^2 + 2a*b*d*e^2*cot(c + dx)) + (atan((((e \cot(c + dx))^{(1/2)}(41*b^{13}e^{16} + 9a^{12}b^*e^{16} - 82a^2b^{11}e^{16} + 1831a^4b^9e^{16} - 4348a^6b^7e^{16} + 1671a^8b^5e^{16} - 210a^{10}b^3e^{16}))/ (a^{16}d^4 + b^{16}d^4 + 8a^2b^{14}d^4 + 28a^4b^{12}d^4 + 56a^6b^{10}d^4 + 70a^8b^8d^4 + 56a^{10}b^6d^4 + 28a^{12}b^4d^4 + 8a^{14}b^2d^4) - (((518a*b^{15}d^2e^{15} - 18a^{15}b*d^2e^{15} - 4494a^3b^{13}d^2e^{15} + 3022a^5b^{11}d^2e^{15} + 17194a^7b^9d^2e^{15} + 5298a^9
\end{aligned}$$

$$\begin{aligned}
& b^7 d^2 e^{15} - 3338 a^{11} b^5 d^2 e^{15} + 506 a^{13} b^3 d^2 e^{15}) / (a^{16} d^5 + \\
& b^{16} d^5 + 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56 a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 + 28 a^{12} b^4 d^5 + 8 a^{14} b^2 d^5) + (((e \cot(c + d \\
& * x))^{(1/2)} * (1544 a^3 b^{18} d^2 e^{13} + 64 a^3 b^{16} d^2 e^{13} - 7456 a^5 b^{14} d^2 \\
& * e^{13} - 576 a^7 b^{12} d^2 e^{13} + 19504 a^9 b^{10} d^2 e^{13} + 18880 a^{11} b^8 d^2 \\
& * e^{13} + 3808 a^{13} b^6 d^2 e^{13} - 960 a^{15} b^4 d^2 e^{13} + 8 a^{17} b^2 d^2 e^{13} \\
& * e^{13})) / (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 \\
& + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4) \\
& + (((4224 a^4 b^{18} d^4 e^{12} - 320 a^2 b^{20} d^4 e^{12} - 192 b^{22} d^4 e^{12} + 2 \\
& 2272 a^6 b^{16} d^4 e^{12} + 51072 a^8 b^{14} d^4 e^{12} + 67200 a^{10} b^{12} d^4 e^{12} \\
& + 53760 a^{12} b^{10} d^4 e^{12} + 25344 a^{14} b^8 d^4 e^{12} + 5952 a^{16} b^6 d^4 e^{12} \\
& + 192 a^{18} b^4 d^4 e^{12} - 128 a^{20} b^2 d^4 e^{12}) / (a^{16} d^5 + b^{16} d^5 + \\
& 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56 a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 \\
& + 28 a^{12} b^4 d^5 + 8 a^{14} b^2 d^5) - ((e \cot(c + d * x))^{(1/2)} * ( \\
& 3 a^4 + 3 b^4 - 26 a^2 b^2) * (-a * b * e^3)^{(1/2)} * (512 b^{25} d^4 e^{10} + 4608 a^2 * \\
& b^{23} d^4 e^{10} + 17920 a^4 b^{21} d^4 e^{10} + 38400 a^6 b^{19} d^4 e^{10} + 46080 a^8 b^{17} d^4 e^{10} \\
& + 21504 a^{10} b^{15} d^4 e^{10} - 21504 a^{12} b^{13} d^4 e^{10} - 46 \\
& 080 a^{14} b^{11} d^4 e^{10} - 38400 a^{16} b^9 d^4 e^{10} - 17920 a^{18} b^7 d^4 e^{10} \\
& - 4608 a^{20} b^5 d^4 e^{10} - 512 a^{22} b^3 d^4 e^{10})) / (8 * (3 a^3 b^5 d + 3 a^5 * \\
& b^3 d + a * b^7 d + a^7 * b * d) * (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 \\
& + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4)) * (3 a^4 + 3 b^4 - 26 a^2 b^2) * (-a * b * e^3)^{(1/2)} / (8 * ( \\
& 3 a^3 b^5 d + 3 a^5 b^3 d + a * b^7 d + a^7 * b * d)) * (3 a^4 + 3 b^4 - 26 a^2 b^2) * (-a * b * e^3)^{(1/2)} / (8 * (3 a^3 b^5 d + 3 a^5 b^3 d + a * b^7 d + a^7 * b * d)) * ( \\
& 3 a^4 + 3 b^4 - 26 a^2 b^2) * (-a * b * e^3)^{(1/2)} / (8 * (3 a^3 b^5 d + 3 a^5 b^3 d + a * b^7 d + a^7 * b * d)) * (-a * b * e^3)^{(1/2)} * i) / ( \\
& 8 * (3 a^3 b^5 d + 3 a^5 b^3 d + a * b^7 d + a^7 * b * d)) + (((e \cot(c + d * x))^{(1/2)} * (41 b^{13} e^{16} + 9 a^{12} b e^{16} - 82 a^2 b^{11} e^{16} + 1831 a^4 b^9 e^{16} - \\
& 4348 a^6 b^7 e^{16} + 1671 a^8 b^5 e^{16} - 210 a^{10} b^3 e^{16})) / (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 \\
& + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4) + (((518 a^3 b^{15} d^2 * \\
& e^{15} - 18 a^{15} b d^2 e^{15} - 4494 a^3 b^{13} d^2 e^{15} + 3022 a^5 b^{11} d^2 e^{15} \\
& + 17194 a^7 b^9 d^2 e^{15} + 5298 a^9 b^7 d^2 e^{15} - 3338 a^{11} b^5 d^2 e^{15} \\
& + 506 a^{13} b^3 d^2 e^{15}) / (a^{16} d^5 + b^{16} d^5 + 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56 a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 + 28 a^{12} b^4 d^5 \\
& + 8 a^{14} b^2 d^5) - (((e \cot(c + d * x))^{(1/2)} * (1544 a^3 b^{18} d^2 e^{13} + 64 a^3 b^{16} d^2 e^{13} - 7456 a^5 b^{14} d^2 e^{13} - 576 a^7 b^{12} d^2 e^{13} + 19504 a^9 b^{10} d^2 e^{13} \\
& + 18880 a^{11} b^8 d^2 e^{13} + 3808 a^{13} b^6 d^2 e^{13} - 960 a^{15} b^4 d^2 e^{13} + 8 a^{17} b^2 d^2 e^{13})) / (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 \\
& + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4) - (((4224 a^4 b^{18} d^4 e^{12} - 320 a^2 b^{20} d^4 e^{12} - 192 b^{22} d^4 e^{12} + 22272 a^6 b^{16} d^4 e^{12} + 51072 a^8 b^{14} d^4 e^{12} + 67200 a^{10} b^{12} d^4 e^{12} \\
& + 53760 a^{12} b^{10} d^4 e^{12} + 25344 a^{14} b^8 d^4 e^{12} + 5952 a^{16} b^6 d^4 e^{12} + 192 a^{18} b^4 d^4 e^{12} - 128 a^{20} b^2 d^4 e^{12}) / (a^{16} d^5 + b^{16} d^5 + 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56
\end{aligned}$$

$$\begin{aligned}
& a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 + 28 a^{12} b^4 d^5 + 8 a^{14} b^2 d^5) + ((e \cot(c + dx))^{1/2} (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2} (512 b^{25} d^4 e^{10} + 4608 a^2 b^{23} d^4 e^{10} + 17920 a^4 b^{21} d^4 e^{10} + 38400 a^6 b^{19} d^4 e^{10} + 46080 a^8 b^{17} d^4 e^{10} + 21504 a^{10} b^{15} d^4 e^{10} - 21504 a^{12} b^{13} d^4 e^{10} - 46080 a^{14} b^{11} d^4 e^{10} - 38400 a^{16} b^9 d^4 e^{10} - 17920 a^{18} b^7 d^4 e^{10} - 4608 a^{20} b^5 d^4 e^{10} - 512 a^{22} b^3 d^4 e^{10})) / (8 (3 a^3 b^5 d + 3 a^5 b^3 d + a b^7 d + a^7 b d) (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4)) (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2} / (8 (3 a^3 b^5 d + 3 a^5 b^3 d + a b^7 d + a^7 b d)) (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2} / (8 (3 a^3 b^5 d + 3 a^5 b^3 d + a b^7 d + a^7 b d)) (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2} / (8 (3 a^3 b^5 d + 3 a^5 b^3 d + a b^7 d + a^7 b d)) (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2} * i) / (8 (3 a^3 b^5 d + 3 a^5 b^3 d + a b^7 d + a^7 b d)) / ((28 a^2 b^8 e^{18} - 15 b^{10} e^{18} + 878 a^4 b^6 e^{18} - 180 a^6 b^4 e^{18} + 9 a^8 b^2 e^{18}) / (a^{16} d^5 + b^{16} d^5 + 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56 a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 + 28 a^{12} b^4 d^5 + 8 a^{14} b^2 d^5) - (((e \cot(c + dx))^{1/2} (41 b^{13} e^{16} + 9 a^{12} b^5 e^{16} - 82 a^2 b^{11} e^{16} + 1831 a^4 b^9 e^{16} - 4348 a^6 b^7 e^{16} + 1671 a^8 b^5 e^{16} - 210 a^{10} b^3 e^{16})) / (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4) - ((518 a b^{15} d^2 e^{15} - 18 a^{15} b d^2 e^{15} - 4494 a^3 b^{13} d^2 e^{15} + 3022 a^5 b^{11} d^2 e^{15} + 17194 a^7 b^9 d^2 e^{15} + 5298 a^9 b^7 d^2 e^{15} - 3338 a^{11} b^5 d^2 e^{15} + 506 a^{13} b^3 d^2 e^{15}) / (a^{16} d^5 + b^{16} d^5 + 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56 a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 + 28 a^{12} b^4 d^5 + 8 a^{14} b^2 d^5) + (((e \cot(c + dx))^{1/2} (1544 a b^{18} d^2 e^{13} + 64 a^3 b^{16} d^2 e^{13} - 7456 a^5 b^{14} d^2 e^{13} - 576 a^7 b^{12} d^2 e^{13} + 19504 a^9 b^{10} d^2 e^{13} + 18880 a^{11} b^8 d^2 e^{13} + 3808 a^{13} b^6 d^2 e^{13} - 960 a^{15} b^4 d^2 e^{13} + 8 a^{17} b^2 d^2 e^{13})) / (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4) + ((4224 a^4 b^{18} d^4 e^{12} - 320 a^2 b^{20} d^4 e^{12} - 192 b^{22} d^4 e^{12} + 22272 a^6 b^{16} d^4 e^{12} + 51072 a^8 b^{14} d^4 e^{12} + 67200 a^{10} b^{12} d^4 e^{12} + 53760 a^{12} b^{10} d^4 e^{12} + 25344 a^{14} b^8 d^4 e^{12} + 5952 a^{16} b^6 d^4 e^{12} + 192 a^{18} b^4 d^4 e^{12} - 128 a^{20} b^2 d^4 e^{12}) / (a^{16} d^5 + b^{16} d^5 + 8 a^2 b^{14} d^5 + 28 a^4 b^{12} d^5 + 56 a^6 b^{10} d^5 + 70 a^8 b^8 d^5 + 56 a^{10} b^6 d^5 + 28 a^{12} b^4 d^5 + 8 a^{14} b^2 d^5) - ((e \cot(c + dx))^{1/2} (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2} (512 b^{25} d^4 e^{10} + 4608 a^2 b^{23} d^4 e^{10} + 17920 a^4 b^{21} d^4 e^{10} + 38400 a^6 b^{19} d^4 e^{10} + 46080 a^8 b^{17} d^4 e^{10} + 21504 a^{10} b^{15} d^4 e^{10} - 21504 a^{12} b^{13} d^4 e^{10} - 46080 a^{14} b^{11} d^4 e^{10} - 38400 a^{16} b^9 d^4 e^{10} - 17920 a^{18} b^7 d^4 e^{10} - 4608 a^{20} b^5 d^4 e^{10} - 512 a^{22} b^3 d^4 e^{10})) / (8 (3 a^3 b^5 d + 3 a^5 b^3 d + a b^7 d + a^7 b d) (a^{16} d^4 + b^{16} d^4 + 8 a^2 b^{14} d^4 + 28 a^4 b^{12} d^4 + 56 a^6 b^{10} d^4 + 70 a^8 b^8 d^4 + 56 a^{10} b^6 d^4 + 28 a^{12} b^4 d^4 + 8 a^{14} b^2 d^4)) (3 a^4 + 3 b^4 - 26 a^2 b^2) (-a b e^3)^{1/2}
\end{aligned}$$

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/2))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 -
26*a^2*b^2)*(-a*b*e^3)^(1/2))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7
*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/((8*(3*a^3*b^5*d + 3
a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1
/2))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)) + (((e*cot(c + d*
x))^(1/2)*(41*b^13*e^16 + 9*a^12*b*e^16 - 82*a^2*b^11*e^16 + 1831*a^4*b^9*e
^16 - 4348*a^6*b^7*e^16 + 1671*a^8*b^5*e^16 - 210*a^10*b^3*e^16))/(a^16*d^4
+ b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b
^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) + (((518*a*b^1
5*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15 + 3022*a^5*b^11*d^
2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15 - 3338*a^11*b^5*d^2
*e^15 + 506*a^13*b^3*d^2*e^15))/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a
^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*
b^4*d^5 + 8*a^14*b^2*d^5) - (((e*cot(c + d*x))^(1/2)*(1544*a*b^18*d^2*e^13
+ 64*a^3*b^16*d^2*e^13 - 7456*a^5*b^14*d^2*e^13 - 576*a^7*b^12*d^2*e^13 +
19504*a^9*b^10*d^2*e^13 + 18880*a^11*b^8*d^2*e^13 + 3808*a^13*b^6*d^2*e^13
- 960*a^15*b^4*d^2*e^13 + 8*a^17*b^2*d^2*e^13))/(a^16*d^4 + b^16*d^4 + 8*a^
2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b
^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4) - (((4224*a^4*b^18*d^4*e^12 - 32
0*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 + 51072*a
^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e^12 + 25
344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*e^12 - 12
8*a^20*b^2*d^4*e^12))/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a^4*b^12*d^
5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12*b^4*d^5 +
8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*
b*e^3)^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 + 17920*a^4*b^21*d
^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^10 + 21504*a^10*b^
15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*d^4*e^10 - 38400*a
^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b^5*d^4*e^10 - 512*a
^22*b^3*d^4*e^10))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d))*(a^16
*d^4 + b^16*d^4 + 8*a^2*b^14*d^4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a
^8*b^8*d^4 + 56*a^10*b^6*d^4 + 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4)))*(3*a^4 +
3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^
7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/((8*(3*a^3*
b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a
*b*e^3)^(1/2))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d)))*(3*a^4 +
3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))/((8*(3*a^3*b^5*d + 3*a^5*b^3*d + a*b^
7*d + a^7*b*d)))*(3*a^4 + 3*b^4 - 26*a^2*b^2)*(-a*b*e^3)^(1/2))*1i)/(4*(3*a
^3*b^5*d + 3*a^5*b^3*d + a*b^7*d + a^7*b*d))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \cot(c + dx))^{\frac{3}{2}}}{(a + b \cot(c + dx))^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
```

```
[Out] Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**3, x)
```

$$3.85 \quad \int \frac{\sqrt{e \cot(c+dx)}}{(a+b \cot(c+dx))^3} dx$$

Optimal. Leaf size=463

$$\frac{b(7a^2 - b^2) \sqrt{e \cot(c+dx)}}{4ad(a^2 + b^2)^2 (a + b \cot(c+dx))} + \frac{b \sqrt{e \cot(c+dx)}}{2d(a^2 + b^2)(a + b \cot(c+dx))^2} - \frac{\sqrt{e}(a+b)(a^2 - 4ab + b^2) \log(\sqrt{e} \cot(c+dx))}{2\sqrt{2}d(a^2 + b^2)}$$

[Out]  $\frac{1}{2}(a-b)(a^2+4ab+b^2) \arctan(1-2^{1/2}(e \cot(dx+c))^{1/2}/e^{1/2}) e^{1/2}/(a^2+b^2)^{3/2} d^{1/2} - \frac{1}{2}(a-b)(a^2+4ab+b^2) \arctan(1+2^{1/2}(e \cot(dx+c))^{1/2}/e^{1/2}) e^{1/2}/(a^2+b^2)^{3/2} d^{1/2} - \frac{1}{4}(a+b)(a^2-4ab+b^2) \ln(e^{1/2} + \cot(dx+c) e^{1/2}) - 2^{1/2}(e \cot(dx+c))^{1/2} e^{1/2}/(a^2+b^2)^{3/2} d^{1/2} + \frac{1}{4}(a+b)(a^2-4ab+b^2) \ln(e^{1/2} + \cot(dx+c) e^{1/2}) + 2^{1/2}(e \cot(dx+c))^{1/2} e^{1/2}/(a^2+b^2)^{3/2} d^{1/2} + \frac{1}{4}(15a^4 - 18a^2b^2 - b^4) \arctan(b^{1/2}(e \cot(dx+c))^{1/2}/a^{1/2}/e^{1/2}) b^{1/2} e^{1/2}/a^{3/2}/(a^2+b^2)^{3/2} d + \frac{1}{2} b (e \cot(dx+c))^{1/2}/(a^2+b^2)/d + (a+b \cot(dx+c))^2 + \frac{1}{4} b (7a^2 - b^2) (e \cot(dx+c))^{1/2}/(a^2+b^2)^2/d + (a+b \cot(dx+c))$

**Rubi [A]** time = 1.15, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3568, 3649, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{b(7a^2 - b^2) \sqrt{e \cot(c+dx)}}{4ad(a^2 + b^2)^2 (a + b \cot(c+dx))} + \frac{b \sqrt{e \cot(c+dx)}}{2d(a^2 + b^2)(a + b \cot(c+dx))^2} - \frac{\sqrt{e}(a+b)(a^2 - 4ab + b^2) \log(\sqrt{e} \cot(c+dx))}{2\sqrt{2}d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e\*Cot[c + d\*x]]/(a + b\*Cot[c + d\*x])^3, x]

[Out]  $(\text{Sqrt}[b] * (15a^4 - 18a^2b^2 - b^4) * \text{Sqrt}[e] * \text{ArcTan}[(\text{Sqrt}[b] * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / (\text{Sqrt}[a] * \text{Sqrt}[e])]) / (4a^{3/2} * (a^2 + b^2)^{3/2} * d) + ((a - b) * (a^2 + 4ab + b^2) * \text{Sqrt}[e] * \text{ArcTan}[1 - (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / \text{Sqrt}[e]]) / (\text{Sqrt}[2] * (a^2 + b^2)^{3/2} * d) - ((a - b) * (a^2 + 4ab + b^2) * \text{Sqrt}[e] * \text{ArcTan}[1 + (\text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / \text{Sqrt}[e]]) / (\text{Sqrt}[2] * (a^2 + b^2)^{3/2} * d) + (b * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / (2 * (a^2 + b^2) * d * (a + b * \text{Cot}[c + d*x])^2) + (b * (7a^2 - b^2) * \text{Sqrt}[e * \text{Cot}[c + d*x]]) / (4 * a * (a^2 + b^2)^2 * d * (a + b * \text{Cot}[c + d*x])) - ((a + b) * (a^2 - 4ab + b^2) * \text{Sqrt}[e] * \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] * \text{Cot}[c + d*x] - \text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]])] / (2 * \text{Sqrt}[2] * (a^2 + b^2)^{3/2} * d) + ((a + b) * (a^2 - 4ab + b^2) * \text{Sqrt}[e] * \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] * \text{Cot}[c + d*x] + \text{Sqrt}[2] * \text{Sqrt}[e * \text{Cot}[c + d*x]])] / (2 * \text{Sqrt}[2] * (a^2 + b^2)^{3/2} * d)$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$eQ[\{a, c, d, e\}, x] \ \&\& \ EqQ[c*d^2 - a*e^2, 0] \ \&\& \ NegQ[d*e]$

### Rule 1168

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] \ :> \ With[\{q = Rt[a*c, 2]\}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] \ ; \ FreeQ[\{a, c, d, e\}, x] \ \&\& \ NeQ[c*d^2 + a*e^2, 0] \ \&\& \ NeQ[c*d^2 - a*e^2, 0] \ \&\& \ NegQ[-(a*c)]$

### Rule 3534

$Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)^2]], x\_Symbol] \ :> \ Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] \ ; \ FreeQ[\{b, c, d, e, f\}, x] \ \&\& \ NeQ[c^2 - d^2, 0] \ \&\& \ NeQ[c^2 + d^2, 0]$

### Rule 3568

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] \ :> \ Simp[(b*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n)/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c*(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e + f*x]^2, x], x], x] \ ; \ FreeQ[\{a, b, c, d, e, f\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ NeQ[a^2 + b^2, 0] \ \&\& \ NeQ[c^2 + d^2, 0] \ \&\& \ LtQ[m, -1] \ \&\& \ GtQ[n, 0] \ \&\& \ IntegerQ[2*m]$

### Rule 3634

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)^2]), x\_Symbol] \ :> \ Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] \ ; \ FreeQ[\{a, b, c, d, e, f, A, C, m, n\}, x] \ \&\& \ EqQ[A, C]$

### Rule 3649

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)] + (C_)*tan[(e_) + (f_)*(x_)]^2), x\_Symbol] \ :> \ Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan$

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

### Rule 3653

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e \cot(c+dx)}}{(a+b \cot(c+dx))^3} dx &= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} - \frac{\int \frac{-\frac{be}{2} - 2ae \cot(c+dx) + \frac{3}{2}be \cot^2(c+dx)}{\sqrt{e \cot(c+dx)}(a+b \cot(c+dx))^2} dx}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{\int \frac{\frac{1}{4}b(9a^2+b^2)e}{(a+b \cot(c+dx))^2} dx}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{\int \frac{2ab(3a^2-b^2)e}{(a+b \cot(c+dx))^2} dx}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{\text{Subst}\left(\int \frac{-2a}{(a+b \cot(c+dx))^2} dx\right)}{2(a^2+b^2)} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{(b(15a^4-18a^2b^2-b^4))\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{(b(15a^4-18a^2b^2-b^4))\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{(b(15a^4-18a^2b^2-b^4))\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} \\
&= \frac{b\sqrt{e \cot(c+dx)}}{2(a^2+b^2)d(a+b \cot(c+dx))^2} + \frac{b(7a^2-b^2)\sqrt{e \cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b \cot(c+dx))} + \frac{(b(15a^4-18a^2b^2-b^4))\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} + \frac{(a-b)(a^2+4ab+b^2)\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)}
\end{aligned}$$

**Mathematica [C]** time = 6.19, size = 483, normalized size = 1.04

$$\sqrt{e \cot(c+dx)} \left( \frac{2a(a^2-3b^2) \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)}{3(a^2+b^2)^3} + \frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{(a^2+b^2)^3} - \frac{2\sqrt{a}\sqrt{b}(3a^2-b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^3} - \frac{(a-b)(a^2+4ab+b^2)\sqrt{e} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{e \cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e\*Cot[c + d\*x]]/(a + b\*Cot[c + d\*x])^3, x]

```
[Out] -((Sqrt[e*Cot[c + d*x]]*((-2*Sqrt[a]*Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(a^2 + b^2)^3 + (2*b*(3*a^2 - b^2)*Sqrt[Cot[c + d*x]])/(a^2 + b^2)^3 - (2*Sqrt[a]*Sqrt[b]*(-a*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sqrt[b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Cot[c + d*x]))/((a^2 + b^2)^2*(a + b*Cot[c + d*x])) + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)^3) + (2*b^2*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -(b*Cot[c + d*x])/a])/(3*a^3*(a^2 + b^2)) - (b*(3*a^2 - b^2)*(2*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]) - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]])] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3))/(d*Sqrt[Cot[c + d*x]])
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(dx + c)}}{(b \cot(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^3, x)
```

**maple** [B] time = 0.97, size = 1187, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x)
```

```
[Out] 7/4/d*e*b^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*a^3*(e*cot(d*x+c))^(3/2)+3/2/d*e*b^4/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*a*(e*cot(d*x+c))^(3/2)-1/4/d*e*b^6/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2/a*(e*cot(d*x+c))^(3/2)+9/4/d*e^2*b/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)*a^4+5/2/d*e^2*b^3/(a
```

$$\begin{aligned} & \frac{1}{4} \frac{b^2 \sqrt{e} \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^7 + 3a^5b^2 + 3a^3b^4 + ab^6) \sqrt{abe}} + \frac{(9a^3b + ab^3)e \sqrt{\frac{e}{\tan(dx+c)}} + (7a^2b^2 - b^4) \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{(a^7 + 2a^5b^2 + a^3b^4)e^2 + \frac{2(a^6b + 2a^4b^3 + a^2b^5)e^2}{\tan(dx+c)} + \frac{(a^5b^2 + 2a^3b^4 + ab^6)e^2}{\tan(dx+c)^2}} - \frac{2\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3) \arctan\left(\frac{\sqrt{2}}{\sqrt{e}}\right)}{\sqrt{e}} \\ & \frac{1}{4} \frac{b^2 \sqrt{e} \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^7 + 3a^5b^2 + 3a^3b^4 + ab^6) \sqrt{abe}} + \frac{(9a^3b + ab^3)e \sqrt{\frac{e}{\tan(dx+c)}} + (7a^2b^2 - b^4) \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{(a^7 + 2a^5b^2 + a^3b^4)e^2 + \frac{2(a^6b + 2a^4b^3 + a^2b^5)e^2}{\tan(dx+c)} + \frac{(a^5b^2 + 2a^3b^4 + ab^6)e^2}{\tan(dx+c)^2}} - \frac{2\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3) \arctan\left(\frac{\sqrt{2}}{\sqrt{e}}\right)}{\sqrt{e}} \end{aligned}$$

**maxima** [A] time = 0.84, size = 496, normalized size = 1.07

$$e \left[ \frac{(15a^4b - 18a^2b^3 - b^5) \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{abe}}\right)}{(a^7 + 3a^5b^2 + 3a^3b^4 + ab^6) \sqrt{abe}} + \frac{(9a^3b + ab^3)e \sqrt{\frac{e}{\tan(dx+c)}} + (7a^2b^2 - b^4) \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{(a^7 + 2a^5b^2 + a^3b^4)e^2 + \frac{2(a^6b + 2a^4b^3 + a^2b^5)e^2}{\tan(dx+c)} + \frac{(a^5b^2 + 2a^3b^4 + ab^6)e^2}{\tan(dx+c)^2}} - \frac{2\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3) \arctan\left(\frac{\sqrt{2}}{\sqrt{e}}\right)}{\sqrt{e}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/4\*e\*((15\*a^4\*b - 18\*a^2\*b^3 - b^5)\*arctan(b\*sqrt(e/tan(d\*x + c))/sqrt(a\*b\*e)))/((a^7 + 3\*a^5\*b^2 + 3\*a^3\*b^4 + a\*b^6)\*sqrt(a\*b\*e)) + ((9\*a^3\*b + a\*b^3)\*e\*sqrt(e/tan(d\*x + c)) + (7\*a^2\*b^2 - b^4)\*(e/tan(d\*x + c))^(3/2))/((a^7 + 2\*a^5\*b^2 + a^3\*b^4)\*e^2 + 2\*(a^6\*b + 2\*a^4\*b^3 + a^2\*b^5)\*e^2/tan(d\*x +



$$c) + (a^5b^2 + 2a^3b^4 + ab^6)e^2/\tan(dx + c)^2 - (2\sqrt{2})(a^3 + 3a^2b - 3ab^2 - b^3)\arctan(1/2\sqrt{2})(\sqrt{2}\sqrt{e} + 2\sqrt{e/\tan(dx + c)})/\sqrt{e})/\sqrt{e} + 2\sqrt{2})(a^3 + 3a^2b - 3ab^2 - b^3)a\arctan(-1/2\sqrt{2})(\sqrt{2}\sqrt{e} - 2\sqrt{e/\tan(dx + c)})/\sqrt{e})/\sqrt{e} - \sqrt{2})(a^3 - 3a^2b - 3ab^2 + b^3)\log(\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx + c)}) + e + e/\tan(dx + c))/\sqrt{e} + \sqrt{2})(a^3 - 3a^2b - 3ab^2 + b^3)\log(-\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx + c)}) + e + e/\tan(dx + c))/\sqrt{e})/(a^6 + 3a^4b^2 + 3a^2b^4 + b^6))/d$$

**mupad [B]** time = 6.13, size = 19534, normalized size = 42.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e \cdot \cot(c + dx))^{1/2} / (a + b \cdot \cot(c + dx))^3, x)$

[Out]  $((e \cdot \cot(c + dx))^{1/2} (b^3 e^2 + 9a^2 b e^2)) / (4(a^4 + b^4 + 2a^2 b^2)) + (b^2 e (e \cdot \cot(c + dx))^{3/2} (7a^2 - b^2)) / (4a(a^4 + b^4 + 2a^2 b^2)) / (a^2 d e^2 + b^2 d e^2 \cot(c + dx)^2 + 2a b d e^2 \cot(c + dx)) - a \tan(\frac{(64a^7 b^{23} d^4 e^{11} + 1472a^3 b^{21} d^4 e^{11} + 8832a^5 b^{19} d^4 e^{11} + 25344a^7 b^{17} d^4 e^{11} + 40320a^9 b^{15} d^4 e^{11} + 34944a^{11} b^{13} d^4 e^{11} + 10752a^{13} b^{11} d^4 e^{11} - 8448a^{15} b^9 d^4 e^{11} - 10176a^{17} b^7 d^4 e^{11} - 4160a^{19} b^5 d^4 e^{11} - 640a^{21} b^3 d^4 e^{11})}{(a^{18} d^5 + a^2 b^{16} d^5 + 8a^4 b^{14} d^5 + 28a^6 b^{12} d^5 + 56a^8 b^{10} d^5 + 70a^{10} b^8 d^5 + 56a^{12} b^6 d^5 + 28a^{14} b^4 d^5 + 8a^{16} b^2 d^5) + ((e \cdot \cot(c + dx))^{1/2} (-e / (4(b^6 d^2 i - a^6 d^2 i + 6a^5 b^5 d^2 + 6a^5 b^5 d^2 - a^2 b^4 d^2 i - 20a^3 b^3 d^2 + a^4 b^2 d^2 i)))^{1/2} (512a^2 b^{25} d^4 e^{10} + 4608a^4 b^{23} d^4 e^{10} + 17920a^6 b^{21} d^4 e^{10} + 38400a^8 b^{19} d^4 e^{10} + 46080a^{10} b^{17} d^4 e^{10} + 21504a^{12} b^{15} d^4 e^{10} - 21504a^{14} b^{13} d^4 e^{10} - 46080a^{16} b^{11} d^4 e^{10} - 38400a^{18} b^9 d^4 e^{10} - 17920a^{20} b^7 d^4 e^{10} - 4608a^{22} b^5 d^4 e^{10} - 512a^{24} b^3 d^4 e^{10}))}{(a^{18} d^4 + a^2 b^{16} d^4 + 8a^4 b^{14} d^4 + 28a^6 b^{12} d^4 + 56a^8 b^{10} d^4 + 70a^{10} b^8 d^4 + 56a^{12} b^6 d^4 + 28a^{14} b^4 d^4 + 8a^{16} b^2 d^4)} (-e / (4(b^6 d^2 i - a^6 d^2 i + 6a^5 b^5 d^2 + 6a^5 b^5 d^2 - a^2 b^4 d^2 i - 20a^3 b^3 d^2 + a^4 b^2 d^2 i)))^{1/2} - ((e \cdot \cot(c + dx))^{1/2} (8a^2 b^{20} d^2 e^{11} - 1152a^3 b^{18} d^2 e^{11} + 2528a^5 b^{16} d^2 e^{11} + 15296a^7 b^{14} d^2 e^{11} + 14128a^9 b^{12} d^2 e^{11} - 5056a^{11} b^{10} d^2 e^{11} - 9248a^{13} b^8 d^2 e^{11} + 64a^{15} b^6 d^2 e^{11} + 1800a^{17} b^4 d^2 e^{11} + 64a^{19} b^2 d^2 e^{11})) / (a^{18} d^4 + a^2 b^{16} d^4 + 8a^4 b^{14} d^4 + 28a^6 b^{12} d^4 + 56a^8 b^{10} d^4 + 70a^{10} b^8 d^4 + 56a^{12} b^6 d^4 + 28a^{14} b^4 d^4 + 8a^{16} b^2 d^4)} (-e / (4(b^6 d^2 i - a^6 d^2 i + 6a^5 b^5 d^2 + 6a^5 b^5 d^2 - a^2 b^4 d^2 i - 20a^3 b^3 d^2 + a^4 b^2 d^2 i)))^{1/2} - (2b^{18} d^2 e^{12} - 138a^2 b^{16} d^2 e^{12} - 3046a^4 b^{14} d^2 e^{12} + 4862a^6 b^{12} d^2 e^{12} + 9222a^8 b^{10} d^2 e^{12} - 5246a^{10} b^8 d^2 e^{12} - 4290a^{12} b^6 d^2 e^{12} + 2442a^{14} b^4 d^2 e^{12} + 32a^{16} b^2 d^2 e^{12}) / (a^{18} d^5 + a^2 b^{16} d^5))$

$$\begin{aligned}
& d^5 + 8a^4b^{14}d^5 + 28a^6b^{12}d^5 + 56a^8b^{10}d^5 + 70a^{10}b^8d^5 \\
& + 56a^{12}b^6d^5 + 28a^{14}b^4d^5 + 8a^{16}b^2d^5) * (-e / (4 * (b^6d^2 * 1i - \\
& a^6d^2 * 1i + 6 * a * b^5d^2 + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 \\
& + a^4 * b^2d^2 * 15i)))^{(1/2)} + ((e * \cot(c + dx))^{(1/2)} * (2 * a^2 * b^{13} * e^{12} - b^{15} * \\
& 5 * e^{12} + 49 * a^4 * b^{11} * e^{12} + 2460 * a^6 * b^9 * e^{12} - 3631 * a^8 * b^7 * e^{12} + 1922 * a^{10} * \\
& b^5 * e^{12} - 225 * a^{12} * b^3 * e^{12})) / (a^{18} * d^4 + a^2 * b^{16} * d^4 + 8 * a^4 * b^{14} * d^4 \\
& + 28 * a^6 * b^{12} * d^4 + 56 * a^8 * b^{10} * d^4 + 70 * a^{10} * b^8 * d^4 + 56 * a^{12} * b^6 * d^4 + \\
& 28 * a^{14} * b^4 * d^4 + 8 * a^{16} * b^2 * d^4)) * (-e / (4 * (b^6d^2 * 1i - a^6d^2 * 1i + 6 * a * b^5d^2 \\
& + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 + a^4 * b^2d^2 * 15i)))^{(1/2)} * 1i - \\
& (((((64 * a * b^{23} * d^4 * e^{11} + 1472 * a^3 * b^{21} * d^4 * e^{11} + 8832 * a^5 * b^{19} * d^4 * e^{11} + 25344 * a^7 * \\
& b^{17} * d^4 * e^{11} + 40320 * a^9 * b^{15} * d^4 * e^{11} + 34944 * a^{11} * b^{13} * d^4 * e^{11} + 10752 * a^{13} * b^{11} * d^4 * e^{11} - \\
& 8448 * a^{15} * b^9 * d^4 * e^{11} - 10176 * a^{17} * b^7 * d^4 * e^{11} - 4160 * a^{19} * b^5 * d^4 * e^{11} - 640 * a^{21} * b^3 * d^4 * e^{11}) / (a^{18} * \\
& d^5 + a^2 * b^{16} * d^5 + 8 * a^4 * b^{14} * d^5 + 28 * a^6 * b^{12} * d^5 + 56 * a^8 * b^{10} * d^5 + 70 * \\
& a^{10} * b^8 * d^5 + 56 * a^{12} * b^6 * d^5 + 28 * a^{14} * b^4 * d^5 + 8 * a^{16} * b^2 * d^5) - ((e * \cot(c + dx))^{(1/2)} * \\
& (-e / (4 * (b^6d^2 * 1i - a^6d^2 * 1i + 6 * a * b^5d^2 + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 \\
& + a^4 * b^2d^2 * 15i)))^{(1/2)} * (512 * a^2 * b^{25} * d^4 * e^{10} + 4608 * a^4 * b^{23} * d^4 * e^{10} + 17920 * a^6 * b^{21} * d^4 * e^{10} + 38400 * a^8 * \\
& b^{19} * d^4 * e^{10} + 46080 * a^{10} * b^{17} * d^4 * e^{10} + 21504 * a^{12} * b^{15} * d^4 * e^{10} - 21504 * a^{14} * b^{13} * d^4 * e^{10} - \\
& 46080 * a^{16} * b^{11} * d^4 * e^{10} - 38400 * a^{18} * b^9 * d^4 * e^{10} - 17920 * a^{20} * b^7 * d^4 * e^{10} - 4608 * a^{22} * b^5 * d^4 * e^{10} - \\
& 512 * a^{24} * b^3 * d^4 * e^{10})) / (a^{18} * d^4 + a^2 * b^{16} * d^4 + 8 * a^4 * b^{14} * d^4 + 28 * a^6 * b^{12} * d^4 + 56 * a^8 * b^{10} * \\
& d^4 + 70 * a^{10} * b^8 * d^4 + 56 * a^{12} * b^6 * d^4 + 28 * a^{14} * b^4 * d^4 + 8 * a^{16} * b^2 * d^4)) * (-e / (4 * (b^6d^2 * 1i - a^6d^2 * 1i + 6 * a * b^5d^2 \\
& + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 + a^4 * b^2d^2 * 15i)))^{(1/2)} + ((e * \cot(c + dx))^{(1/2)} * \\
& (8 * a * b^{20} * d^2 * e^{11} - 1152 * a^3 * b^{18} * d^2 * e^{11} + 2528 * a^5 * b^{16} * d^2 * e^{11} + 15296 * a^7 * b^{14} * d^2 * e^{11} + \\
& 14128 * a^9 * b^{12} * d^2 * e^{11} - 5056 * a^{11} * b^{10} * d^2 * e^{11} - 9248 * a^{13} * b^8 * d^2 * e^{11} + 64 * a^{15} * b^6 * d^2 * e^{11} + \\
& 1800 * a^{17} * b^4 * d^2 * e^{11} + 64 * a^{19} * b^2 * d^2 * e^{11})) / (a^{18} * d^4 + a^2 * b^{16} * d^4 + 8 * a^4 * b^{14} * d^4 + 28 * a^6 * b^{12} * \\
& d^4 + 56 * a^8 * b^{10} * d^4 + 70 * a^{10} * b^8 * d^4 + 56 * a^{12} * b^6 * d^4 + 28 * a^{14} * b^4 * d^4 + 8 * a^{16} * b^2 * d^4)) * (-e / (4 * (b^6d^2 * 1i - a^6d^2 * 1i + 6 * a * b^5d^2 \\
& + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 + a^4 * b^2d^2 * 15i)))^{(1/2)} - (2 * b^{18} * d^2 * e^{12} - 138 * a^2 * b^{16} * d^2 * e^{12} - \\
& 3046 * a^4 * b^{14} * d^2 * e^{12} + 4862 * a^6 * b^{12} * d^2 * e^{12} + 9222 * a^8 * b^{10} * d^2 * e^{12} - 5246 * a^{10} * b^8 * d^2 * e^{12} - 4290 * a^{12} * b^6 * \\
& d^2 * e^{12} + 2442 * a^{14} * b^4 * d^2 * e^{12} + 32 * a^{16} * b^2 * d^2 * e^{12}) / (a^{18} * d^5 + a^2 * b^{16} * d^5 + 8 * a^4 * b^{14} * d^5 + \\
& 28 * a^6 * b^{12} * d^5 + 56 * a^8 * b^{10} * d^5 + 70 * a^{10} * b^8 * d^5 + 56 * a^{12} * b^6 * d^5 + 28 * a^{14} * b^4 * d^5 + 8 * a^{16} * b^2 * d^5)) * (-e / (4 * (b^6d^2 * 1i - a^6d^2 * 1i + 6 * a * b^5d^2 \\
& + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 + a^4 * b^2d^2 * 15i)))^{(1/2)} - ((e * \cot(c + dx))^{(1/2)} * (2 * a^2 * b^{13} * e^{12} - \\
& b^{15} * e^{12} + 49 * a^4 * b^{11} * e^{12} + 2460 * a^6 * b^9 * e^{12} - 3631 * a^8 * b^7 * e^{12} + 1922 * a^{10} * b^5 * e^{12} - 225 * a^{12} * b^3 * e^{12})) / (a^{18} * d^4 + a^2 * b^{16} * d^4 + 8 * a^4 * b^{14} * d^4 + \\
& 28 * a^6 * b^{12} * d^4 + 56 * a^8 * b^{10} * d^4 + 70 * a^{10} * b^8 * d^4 + 56 * a^{12} * b^6 * d^4 + 28 * a^{14} * b^4 * d^4 + 8 * a^{16} * b^2 * d^4)) * (-e / (4 * (b^6d^2 * 1i - a^6d^2 * 1i + \\
& 6 * a * b^5d^2 + 6 * a^5 * b * d^2 - a^2 * b^4d^2 * 15i - 20 * a^3 * b^3d^2 + a^4 * b^2d^2 * 15i)))^{(1/2)} * 1i) / (((7 * a * b^{11} * e^{13} + 116 * a^3 * b^9 * e^{13} - 270 * a^5 * b^7 * e^{13} + 4
\end{aligned}$$





$$\begin{aligned}
& t(c + d*x)^{(1/2)} * (8*a*b^{20}*d^2*e^{11} - 1152*a^3*b^{18}*d^2*e^{11} + 2528*a^5*b^{16}*d^2*e^{11} + 15296*a^7*b^{14}*d^2*e^{11} + 14128*a^9*b^{12}*d^2*e^{11} - 5056*a^{11}*b^{10}*d^2*e^{11} - 9248*a^{13}*b^8*d^2*e^{11} + 64*a^{15}*b^6*d^2*e^{11} + 1800*a^{17}*b^4*d^2*e^{11} + 64*a^{19}*b^2*d^2*e^{11}) / (a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4) * (-e^{1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))^{(1/2)} - (2*b^{18}*d^2*e^{12} - 138*a^2*b^{16}*d^2*e^{12} - 3046*a^4*b^{14}*d^2*e^{12} + 4862*a^6*b^{12}*d^2*e^{12} + 9222*a^8*b^{10}*d^2*e^{12} - 5246*a^{10}*b^8*d^2*e^{12} - 4290*a^{12}*b^6*d^2*e^{12} + 2442*a^{14}*b^4*d^2*e^{12} + 32*a^{16}*b^2*d^2*e^{12}) / (a^{18}*d^5 + a^2*b^{16}*d^5 + 8*a^4*b^{14}*d^5 + 28*a^6*b^{12}*d^5 + 56*a^8*b^{10}*d^5 + 70*a^{10}*b^8*d^5 + 56*a^{12}*b^6*d^5 + 28*a^{14}*b^4*d^5 + 8*a^{16}*b^2*d^5) * (-e^{1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))^{(1/2)} + ((e*cot(c + d*x))^{(1/2)}) * (2*a^2*b^{13}*e^{12} - b^{15}*e^{12} + 49*a^4*b^{11}*e^{12} + 2460*a^6*b^9*e^{12} - 3631*a^8*b^7*e^{12} + 1922*a^{10}*b^5*e^{12} - 225*a^{12}*b^3*e^{12}) / (a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4) * (-e^{1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))^{(1/2)} * 1i - ((((((64*a*b^{23}*d^4*e^{11} + 1472*a^3*b^{21}*d^4*e^{11} + 8832*a^5*b^{19}*d^4*e^{11} + 25344*a^7*b^{17}*d^4*e^{11} + 40320*a^9*b^{15}*d^4*e^{11} + 34944*a^{11}*b^{13}*d^4*e^{11} + 10752*a^{13}*b^{11}*d^4*e^{11} - 8448*a^{15}*b^9*d^4*e^{11} - 10176*a^{17}*b^7*d^4*e^{11} - 4160*a^{19}*b^5*d^4*e^{11} - 640*a^{21}*b^3*d^4*e^{11}) / (a^{18}*d^5 + a^2*b^{16}*d^5 + 8*a^4*b^{14}*d^5 + 28*a^6*b^{12}*d^5 + 56*a^8*b^{10}*d^5 + 70*a^{10}*b^8*d^5 + 56*a^{12}*b^6*d^5 + 28*a^{14}*b^4*d^5 + 8*a^{16}*b^2*d^5) - ((e*cot(c + d*x))^{(1/2)}) * (-e^{1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))^{(1/2)} * (512*a^2*b^{25}*d^4*e^{10} + 4608*a^4*b^{23}*d^4*e^{10} + 17920*a^6*b^{21}*d^4*e^{10} + 38400*a^8*b^{19}*d^4*e^{10} + 46080*a^{10}*b^{17}*d^4*e^{10} + 21504*a^{12}*b^{15}*d^4*e^{10} - 21504*a^{14}*b^{13}*d^4*e^{10} - 46080*a^{16}*b^{11}*d^4*e^{10} - 38400*a^{18}*b^9*d^4*e^{10} - 17920*a^{20}*b^7*d^4*e^{10} - 4608*a^{22}*b^5*d^4*e^{10} - 512*a^{24}*b^3*d^4*e^{10}) / (a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4) * (-e^{1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))^{(1/2)} + ((e*cot(c + d*x))^{(1/2)}) * (8*a*b^{20}*d^2*e^{11} - 1152*a^3*b^{18}*d^2*e^{11} + 2528*a^5*b^{16}*d^2*e^{11} + 15296*a^7*b^{14}*d^2*e^{11} + 14128*a^9*b^{12}*d^2*e^{11} - 5056*a^{11}*b^{10}*d^2*e^{11} - 9248*a^{13}*b^8*d^2*e^{11} + 64*a^{15}*b^6*d^2*e^{11} + 1800*a^{17}*b^4*d^2*e^{11} + 64*a^{19}*b^2*d^2*e^{11}) / (a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4) * (-e^{1i}) / (4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2))^{(1/2)} - (2*b^{18}*d^2*e^{12} - 138*a^2*b^{16}*d^2*e^{12} - 3046*a^4*b^{14}*d^2*e^{12} + 4862*a^6*b^{12}*d^2*e^{12} + 9222*a^8*b^{10}*d^2*e^{12} - 5246*a^{10}*b^8*d^2*e^{12} - 4290*a^{12}*b^6*d^2*e^{12} + 2442*a^{14}*b^4*d^2*e^{12} + 32*a^{16}*b^2*d^2*e^{12} - 4290*a^{12}*b^6*d^2*e^{12} + 2442*a^{14}*b^4*d^2*e^{12} + 32*a^{16}*b^2*d^2*e^{12}
\end{aligned}$$

$$\begin{aligned}
& d^2 e^{12} / (a^{18} d^5 + a^2 b^{16} d^5 + 8 a^4 b^{14} d^5 + 28 a^6 b^{12} d^5 + 56 a^8 b^{10} d^5 + 70 a^{10} b^8 d^5 + 56 a^{12} b^6 d^5 + 28 a^{14} b^4 d^5 + 8 a^{16} b^2 d^5) * (- (e^{1i}) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 * 6i + a^5 b d^2 * 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 * 20i + 15 a^4 b^2 d^2)))^{(1/2)} - ((e \cot(c + d x))^{(1/2)} * (2 a^2 b^{13} e^{12} - b^{15} e^{12} + 49 a^4 b^{11} e^{12} + 2460 a^6 b^9 e^{12} - 3631 a^8 b^7 e^{12} + 1922 a^{10} b^5 e^{12} - 225 a^{12} b^3 e^{12})) / (a^{18} d^4 + a^2 b^{16} d^4 + 8 a^4 b^{14} d^4 + 28 a^6 b^{12} d^4 + 56 a^8 b^{10} d^4 + 70 a^{10} b^8 d^4 + 56 a^{12} b^6 d^4 + 28 a^{14} b^4 d^4 + 8 a^{16} b^2 d^4)) * (- (e^{1i}) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 * 6i + a^5 b d^2 * 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 * 20i + 15 a^4 b^2 d^2)))^{(1/2)} * 1i) / ((7 a b^{11} e^{13} + 116 a^3 b^9 e^{13} - 270 a^5 b^7 e^{13} + 420 a^7 b^5 e^{13} - 225 a^9 b^3 e^{13}) / (a^{18} d^5 + a^2 b^{16} d^5 + 8 a^4 b^{14} d^5 + 28 a^6 b^{12} d^5 + 56 a^8 b^{10} d^5 + 70 a^{10} b^8 d^5 + 56 a^{12} b^6 d^5 + 28 a^{14} b^4 d^5 + 8 a^{16} b^2 d^5) + (((((64 a b^2 3 d^4 e^{11} + 1472 a^3 b^2 1 d^4 e^{11} + 8832 a^5 b^1 9 d^4 e^{11} + 25344 a^7 b^1 7 d^4 e^{11} + 40320 a^9 b^1 5 d^4 e^{11} + 34944 a^{11} b^1 3 d^4 e^{11} + 10752 a^{13} b^1 1 d^4 e^{11} - 8448 a^{15} b^1 9 d^4 e^{11} - 10176 a^{17} b^1 7 d^4 e^{11} - 4160 a^{19} b^1 5 d^4 e^{11} - 640 a^{21} b^1 3 d^4 e^{11}) / (a^{18} d^5 + a^2 b^{16} d^5 + 8 a^4 b^{14} d^5 + 28 a^6 b^{12} d^5 + 56 a^8 b^{10} d^5 + 70 a^{10} b^8 d^5 + 56 a^{12} b^6 d^5 + 28 a^{14} b^4 d^5 + 8 a^{16} b^2 d^5) + ((e \cot(c + d x))^{(1/2)} * (- (e^{1i}) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 * 6i + a^5 b d^2 * 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 * 20i + 15 a^4 b^2 d^2)))^{(1/2)} * (512 a^2 b^{25} d^4 e^{10} + 4608 a^4 b^{23} d^4 e^{10} + 17920 a^6 b^{21} d^4 e^{10} + 38400 a^8 b^{19} d^4 e^{10} + 46080 a^{10} b^{17} d^4 e^{10} + 21504 a^{12} b^{15} d^4 e^{10} - 21504 a^{14} b^{13} d^4 e^{10} - 46080 a^{16} b^{11} d^4 e^{10} - 38400 a^{18} b^9 d^4 e^{10} - 17920 a^{20} b^7 d^4 e^{10} - 4608 a^{22} b^5 d^4 e^{10} - 512 a^{24} b^3 d^4 e^{10}))) / (a^{18} d^4 + a^2 b^{16} d^4 + 8 a^4 b^{14} d^4 + 28 a^6 b^{12} d^4 + 56 a^8 b^{10} d^4 + 70 a^{10} b^8 d^4 + 56 a^{12} b^6 d^4 + 28 a^{14} b^4 d^4 + 8 a^{16} b^2 d^4)) * (- (e^{1i}) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 * 6i + a^5 b d^2 * 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 * 20i + 15 a^4 b^2 d^2)))^{(1/2)} - ((e \cot(c + d x))^{(1/2)} * (8 a b^{20} d^2 e^{11} - 115 2 a^3 b^{18} d^2 e^{11} + 2528 a^5 b^{16} d^2 e^{11} + 15296 a^7 b^{14} d^2 e^{11} + 14 128 a^9 b^{12} d^2 e^{11} - 5056 a^{11} b^{10} d^2 e^{11} - 9248 a^{13} b^8 d^2 e^{11} + 64 a^{15} b^6 d^2 e^{11} + 1800 a^{17} b^4 d^2 e^{11} + 64 a^{19} b^2 d^2 e^{11})) / (a^{18} d^4 + a^2 b^{16} d^4 + 8 a^4 b^{14} d^4 + 28 a^6 b^{12} d^4 + 56 a^8 b^{10} d^4 + 70 a^{10} b^8 d^4 + 56 a^{12} b^6 d^4 + 28 a^{14} b^4 d^4 + 8 a^{16} b^2 d^4)) * (- (e^{1i}) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 * 6i + a^5 b d^2 * 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 * 20i + 15 a^4 b^2 d^2)))^{(1/2)} - (2 b^{18} d^2 e^{12} - 138 a^2 b^{16} d^2 e^{12} - 3046 a^4 b^{14} d^2 e^{12} + 4862 a^6 b^{12} d^2 e^{12} + 9222 a^8 b^{10} d^2 e^{12} - 5246 a^{10} b^8 d^2 e^{12} - 4290 a^{12} b^6 d^2 e^{12} + 2442 a^{14} b^4 d^2 e^{12} + 32 a^{16} b^2 d^2 e^{12}) / (a^{18} d^5 + a^2 b^{16} d^5 + 8 a^4 b^{14} d^5 + 28 a^6 b^{12} d^5 + 56 a^8 b^{10} d^5 + 70 a^{10} b^8 d^5 + 56 a^{12} b^6 d^5 + 28 a^{14} b^4 d^5 + 8 a^{16} b^2 d^5)) * (- (e^{1i}) / (4 (b^6 d^2 - a^6 d^2 + a b^5 d^2 * 6i + a^5 b d^2 * 6i - 15 a^2 b^4 d^2 - a^3 b^3 d^2 * 20i + 15 a^4 b^2 d^2)))^{(1/2)} + ((e \cot(c + d x))^{(1/2)} * (2 a^2 b^{13} e^{12} - b^{15} e^{12} + 49 a^4 b^{11} e^{12} + 2460 a^6 b^9 e^{12} - 3631 a^8 b^7 e^{12} + 1922 a^{10} b^5 e^{12} - 225 a^{12} b^3 e^{12})) / (a^{18} d^4 + a^2 b^{16} d^4 + 8 a^4 b^{14} d^4 + 28 a^6 b^{12} d^4
\end{aligned}$$



$$\begin{aligned}
& *e^{12} + 9222*a^8*b^{10}*d^2*e^{12} - 5246*a^{10}*b^8*d^2*e^{12} - 4290*a^{12}*b^6*d^2 \\
& *e^{12} + 2442*a^{14}*b^4*d^2*e^{12} + 32*a^{16}*b^2*d^2*e^{12})/(a^{18}*d^5 + a^2*b^{16} \\
& *d^5 + 8*a^4*b^{14}*d^5 + 28*a^6*b^{12}*d^5 + 56*a^8*b^{10}*d^5 + 70*a^{10}*b^8*d^5 \\
& + 56*a^{12}*b^6*d^5 + 28*a^{14}*b^4*d^5 + 8*a^{16}*b^2*d^5) - (((e*\cot(c + d*x) \\
& )^{(1/2)}*(8*a*b^{20}*d^2*e^{11} - 1152*a^3*b^{18}*d^2*e^{11} + 2528*a^5*b^{16}*d^2*e^{11} \\
& + 15296*a^7*b^{14}*d^2*e^{11} + 14128*a^9*b^{12}*d^2*e^{11} - 5056*a^{11}*b^{10}*d^2* \\
& e^{11} - 9248*a^{13}*b^8*d^2*e^{11} + 64*a^{15}*b^6*d^2*e^{11} + 1800*a^{17}*b^4*d^2*e^{11} \\
& + 64*a^{19}*b^2*d^2*e^{11}))/ (a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28* \\
& a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14} \\
& *b^4*d^4 + 8*a^{16}*b^2*d^4) + (((64*a*b^{23}*d^4*e^{11} + 1472*a^3*b^{21}*d^4*e^{11} \\
& + 8832*a^5*b^{19}*d^4*e^{11} + 25344*a^7*b^{17}*d^4*e^{11} + 40320*a^9*b^{15}*d^4*e \\
& ^{11} + 34944*a^{11}*b^{13}*d^4*e^{11} + 10752*a^{13}*b^{11}*d^4*e^{11} - 8448*a^{15}*b^9*d \\
& ^4*e^{11} - 10176*a^{17}*b^7*d^4*e^{11} - 4160*a^{19}*b^5*d^4*e^{11} - 640*a^{21}*b^3*d \\
& ^4*e^{11}))/ (a^{18}*d^5 + a^2*b^{16}*d^5 + 8*a^4*b^{14}*d^5 + 28*a^6*b^{12}*d^5 + 56*a \\
& ^8*b^{10}*d^5 + 70*a^{10}*b^8*d^5 + 56*a^{12}*b^6*d^5 + 28*a^{14}*b^4*d^5 + 8*a^{16} \\
& *b^2*d^5) - ((e*\cot(c + d*x))^{(1/2)}*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)} \\
& *(512*a^2*b^{25}*d^4*e^{10} + 4608*a^4*b^{23}*d^4*e^{10} + 17920*a^6*b^{21}*d^4*e \\
& ^{10} + 38400*a^8*b^{19}*d^4*e^{10} + 46080*a^{10}*b^{17}*d^4*e^{10} + 21504*a^{12}*b^{15} \\
& *d^4*e^{10} - 21504*a^{14}*b^{13}*d^4*e^{10} - 46080*a^{16}*b^{11}*d^4*e^{10} - 38400*a^{18} \\
& *b^9*d^4*e^{10} - 17920*a^{20}*b^7*d^4*e^{10} - 4608*a^{22}*b^5*d^4*e^{10} - 512*a^{24} \\
& *b^3*d^4*e^{10}))/ (8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)*(a^{18}*d^4 \\
& + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70* \\
& a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4)))*(b^4 - \\
& 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2}))/ (8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d \\
& + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2}))/ (8*(a^9*d + \\
& a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3 \\
& *b*e)^{(1/2}))/ (8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15 \\
& *a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)}*i)/ (8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d \\
& + 3*a^7*b^2*d)) + (((e*\cot(c + d*x))^{(1/2)}*(2*a^2*b^{13}*e^{12} - b^{15}*e^{12} + \\
& 49*a^4*b^{11}*e^{12} + 2460*a^6*b^9*e^{12} - 3631*a^8*b^7*e^{12} + 1922*a^{10}*b^5*e \\
& ^{12} - 225*a^{12}*b^3*e^{12}))/ (a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6 \\
& *b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14} \\
& *b^4*d^4 + 8*a^{16}*b^2*d^4) - (((2*b^{18}*d^2*e^{12} - 138*a^2*b^{16}*d^2*e^{12} - 30 \\
& 46*a^4*b^{14}*d^2*e^{12} + 4862*a^6*b^{12}*d^2*e^{12} + 9222*a^8*b^{10}*d^2*e^{12} - 52 \\
& 46*a^{10}*b^8*d^2*e^{12} - 4290*a^{12}*b^6*d^2*e^{12} + 2442*a^{14}*b^4*d^2*e^{12} + 32 \\
& *a^{16}*b^2*d^2*e^{12}))/ (a^{18}*d^5 + a^2*b^{16}*d^5 + 8*a^4*b^{14}*d^5 + 28*a^6*b^{12} \\
& *d^5 + 56*a^8*b^{10}*d^5 + 70*a^{10}*b^8*d^5 + 56*a^{12}*b^6*d^5 + 28*a^{14}*b^4*d^5 \\
& + 8*a^{16}*b^2*d^5) + (((e*\cot(c + d*x))^{(1/2)}*(8*a*b^{20}*d^2*e^{11} - 1152*a \\
& ^3*b^{18}*d^2*e^{11} + 2528*a^5*b^{16}*d^2*e^{11} + 15296*a^7*b^{14}*d^2*e^{11} + 14128 \\
& *a^9*b^{12}*d^2*e^{11} - 5056*a^{11}*b^{10}*d^2*e^{11} - 9248*a^{13}*b^8*d^2*e^{11} + 64* \\
& a^{15}*b^6*d^2*e^{11} + 1800*a^{17}*b^4*d^2*e^{11} + 64*a^{19}*b^2*d^2*e^{11}))/ (a^{18}*d \\
& ^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70 \\
& *a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4) - (((64 \\
& *a*b^{23}*d^4*e^{11} + 1472*a^3*b^{21}*d^4*e^{11} + 8832*a^5*b^{19}*d^4*e^{11} + 25344* \\
& a^7*b^{17}*d^4*e^{11} + 40320*a^9*b^{15}*d^4*e^{11} + 34944*a^{11}*b^{13}*d^4*e^{11} + 10
\end{aligned}$$





$$\begin{aligned}
& *b^5*d^4*e^{10} - 512*a^{24}*b^3*d^4*e^{10})/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d \\
& + 3*a^7*b^2*d)*(a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 \\
& + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + \\
& 8*a^{16}*b^2*d^4)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)})/(8*(a^9*d + \\
& a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3 \\
& *b*e)^{(1/2)})/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15 \\
& *a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)})/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + \\
& 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)})/(8*(a^9*d + a^ \\
& 3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)) + (((e*cot(c + d*x))^{(1/2)}*(2*a^2*b^ \\
& 13*e^{12} - b^{15}*e^{12} + 49*a^4*b^{11}*e^{12} + 2460*a^6*b^9*e^{12} - 3631*a^8*b^7*e \\
& ^{12} + 1922*a^{10}*b^5*e^{12} - 225*a^{12}*b^3*e^{12}))/((a^{18}*d^4 + a^2*b^{16}*d^4 + 8 \\
& *a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^ \\
& 12*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4) - (((2*b^{18}*d^2*e^{12} - 138*a \\
& ^2*b^{16}*d^2*e^{12} - 3046*a^4*b^{14}*d^2*e^{12} + 4862*a^6*b^{12}*d^2*e^{12} + 9222*a \\
& ^8*b^{10}*d^2*e^{12} - 5246*a^{10}*b^8*d^2*e^{12} - 4290*a^{12}*b^6*d^2*e^{12} + 2442*a \\
& ^14*b^4*d^2*e^{12} + 32*a^{16}*b^2*d^2*e^{12}))/((a^{18}*d^5 + a^2*b^{16}*d^5 + 8*a^4*b \\
& ^14*d^5 + 28*a^6*b^{12}*d^5 + 56*a^8*b^{10}*d^5 + 70*a^{10}*b^8*d^5 + 56*a^{12}*b^6 \\
& *d^5 + 28*a^{14}*b^4*d^5 + 8*a^{16}*b^2*d^5) + (((e*cot(c + d*x))^{(1/2)}*(8*a*b \\
& ^20*d^2*e^{11} - 1152*a^3*b^{18}*d^2*e^{11} + 2528*a^5*b^{16}*d^2*e^{11} + 15296*a^7* \\
& b^{14}*d^2*e^{11} + 14128*a^9*b^{12}*d^2*e^{11} - 5056*a^{11}*b^{10}*d^2*e^{11} - 9248*a^ \\
& 13*b^8*d^2*e^{11} + 64*a^{15}*b^6*d^2*e^{11} + 1800*a^{17}*b^4*d^2*e^{11} + 64*a^{19}*b \\
& ^2*d^2*e^{11}))/((a^{18}*d^4 + a^2*b^{16}*d^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + \\
& 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8 \\
& *a^{16}*b^2*d^4) - (((64*a*b^{23}*d^4*e^{11} + 1472*a^3*b^{21}*d^4*e^{11} + 8832*a^5*b \\
& ^19*d^4*e^{11} + 25344*a^7*b^{17}*d^4*e^{11} + 40320*a^9*b^{15}*d^4*e^{11} + 34944*a^ \\
& 11*b^{13}*d^4*e^{11} + 10752*a^{13}*b^{11}*d^4*e^{11} - 8448*a^{15}*b^9*d^4*e^{11} - 1017 \\
& 6*a^{17}*b^7*d^4*e^{11} - 4160*a^{19}*b^5*d^4*e^{11} - 640*a^{21}*b^3*d^4*e^{11}))/((a^{18} \\
& *d^5 + a^2*b^{16}*d^5 + 8*a^4*b^{14}*d^5 + 28*a^6*b^{12}*d^5 + 56*a^8*b^{10}*d^5 + \\
& 70*a^{10}*b^8*d^5 + 56*a^{12}*b^6*d^5 + 28*a^{14}*b^4*d^5 + 8*a^{16}*b^2*d^5) + ((e \\
& *cot(c + d*x))^{(1/2)}*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)}*(512*a^2* \\
& b^{25}*d^4*e^{10} + 4608*a^4*b^{23}*d^4*e^{10} + 17920*a^6*b^{21}*d^4*e^{10} + 38400*a^ \\
& 8*b^{19}*d^4*e^{10} + 46080*a^{10}*b^{17}*d^4*e^{10} + 21504*a^{12}*b^{15}*d^4*e^{10} - 215 \\
& 04*a^{14}*b^{13}*d^4*e^{10} - 46080*a^{16}*b^{11}*d^4*e^{10} - 38400*a^{18}*b^9*d^4*e^{10} \\
& - 17920*a^{20}*b^7*d^4*e^{10} - 4608*a^{22}*b^5*d^4*e^{10} - 512*a^{24}*b^3*d^4*e^{10} \\
& ))/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)*(a^{18}*d^4 + a^2*b^{16}*d \\
& ^4 + 8*a^4*b^{14}*d^4 + 28*a^6*b^{12}*d^4 + 56*a^8*b^{10}*d^4 + 70*a^{10}*b^8*d^4 + \\
& 56*a^{12}*b^6*d^4 + 28*a^{14}*b^4*d^4 + 8*a^{16}*b^2*d^4)))*(b^4 - 15*a^4 + 18*a \\
& ^2*b^2)*(-a^3*b*e)^{(1/2)})/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d \\
& ))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)})/(8*(a^9*d + a^3*b^6*d + 3 \\
& *a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)})/( \\
& 8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)))*(b^4 - 15*a^4 + 18*a^2* \\
& b^2)*(-a^3*b*e)^{(1/2)})/(8*(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d)) \\
& )*(b^4 - 15*a^4 + 18*a^2*b^2)*(-a^3*b*e)^{(1/2)}*1i)/(4*(a^9*d + a^3*b^6*d + \\
& 3*a^5*b^4*d + 3*a^7*b^2*d))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \cot(c + dx)}}{(a + b \cot(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3, x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**3, x)
```

$$3.86 \quad \int \frac{1}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=476

$$\frac{b^2 (11a^2 + 3b^2) \sqrt{e \cot(c+dx)}}{4a^2 d e (a^2 + b^2)^2 (a + b \cot(c+dx))} - \frac{b^2 \sqrt{e \cot(c+dx)}}{2a d e (a^2 + b^2) (a + b \cot(c+dx))^2} + \frac{(a-b) (a^2 + 4ab + b^2) \log(\sqrt{e \cot(c+dx)})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)}$$

[Out]  $-1/4*b^{(3/2)}*(35*a^4+6*a^2*b^2+3*b^4)*\arctan(b^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/a^{(1/2)}/e^{(1/2)})/a^{(5/2)}/(a^2+b^2)^3/d/e^{(1/2)}+1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}/e^{(1/2)}-1/2*(a+b)*(a^2-4*a*b+b^2)*\arctan(1+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/e^{(1/2)})/(a^2+b^2)^3/d*2^{(1/2)}/e^{(1/2)}+1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}-2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)^3/d*2^{(1/2)}/e^{(1/2)}-1/4*(a-b)*(a^2+4*a*b+b^2)*\ln(e^{(1/2)}+\cot(d*x+c))*e^{(1/2)}+2^{(1/2)}*(e*\cot(d*x+c))^{(1/2)}/(a^2+b^2)^3/d*2^{(1/2)}/e^{(1/2)}-1/2*b^2*(e*\cot(d*x+c))^{(1/2)}/a/(a^2+b^2)/d/e/(a+b*\cot(d*x+c))^2-1/4*b^2*(11*a^2+3*b^2)*(e*\cot(d*x+c))^{(1/2)}/a^2/(a^2+b^2)^2/d/e/(a+b*\cot(d*x+c))$

**Rubi [A]** time = 1.24, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3569, 3649, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{b^2 (11a^2 + 3b^2) \sqrt{e \cot(c+dx)}}{4a^2 d e (a^2 + b^2)^2 (a + b \cot(c+dx))} - \frac{b^2 \sqrt{e \cot(c+dx)}}{2a d e (a^2 + b^2) (a + b \cot(c+dx))^2} + \frac{(a-b) (a^2 + 4ab + b^2) \log(\sqrt{e \cot(c+dx)})}{2\sqrt{2} d \sqrt{e} (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^3), x]

[Out]  $-(b^{(3/2)}*(35*a^4 + 6*a^2*b^2 + 3*b^4)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(\text{Sqrt}[a]*\text{Sqrt}[e])])/(4*a^{(5/2)}*(a^2 + b^2)^3*d*\text{Sqrt}[e]) + ((a + b)*(a^2 - 4*a*b + b^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)^3*d*\text{Sqrt}[e]) - ((a + b)*(a^2 - 4*a*b + b^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/\text{Sqrt}[e]])/(\text{Sqrt}[2]*(a^2 + b^2)^3*d*\text{Sqrt}[e]) - (b^2*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*a*(a^2 + b^2)*d*e*(a + b*\text{Cot}[c + d*x])^2) - (b^2*(11*a^2 + 3*b^2)*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(4*a^2*(a^2 + b^2)^2*d*e*(a + b*\text{Cot}[c + d*x])) + ((a - b)*(a^2 + 4*a*b + b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^3*d*\text{Sqrt}[e]) - ((a - b)*(a^2 + 4*a*b + b^2)*\text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e]*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[e*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*(a^2 + b^2)^3*d*\text{Sqrt}[e]))$

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

$eQ[\{a, c, d, e\}, x] \ \&\& \ EqQ[c*d^2 - a*e^2, 0] \ \&\& \ NegQ[d*e]$

### Rule 1168

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] \ :> \ With[\{q = Rt[a*c, 2]\}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] \ /; \ FreeQ[\{a, c, d, e\}, x] \ \&\& \ NeQ[c*d^2 + a*e^2, 0] \ \&\& \ NeQ[c*d^2 - a*e^2, 0] \ \&\& \ NegQ[-(a*c)]$

### Rule 3534

$Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/Sqrt[(b_)*tan[(e_) + (f_)*(x_)]], x\_Symbol] \ :> \ Dist[2/f, Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sqrt[b*Tan[e + f*x]]], x] \ /; \ FreeQ[\{b, c, d, e, f\}, x] \ \&\& \ NeQ[c^2 - d^2, 0] \ \&\& \ NeQ[c^2 + d^2, 0]$

### Rule 3569

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_), x\_Symbol] \ :> \ Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] \ /; \ FreeQ[\{a, b, c, d, e, f, n\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ NeQ[a^2 + b^2, 0] \ \&\& \ NeQ[c^2 + d^2, 0] \ \&\& \ IntegerQ[2*m] \ \&\& \ LtQ[m, -1] \ \&\& \ (LtQ[n, 0] \ || \ IntegerQ[m]) \ \&\& \ !(ILtQ[n, -1] \ \&\& \ (!IntegerQ[m] \ || \ (EqQ[c, 0] \ \&\& \ NeQ[a, 0])))$

### Rule 3634

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*tan[(e_) + (f_)*(x_)^2]), x\_Symbol] \ :> \ Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] \ /; \ FreeQ[\{a, b, c, d, e, f, A, C, m, n\}, x] \ \&\& \ EqQ[A, C]$

### Rule 3649

$Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*tan[(e_) + (f_)*(x_)^2] + (C_)*tan[(e_) + (f_)*(x_)^2]), x\_Symbol] \ :> \ Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)$

```

*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))^3} dx &= -\frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}(4a^2+3b^2)e+2abe \cot(c+dx)-\frac{3}{2}b^2e}{\sqrt{e \cot(c+dx)} (a+b \cot(c+dx))} dx}{2a(a^2+b^2)e} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))^2} - \frac{b^2(11a^2+3b^2) \sqrt{e \cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b \cot(c+dx))} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))^2} - \frac{b^2(11a^2+3b^2) \sqrt{e \cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b \cot(c+dx))} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))^2} - \frac{b^2(11a^2+3b^2) \sqrt{e \cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b \cot(c+dx))} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))^2} - \frac{b^2(11a^2+3b^2) \sqrt{e \cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b \cot(c+dx))} \\
&= -\frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))^2} - \frac{b^2(11a^2+3b^2) \sqrt{e \cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b \cot(c+dx))} \\
&= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} - \frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))} \\
&= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} - \frac{b^2 \sqrt{e \cot(c+dx)}}{2a(a^2+b^2) de(a+b \cot(c+dx))} \\
&= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4) \tan^{-1}\left(\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} + \frac{(a+b)(a^2-4ab)}{\sqrt{e}}
\end{aligned}$$

**Mathematica** [C] time = 6.13, size = 411, normalized size = 0.86

$$\sqrt{\cot(c+dx)} \left( -\frac{2b(3a^2-b^2) \cot^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\cot^2(c+dx)\right)}{3(a^2+b^2)^3} + \frac{2b^2 \sqrt{\cot(c+dx)} \left( \frac{a}{a+b \cot(c+dx)} + \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b} \sqrt{\cot(c+dx)}} \right)}{a(a^2+b^2)^2} - \frac{a(a^2-3b^2)(2)}{\sqrt{e}} \right)$$

Antiderivative was successfully verified.



[In] Integrate[1/(Sqrt[e\*Cot[c + d\*x]]\*(a + b\*Cot[c + d\*x])^3),x]

[Out]  $-\left(\frac{\sqrt{\cot[c + dx]} \left( (2b^{3/2} (3a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot[c + dx]}}{\sqrt{a}}\right] \right) / \sqrt{a}}{\sqrt{a} (a^2 + b^2)^3} + (2b^2 \sqrt{\cot[c + dx]} \left( \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot[c + dx]}}{\sqrt{a}}\right]}{\sqrt{a}} \right) / (\sqrt{b} \sqrt{\cot[c + dx]}) + a / (a + b \cot[c + dx]) \right) / (a (a^2 + b^2)^2} + (2b^2 \sqrt{\cot[c + dx]} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 3, \frac{3}{2}, -\frac{b \cot[c + dx]}{a}\right] / (a^3 (a^2 + b^2)) - (2b (3a^2 - b^2) \cot[c + dx]^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -\cot[c + dx]^2\right] / (3 (a^2 + b^2)^3} - (a (a^2 - 3b^2) (4 \sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\cot[c + dx]}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\cot[c + dx]}\right]) + 2 \sqrt{2} \log\left[1 - \sqrt{2} \sqrt{\cot[c + dx]} + \cot[c + dx]\right] - 2 \sqrt{2} \log\left[1 + \sqrt{2} \sqrt{\cot[c + dx]} + \cot[c + dx]\right]) / (8 (a^2 + b^2)^3)}\right) / (d \sqrt{e \cot[c + dx]})$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(dx+c))^(1/2)/(a+b\*cot(dx+c))^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cot(dx + c) + a)^3 \sqrt{e \cot(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(dx+c))^(1/2)/(a+b\*cot(dx+c))^3,x, algorithm="giac")

[Out] integrate(1/((b\*cot(dx + c) + a)^3\*sqrt(e\*cot(dx + c))), x)

**maple** [B] time = 0.84, size = 1190, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*cot(dx+c))^(1/2)/(a+b\*cot(dx+c))^3,x)

[Out]  $-11/4/d*b^3/(a^2+b^2)^3/(e*\cot(dx+c)*b+a*e)^2*a^2*(e*\cot(dx+c))^{3/2}-7/2/d*b^5/(a^2+b^2)^3/(e*\cot(dx+c)*b+a*e)^2*(e*\cot(dx+c))^{3/2}-3/4/d*b^7/(a^2+b^2)^3/(e*\cot(dx+c)*b+a*e)^2/a^2*(e*\cot(dx+c))^{3/2}-13/4/d*e*b^2/(a^2+b^2)^3/(e*\cot(dx+c)*b+a*e)^2*a^3*(e*\cot(dx+c))^{1/2}-9/2/d*e*b^4/(a^2+b^2)^3/(e*\cot(dx+c)*b+a*e)^2*a*(e*\cot(dx+c))^{1/2}-5/4/d*e*b^6/(a^2+b^2)^3/$

$$\begin{aligned} & (e \cot(dx+c) * b + a * e)^2 / a * (e \cot(dx+c))^{1/2} - 35/4 / d * b^2 / (a^2 + b^2)^3 * a^2 / (a * e * b)^{1/2} * \arctan((e \cot(dx+c))^{1/2} * b / (a * e * b)^{1/2}) - 3/2 / d * b^4 / (a^2 + b^2)^3 / (a * e * b)^{1/2} * \arctan((e \cot(dx+c))^{1/2} * b / (a * e * b)^{1/2}) - 3/4 / d * b^6 / (a^2 + b^2)^3 / a^2 / (a * e * b)^{1/2} * \arctan((e \cot(dx+c))^{1/2} * b / (a * e * b)^{1/2}) + 1/2 / d / e / (a^2 + b^2)^3 * (e^2)^{1/4} * 2^{1/2} * \arctan(-2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * a^3 - 3/2 / d / e / (a^2 + b^2)^3 * (e^2)^{1/4} * 2^{1/2} * \arctan(-2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * a * b^2 - 1/2 / d / e / (a^2 + b^2)^3 * (e^2)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * a^3 + 3/2 / d / e / (a^2 + b^2)^3 * (e^2)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * a * b^2 - 1/4 / d / e / (a^2 + b^2)^3 * (e^2)^{1/4} * 2^{1/2} * \ln((e \cot(dx+c) + (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) - (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) * a^3 + 3/4 / d / e / (a^2 + b^2)^3 * (e^2)^{1/4} * 2^{1/2} * \ln((e \cot(dx+c) + (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) - (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) * a * b^2 - 3/2 / d / (a^2 + b^2)^3 * 2^{1/2} / (e^2)^{1/4} * \arctan(-2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * a^2 * b + 1/2 / d / (a^2 + b^2)^3 * 2^{1/2} / (e^2)^{1/4} * \arctan(-2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * a^2 * b - 1/2 / d / (a^2 + b^2)^3 * 2^{1/2} / (e^2)^{1/4} * \arctan(2^{1/2} / (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} + 1 * b^3 + 3/4 / d / (a^2 + b^2)^3 * 2^{1/2} / (e^2)^{1/4} * \ln((e \cot(dx+c) - (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) + (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) * a^2 * b - 1/4 / d / (a^2 + b^2)^3 * 2^{1/2} / (e^2)^{1/4} * \ln((e \cot(dx+c) - (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) / (e \cot(dx+c) + (e^2)^{1/4}) * (e \cot(dx+c))^{1/2} * 2^{1/2} + (e^2)^{1/2}) * b^3 \end{aligned}$$

**maxima** [A] time = 0.48, size = 510, normalized size = 1.07

$$e \left( \frac{(13 a^3 b^2 + 5 a b^4) e \sqrt{\frac{e}{\tan(dx+c)}} + (11 a^2 b^3 + 3 b^5) \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}}}{(a^8 + 2 a^6 b^2 + a^4 b^4) e^3 + \frac{2(a^7 b + 2 a^5 b^3 + a^3 b^5) e^3}{\tan(dx+c)} + \frac{(a^6 b^2 + 2 a^4 b^4 + a^2 b^6) e^3}{\tan(dx+c)^2}} + \frac{(35 a^4 b^2 + 6 a^2 b^4 + 3 b^6) \arctan\left(\frac{b \sqrt{\frac{e}{\tan(dx+c)}}}{\sqrt{a b e}}\right)}{(a^8 + 3 a^6 b^2 + 3 a^4 b^4 + a^2 b^6) \sqrt{a b e}} + \frac{2 \sqrt{2} (a^3 - 3 a^2 b - 3 a b^2 + b^3) \arctan\left(\frac{1}{2} \sqrt{2} * \sqrt{2} * \sqrt{2} * \sqrt{2}\right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(1/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] -1/4\*e\*(((13\*a^3\*b^2 + 5\*a\*b^4)\*e\*sqrt(e/tan(d\*x + c)) + (11\*a^2\*b^3 + 3\*b^5)\*(e/tan(d\*x + c))^(3/2))/((a^8 + 2\*a^6\*b^2 + a^4\*b^4)\*e^3 + 2\*(a^7\*b + 2\*a^5\*b^3 + a^3\*b^5)\*e^3/tan(d\*x + c) + (a^6\*b^2 + 2\*a^4\*b^4 + a^2\*b^6)\*e^3/tan(d\*x + c)^2) + (35\*a^4\*b^2 + 6\*a^2\*b^4 + 3\*b^6)\*arctan(b\*sqrt(e/tan(d\*x + c))/sqrt(a\*b\*e))/((a^8 + 3\*a^6\*b^2 + 3\*a^4\*b^4 + a^2\*b^6)\*sqrt(a\*b\*e)\*e) + (2\*sqrt(2)\*(a^3 - 3\*a^2\*b - 3\*a\*b^2 + b^3)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*sqrt(2)\*sqrt(2))))

$$\frac{t(e) + 2\sqrt{e/\tan(dx + c)}}{\sqrt{e}}/\sqrt{e} + 2\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3)\arctan(-1/2\sqrt{2}(\sqrt{2}\sqrt{e} - 2\sqrt{e/\tan(dx + c)}))/\sqrt{e}}/\sqrt{e} + \sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3)\log(\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx + c)} + e + e/\tan(dx + c))/\sqrt{e} - \sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3)\log(-\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx + c)} + e + e/\tan(dx + c))/\sqrt{e}}/((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)e)/d$$

**mupad [B]** time = 6.79, size = 20155, normalized size = 42.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/((e*\cot(c + d*x))^{1/2}*(a + b*\cot(c + d*x))^3), x)$

[Out]  $\text{atan}\left(\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{192a^2b^{24}d^4e^{10} + 1728a^4b^{22}d^4e^{10} + 8320a^6b^{20}d^4e^{10} + 27264a^8b^{18}d^4e^{10} + 62592a^{10}b^{16}d^4e^{10} + 99456a^{12}b^{14}d^4e^{10} + 107520a^{14}b^{12}d^4e^{10} + 76800a^{16}b^{10}d^4e^{10} + 33984a^{18}b^8d^4e^{10} + 7872a^{20}b^6d^4e^{10} + 384a^{22}b^4d^4e^{10} - 128a^{24}b^2d^4e^{10}\right)}{(a^{20}d^5 + a^4b^{16}d^5 + 8a^6b^{14}d^5 + 28a^8b^{12}d^5 + 56a^{10}b^{10}d^5 + 70a^{12}b^8d^5 + 56a^{14}b^6d^5 + 28a^{16}b^4d^5 + 8a^{18}b^2d^5) - \left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{512a^4b^{25}d^4e^{10} + 4608a^6b^{23}d^4e^{10} + 17920a^8b^{21}d^4e^{10} + 38400a^{10}b^{19}d^4e^{10} + 46080a^{12}b^{17}d^4e^{10} + 21504a^{14}b^{15}d^4e^{10} - 21504a^{16}b^{13}d^4e^{10} - 46080a^{18}b^{11}d^4e^{10} - 38400a^{20}b^9d^4e^{10} - 17920a^{22}b^7d^4e^{10} - 4608a^{24}b^5d^4e^{10} - 512a^{26}b^3d^4e^{10}\right)}{(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4)} + \left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{1}{4}(b^6d^2e - a^6d^2e - 15a^2b^4d^2e - a^3b^3d^2e*20i + 15a^4b^2d^2e + ab^5d^2e*6i + a^5bd^2e*6i)\right)^{1/2}\left(\frac{72a^3b^{22}d^2e^9 + 576a^5b^{20}d^2e^9 + 5024a^7b^{18}d^2e^9 + 14272a^9b^{16}d^2e^9 + 27824a^{11}b^{14}d^2e^9 + 53184a^{13}b^{12}d^2e^9 + 70240a^{15}b^{10}d^2e^9 + 47680a^{17}b^8d^2e^9 + 12616a^{19}b^6d^2e^9 - 64a^{21}b^4d^2e^9 + 47680a^{23}b^2d^2e^9 - 12616a^{25}d^2e^9 - 5024a^{27}d^2e^9 - 576a^{29}d^2e^9 - 72a^{31}d^2e^9}{(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4)} - \left(\frac{90a^3b^{19}d^2e^9 + 846a^5b^{17}d^2e^9 + 1714a^7b^{15}d^2e^9 + 3606a^9b^{13}d^2e^9 - 14578a^{11}b^{11}d^2e^9 - 34486a^{13}b^9d^2e^9 - 14970a^{15}b^7d^2e^9 + 2258a^{17}b^5d^2e^9 - 32a^{19}b^3d^2e^9}{(a^{20}d^5 + a^4b^{16}d^5 + 8a^6b^{14}d^5 + 28a^8b^{12}d^5 + 56a^{10}b^{10}d^5 + 70a^{12}b^8d^5 + 56a^{14}b^6d^5 + 28a^{16}b^4d^5 + 8a^{18}b^2d^5)}\right)$

$$\begin{aligned}
& 5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5)) \\
& + ((e*\cot(c + d*x))^{(1/2)}*(18*a^2*b^{15}*e^8 - 9*b^{17}*e^8 - 71*a^4*b^{13}*e^8 + \\
& 892*a^6*b^{11}*e^8 + 857*a^8*b^9*e^8 + 6802*a^{10}*b^7*e^8 - 1257*a^{12}*b^5*e^8 \\
& ))/(a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 \\
& + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4) \\
& ))*1i - (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e* \\
& 20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^{(1/2)}*((1i/(4*( \\
& b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2* \\
& e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^{(1/2)}*((1i/(4*(b^6*d^2*e - a^6*d^ \\
& 2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e \\
& *6i + a^5*b*d^2*e*6i)))^{(1/2)}*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d \\
& ^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e* \\
& 6i)))^{(1/2)}*((192*a^2*b^{24}*d^4*e^{10} + 1728*a^4*b^{22}*d^4*e^{10} + 8320*a^6*b^2 \\
& 0*d^4*e^{10} + 27264*a^8*b^{18}*d^4*e^{10} + 62592*a^{10}*b^{16}*d^4*e^{10} + 99456*a^{1 \\
& 2*b^{14}*d^4*e^{10} + 107520*a^{14}*b^{12}*d^4*e^{10} + 76800*a^{16}*b^{10}*d^4*e^{10} + 33 \\
& 984*a^{18}*b^8*d^4*e^{10} + 7872*a^{20}*b^6*d^4*e^{10} + 384*a^{22}*b^4*d^4*e^{10} - 12 \\
& 8*a^{24}*b^2*d^4*e^{10})/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{1 \\
& 2*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4* \\
& d^5 + 8*a^{18}*b^2*d^5) + ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - \\
& a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^{( \\
& 1/2)}*(e*\cot(c + d*x))^{(1/2)}*(512*a^4*b^{25}*d^4*e^{10} + 4608*a^6*b^{23}*d^4*e^{1 \\
& 0 + 17920*a^8*b^{21}*d^4*e^{10} + 38400*a^{10}*b^{19}*d^4*e^{10} + 46080*a^{12}*b^{17}*d^ \\
& 4*e^{10} + 21504*a^{14}*b^{15}*d^4*e^{10} - 21504*a^{16}*b^{13}*d^4*e^{10} - 46080*a^{18}*b \\
& ^{11}*d^4*e^{10} - 38400*a^{20}*b^9*d^4*e^{10} - 17920*a^{22}*b^7*d^4*e^{10} - 4608*a^{2 \\
& 4*b^5*d^4*e^{10} - 512*a^{26}*b^3*d^4*e^{10}))/ (a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b \\
& ^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^ \\
& 6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4) - ((e*\cot(c + d*x))^{(1/2)}*(72*a* \\
& b^{22}*d^2*e^9 + 576*a^3*b^{20}*d^2*e^9 + 5024*a^5*b^{18}*d^2*e^9 + 14272*a^7*b^{1 \\
& 6*d^2*e^9 + 27824*a^9*b^{14}*d^2*e^9 + 53184*a^{11}*b^{12}*d^2*e^9 + 70240*a^{13}*b \\
& ^{10}*d^2*e^9 + 47680*a^{15}*b^8*d^2*e^9 + 12616*a^{17}*b^6*d^2*e^9 - 64*a^{21}*b^2 \\
& *d^2*e^9))/ (a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56 \\
& *a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^ \\
& 18*b^2*d^4) - (90*a*b^{19}*d^2*e^9 + 846*a^3*b^{17}*d^2*e^9 + 1714*a^5*b^{15}*d^ \\
& 2*e^9 + 3606*a^7*b^{13}*d^2*e^9 - 14578*a^9*b^{11}*d^2*e^9 - 34486*a^{11}*b^9*d^2 \\
& *e^9 - 14970*a^{13}*b^7*d^2*e^9 + 2258*a^{15}*b^5*d^2*e^9 - 32*a^{17}*b^3*d^2*e^9 \\
& )/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{1 \\
& 0*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^ \\
& 5) - ((e*\cot(c + d*x))^{(1/2)}*(18*a^2*b^{15}*e^8 - 9*b^{17}*e^8 - 71*a^4*b^{13}*e \\
& ^8 + 892*a^6*b^{11}*e^8 + 857*a^8*b^9*e^8 + 6802*a^{10}*b^7*e^8 - 1257*a^{12}*b^5 \\
& *e^8))/ (a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{1 \\
& 0*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b \\
& ^2*d^4))*1i)/((9*b^{14}*e^8 + 60*a^2*b^{12}*e^8 + 318*a^4*b^{10}*e^8 + 748*a^6*b^ \\
& 8*e^8 + 1505*a^8*b^6*e^8)/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^ \\
& 8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16} \\
& *b^4*d^5 + 8*a^{18}*b^2*d^5) + (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2
\end{aligned}$$

$$\begin{aligned}
& *e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2*d^2*e + a*b^5*d^2*e^{6i} + a^5*b*d^2*e^{6i} \\
& ))^{(1/2)}*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e \\
& *20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e^{6i} + a^5*b*d^2*e^{6i})))^{(1/2)}*((1i/(4* \\
& (b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2* \\
& d^2*e + a*b^5*d^2*e^{6i} + a^5*b*d^2*e^{6i})))^{(1/2)}*((1i/(4*(b^6*d^2*e - a^6*d \\
& ^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2*d^2*e + a*b^5*d^2* \\
& e^{6i} + a^5*b*d^2*e^{6i})))^{(1/2)}*((192*a^2*b^24*d^4*e^{10} + 1728*a^4*b^22*d^4* \\
& e^{10} + 8320*a^6*b^20*d^4*e^{10} + 27264*a^8*b^18*d^4*e^{10} + 62592*a^10*b^16*d \\
& ^4*e^{10} + 99456*a^12*b^14*d^4*e^{10} + 107520*a^14*b^12*d^4*e^{10} + 76800*a^16 \\
& *b^10*d^4*e^{10} + 33984*a^18*b^8*d^4*e^{10} + 7872*a^20*b^6*d^4*e^{10} + 384*a^2 \\
& 2*b^4*d^4*e^{10} - 128*a^24*b^2*d^4*e^{10}))/ (a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^ \\
& 14*d^5 + 28*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6 \\
& *d^5 + 28*a^16*b^4*d^5 + 8*a^18*b^2*d^5) - ((1i/(4*(b^6*d^2*e - a^6*d^2*e - \\
& 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2*d^2*e + a*b^5*d^2*e^{6i} + \\
& a^5*b*d^2*e^{6i})))^{(1/2)}*(e*cot(c + d*x))^{(1/2)}*(512*a^4*b^25*d^4*e^{10} + 46 \\
& 08*a^6*b^23*d^4*e^{10} + 17920*a^8*b^21*d^4*e^{10} + 38400*a^10*b^19*d^4*e^{10} + \\
& 46080*a^12*b^17*d^4*e^{10} + 21504*a^14*b^15*d^4*e^{10} - 21504*a^16*b^13*d^4* \\
& e^{10} - 46080*a^18*b^11*d^4*e^{10} - 38400*a^20*b^9*d^4*e^{10} - 17920*a^22*b^7* \\
& d^4*e^{10} - 4608*a^24*b^5*d^4*e^{10} - 512*a^26*b^3*d^4*e^{10}))/ (a^20*d^4 + a^4 \\
& *b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^ \\
& ^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^2*d^4)) + ((e*cot(c + \\
& d*x))^{(1/2)}*(72*a*b^22*d^2*e^9 + 576*a^3*b^20*d^2*e^9 + 5024*a^5*b^18*d^2* \\
& e^9 + 14272*a^7*b^16*d^2*e^9 + 27824*a^9*b^14*d^2*e^9 + 53184*a^11*b^12*d^2 \\
& *e^9 + 70240*a^13*b^10*d^2*e^9 + 47680*a^15*b^8*d^2*e^9 + 12616*a^17*b^6*d^ \\
& 2*e^9 - 64*a^21*b^2*d^2*e^9))/ (a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 2 \\
& 8*a^8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28* \\
& a^16*b^4*d^4 + 8*a^18*b^2*d^4)) - (90*a*b^19*d^2*e^9 + 846*a^3*b^17*d^2*e^9 \\
& + 1714*a^5*b^15*d^2*e^9 + 3606*a^7*b^13*d^2*e^9 - 14578*a^9*b^11*d^2*e^9 - \\
& 34486*a^11*b^9*d^2*e^9 - 14970*a^13*b^7*d^2*e^9 + 2258*a^15*b^5*d^2*e^9 - \\
& 32*a^17*b^3*d^2*e^9))/ (a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28*a^8*b^1 \\
& 2*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*a^16*b^4* \\
& d^5 + 8*a^18*b^2*d^5)) + ((e*cot(c + d*x))^{(1/2)}*(18*a^2*b^15*e^8 - 9*b^17* \\
& e^8 - 71*a^4*b^13*e^8 + 892*a^6*b^11*e^8 + 857*a^8*b^9*e^8 + 6802*a^10*b^7* \\
& e^8 - 1257*a^12*b^5*e^8))/ (a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^ \\
& 8*b^12*d^4 + 56*a^10*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16 \\
& *b^4*d^4 + 8*a^18*b^2*d^4)) + (1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^ \\
& 2*e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2*d^2*e + a*b^5*d^2*e^{6i} + a^5*b*d^2*e^{6i} \\
& i)))^{(1/2)}*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2* \\
& e^{20i} + 15*a^4*b^2*d^2*e + a*b^5*d^2*e^{6i} + a^5*b*d^2*e^{6i})))^{(1/2)}*((1i/(4 \\
& *(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2 \\
& *d^2*e + a*b^5*d^2*e^{6i} + a^5*b*d^2*e^{6i})))^{(1/2)}*((1i/(4*(b^6*d^2*e - a^6* \\
& d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e^{20i} + 15*a^4*b^2*d^2*e + a*b^5*d^2 \\
& *e^{6i} + a^5*b*d^2*e^{6i})))^{(1/2)}*((192*a^2*b^24*d^4*e^{10} + 1728*a^4*b^22*d^4 \\
& *e^{10} + 8320*a^6*b^20*d^4*e^{10} + 27264*a^8*b^18*d^4*e^{10} + 62592*a^10*b^16* \\
& d^4*e^{10} + 99456*a^12*b^14*d^4*e^{10} + 107520*a^14*b^12*d^4*e^{10} + 76800*a^1
\end{aligned}$$

$$\begin{aligned}
& 6*b^{10}*d^4*e^{10} + 33984*a^{18}*b^8*d^4*e^{10} + 7872*a^{20}*b^6*d^4*e^{10} + 384*a^{22}*b^4*d^4*e^{10} - 128*a^{24}*b^2*d^4*e^{10})/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5) + ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^{(1/2)}*(e*cot(c + d*x))^{(1/2)}*(512*a^4*b^{25}*d^4*e^{10} + 4608*a^6*b^{23}*d^4*e^{10} + 17920*a^8*b^{21}*d^4*e^{10} + 38400*a^{10}*b^{19}*d^4*e^{10} + 46080*a^{12}*b^{17}*d^4*e^{10} + 21504*a^{14}*b^{15}*d^4*e^{10} - 21504*a^{16}*b^{13}*d^4*e^{10} - 46080*a^{18}*b^{11}*d^4*e^{10} - 38400*a^{20}*b^9*d^4*e^{10} - 17920*a^{22}*b^7*d^4*e^{10} - 4608*a^{24}*b^5*d^4*e^{10} - 512*a^{26}*b^3*d^4*e^{10}))/((a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4)) - ((e*cot(c + d*x))^{(1/2)}*(72*a*b^{22}*d^2*e^9 + 576*a^3*b^{20}*d^2*e^9 + 5024*a^5*b^{18}*d^2*e^9 + 14272*a^7*b^{16}*d^2*e^9 + 27824*a^9*b^{14}*d^2*e^9 + 53184*a^{11}*b^{12}*d^2*e^9 + 70240*a^{13}*b^{10}*d^2*e^9 + 47680*a^{15}*b^8*d^2*e^9 + 12616*a^{17}*b^6*d^2*e^9 - 64*a^{21}*b^2*d^2*e^9))/((a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4)) - (90*a*b^{19}*d^2*e^9 + 846*a^3*b^{17}*d^2*e^9 + 1714*a^5*b^{15}*d^2*e^9 + 3606*a^7*b^{13}*d^2*e^9 - 14578*a^9*b^{11}*d^2*e^9 - 34486*a^{11}*b^9*d^2*e^9 - 14970*a^{13}*b^7*d^2*e^9 + 2258*a^{15}*b^5*d^2*e^9 - 32*a^{17}*b^3*d^2*e^9)/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5)) - ((e*cot(c + d*x))^{(1/2)}*(18*a^2*b^{15}*e^8 - 9*b^{17}*e^8 - 71*a^4*b^{13}*e^8 + 892*a^6*b^{11}*e^8 + 857*a^8*b^9*e^8 + 6802*a^{10}*b^7*e^8 - 1257*a^{12}*b^5*e^8))/((a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4))))*(1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^{(1/2)}*2i - ((b^3*(e*cot(c + d*x))^{(3/2)}*(11*a^2 + 3*b^2))/(4*a^2*(a^4 + b^4 + 2*a^2*b^2)) + (b^2*e*(e*cot(c + d*x))^{(1/2)}*(13*a^2 + 5*b^2))/(4*a*(a^2 + b^2)^2))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c + d*x)) + atan((((((((1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e)))^{(1/2)}*((192*a^2*b^{24}*d^4*e^{10} + 1728*a^4*b^{22}*d^4*e^{10} + 8320*a^6*b^{20}*d^4*e^{10} + 27264*a^8*b^{18}*d^4*e^{10} + 62592*a^{10}*b^{16}*d^4*e^{10} + 99456*a^{12}*b^{14}*d^4*e^{10} + 107520*a^{14}*b^{12}*d^4*e^{10} + 76800*a^{16}*b^{10}*d^4*e^{10} + 33984*a^{18}*b^8*d^4*e^{10} + 7872*a^{20}*b^6*d^4*e^{10} + 384*a^{22}*b^4*d^4*e^{10} - 128*a^{24}*b^2*d^4*e^{10}))/((2*(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5)) - ((e*cot(c + d*x))^{(1/2)}*(1/(b^6*d^2*e*1i - a^6*d^2*e*1i - a^2*b^4*d^2*e*15i - 20*a^3*b^3*d^2*e + a^4*b^2*d^2*e*15i + 6*a*b^5*d^2*e + 6*a^5*b*d^2*e)))^{(1/2)}*(512*a^4*b^{25}*d^4*e^{10} + 4608*a^6*b^{23}*d^4*e^{10} + 17920*a^8*b^{21}*d^4*e^{10} + 38400*a^{10}*b^{19}*d^4*e^{10} + 46080*a^{12}*b^{17}*d^4*e^{10} + 21504*a^{14}*b^{15}*d^4*e^{10} - 21504*a^{16}*b^{13}*d^4*e^{10} - 46080*a^{18}*b^{11}*d^4*e^{10} - 38400*a^{20}*b^9*d^4*e^{10} - 17920*a^{22}*b^7*d^4*e^{10} - 4608*a^{24}*b^5
\end{aligned}$$

$$\begin{aligned}
& *d^4e^{10} - 512a^{26}b^3d^4e^{10})) / (4*(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4))) / 2 + ((e*\cot(c + dx))^{(1/2)}*(72a*b^{22}d^2e^9 + 576a^3b^{20}d^2e^9 + 5024a^5b^{18}d^2e^9 + 14272a^7b^{16}d^2e^9 + 27824a^9b^{14}d^2e^9 + 53184a^{11}b^{12}d^2e^9 + 70240a^{13}b^{10}d^2e^9 + 47680a^{15}b^8d^2e^9 + 12616a^{17}b^6d^2e^9 - 64a^{21}b^2d^2e^9)) / (2*(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4))) * (1/(b^6d^2e^{11} - a^6d^2e^{11} - a^2b^4d^2e^{15} - 20a^3b^3d^2e + a^4b^2d^2e^{15} + 6a*b^5d^2e + 6a^5b*d^2e))^{(1/2)}) / 2 - (90a*b^{19}d^2e^9 + 846a^3b^{17}d^2e^9 + 1714a^5b^{15}d^2e^9 + 3606a^7b^{13}d^2e^9 - 14578a^9b^{11}d^2e^9 - 34486a^{11}b^9d^2e^9 - 14970a^{13}b^7d^2e^9 + 2258a^{15}b^5d^2e^9 - 32a^{17}b^3d^2e^9) / (2*(a^{20}d^5 + a^4b^{16}d^5 + 8a^6b^{14}d^5 + 28a^8b^{12}d^5 + 56a^{10}b^{10}d^5 + 70a^{12}b^8d^5 + 56a^{14}b^6d^5 + 28a^{16}b^4d^5 + 8a^{18}b^2d^5))) * (1/(b^6d^2e^{11} - a^6d^2e^{11} - a^2b^4d^2e^{15} - 20a^3b^3d^2e + a^4b^2d^2e^{15} + 6a*b^5d^2e + 6a^5b*d^2e))^{(1/2)}) / 2 + ((e*\cot(c + dx))^{(1/2)}*(18a^2b^{15}e^8 - 9b^{17}e^8 - 71a^4b^{13}e^8 + 892a^6b^{11}e^8 + 857a^8b^9e^8 + 6802a^{10}b^7e^8 - 1257a^{12}b^5e^8)) / (2*(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4))) * (1/(b^6d^2e^{11} - a^6d^2e^{11} - a^2b^4d^2e^{15} - 20a^3b^3d^2e + a^4b^2d^2e^{15} + 6a*b^5d^2e + 6a^5b*d^2e))^{(1/2)}) * 1 - (((((((1/(b^6d^2e^{11} - a^6d^2e^{11} - a^2b^4d^2e^{15} - 20a^3b^3d^2e + a^4b^2d^2e^{15} + 6a*b^5d^2e + 6a^5b*d^2e))^{(1/2)}*((192a^2b^{24}d^4e^{10} + 1728a^4b^{22}d^4e^{10} + 8320a^6b^{20}d^4e^{10} + 27264a^8b^{18}d^4e^{10} + 62592a^{10}b^{16}d^4e^{10} + 99456a^{12}b^{14}d^4e^{10} + 107520a^{14}b^{12}d^4e^{10} + 76800a^{16}b^{10}d^4e^{10} + 33984a^{18}b^8d^4e^{10} + 7872a^{20}b^6d^4e^{10} + 384a^{22}b^4d^4e^{10} - 128a^{24}b^2d^4e^{10})) / (2*(a^{20}d^5 + a^4b^{16}d^5 + 8a^6b^{14}d^5 + 28a^8b^{12}d^5 + 56a^{10}b^{10}d^5 + 70a^{12}b^8d^5 + 56a^{14}b^6d^5 + 28a^{16}b^4d^5 + 8a^{18}b^2d^5))) + ((e*\cot(c + dx))^{(1/2)}*(1/(b^6d^2e^{11} - a^6d^2e^{11} - a^2b^4d^2e^{15} - 20a^3b^3d^2e + a^4b^2d^2e^{15} + 6a*b^5d^2e + 6a^5b*d^2e))^{(1/2)}*(512a^4b^{25}d^4e^{10} + 4608a^6b^{23}d^4e^{10} + 17920a^8b^{21}d^4e^{10} + 38400a^{10}b^{19}d^4e^{10} + 46080a^{12}b^{17}d^4e^{10} + 21504a^{14}b^{15}d^4e^{10} - 21504a^{16}b^{13}d^4e^{10} - 46080a^{18}b^{11}d^4e^{10} - 38400a^{20}b^9d^4e^{10} - 17920a^{22}b^7d^4e^{10} - 4608a^{24}b^5d^4e^{10} - 512a^{26}b^3d^4e^{10})) / (4*(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4)))))) / 2 - ((e*\cot(c + dx))^{(1/2)}*(72a*b^{22}d^2e^9 + 576a^3b^{20}d^2e^9 + 5024a^5b^{18}d^2e^9 + 14272a^7b^{16}d^2e^9 + 27824a^9b^{14}d^2e^9 + 53184a^{11}b^{12}d^2e^9 + 70240a^{13}b^{10}d^2e^9 + 47680a^{15}b^8d^2e^9 + 12616a^{17}b^6d^2e^9 - 64a^{21}b^2d^2e^9)) / (2*(a^{20}d^4 + a^4b^{16}d^4 + 8a^6b^{14}d^4 + 28a^8b^{12}d^4 + 56a^{10}b^{10}d^4 + 70a^{12}b^8d^4 + 56a^{14}b^6d^4 + 28a^{16}b^4d^4 + 8a^{18}b^2d^4))) * (1/(b^6d^2e^{11} - a
\end{aligned}$$

$$\begin{aligned}
& \left( b^6 d^2 e^{11i} - a^2 b^4 d^2 e^{15i} - 20 a^3 b^3 d^2 e + a^4 b^2 d^2 e^{15i} + 6 a b^5 d^2 e + 6 a^5 b d^2 e \right)^{(1/2)} / 2 - (90 a^3 b^{19} d^2 e^9 + 846 a^3 b^{17} d^2 e^9 + 1714 a^5 b^{15} d^2 e^9 + 3606 a^7 b^{13} d^2 e^9 - 14578 a^9 b^{11} d^2 e^9 - 34486 a^{11} b^9 d^2 e^9 - 14970 a^{13} b^7 d^2 e^9 + 2258 a^{15} b^5 d^2 e^9 - 32 a^{17} b^3 d^2 e^9) / (2 (a^{20} d^5 + a^4 b^{16} d^5 + 8 a^6 b^{14} d^5 + 28 a^8 b^{12} d^5 + 56 a^{10} b^{10} d^5 + 70 a^{12} b^8 d^5 + 56 a^{14} b^6 d^5 + 28 a^{16} b^4 d^5 + 8 a^{18} b^2 d^5)) * (1 / (b^6 d^2 e^{11i} - a^6 d^2 e^{11i} - a^2 b^4 d^2 e^{15i} - 20 a^3 b^3 d^2 e + a^4 b^2 d^2 e^{15i} + 6 a b^5 d^2 e + 6 a^5 b d^2 e))^{(1/2)} / 2 - ((e \cot(c + dx))^{(1/2)} * (18 a^2 b^{15} e^8 - 9 b^{17} e^8 - 71 a^4 b^{13} e^8 + 892 a^6 b^{11} e^8 + 857 a^8 b^9 e^8 + 6802 a^{10} b^7 e^8 - 1257 a^{12} b^5 e^8)) / (2 (a^{20} d^4 + a^4 b^{16} d^4 + 8 a^6 b^{14} d^4 + 28 a^8 b^{12} d^4 + 56 a^{10} b^{10} d^4 + 70 a^{12} b^8 d^4 + 56 a^{14} b^6 d^4 + 28 a^{16} b^4 d^4 + 8 a^{18} b^2 d^4)) * (1 / (b^6 d^2 e^{11i} - a^6 d^2 e^{11i} - a^2 b^4 d^2 e^{15i} - 20 a^3 b^3 d^2 e + a^4 b^2 d^2 e^{15i} + 6 a b^5 d^2 e + 6 a^5 b d^2 e))^{(1/2)} * 1i) / ((9 b^{14} e^8 + 60 a^2 b^{12} e^8 + 318 a^4 b^{10} e^8 + 748 a^6 b^8 e^8 + 1505 a^8 b^6 e^8) / (a^{20} d^5 + a^4 b^{16} d^5 + 8 a^6 b^{14} d^5 + 28 a^8 b^{12} d^5 + 56 a^{10} b^{10} d^5 + 70 a^{12} b^8 d^5 + 56 a^{14} b^6 d^5 + 28 a^{16} b^4 d^5 + 8 a^{18} b^2 d^5) + ((((((1 / (b^6 d^2 e^{11i} - a^6 d^2 e^{11i} - a^2 b^4 d^2 e^{15i} - 20 a^3 b^3 d^2 e + a^4 b^2 d^2 e^{15i} + 6 a b^5 d^2 e + 6 a^5 b d^2 e))^{(1/2)} * ((192 a^2 b^{24} d^4 e^{10} + 1728 a^4 b^{22} d^4 e^{10} + 8320 a^6 b^{20} d^4 e^{10} + 27264 a^8 b^{18} d^4 e^{10} + 62592 a^{10} b^{16} d^4 e^{10} + 99456 a^{12} b^{14} d^4 e^{10} + 107520 a^{14} b^{12} d^4 e^{10} + 76800 a^{16} b^{10} d^4 e^{10} + 33984 a^{18} b^8 d^4 e^{10} + 7872 a^{20} b^6 d^4 e^{10} + 384 a^{22} b^4 d^4 e^{10} - 128 a^{24} b^2 d^4 e^{10}) / (2 (a^{20} d^5 + a^4 b^{16} d^5 + 8 a^6 b^{14} d^5 + 28 a^8 b^{12} d^5 + 56 a^{10} b^{10} d^5 + 70 a^{12} b^8 d^5 + 56 a^{14} b^6 d^5 + 28 a^{16} b^4 d^5 + 8 a^{18} b^2 d^5)) - ((e \cot(c + dx))^{(1/2)} * (1 / (b^6 d^2 e^{11i} - a^6 d^2 e^{11i} - a^2 b^4 d^2 e^{15i} - 20 a^3 b^3 d^2 e + a^4 b^2 d^2 e^{15i} + 6 a b^5 d^2 e + 6 a^5 b d^2 e))^{(1/2)} * (512 a^4 b^{25} d^4 e^{10} + 4608 a^6 b^{23} d^4 e^{10} + 17920 a^8 b^{21} d^4 e^{10} + 38400 a^{10} b^{19} d^4 e^{10} + 46080 a^{12} b^{17} d^4 e^{10} + 21504 a^{14} b^{15} d^4 e^{10} - 21504 a^{16} b^{13} d^4 e^{10} - 46080 a^{18} b^{11} d^4 e^{10} - 38400 a^{20} b^9 d^4 e^{10} - 17920 a^{22} b^7 d^4 e^{10} - 46080 a^{24} b^5 d^4 e^{10} - 512 a^{26} b^3 d^4 e^{10})) / (4 (a^{20} d^4 + a^4 b^{16} d^4 + 8 a^6 b^{14} d^4 + 28 a^8 b^{12} d^4 + 56 a^{10} b^{10} d^4 + 70 a^{12} b^8 d^4 + 56 a^{14} b^6 d^4 + 28 a^{16} b^4 d^4 + 8 a^{18} b^2 d^4)))) / 2 + ((e \cot(c + dx))^{(1/2)} * (72 a^3 b^{22} d^2 e^9 + 576 a^3 b^{20} d^2 e^9 + 5024 a^5 b^{18} d^2 e^9 + 14272 a^7 b^{16} d^2 e^9 + 27824 a^9 b^{14} d^2 e^9 + 53184 a^{11} b^{12} d^2 e^9 + 70240 a^{13} b^{10} d^2 e^9 + 47680 a^{15} b^8 d^2 e^9 + 12616 a^{17} b^6 d^2 e^9 - 64 a^{21} b^2 d^2 e^9)) / (2 (a^{20} d^4 + a^4 b^{16} d^4 + 8 a^6 b^{14} d^4 + 28 a^8 b^{12} d^4 + 56 a^{10} b^{10} d^4 + 70 a^{12} b^8 d^4 + 56 a^{14} b^6 d^4 + 28 a^{16} b^4 d^4 + 8 a^{18} b^2 d^4)) * (1 / (b^6 d^2 e^{11i} - a^6 d^2 e^{11i} - a^2 b^4 d^2 e^{15i} - 20 a^3 b^3 d^2 e + a^4 b^2 d^2 e^{15i} + 6 a b^5 d^2 e + 6 a^5 b d^2 e))^{(1/2)} / 2 - (90 a^3 b^{19} d^2 e^9 + 846 a^3 b^{17} d^2 e^9 + 1714 a^5 b^{15} d^2 e^9 + 3606 a^7 b^{13} d^2 e^9 - 14578 a^9 b^{11} d^2 e^9 - 34486 a^{11} b^9 d^2 e^9 - 14970 a^{13} b^7 d^2 e^9 + 2258 a^{15} b^5 d^2 e^9 - 32 a^{17} b^3 d^2 e^9) / (2 (a^{20} d^5 + a^4 b^{16} d^5 + 8 a^6 b^{14} d^5 + 28 a^8 b^{12} d^5 + 56 a^{10} b^{10} d^5 + 70 a^{12} b^8 d^5 + 56 a^{14} b^6 d^5 + 28 a^{16} b^4 d^5 + 8 a^{18} b^2 d^5))
\end{aligned}$$







$$\begin{aligned}
& 8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5) + (((e*\cot(c + \\
& d*x))^{(1/2)}*(72*a*b^{22}*d^2*e^9 + 576*a^3*b^{20}*d^2*e^9 + 5024*a^5*b^{18}*d^2* \\
& e^9 + 14272*a^7*b^{16}*d^2*e^9 + 27824*a^9*b^{14}*d^2*e^9 + 53184*a^{11}*b^{12}*d^2 \\
& *e^9 + 70240*a^{13}*b^{10}*d^2*e^9 + 47680*a^{15}*b^8*d^2*e^9 + 12616*a^{17}*b^6*d^ \\
& 2*e^9 - 64*a^{21}*b^2*d^2*e^9))/(a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 2 \\
& 8*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28* \\
& a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4) - (((192*a^2*b^{24}*d^4*e^{10} + 1728*a^4*b^{22}*d \\
& ^4*e^{10} + 8320*a^6*b^{20}*d^4*e^{10} + 27264*a^8*b^{18}*d^4*e^{10} + 62592*a^{10}*b^{1 \\
& 6}*d^4*e^{10} + 99456*a^{12}*b^{14}*d^4*e^{10} + 107520*a^{14}*b^{12}*d^4*e^{10} + 76800*a \\
& ^{16}*b^{10}*d^4*e^{10} + 33984*a^{18}*b^8*d^4*e^{10} + 7872*a^{20}*b^6*d^4*e^{10} + 384* \\
& a^{22}*b^4*d^4*e^{10} - 128*a^{24}*b^2*d^4*e^{10}))/ (a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6 \\
& *b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}* \\
& b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5) + ((e*\cot(c + d*x))^{(1/2)}*(-a^5 \\
& *b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a^2*b^2)*(512*a^4*b^{25}*d^4*e^{10} + 4608*a^ \\
& 6*b^{23}*d^4*e^{10} + 17920*a^8*b^{21}*d^4*e^{10} + 38400*a^{10}*b^{19}*d^4*e^{10} + 4608 \\
& 0*a^{12}*b^{17}*d^4*e^{10} + 21504*a^{14}*b^{15}*d^4*e^{10} - 21504*a^{16}*b^{13}*d^4*e^{10} \\
& - 46080*a^{18}*b^{11}*d^4*e^{10} - 38400*a^{20}*b^9*d^4*e^{10} - 17920*a^{22}*b^7*d^4*e \\
& ^{10} - 4608*a^{24}*b^5*d^4*e^{10} - 512*a^{26}*b^3*d^4*e^{10}))/ (8*(a^{11}*d*e + a^5*b^ \\
& ^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)*(a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^ \\
& ^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6 \\
& *d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4)))*(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b \\
& ^4 + 6*a^2*b^2))/ (8*(a^{11}*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e \\
& )))*(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a^2*b^2))/ (8*(a^{11}*d*e + a^5*b^6 \\
& *d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 \\
& + 6*a^2*b^2))/ (8*(a^{11}*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e))) \\
& *(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a^2*b^2)*i)/ (8*(a^{11}*d*e + a^5*b^6 \\
& *d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))/ ((9*b^{14}*e^8 + 60*a^2*b^{12}*e^8 + 31 \\
& 8*a^4*b^{10}*e^8 + 748*a^6*b^8*e^8 + 1505*a^8*b^6*e^8)/(a^{20}*d^5 + a^4*b^{16}*d \\
& ^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 \\
& + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5) + (((e*\cot(c + d*x)) \\
& ^{(1/2)}*(18*a^2*b^{15}*e^8 - 9*b^{17}*e^8 - 71*a^4*b^{13}*e^8 + 892*a^6*b^{11}*e^8 + \\
& 857*a^8*b^9*e^8 + 6802*a^{10}*b^7*e^8 - 1257*a^{12}*b^5*e^8))/(a^{20}*d^4 + a^4* \\
& b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^ \\
& 8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4) - (((90*a*b^{19} \\
& d^2*e^9 + 846*a^3*b^{17}*d^2*e^9 + 1714*a^5*b^{15}*d^2*e^9 + 3606*a^7*b^{13}*d^2* \\
& e^9 - 14578*a^9*b^{11}*d^2*e^9 - 34486*a^{11}*b^9*d^2*e^9 - 14970*a^{13}*b^7*d^2* \\
& e^9 + 2258*a^{15}*b^5*d^2*e^9 - 32*a^{17}*b^3*d^2*e^9)/(a^{20}*d^5 + a^4*b^{16}*d^5 \\
& + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + \\
& 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5) - (((e*\cot(c + d*x))^{( \\
& 1/2)}*(72*a*b^{22}*d^2*e^9 + 576*a^3*b^{20}*d^2*e^9 + 5024*a^5*b^{18}*d^2*e^9 + 14 \\
& 272*a^7*b^{16}*d^2*e^9 + 27824*a^9*b^{14}*d^2*e^9 + 53184*a^{11}*b^{12}*d^2*e^9 + 7 \\
& 0240*a^{13}*b^{10}*d^2*e^9 + 47680*a^{15}*b^8*d^2*e^9 + 12616*a^{17}*b^6*d^2*e^9 - \\
& 64*a^{21}*b^2*d^2*e^9))/(a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^ \\
& ^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4 \\
& *d^4 + 8*a^{18}*b^2*d^4) + (((192*a^2*b^{24}*d^4*e^{10} + 1728*a^4*b^{22}*d^4*e^{10}
\end{aligned}$$

$$\begin{aligned}
& + 8320*a^6*b^{20}*d^4*e^{10} + 27264*a^8*b^{18}*d^4*e^{10} + 62592*a^{10}*b^{16}*d^4*e^{10} \\
& + 99456*a^{12}*b^{14}*d^4*e^{10} + 107520*a^{14}*b^{12}*d^4*e^{10} + 76800*a^{16}*b^{10} \\
& *d^4*e^{10} + 33984*a^{18}*b^8*d^4*e^{10} + 7872*a^{20}*b^6*d^4*e^{10} + 384*a^{22}*b^4 \\
& *d^4*e^{10} - 128*a^{24}*b^2*d^4*e^{10})/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 \\
& + 28*a^8*b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 \\
& + 28*a^{16}*b^4*d^5 + 8*a^{18}*b^2*d^5) - ((e*cot(c + d*x))^{(1/2)}*(-a^5*b^3*e)^{ \\
& (1/2)}*(35*a^4 + 3*b^4 + 6*a^2*b^2)*(512*a^4*b^{25}*d^4*e^{10} + 4608*a^6*b^{23}*d \\
& ^4*e^{10} + 17920*a^8*b^{21}*d^4*e^{10} + 38400*a^{10}*b^{19}*d^4*e^{10} + 46080*a^{12}*b \\
& ^{17}*d^4*e^{10} + 21504*a^{14}*b^{15}*d^4*e^{10} - 21504*a^{16}*b^{13}*d^4*e^{10} - 46080* \\
& a^{18}*b^{11}*d^4*e^{10} - 38400*a^{20}*b^9*d^4*e^{10} - 17920*a^{22}*b^7*d^4*e^{10} - 46 \\
& 08*a^{24}*b^5*d^4*e^{10} - 512*a^{26}*b^3*d^4*e^{10}))/((8*(a^{11}*d*e + a^5*b^6*d*e + \\
& 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)*(a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + \\
& 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 2 \\
& 8*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4)))*(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a \\
& ^2*b^2))/((8*(a^{11}*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^ \\
& 5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a^2*b^2))/((8*(a^{11}*d*e + a^5*b^6*d*e + 3 \\
& *a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a^2* \\
& b^2))/((8*(a^{11}*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b \\
& ^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6*a^2*b^2))/((8*(a^{11}*d*e + a^5*b^6*d*e + 3*a^ \\
& 7*b^4*d*e + 3*a^9*b^2*d*e)) - (((e*cot(c + d*x))^{(1/2)}*(18*a^2*b^{15}*e^8 - \\
& 9*b^{17}*e^8 - 71*a^4*b^{13}*e^8 + 892*a^6*b^{11}*e^8 + 857*a^8*b^9*e^8 + 6802*a^ \\
& 10*b^7*e^8 - 1257*a^{12}*b^5*e^8))/(a^{20}*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 \\
& + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + \\
& 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4) + (((90*a*b^{19}*d^2*e^9 + 846*a^3*b^{17}*d^2 \\
& *e^9 + 1714*a^5*b^{15}*d^2*e^9 + 3606*a^7*b^{13}*d^2*e^9 - 14578*a^9*b^{11}*d^2*e \\
& ^9 - 34486*a^{11}*b^9*d^2*e^9 - 14970*a^{13}*b^7*d^2*e^9 + 2258*a^{15}*b^5*d^2*e^ \\
& 9 - 32*a^{17}*b^3*d^2*e^9))/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8 \\
& *b^{12}*d^5 + 56*a^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16} \\
& *b^4*d^5 + 8*a^{18}*b^2*d^5) + (((e*cot(c + d*x))^{(1/2)}*(72*a*b^{22}*d^2*e^9 + \\
& 576*a^3*b^{20}*d^2*e^9 + 5024*a^5*b^{18}*d^2*e^9 + 14272*a^7*b^{16}*d^2*e^9 + 278 \\
& 24*a^9*b^{14}*d^2*e^9 + 53184*a^{11}*b^{12}*d^2*e^9 + 70240*a^{13}*b^{10}*d^2*e^9 + 4 \\
& 7680*a^{15}*b^8*d^2*e^9 + 12616*a^{17}*b^6*d^2*e^9 - 64*a^{21}*b^2*d^2*e^9))/(a^2 \\
& 0*d^4 + a^4*b^{16}*d^4 + 8*a^6*b^{14}*d^4 + 28*a^8*b^{12}*d^4 + 56*a^{10}*b^{10}*d^4 \\
& + 70*a^{12}*b^8*d^4 + 56*a^{14}*b^6*d^4 + 28*a^{16}*b^4*d^4 + 8*a^{18}*b^2*d^4) - ( \\
& ((192*a^2*b^{24}*d^4*e^{10} + 1728*a^4*b^{22}*d^4*e^{10} + 8320*a^6*b^{20}*d^4*e^{10} + \\
& 27264*a^8*b^{18}*d^4*e^{10} + 62592*a^{10}*b^{16}*d^4*e^{10} + 99456*a^{12}*b^{14}*d^4*e \\
& ^{10} + 107520*a^{14}*b^{12}*d^4*e^{10} + 76800*a^{16}*b^{10}*d^4*e^{10} + 33984*a^{18}*b^8 \\
& *d^4*e^{10} + 7872*a^{20}*b^6*d^4*e^{10} + 384*a^{22}*b^4*d^4*e^{10} - 128*a^{24}*b^2*d \\
& ^4*e^{10})/(a^{20}*d^5 + a^4*b^{16}*d^5 + 8*a^6*b^{14}*d^5 + 28*a^8*b^{12}*d^5 + 56*a \\
& ^{10}*b^{10}*d^5 + 70*a^{12}*b^8*d^5 + 56*a^{14}*b^6*d^5 + 28*a^{16}*b^4*d^5 + 8*a^{18} \\
& *b^2*d^5) + ((e*cot(c + d*x))^{(1/2)}*(-a^5*b^3*e)^{(1/2)}*(35*a^4 + 3*b^4 + 6* \\
& a^2*b^2)*(512*a^4*b^{25}*d^4*e^{10} + 4608*a^6*b^{23}*d^4*e^{10} + 17920*a^8*b^{21}*d \\
& ^4*e^{10} + 38400*a^{10}*b^{19}*d^4*e^{10} + 46080*a^{12}*b^{17}*d^4*e^{10} + 21504*a^{14} \\
& *b^{15}*d^4*e^{10} - 21504*a^{16}*b^{13}*d^4*e^{10} - 46080*a^{18}*b^{11}*d^4*e^{10} - 38400 \\
& *a^{20}*b^9*d^4*e^{10} - 17920*a^{22}*b^7*d^4*e^{10} - 4608*a^{24}*b^5*d^4*e^{10} - 512
\end{aligned}$$

```

*a^26*b^3*d^4*e^10))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2
*d*e)*(a^20*d^4 + a^4*b^16*d^4 + 8*a^6*b^14*d^4 + 28*a^8*b^12*d^4 + 56*a^10
*b^10*d^4 + 70*a^12*b^8*d^4 + 56*a^14*b^6*d^4 + 28*a^16*b^4*d^4 + 8*a^18*b^
2*d^4)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^
5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3
*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d
*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b
^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^
4 + 6*a^2*b^2))/(8*(a^11*d*e + a^5*b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e)
)))*(-a^5*b^3*e)^(1/2)*(35*a^4 + 3*b^4 + 6*a^2*b^2)*1i)/(4*(a^11*d*e + a^5*
b^6*d*e + 3*a^7*b^4*d*e + 3*a^9*b^2*d*e))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(1/2)/(a+b\*cot(d\*x+c))\*\*3,x)

[Out] Integral(1/(sqrt(e\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))\*\*3), x)

$$3.87 \quad \int \frac{1}{(e \cot(c+dx))^{3/2} (a+b \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=529

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2+b^2)^3} - \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2+b^2)^3}$$

[Out]  $\frac{1}{4} b^{5/2} (63 a^4 + 46 a^2 b^2 + 15 b^4) \arctan(b^{1/2} (e \cot(dx+c))^{1/2} / a^{1/2} / e^{1/2}) / a^{7/2} / (a^2 + b^2)^3 / d / e^{3/2} - 1/2 (a-b) (a^2 + 4 a b + b^2) \arctan(1 - 2^{1/2} (e \cot(dx+c))^{1/2} / e^{1/2}) / (a^2 + b^2)^3 / d / e^{3/2} * 2^{1/2} + 1/2 (a-b) (a^2 + 4 a b + b^2) \arctan(1 + 2^{1/2} (e \cot(dx+c))^{1/2} / e^{1/2}) / (a^2 + b^2)^3 / d / e^{3/2} * 2^{1/2} + 1/4 (a+b) (a^2 - 4 a b + b^2) \ln(e^{1/2} + \cot(dx+c)) * e^{1/2} - 2^{1/2} (e \cot(dx+c))^{1/2} / (a^2 + b^2)^3 / d / e^{3/2} * 2^{1/2} - 1/4 (a+b) (a^2 - 4 a b + b^2) \ln(e^{1/2} + \cot(dx+c)) * e^{1/2} + 2^{1/2} (e \cot(dx+c))^{1/2} / (a^2 + b^2)^3 / d / e^{3/2} * 2^{1/2} + 1/4 (8 a^4 + 31 a^2 b^2 + 15 b^4) / a^3 / (a^2 + b^2)^2 / d / e / (e \cot(dx+c))^{1/2} - 1/2 b^2 / a / (a^2 + b^2) / d / e / (a + b \cot(dx+c))^2 / (e \cot(dx+c))^{1/2} - 1/4 b^2 (13 a^2 + 5 b^2) / a^2 / (a^2 + b^2)^2 / d / e / (a + b \cot(dx+c)) / (e \cot(dx+c))^{1/2}$

**Rubi [A]** time = 1.66, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$ , Rules used = {3569, 3649, 3653, 3534, 1168, 1162, 617, 204, 1165, 628, 3634, 63, 205}

$$\frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) - \sqrt{2} \sqrt{e \cot(c+dx)} + \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2+b^2)^3} - \frac{(a+b)(a^2-4ab+b^2) \log(\sqrt{e} \cot(c+dx) + \sqrt{2} \sqrt{e \cot(c+dx)} - \sqrt{e})}{2\sqrt{2} de^{3/2} (a^2+b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((e\*Cot[c + d\*x])^(3/2)\*(a + b\*Cot[c + d\*x])^3),x]

[Out]  $(b^{5/2} (63 a^4 + 46 a^2 b^2 + 15 b^4) \text{ArcTan}[\frac{\sqrt{b} \sqrt{e \cot(c+dx)}}{\sqrt{a} \sqrt{e}}]) / (4 a^{7/2} (a^2 + b^2)^3 d e^{3/2}) - ((a-b) (a^2 + 4 a b + b^2) \text{ArcTan}[1 - \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}]) / (\sqrt{2} (a^2 + b^2)^3 d e^{3/2}) + ((a-b) (a^2 + 4 a b + b^2) \text{ArcTan}[1 + \frac{\sqrt{2} \sqrt{e \cot(c+dx)}}{\sqrt{e}}]) / (\sqrt{2} (a^2 + b^2)^3 d e^{3/2}) + (8 a^4 + 31 a^2 b^2 + 15 b^4) / (4 a^3 (a^2 + b^2)^2 d e \sqrt{e \cot(c+dx)}) - b^2 / (2 a (a^2 + b^2) d e \sqrt{e \cot(c+dx)}) * (a + b \cot(c+dx))^2 - (b^2 (13 a^2 + 5 b^2)) / (4 a^2 (a^2 + b^2)^2 d e \sqrt{e \cot(c+dx)}) * (a + b \cot(c+dx)) + ((a+b) (a^2 - 4 a b + b^2) \text{Log}[\sqrt{e} + \sqrt{e} \cot(c+dx)] - \sqrt{2} \sqrt{e \cot(c+dx)}) / (2 \sqrt{2} (a^2 + b^2)^3 d e^{3/2}) - ((a-b) (a^2 - 4 a b + b^2) \text{Log}[\sqrt{e} - \sqrt{e} \cot(c+dx)] - \sqrt{2} \sqrt{e \cot(c+dx)}) / (2 \sqrt{2} (a^2 + b^2)^3 d e^{3/2})$

$$(a + b)(a^2 - 4ab + b^2) \text{Log}[\text{Sqrt}[e] + \text{Sqrt}[e] \text{Cot}[c + dx] + \text{Sqrt}[2] \text{Sqrt}[e \text{Cot}[c + dx]]] / (2 \text{Sqrt}[2] (a^2 + b^2)^{3/2} d e^{3/2})$$
Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

$x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 1168

$\text{Int}[(d + (e_*)*(x_)^2)/((a_) + (c_*)*(x_)^4), x\_Symbol] \text{:>} \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

### Rule 3534

$\text{Int}[(c + (d_*)*\tan[(e_*) + (f_*)*(x_)])/ \text{Sqrt}[(b_*)*\tan[(e_*) + (f_*)*(x_)]], x\_Symbol] \text{:>} \text{Dist}[2/f, \text{Subst}[\text{Int}[(b*c + d*x^2)/(b^2 + x^4), x], x, \text{Sqrt}[b*\tan[e + f*x]]], x] /; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0]$

### Rule 3569

$\text{Int}[(a + (b_*)*\tan[(e_*) + (f_*)*(x_)])^{(m_*)} * ((c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)])^{(n_*)}, x\_Symbol] \text{:>} \text{Simp}[(b^2*(a + b*\tan[e + f*x])^{(m+1)} * (c + d*\tan[e + f*x])^{(n+1)}) / (f*(m+1)*(a^2 + b^2)*(b*c - a*d)), x] + \text{Dist}[1 / ((m+1)*(a^2 + b^2)*(b*c - a*d)), \text{Int}[(a + b*\tan[e + f*x])^{(m+1)} * (c + d*\tan[e + f*x])^n * \text{Simp}[a*(b*c - a*d)*(m+1) - b^2*d*(m+n+2) - b*(b*c - a*d)*(m+1)*\tan[e + f*x] - b^2*d*(m+n+2)*\tan[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{NeQ}[c^2 + d^2, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ (\text{LtQ}[n, 0] \ || \ \text{IntegerQ}[m]) \ \&\& \ !(\text{ILtQ}[n, -1] \ \&\& \ (!\text{IntegerQ}[m] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{NeQ}[a, 0])))$

### Rule 3634

$\text{Int}[(a + (b_*)*\tan[(e_*) + (f_*)*(x_)])^{(m_*)} * ((c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)])^{(n_*)} * ((A_*) + (C_*)*\tan[(e_*) + (f_*)*(x_)])^2), x\_Symbol] \text{:>} \text{Dist}[A/f, \text{Subst}[\text{Int}[(a + b*x)^m * (c + d*x)^n, x], x, \tan[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C, m, n, x\} \ \&\& \ \text{EqQ}[A, C]$

### Rule 3649

$\text{Int}[(a + (b_*)*\tan[(e_*) + (f_*)*(x_)])^{(m_*)} * ((c_*) + (d_*)*\tan[(e_*) + (f_*)*(x_)])^{(n_*)} * ((A_*) + (B_*)*\tan[(e_*) + (f_*)*(x_)]) + (C_*)*\tan[(e_*) + (f_*)*(x_)])^2), x\_Symbol] \text{:>} \text{Simp}[(A*b^2 - a*(b*B - a*C)) * (a + b*\tan[e + f*x])^{(m+1)} * (c + d*\tan[e + f*x])^{(n+1)}) / (f*(m+1)*(b*c - a*d)*(a^2 + b^2)), x] + \text{Dist}[1 / ((m+1)*(b*c - a*d)*(a^2 + b^2)), \text{Int}[(a + b*\tan[e + f*x])^{(m+1)} * (c + d*\tan[e + f*x])^n * \text{Simp}[A*(a*(b*c - a*d)*(m+1) - b^2*d*($



```

m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

### Rule 3653

```

Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[((c + d*Tan[e + f*x])^n*(1 + Tan[e
+ f*x]^2))/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
!GtQ[n, 0] && !LeQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(e \cot(c + dx))^{3/2} (a + b \cot(c + dx))^3} dx &= -\frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2} - \int \frac{-\frac{1}{2}(4a^2 + 5b^2)e}{(e \cot(c + dx))^{3/2} (a + b \cot(c + dx))^3} dx \\
&= -\frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))^2} - \frac{b^2}{4a^2(a^2 + b^2)^2} \\
&= \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2 de \sqrt{e \cot(c + dx)}} - \frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} \\
&= \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2 de \sqrt{e \cot(c + dx)}} - \frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} \\
&= \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2 de \sqrt{e \cot(c + dx)}} - \frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} \\
&= \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2 de \sqrt{e \cot(c + dx)}} - \frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} \\
&= \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2 de \sqrt{e \cot(c + dx)}} - \frac{b^2}{2a(a^2 + b^2) de \sqrt{e \cot(c + dx)} (a + b \cot(c + dx))} \\
&= \frac{b^{5/2} (63a^4 + 46a^2b^2 + 15b^4) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right)}{4a^{7/2} (a^2 + b^2)^3 de^{3/2}} + \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2} \\
&= \frac{b^{5/2} (63a^4 + 46a^2b^2 + 15b^4) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right)}{4a^{7/2} (a^2 + b^2)^3 de^{3/2}} + \frac{8a^4 + 31a^2b^2 + 15b^4}{4a^3(a^2 + b^2)^2} \\
&= \frac{b^{5/2} (63a^4 + 46a^2b^2 + 15b^4) \tan^{-1} \left( \frac{\sqrt{b} \sqrt{e \cot(c + dx)}}{\sqrt{a} \sqrt{e}} \right)}{4a^{7/2} (a^2 + b^2)^3 de^{3/2}} - \frac{(a - b)(a^2 + b^2)}{4a^3(a^2 + b^2)^2}
\end{aligned}$$

**Mathematica [C]** time = 1.79, size = 303, normalized size = 0.57

$$-8a^2b^2(3a^2 - b^2) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\frac{b \cot(c + dx)}{a}\right) - 16a^2b^2(a^2 + b^2) {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; -\frac{b \cot(c + dx)}{a}\right) - 8b^2(a^2 + b^2)^2 {}_2F_1\left(-\frac{1}{2}, 3; \frac{1}{2}; -\frac{b \cot(c + dx)}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
[Out] -1/4*(-8*a^2*b^2*(3*a^2 - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c +
d*x])/a)] - 16*a^2*b^2*(a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*Co
t[c + d*x])/a)] - 8*b^2*(a^2 + b^2)^2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*
Cot[c + d*x])/a)] - 8*a^4*(a^2 - 3*b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Co
t[c + d*x]^2] + Sqrt[2]*a^3*b*(3*a^2 - b^2)*Sqrt[Cot[c + d*x]]*(2*ArcTan[1
- Sqrt[2]*Sqrt[Cot[c + d*x]])] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]])] +
Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]])/(a^3*(a^2 + b^2)^3*d*e*Sqrt[e*Cot[c + d*x]])
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{(b \cot(dx + c) + a)^3 (e \cot(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)
maple [B] time = 0.80, size = 1245, normalized size = 2.35
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x)
[Out] 15/4/d/e*b^4*a/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)+11/2
/d/e*b^6/a/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)+7/4/d/e*
b^8/a^3/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(3/2)+17/4/d*b^3*
a^2/(a^2+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)+13/2/d*b^5/(a^2
+b^2)^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)+9/4/d*b^7/a^2/(a^2+b^2)
^3/(e*cot(d*x+c)*b+a*e)^2*(e*cot(d*x+c))^(1/2)+63/4/d/e*b^3*a/(a^2+b^2)^3/(
a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))+23/2/d/e*b^5/a/(a
```

$$\begin{aligned} & \sqrt{a^2+b^2}^3/(a*e*b)^{(1/2)}*\arctan((e*\cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)})+15/4/d \\ & /e*b^7/a^3/\sqrt{a^2+b^2}^3/(a*e*b)^{(1/2)}*\arctan((e*\cot(d*x+c))^{(1/2)}*b/(a*e*b)^{(1/2)}) \\ & -3/2/d/e^2/\sqrt{a^2+b^2}^3*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(-2^{(1/2)}/(e^2)^{(1/4)} \\ & )*(e*\cot(d*x+c))^{(1/2)}+1)*a^2*b+1/2/d/e^2/\sqrt{a^2+b^2}^3*(e^2)^{(1/4)}*2^{(1/2)}*a \\ & rctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*b^3+3/4/d/e^2/\sqrt{a^2+b^2}^3 \\ & *3*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)} \\ & +1)/(e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})) \\ & )*a^2*b-1/4/d/e^2/\sqrt{a^2+b^2}^3*(e^2)^{(1/4)}*2^{(1/2)}*\ln((e*\cot(d*x+c) \\ & )+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))/(e*\cot(d*x+c)-(e^2)^{(1/4)} \\ & *(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})))*b^3+3/2/d/e^2/\sqrt{a^2+b^2}^3 \\ & *(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^2 \\ & *b-1/2/d/e^2/\sqrt{a^2+b^2}^3*(e^2)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e* \\ & \cot(d*x+c))^{(1/2)}+1)*b^3+1/4/d/e/\sqrt{a^2+b^2}^3*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot( \\ & d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)}))/(e*\cot(d*x+c)+ \\ & (e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+(e^2)^{(1/2)})))*a^3-3/4/d/e/\sqrt{a^2+b^2} \\ & )^3*2^{(1/2)}/(e^2)^{(1/4)}*\ln((e*\cot(d*x+c)-(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2 \\ & ^{(1/2)}+(e^2)^{(1/2)}))/(e*\cot(d*x+c)+(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}*2^{(1/2)}+ \\ & (e^2)^{(1/2)})))*a*b^2-1/2/d/e/\sqrt{a^2+b^2}^3*2^{(1/2)}/(e^2)^{(1/4)}*\arctan(-2^{(1/2)} \\ & /((e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^3+3/2/d/e/\sqrt{a^2+b^2}^3*2^{(1/2)}/(e^2)^{(1/4)} \\ & *\arctan(-2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a*b^2+1/2/d/e/\sqrt{a^2+b^2}^3 \\ & *2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a^3-3/2/d/e/\sqrt{a^2+b^2}^3 \\ & *2^{(1/2)}/(e^2)^{(1/4)}*\arctan(2^{(1/2)}/(e^2)^{(1/4)}*(e*\cot(d*x+c))^{(1/2)}+1)*a*b^2+2/a^3/d/e/ \\ & (e*\cot(d*x+c))^{(1/2)} \end{aligned}$$

**maxima** [A] time = 0.45, size = 565, normalized size = 1.07

$$e^{\left( \frac{8(a^6+2a^4b^2+a^2b^4)e^2 + \frac{(16a^5b+49a^3b^3+25ab^5)e^2}{\tan(dx+c)} + \frac{(8a^4b^2+31a^2b^4+15b^6)e^2}{\tan(dx+c)^2}}{(a^9+2a^7b^2+a^5b^4)e^4 \sqrt{\frac{e}{\tan(dx+c)}} + 2(a^8b+2a^6b^3+a^4b^5)e^3 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{3}{2}} + (a^7b^2+2a^5b^4+a^3b^6)e^2 \left(\frac{e}{\tan(dx+c)}\right)^{\frac{5}{2}}} \right)} + \frac{(63a^4b^3+46a^2b^5+15b^7) \arctan\left(\frac{b\sqrt{e/\tan(dx+c)}}{\sqrt{a*b*e}}\right)}{(a^9+3a^7b^2+3a^5b^4+a^3b^6)\sqrt{a*b*e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))^(3/2)/(a+b\*cot(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/4\*e\*((8\*(a^6 + 2\*a^4\*b^2 + a^2\*b^4)\*e^2 + (16\*a^5\*b + 49\*a^3\*b^3 + 25\*a\*b^5)\*e^2/tan(d\*x + c) + (8\*a^4\*b^2 + 31\*a^2\*b^4 + 15\*b^6)\*e^2/tan(d\*x + c)^2)/((a^9 + 2\*a^7\*b^2 + a^5\*b^4)\*e^4\*sqrt(e/tan(d\*x + c)) + 2\*(a^8\*b + 2\*a^6\*b^3 + a^4\*b^5)\*e^3\*(e/tan(d\*x + c))^(3/2) + (a^7\*b^2 + 2\*a^5\*b^4 + a^3\*b^6)\*e^2\*(e/tan(d\*x + c))^(5/2)) + (63\*a^4\*b^3 + 46\*a^2\*b^5 + 15\*b^7)\*arctan(b\*sqrt(e/tan(d\*x + c))/sqrt(a\*b\*e))/((a^9 + 3\*a^7\*b^2 + 3\*a^5\*b^4 + a^3\*b^6)\*sqrt(a\*b\*e)\*e^2) + (2\*sqrt(2))\*(a^3 + 3\*a^2\*b - 3\*a\*b^2 - b^3)\*arctan(1/2\*sq

$$\frac{\sqrt{2}(\sqrt{2}\sqrt{e} + 2\sqrt{e/\tan(dx+c)})/\sqrt{e}}{\sqrt{e} + 2\sqrt{2}(a^3 + 3a^2b - 3ab^2 - b^3)\arctan(-1/2\sqrt{2}\sqrt{e}/\sqrt{e/\tan(dx+c)})/\sqrt{e}} - \frac{\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3)\log(\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))/\sqrt{e}}{\sqrt{2}(a^3 - 3a^2b - 3ab^2 + b^3)\log(-\sqrt{2}\sqrt{e}\sqrt{e/\tan(dx+c)} + e + e/\tan(dx+c))/\sqrt{e}}}{((a^6 + 3a^4b^2 + 3a^2b^4 + b^6)e^2)/d}$$

mupad [B] time = 10.00, size = 21158, normalized size = 40.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e\*cot(c + d\*x))^(3/2)\*(a + b\*cot(c + d\*x))^3),x)

[Out] 
$$\frac{((2e)/a + (e\cot(c + dx))(16a^4b + 25b^5 + 49a^2b^3))/(4a^2(a^4 + b^4 + 2a^2b^2)) + (b^2e^2\cot(c + dx)^2(8a^4 + 15b^4 + 31a^2b^2))/(4a^3(a^4e + b^4e + 2a^2b^2e))}{(b^2d(e\cot(c + dx))^{5/2} + a^2de^2(e\cot(c + dx))^{1/2} + 2abde(e\cot(c + dx))^{3/2}) + \operatorname{atan}\left(\frac{-1i/(4(b^6d^2e^3 - a^6d^2e^3 + ab^5d^2e^3 + a^5bd^2e^3 - 15a^2b^4d^2e^3 - a^3b^3d^2e^3 + 15a^4b^2d^2e^3))^{1/2}}{(e\cot(c + dx))^{1/2}(471859200a^{22}b^{44}d^7e^{16} + 9500098560a^{24}b^{42}d^7e^{16} + 91857354752a^{26}b^{40}d^7e^{16} + 564502986752a^{28}b^{38}d^7e^{16} + 2464648527872a^{30}b^{36}d^7e^{16} + 8104469069824a^{32}b^{34}d^7e^{16} + 20769933361152a^{34}b^{32}d^7e^{16} + 42351565209600a^{36}b^{30}d^7e^{16} + 69534945902592a^{38}b^{28}d^7e^{16} + 92434029608960a^{40}b^{26}d^7e^{16} + 99508717355008a^{42}b^{24}d^7e^{16} + 86342935511040a^{44}b^{22}d^7e^{16} + 59767095558144a^{46}b^{20}d^7e^{16} + 32432589897728a^{48}b^{18}d^7e^{16} + 13411815522304a^{50}b^{16}d^7e^{16} + 4030457708544a^{52}b^{14}d^7e^{16} + 805425905664a^{54}b^{12}d^7e^{16} + 86608183296a^{56}b^{10}d^7e^{16} + 1612709888a^{58}b^8d^7e^{16} + 16777216a^{60}b^6d^7e^{16} + 167772160a^{62}b^4d^7e^{16} + 16777216a^{64}b^2d^7e^{16}) + (-1i/(4(b^6d^2e^3 - a^6d^2e^3 + ab^5d^2e^3 + a^5bd^2e^3 - 15a^2b^4d^2e^3 - a^3b^3d^2e^3 + 15a^4b^2d^2e^3))^{1/2}}{(251658240a^{24}b^{45}d^8e^{18} - (e\cot(c + dx))^{1/2}(-1i/(4(b^6d^2e^3 - a^6d^2e^3 + ab^5d^2e^3 + a^5bd^2e^3 - 15a^2b^4d^2e^3 - a^3b^3d^2e^3 + 15a^4b^2d^2e^3))^{1/2}}(134217728a^{27}b^{45}d^9e^{19} + 2550136832a^{29}b^{43}d^9e^{19} + 22817013760a^{31}b^{41}d^9e^{19} + 127506841600a^{33}b^{39}d^9e^{19} + 497276682240a^{35}b^{37}d^9e^{19} + 1430626762752a^{37}b^{35}d^9e^{19} + 3121367482368a^{39}b^{33}d^9e^{19} + 5202279137280a^{41}b^{31}d^9e^{19} + 6502848921600a^{43}b^{29}d^9e^{19} + 5635802398720a^{45}b^{27}d^9e^{19} + 2254320959488a^{47}b^{25}d^9e^{19} - 2254320959488a^{49}b^{23}d^9e^{19} - 5635802398720a^{51}b^{21}d^9e^{19} - 6502848921600a^{53}b^{19}d^9e^{19} - 5202279137280a^{55}b^{17}d^9e^{19} - 3121367482368a^{57}b^{15}d^9e^{19} - 1430626762752a^{59}b^{13}d^9e^{19} - 497276682240a^{61}b^{11}d^9e^{19} - 127506841600a^{63}b^9d^9e^{19} - 22817013760a^{65}b^7d^9e^{19} - 25$$

$$\begin{aligned}
& 50136832*a^{67}*b^5*d^9*e^{19} - 134217728*a^{69}*b^3*d^9*e^{19}) + 5049942016*a^{26} \\
& *b^{43}*d^8*e^{18} + 48368713728*a^{28}*b^{41}*d^8*e^{18} + 293819383808*a^{30}*b^{39}*d^8 \\
& *e^{18} + 1268458192896*a^{32}*b^{37}*d^8*e^{18} + 4132731617280*a^{34}*b^{35}*d^8*e^{18} \\
& + 10531192700928*a^{36}*b^{33}*d^8*e^{18} + 21462823993344*a^{38}*b^{31}*d^8*e^{18} + \\
& 35469618315264*a^{40}*b^{29}*d^8*e^{18} + 47896904859648*a^{42}*b^{27}*d^8*e^{18} + 52 \\
& 983958077440*a^{44}*b^{25}*d^8*e^{18} + 47896904859648*a^{46}*b^{23}*d^8*e^{18} + 35090 \\
& 285461504*a^{48}*b^{21}*d^8*e^{18} + 20487396655104*a^{50}*b^{19}*d^8*e^{18} + 92306229 \\
& 16608*a^{52}*b^{17}*d^8*e^{18} + 2994733056000*a^{54}*b^{15}*d^8*e^{18} + 565576728576* \\
& a^{56}*b^{13}*d^8*e^{18} - 18572378112*a^{58}*b^{11}*d^8*e^{18} - 50281316352*a^{60}*b^9* \\
& d^8*e^{18} - 16089350144*a^{62}*b^7*d^8*e^{18} - 2516582400*a^{64}*b^5*d^8*e^{18} - 1 \\
& 67772160*a^{66}*b^3*d^8*e^{18}))*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2 \\
& *e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15* \\
& a^4*b^2*d^2*e^3)))^{(1/2)} - 117964800*a^{21}*b^{42}*d^6*e^{15} - 841482240*a^{23}*b^{40} \\
& *d^6*e^{15} + 3829399552*a^{25}*b^{38}*d^6*e^{15} + 78068580352*a^{27}*b^{36}*d^6*e^{15} \\
& + 497438162944*a^{29}*b^{34}*d^6*e^{15} + 1899895980032*a^{31}*b^{32}*d^6*e^{15} + 49 \\
& 72695519232*a^{33}*b^{30}*d^6*e^{15} + 9371195015168*a^{35}*b^{28}*d^6*e^{15} + 1289072 \\
& 0436224*a^{37}*b^{26}*d^6*e^{15} + 12726089809920*a^{39}*b^{24}*d^6*e^{15} + 8366961197 \\
& 056*a^{41}*b^{22}*d^6*e^{15} + 2597662490624*a^{43}*b^{20}*d^6*e^{15} - 1171836108800*a \\
& ^{45}*b^{18}*d^6*e^{15} - 1986881650688*a^{47}*b^{16}*d^6*e^{15} - 1237583921152*a^{49}*b \\
& ^{14}*d^6*e^{15} - 449507753984*a^{51}*b^{12}*d^6*e^{15} - 97476149248*a^{53}*b^{10}*d^6* \\
& e^{15} - 11931222016*a^{55}*b^8*d^6*e^{15} - 1006632960*a^{57}*b^6*d^6*e^{15} - 13421 \\
& 7728*a^{59}*b^4*d^6*e^{15} - 8388608*a^{61}*b^2*d^6*e^{15}) - (e*cot(c + d*x))^{(1/2)} \\
& )*(7610564608*a^{27}*b^{33}*d^5*e^{13} - 597688320*a^{23}*b^{37}*d^5*e^{13} - 167143014 \\
& 4*a^{25}*b^{35}*d^5*e^{13} - 58982400*a^{21}*b^{39}*d^5*e^{13} + 85774565376*a^{29}*b^{31}* \\
& d^5*e^{13} + 385487994880*a^{31}*b^{29}*d^5*e^{13} + 1104303620096*a^{33}*b^{27}*d^5*e^{13} \\
& + 2240523796480*a^{35}*b^{25}*d^5*e^{13} + 3345249468416*a^{37}*b^{23}*d^5*e^{13} + \\
& 3717287903232*a^{39}*b^{21}*d^5*e^{13} + 3053967114240*a^{41}*b^{19}*d^5*e^{13} + 18074 \\
& 74491392*a^{43}*b^{17}*d^5*e^{13} + 726513221632*a^{45}*b^{15}*d^5*e^{13} + 17076899020 \\
& 8*a^{47}*b^{13}*d^5*e^{13} + 10492051456*a^{49}*b^{11}*d^5*e^{13} - 4917821440*a^{51}*b^9 \\
& *d^5*e^{13} - 923009024*a^{53}*b^7*d^5*e^{13} + 8388608*a^{55}*b^5*d^5*e^{13}))*(-1i/ \\
& (4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2 \\
& *b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^{(1/2)}*1i + ((-1 \\
& i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15* \\
& a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^{(1/2)}*((e*co \\
& t(c + d*x))^{(1/2)}*(471859200*a^{22}*b^{44}*d^7*e^{16} + 9500098560*a^{24}*b^{42}*d^7* \\
& e^{16} + 91857354752*a^{26}*b^{40}*d^7*e^{16} + 564502986752*a^{28}*b^{38}*d^7*e^{16} + 2 \\
& 464648527872*a^{30}*b^{36}*d^7*e^{16} + 8104469069824*a^{32}*b^{34}*d^7*e^{16} + 207699 \\
& 33361152*a^{34}*b^{32}*d^7*e^{16} + 42351565209600*a^{36}*b^{30}*d^7*e^{16} + 695349459 \\
& 02592*a^{38}*b^{28}*d^7*e^{16} + 92434029608960*a^{40}*b^{26}*d^7*e^{16} + 995087173550 \\
& 08*a^{42}*b^{24}*d^7*e^{16} + 86342935511040*a^{44}*b^{22}*d^7*e^{16} + 59767095558144* \\
& a^{46}*b^{20}*d^7*e^{16} + 32432589897728*a^{48}*b^{18}*d^7*e^{16} + 13411815522304*a^5 \\
& 0*b^{16}*d^7*e^{16} + 4030457708544*a^{52}*b^{14}*d^7*e^{16} + 805425905664*a^{54}*b^{12} \\
& *d^7*e^{16} + 86608183296*a^{56}*b^{10}*d^7*e^{16} + 1612709888*a^{58}*b^8*d^7*e^{16} + \\
& 16777216*a^{60}*b^6*d^7*e^{16} + 167772160*a^{62}*b^4*d^7*e^{16} + 16777216*a^{64}*b \\
& ^2*d^7*e^{16}) - (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*
\end{aligned}$$

$$\begin{aligned}
& b^2 d^2 e^3 6i - 15 a^2 b^4 d^2 e^3 - a^3 b^3 d^2 e^3 20i + 15 a^4 b^2 d^2 e^3 \\
& 3))^{(1/2)} * ((e * \cot(c + d * x))^{(1/2)} * (-1i / (4 * (b^6 d^2 e^3 - a^6 d^2 e^3 + a * b \\
& ^5 d^2 e^3 6i + a^5 b * d^2 e^3 6i - 15 a^2 b^4 d^2 e^3 - a^3 b^3 d^2 e^3 20i \\
& + 15 a^4 b^2 d^2 e^3)))^{(1/2)} * (134217728 a^{27} b^{45} d^9 e^{19} + 2550136832 a \\
& ^{29} b^{43} d^9 e^{19} + 22817013760 a^{31} b^{41} d^9 e^{19} + 127506841600 a^{33} b^{39} \\
& * d^9 e^{19} + 497276682240 a^{35} b^{37} d^9 e^{19} + 1430626762752 a^{37} b^{35} d^9 e \\
& ^{19} + 3121367482368 a^{39} b^{33} d^9 e^{19} + 5202279137280 a^{41} b^{31} d^9 e^{19} + \\
& 6502848921600 a^{43} b^{29} d^9 e^{19} + 5635802398720 a^{45} b^{27} d^9 e^{19} + 2254 \\
& 320959488 a^{47} b^{25} d^9 e^{19} - 2254320959488 a^{49} b^{23} d^9 e^{19} - 563580239 \\
& 8720 a^{51} b^{21} d^9 e^{19} - 6502848921600 a^{53} b^{19} d^9 e^{19} - 5202279137280 * \\
& a^{55} b^{17} d^9 e^{19} - 3121367482368 a^{57} b^{15} d^9 e^{19} - 1430626762752 a^{59} * \\
& b^{13} d^9 e^{19} - 497276682240 a^{61} b^{11} d^9 e^{19} - 127506841600 a^{63} b^9 d^9 \\
& * e^{19} - 22817013760 a^{65} b^7 d^9 e^{19} - 2550136832 a^{67} b^5 d^9 e^{19} - 1342 \\
& 17728 a^{69} b^3 d^9 e^{19}) + 251658240 a^{24} b^{45} d^8 e^{18} + 5049942016 a^{26} b \\
& ^{43} d^8 e^{18} + 48368713728 a^{28} b^{41} d^8 e^{18} + 293819383808 a^{30} b^{39} d^8 * \\
& e^{18} + 1268458192896 a^{32} b^{37} d^8 e^{18} + 4132731617280 a^{34} b^{35} d^8 e^{18} \\
& + 10531192700928 a^{36} b^{33} d^8 e^{18} + 21462823993344 a^{38} b^{31} d^8 e^{18} + 3 \\
& 5469618315264 a^{40} b^{29} d^8 e^{18} + 47896904859648 a^{42} b^{27} d^8 e^{18} + 5298 \\
& 3958077440 a^{44} b^{25} d^8 e^{18} + 47896904859648 a^{46} b^{23} d^8 e^{18} + 3509028 \\
& 5461504 a^{48} b^{21} d^8 e^{18} + 20487396655104 a^{50} b^{19} d^8 e^{18} + 9230622916 \\
& 608 a^{52} b^{17} d^8 e^{18} + 2994733056000 a^{54} b^{15} d^8 e^{18} + 565576728576 a^ \\
& 56 b^{13} d^8 e^{18} - 18572378112 a^{58} b^{11} d^8 e^{18} - 50281316352 a^{60} b^9 d^ \\
& 8 e^{18} - 16089350144 a^{62} b^7 d^8 e^{18} - 2516582400 a^{64} b^5 d^8 e^{18} - 167 \\
& 772160 a^{66} b^3 d^8 e^{18})) * (-1i / (4 * (b^6 d^2 e^3 - a^6 d^2 e^3 + a * b^5 d^2 e \\
& ^3 6i + a^5 b * d^2 e^3 6i - 15 a^2 b^4 d^2 e^3 - a^3 b^3 d^2 e^3 20i + 15 a^ \\
& 4 b^2 d^2 e^3)))^{(1/2)} + 117964800 a^{21} b^{42} d^6 e^{15} + 841482240 a^{23} b^{40} \\
& * d^6 e^{15} - 3829399552 a^{25} b^{38} d^6 e^{15} - 78068580352 a^{27} b^{36} d^6 e^{15} \\
& - 497438162944 a^{29} b^{34} d^6 e^{15} - 1899895980032 a^{31} b^{32} d^6 e^{15} - 4972 \\
& 695519232 a^{33} b^{30} d^6 e^{15} - 9371195015168 a^{35} b^{28} d^6 e^{15} - 128907204 \\
& 36224 a^{37} b^{26} d^6 e^{15} - 12726089809920 a^{39} b^{24} d^6 e^{15} - 836696119705 \\
& 6 a^{41} b^{22} d^6 e^{15} - 2597662490624 a^{43} b^{20} d^6 e^{15} + 1171836108800 a^4 \\
& 5 b^{18} d^6 e^{15} + 1986881650688 a^{47} b^{16} d^6 e^{15} + 1237583921152 a^{49} b^{14} \\
& * d^6 e^{15} + 449507753984 a^{51} b^{12} d^6 e^{15} + 97476149248 a^{53} b^{10} d^6 e^ \\
& 15 + 11931222016 a^{55} b^8 d^6 e^{15} + 1006632960 a^{57} b^6 d^6 e^{15} + 1342177 \\
& 28 a^{59} b^4 d^6 e^{15} + 8388608 a^{61} b^2 d^6 e^{15}) - (e * \cot(c + d * x))^{(1/2)} * \\
& (7610564608 a^{27} b^{33} d^5 e^{13} - 597688320 a^{23} b^{37} d^5 e^{13} - 1671430144 * \\
& a^{25} b^{35} d^5 e^{13} - 58982400 a^{21} b^{39} d^5 e^{13} + 85774565376 a^{29} b^{31} d^ \\
& 5 e^{13} + 385487994880 a^{31} b^{29} d^5 e^{13} + 1104303620096 a^{33} b^{27} d^5 e^{13} \\
& + 2240523796480 a^{35} b^{25} d^5 e^{13} + 3345249468416 a^{37} b^{23} d^5 e^{13} + 37 \\
& 17287903232 a^{39} b^{21} d^5 e^{13} + 3053967114240 a^{41} b^{19} d^5 e^{13} + 1807474 \\
& 491392 a^{43} b^{17} d^5 e^{13} + 726513221632 a^{45} b^{15} d^5 e^{13} + 170768990208 * \\
& a^{47} b^{13} d^5 e^{13} + 10492051456 a^{49} b^{11} d^5 e^{13} - 4917821440 a^{51} b^9 d^ \\
& ^5 e^{13} - 923009024 a^{53} b^7 d^5 e^{13} + 8388608 a^{55} b^5 d^5 e^{13})) * (-1i / (4 \\
& * (b^6 d^2 e^3 - a^6 d^2 e^3 + a * b^5 d^2 e^3 6i + a^5 b * d^2 e^3 6i - 15 a^2 * \\
& b^4 d^2 e^3 - a^3 b^3 d^2 e^3 20i + 15 a^4 b^2 d^2 e^3)))^{(1/2)} * 1i) / (((-1i /
\end{aligned}$$

$$\begin{aligned}
& (4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3))^{(1/2)} * (((e*\cot(c + d*x))^{(1/2)} * (471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*b^42*d^7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^7*e^16 + 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^16 + 20769933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16 + 69534945902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 + 99508717355008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 59767095558144*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 13411815522304*a^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425905664*a^54*b^12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^58*b^8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16 + 16777216*a^64*b^2*d^7*e^16) - (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^{(1/2)} * ((e*\cot(c + d*x))^{(1/2)} * (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^{(1/2)} * (134217728*a^27*b^45*d^9*e^19 + 2550136832*a^29*b^43*d^9*e^19 + 22817013760*a^31*b^41*d^9*e^19 + 127506841600*a^33*b^39*d^9*e^19 + 497276682240*a^35*b^37*d^9*e^19 + 1430626762752*a^37*b^35*d^9*e^19 + 3121367482368*a^39*b^33*d^9*e^19 + 5202279137280*a^41*b^31*d^9*e^19 + 6502848921600*a^43*b^29*d^9*e^19 + 5635802398720*a^45*b^27*d^9*e^19 + 2254320959488*a^47*b^25*d^9*e^19 - 2254320959488*a^49*b^23*d^9*e^19 - 5635802398720*a^51*b^21*d^9*e^19 - 6502848921600*a^53*b^19*d^9*e^19 - 5202279137280*a^55*b^17*d^9*e^19 - 3121367482368*a^57*b^15*d^9*e^19 - 1430626762752*a^59*b^13*d^9*e^19 - 497276682240*a^61*b^11*d^9*e^19 - 127506841600*a^63*b^9*d^9*e^19 - 22817013760*a^65*b^7*d^9*e^19 - 2550136832*a^67*b^5*d^9*e^19 - 134217728*a^69*b^3*d^9*e^19) + 251658240*a^24*b^45*d^8*e^18 + 5049942016*a^26*b^43*d^8*e^18 + 48368713728*a^28*b^41*d^8*e^18 + 293819383808*a^30*b^39*d^8*e^18 + 1268458192896*a^32*b^37*d^8*e^18 + 4132731617280*a^34*b^35*d^8*e^18 + 10531192700928*a^36*b^33*d^8*e^18 + 21462823993344*a^38*b^31*d^8*e^18 + 35469618315264*a^40*b^29*d^8*e^18 + 47896904859648*a^42*b^27*d^8*e^18 + 52983958077440*a^44*b^25*d^8*e^18 + 47896904859648*a^46*b^23*d^8*e^18 + 35090285461504*a^48*b^21*d^8*e^18 + 20487396655104*a^50*b^19*d^8*e^18 + 9230622916608*a^52*b^17*d^8*e^18 + 2994733056000*a^54*b^15*d^8*e^18 + 565576728576*a^56*b^13*d^8*e^18 - 18572378112*a^58*b^11*d^8*e^18 - 50281316352*a^60*b^9*d^8*e^18 - 16089350144*a^62*b^7*d^8*e^18 - 2516582400*a^64*b^5*d^8*e^18 - 167772160*a^66*b^3*d^8*e^18)) * (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^{(1/2)} + 117964800*a^21*b^42*d^6*e^15 + 841482240*a^23*b^40*d^6*e^15 - 3829399552*a^25*b^38*d^6*e^15 - 78068580352*a^27*b^36*d^6*e^15 - 497438162944*a^29*b^34*d^6*e^15 - 1899895980032*a^31*b^32*d^6*e^15 - 4972695519232*a^33*b^30*d^6*e^15 - 9371195015168*a^35*b^28*d^6*e^15 - 12890720436224*a^37*b^26*d^6*e^15 - 12726089809920*a^39*b^24*d^6*e^15 - 8366961197056*a^41*b^22*d^6*e^15 - 2597662490624*a^43*b^20*d^6*e^15 + 1171836108800*a^45*b^18*d^6*e^15 + 1986881650688*a^47*b^16*d^6*e^15 + 1237583921152*a^49*b^14*
\end{aligned}$$



$$\begin{aligned}
& d^6 e^{15} + 449507753984 a^{51} b^{12} d^6 e^{15} + 97476149248 a^{53} b^{10} d^6 e^{15} \\
& + 11931222016 a^{55} b^8 d^6 e^{15} + 1006632960 a^{57} b^6 d^6 e^{15} + 134217728 \\
& a^{59} b^4 d^6 e^{15} + 8388608 a^{61} b^2 d^6 e^{15}) - (e \cot(c + dx))^{1/2} (7 \\
& 610564608 a^{27} b^{33} d^5 e^{13} - 597688320 a^{23} b^{37} d^5 e^{13} - 1671430144 a^{25} \\
& b^{35} d^5 e^{13} - 58982400 a^{21} b^{39} d^5 e^{13} + 85774565376 a^{29} b^{31} d^5 e^{13} \\
& + 385487994880 a^{31} b^{29} d^5 e^{13} + 1104303620096 a^{33} b^{27} d^5 e^{13} + \\
& 2240523796480 a^{35} b^{25} d^5 e^{13} + 3345249468416 a^{37} b^{23} d^5 e^{13} + 3717 \\
& 287903232 a^{39} b^{21} d^5 e^{13} + 3053967114240 a^{41} b^{19} d^5 e^{13} + 180747449 \\
& 1392 a^{43} b^{17} d^5 e^{13} + 726513221632 a^{45} b^{15} d^5 e^{13} + 170768990208 a^{47} \\
& b^{13} d^5 e^{13} + 10492051456 a^{49} b^{11} d^5 e^{13} - 4917821440 a^{51} b^9 d^5 \\
& e^{13} - 923009024 a^{53} b^7 d^5 e^{13} + 8388608 a^{55} b^5 d^5 e^{13})) (-1i / (4 * ( \\
& b^6 d^2 e^3 - a^6 d^2 e^3 + a b^5 d^2 e^3 * 6i + a^5 b d^2 e^3 * 6i - 15 a^2 b^4 \\
& d^2 e^3 - a^3 b^3 d^2 e^3 * 20i + 15 a^4 b^2 d^2 e^3)))^{1/2} - ((-1i / (4 * (b \\
& ^6 d^2 e^3 - a^6 d^2 e^3 + a b^5 d^2 e^3 * 6i + a^5 b d^2 e^3 * 6i - 15 a^2 b^4 \\
& * d^2 e^3 - a^3 b^3 d^2 e^3 * 20i + 15 a^4 b^2 d^2 e^3)))^{1/2} * ((e \cot(c + d \\
& * x))^{1/2} * (471859200 a^{22} b^{44} d^7 e^{16} + 9500098560 a^{24} b^{42} d^7 e^{16} + \\
& 91857354752 a^{26} b^{40} d^7 e^{16} + 564502986752 a^{28} b^{38} d^7 e^{16} + 24646485 \\
& 27872 a^{30} b^{36} d^7 e^{16} + 8104469069824 a^{32} b^{34} d^7 e^{16} + 2076993336115 \\
& 2 a^{34} b^{32} d^7 e^{16} + 42351565209600 a^{36} b^{30} d^7 e^{16} + 69534945902592 a \\
& ^{38} b^{28} d^7 e^{16} + 92434029608960 a^{40} b^{26} d^7 e^{16} + 99508717355008 a^{42} \\
& * b^{24} d^7 e^{16} + 86342935511040 a^{44} b^{22} d^7 e^{16} + 59767095558144 a^{46} b^{20} \\
& d^7 e^{16} + 32432589897728 a^{48} b^{18} d^7 e^{16} + 13411815522304 a^{50} b^{16} d^7 \\
& e^{16} + 4030457708544 a^{52} b^{14} d^7 e^{16} + 805425905664 a^{54} b^{12} d^7 e^{16} \\
& + 86608183296 a^{56} b^{10} d^7 e^{16} + 1612709888 a^{58} b^8 d^7 e^{16} + 167772 \\
& 16 a^{60} b^6 d^7 e^{16} + 167772160 a^{62} b^4 d^7 e^{16} + 16777216 a^{64} b^2 d^7 e^{16} \\
& + (-1i / (4 * (b^6 d^2 e^3 - a^6 d^2 e^3 + a b^5 d^2 e^3 * 6i + a^5 b d^2 e^3 * 6i - 15 a^2 b^4 \\
& d^2 e^3 - a^3 b^3 d^2 e^3 * 20i + 15 a^4 b^2 d^2 e^3)))^{1/2} * (251658240 a^{24} b^{45} d^8 e^{18} - (e \cot(c + dx))^{1/2} * (-1i / (4 * (b^6 d^2 \\
& * e^3 - a^6 d^2 e^3 + a b^5 d^2 e^3 * 6i + a^5 b d^2 e^3 * 6i - 15 a^2 b^4 d^2 e^3 \\
& ^3 - a^3 b^3 d^2 e^3 * 20i + 15 a^4 b^2 d^2 e^3)))^{1/2} * (134217728 a^{27} b^{45} \\
& * d^9 e^{19} + 2550136832 a^{29} b^{43} d^9 e^{19} + 22817013760 a^{31} b^{41} d^9 e^{19} \\
& + 127506841600 a^{33} b^{39} d^9 e^{19} + 497276682240 a^{35} b^{37} d^9 e^{19} + 14306 \\
& 26762752 a^{37} b^{35} d^9 e^{19} + 3121367482368 a^{39} b^{33} d^9 e^{19} + 5202279137 \\
& 280 a^{41} b^{31} d^9 e^{19} + 6502848921600 a^{43} b^{29} d^9 e^{19} + 5635802398720 a \\
& ^{45} b^{27} d^9 e^{19} + 2254320959488 a^{47} b^{25} d^9 e^{19} - 2254320959488 a^{49} b^{23} \\
& d^9 e^{19} - 5635802398720 a^{51} b^{21} d^9 e^{19} - 6502848921600 a^{53} b^{19} d^9 \\
& e^{19} - 5202279137280 a^{55} b^{17} d^9 e^{19} - 3121367482368 a^{57} b^{15} d^9 e^{19} \\
& - 1430626762752 a^{59} b^{13} d^9 e^{19} - 497276682240 a^{61} b^{11} d^9 e^{19} - 1 \\
& 27506841600 a^{63} b^9 d^9 e^{19} - 22817013760 a^{65} b^7 d^9 e^{19} - 2550136832 a^{67} \\
& b^5 d^9 e^{19} - 134217728 a^{69} b^3 d^9 e^{19}) + 5049942016 a^{26} b^{43} d^8 \\
& e^{18} + 48368713728 a^{28} b^{41} d^8 e^{18} + 293819383808 a^{30} b^{39} d^8 e^{18} + \\
& 1268458192896 a^{32} b^{37} d^8 e^{18} + 4132731617280 a^{34} b^{35} d^8 e^{18} + 10531 \\
& 192700928 a^{36} b^{33} d^8 e^{18} + 21462823993344 a^{38} b^{31} d^8 e^{18} + 35469618 \\
& 315264 a^{40} b^{29} d^8 e^{18} + 47896904859648 a^{42} b^{27} d^8 e^{18} + 52983958077 \\
& 440 a^{44} b^{25} d^8 e^{18} + 47896904859648 a^{46} b^{23} d^8 e^{18} + 35090285461504
\end{aligned}$$

$$\begin{aligned}
& *a^{48}b^{21}d^8e^{18} + 20487396655104a^{50}b^{19}d^8e^{18} + 9230622916608a^5 \\
& 2b^{17}d^8e^{18} + 2994733056000a^{54}b^{15}d^8e^{18} + 565576728576a^{56}b^{13} \\
& *d^8e^{18} - 18572378112a^{58}b^{11}d^8e^{18} - 50281316352a^{60}b^9d^8e^{18} \\
& - 16089350144a^{62}b^7d^8e^{18} - 2516582400a^{64}b^5d^8e^{18} - 167772160* \\
& a^{66}b^3d^8e^{18})) * (-1i / (4*(b^6d^2e^3 - a^6d^2e^3 + a*b^5d^2e^3*6i + \\
& a^5*b*d^2e^3*6i - 15*a^2*b^4*d^2e^3 - a^3*b^3*d^2e^3*20i + 15*a^4*b^2*d^2e^3)))^{(1/2)} - 117964800*a^{21}b^{42}d^6e^{15} - 841482240*a^{23}b^{40}d^6e^{15} \\
& + 3829399552*a^{25}b^{38}d^6e^{15} + 78068580352*a^{27}b^{36}d^6e^{15} + 49743 \\
& 8162944*a^{29}b^{34}d^6e^{15} + 1899895980032*a^{31}b^{32}d^6e^{15} + 49726955192 \\
& 32*a^{33}b^{30}d^6e^{15} + 9371195015168*a^{35}b^{28}d^6e^{15} + 12890720436224*a \\
& ^{37}b^{26}d^6e^{15} + 12726089809920*a^{39}b^{24}d^6e^{15} + 8366961197056*a^{41}* \\
& b^{22}d^6e^{15} + 2597662490624*a^{43}b^{20}d^6e^{15} - 1171836108800*a^{45}b^{18}* \\
& d^6e^{15} - 1986881650688*a^{47}b^{16}d^6e^{15} - 1237583921152*a^{49}b^{14}d^6e \\
& ^{15} - 449507753984*a^{51}b^{12}d^6e^{15} - 97476149248*a^{53}b^{10}d^6e^{15} - 11 \\
& 931222016*a^{55}b^8d^6e^{15} - 1006632960*a^{57}b^6d^6e^{15} - 134217728*a^{59} \\
& *b^4d^6e^{15} - 8388608*a^{61}b^2d^6e^{15}) - (e*cot(c + d*x))^{(1/2)}*(761056 \\
& 4608*a^{27}b^{33}d^5e^{13} - 597688320*a^{23}b^{37}d^5e^{13} - 1671430144*a^{25}b^ \\
& 35*d^5e^{13} - 58982400*a^{21}b^{39}d^5e^{13} + 85774565376*a^{29}b^{31}d^5e^{13} \\
& + 385487994880*a^{31}b^{29}d^5e^{13} + 1104303620096*a^{33}b^{27}d^5e^{13} + 2240 \\
& 523796480*a^{35}b^{25}d^5e^{13} + 3345249468416*a^{37}b^{23}d^5e^{13} + 371728790 \\
& 3232*a^{39}b^{21}d^5e^{13} + 3053967114240*a^{41}b^{19}d^5e^{13} + 1807474491392* \\
& a^{43}b^{17}d^5e^{13} + 726513221632*a^{45}b^{15}d^5e^{13} + 170768990208*a^{47}b^ \\
& 13*d^5e^{13} + 10492051456*a^{49}b^{11}d^5e^{13} - 4917821440*a^{51}b^9d^5e^{13} \\
& - 923009024*a^{53}b^7d^5e^{13} + 8388608*a^{55}b^5d^5e^{13})) * (-1i / (4*(b^6d \\
& ^2e^3 - a^6d^2e^3 + a*b^5d^2e^3*6i + a^5*b*d^2e^3*6i - 15*a^2*b^4*d^2e^3 \\
& *e^3 - a^3*b^3*d^2e^3*20i + 15*a^4*b^2*d^2e^3)))^{(1/2)} + 58982400*a^{22}b^ \\
& 35*d^4e^{12} + 920125440*a^{24}b^{33}d^4e^{12} + 6879444992*a^{26}b^{31}d^4e^{12} \\
& + 32454475776*a^{28}b^{29}d^4e^{12} + 107338792960*a^{30}b^{27}d^4e^{12} + 262062 \\
& 735360*a^{32}b^{25}d^4e^{12} + 485059461120*a^{34}b^{23}d^4e^{12} + 688908140544* \\
& a^{36}b^{21}d^4e^{12} + 751987064832*a^{38}b^{19}d^4e^{12} + 626086379520*a^{40}b^ \\
& 17*d^4e^{12} + 390506741760*a^{42}b^{15}d^4e^{12} + 176637870080*a^{44}b^{13}d^4* \\
& e^{12} + 54704996352*a^{46}b^{11}d^4e^{12} + 10374086656*a^{48}b^9d^4e^{12} + 908 \\
& 328960*a^{50}b^7d^4e^{12})) * (-1i / (4*(b^6d^2e^3 - a^6d^2e^3 + a*b^5d^2e^3*6i + \\
& a^5*b*d^2e^3*6i - 15*a^2*b^4*d^2e^3 - a^3*b^3*d^2e^3*20i + 15*a^4*b^2*d^2e^3)))^{(1/2)} * 2i + (\log(((((-1/(b^6d^2e^3*1i - a^6d^2e^3*1i + \\
& 6*a*b^5d^2e^3 + 6*a^5*b*d^2e^3 - a^2*b^4*d^2e^3*15i - 20*a^3*b^3*d^2e^3 \\
& + a^4*b^2*d^2e^3*15i)))^{(1/2)} * (((e*cot(c + d*x))^{(1/2)} * (471859200*a^{22}b^ \\
& ^{44}d^7e^{16} + 9500098560*a^{24}b^{42}d^7e^{16} + 91857354752*a^{26}b^{40}d^7e^ \\
& ^{16} + 564502986752*a^{28}b^{38}d^7e^{16} + 2464648527872*a^{30}b^{36}d^7e^{16} + 8 \\
& 104469069824*a^{32}b^{34}d^7e^{16} + 20769933361152*a^{34}b^{32}d^7e^{16} + 42351 \\
& 565209600*a^{36}b^{30}d^7e^{16} + 69534945902592*a^{38}b^{28}d^7e^{16} + 92434029 \\
& 608960*a^{40}b^{26}d^7e^{16} + 99508717355008*a^{42}b^{24}d^7e^{16} + 86342935511 \\
& 040*a^{44}b^{22}d^7e^{16} + 59767095558144*a^{46}b^{20}d^7e^{16} + 32432589897728 \\
& *a^{48}b^{18}d^7e^{16} + 13411815522304*a^{50}b^{16}d^7e^{16} + 4030457708544*a^5 \\
& 2*b^{14}d^7e^{16} + 805425905664*a^{54}b^{12}d^7e^{16} + 86608183296*a^{56}b^{10}d
\end{aligned}$$

$$\begin{aligned}
& 7e^{16} + 1612709888a^{58}b^8d^7e^{16} + 16777216a^{60}b^6d^7e^{16} + 167772160a^{62}b^4d^7e^{16} + 16777216a^{64}b^2d^7e^{16}) + ((-1/(b^6d^2e^3i \\
& - a^6d^2e^3i + 6a*b^5d^2e^3 + 6a^5*b*d^2e^3 - a^2*b^4*d^2e^3*15i \\
& - 20a^3*b^3*d^2e^3 + a^4*b^2*d^2e^3*15i))^{(1/2)}*(251658240a^{24}b^45d^8e^{18} - ((e*\cot(c + d*x))^{(1/2)}*(-1/(b^6d^2e^3i - a^6d^2e^3i + 6a \\
& *b^5d^2e^3 + 6a^5*b*d^2e^3 - a^2*b^4*d^2e^3*15i - 20a^3*b^3*d^2e^3 + \\
& a^4*b^2*d^2e^3*15i))^{(1/2)}*(134217728a^{27}b^45d^9e^{19} + 2550136832a^{29}b^43d^9e^{19} + 22817013760a^{31}b^41d^9e^{19} + 127506841600a^{33}b^39d \\
& ^9e^{19} + 497276682240a^{35}b^37d^9e^{19} + 1430626762752a^{37}b^35d^9e^{19} + 3121367482368a^{39}b^33d^9e^{19} + 5202279137280a^{41}b^31d^9e^{19} + 6 \\
& 502848921600a^{43}b^29d^9e^{19} + 5635802398720a^{45}b^27d^9e^{19} + 2254320959488a^{47}b^25d^9e^{19} - 2254320959488a^{49}b^23d^9e^{19} - 56358023987 \\
& 20a^{51}b^21d^9e^{19} - 6502848921600a^{53}b^19d^9e^{19} - 5202279137280a^{55}b^17d^9e^{19} - 3121367482368a^{57}b^15d^9e^{19} - 1430626762752a^{59}b^13d^9e^{19} - 497276682240a^{61}b^11d^9e^{19} - 127506841600a^{63}b^9d^9e \\
& ^19 - 22817013760a^{65}b^7d^9e^{19} - 2550136832a^{67}b^5d^9e^{19} - 134217728a^{69}b^3d^9e^{19}))/2 + 5049942016a^{26}b^43d^8e^{18} + 48368713728a^{28}b^41d^8e^{18} + 293819383808a^{30}b^39d^8e^{18} + 1268458192896a^{32}b^37 \\
& *d^8e^{18} + 4132731617280a^{34}b^35d^8e^{18} + 10531192700928a^{36}b^33d^8e^{18} + 21462823993344a^{38}b^31d^8e^{18} + 35469618315264a^{40}b^29d^8e^{18} + 47896904859648a^{42}b^27d^8e^{18} + 52983958077440a^{44}b^25d^8e^{18} \\
& + 47896904859648a^{46}b^23d^8e^{18} + 35090285461504a^{48}b^21d^8e^{18} + 20487396655104a^{50}b^19d^8e^{18} + 9230622916608a^{52}b^17d^8e^{18} + 2994733056000a^{54}b^15d^8e^{18} + 565576728576a^{56}b^13d^8e^{18} - 18572378112 \\
& *a^{58}b^11d^8e^{18} - 50281316352a^{60}b^9d^8e^{18} - 16089350144a^{62}b^7d^8e^{18} - 2516582400a^{64}b^5d^8e^{18} - 167772160a^{66}b^3d^8e^{18}))/2)* \\
& (-1/(b^6d^2e^3i - a^6d^2e^3i + 6a*b^5d^2e^3 + 6a^5*b*d^2e^3 - a^2*b^4*d^2e^3*15i - 20a^3*b^3*d^2e^3 + a^4*b^2*d^2e^3*15i))^{(1/2)})/2 - \\
& 117964800a^{21}b^42d^6e^{15} - 841482240a^{23}b^40d^6e^{15} + 3829399552a^{25}b^38d^6e^{15} + 78068580352a^{27}b^36d^6e^{15} + 497438162944a^{29}b^34 \\
& *d^6e^{15} + 1899895980032a^{31}b^32d^6e^{15} + 4972695519232a^{33}b^30d^6e^{15} + 9371195015168a^{35}b^28d^6e^{15} + 12890720436224a^{37}b^26d^6e^{15} \\
& + 12726089809920a^{39}b^24d^6e^{15} + 8366961197056a^{41}b^22d^6e^{15} + 2597662490624a^{43}b^20d^6e^{15} - 1171836108800a^{45}b^18d^6e^{15} - 198688 \\
& 1650688a^{47}b^16d^6e^{15} - 1237583921152a^{49}b^14d^6e^{15} - 449507753984a^{51}b^12d^6e^{15} - 97476149248a^{53}b^10d^6e^{15} - 11931222016a^{55}b^8d^6e^{15} - 1006632960a^{57}b^6d^6e^{15} - 134217728a^{59}b^4d^6e^{15} - 8 \\
& 388608a^{61}b^2d^6e^{15}))/2 - (e*\cot(c + d*x))^{(1/2)}*(7610564608a^{27}b^33d^5e^{13} - 597688320a^{23}b^37d^5e^{13} - 1671430144a^{25}b^35d^5e^{13} - \\
& 58982400a^{21}b^39d^5e^{13} + 85774565376a^{29}b^31d^5e^{13} + 385487994880a^{31}b^29d^5e^{13} + 1104303620096a^{33}b^27d^5e^{13} + 2240523796480a^{35} \\
& *b^25d^5e^{13} + 3345249468416a^{37}b^23d^5e^{13} + 3717287903232a^{39}b^21d^5e^{13} + 3053967114240a^{41}b^19d^5e^{13} + 1807474491392a^{43}b^17d^5e^{13} \\
& e^{13} + 726513221632a^{45}b^15d^5e^{13} + 170768990208a^{47}b^13d^5e^{13} + 10492051456a^{49}b^11d^5e^{13} - 4917821440a^{51}b^9d^5e^{13} - 923009024a
\end{aligned}$$

$$\begin{aligned}
& \left( ^53*b^7*d^5*e^{13} + 8388608*a^{55}*b^5*d^5*e^{13} \right) * \left( -1 / \left( b^6*d^2*e^{3*1i} - a^6*d^2*e^{3*1i} + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^{3*15i} - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^{3*15i} \right) \right)^{(1/2)} / 2 - 29491200*a^{22}*b^{35}*d^4*e^{12} \\
& - 460062720*a^{24}*b^{33}*d^4*e^{12} - 3439722496*a^{26}*b^{31}*d^4*e^{12} - 16227237888*a^{28}*b^{29}*d^4*e^{12} - 53669396480*a^{30}*b^{27}*d^4*e^{12} - 131031367680*a^{32}*b^{25}*d^4*e^{12} \\
& - 242529730560*a^{34}*b^{23}*d^4*e^{12} - 344454070272*a^{36}*b^{21}*d^4*e^{12} - 375993532416*a^{38}*b^{19}*d^4*e^{12} - 313043189760*a^{40}*b^{17}*d^4*e^{12} \\
& - 195253370880*a^{42}*b^{15}*d^4*e^{12} - 88318935040*a^{44}*b^{13}*d^4*e^{12} - 27352498176*a^{46}*b^{11}*d^4*e^{12} - 5187043328*a^{48}*b^9*d^4*e^{12} - 454164480*a^{50}*b^7*d^4*e^{12} \\
& * \left( -1 / \left( b^6*d^2*e^{3*1i} - a^6*d^2*e^{3*1i} + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^{3*15i} - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^{3*15i} \right) \right)^{(1/2)} / 2 - \log \left( - \left( -1 / \left( 4 * \left( b^6*d^2*e^{3*1i} - a^6*d^2*e^{3*1i} + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^{3*15i} - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^{3*15i} \right) \right) \right)^{(1/2)} * \left( \left( e^{\cot(c + d*x)} \right)^{(1/2)} * \left( 471859200*a^{22}*b^{44}*d^7*e^{16} \right. \right. \\
& + 9500098560*a^{24}*b^{42}*d^7*e^{16} + 91857354752*a^{26}*b^{40}*d^7*e^{16} + 564502986752*a^{28}*b^{38}*d^7*e^{16} + 2464648527872*a^{30}*b^{36}*d^7*e^{16} + 8104469069824*a^{32}*b^{34}*d^7*e^{16} \\
& + 20769933361152*a^{34}*b^{32}*d^7*e^{16} + 42351565209600*a^{36}*b^{30}*d^7*e^{16} + 69534945902592*a^{38}*b^{28}*d^7*e^{16} + 92434029608960*a^{40}*b^{26}*d^7*e^{16} \\
& + 99508717355008*a^{42}*b^{24}*d^7*e^{16} + 86342935511040*a^{44}*b^{22}*d^7*e^{16} + 59767095558144*a^{46}*b^{20}*d^7*e^{16} + 32432589897728*a^{48}*b^{18}*d^7*e^{16} \\
& + 13411815522304*a^{50}*b^{16}*d^7*e^{16} + 4030457708544*a^{52}*b^{14}*d^7*e^{16} + 805425905664*a^{54}*b^{12}*d^7*e^{16} + 86608183296*a^{56}*b^{10}*d^7*e^{16} + 1612709888*a^{58}*b^8*d^7*e^{16} \\
& + 16777216*a^{60}*b^6*d^7*e^{16} + 167772160*a^{62}*b^4*d^7*e^{16} + 16777216*a^{64}*b^2*d^7*e^{16} \left. \right) - \left( -1 / \left( 4 * \left( b^6*d^2*e^{3*1i} - a^6*d^2*e^{3*1i} + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^{3*15i} - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^{3*15i} \right) \right) \right)^{(1/2)} * \left( \left( e^{\cot(c + d*x)} \right)^{(1/2)} * \left( -1 / \left( 4 * \left( b^6*d^2*e^{3*1i} - a^6*d^2*e^{3*1i} + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^{3*15i} - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^{3*15i} \right) \right) \right)^{(1/2)} * \left( 134217728*a^{27}*b^{45}*d^9*e^{19} \right. \\
& + 2550136832*a^{29}*b^{43}*d^9*e^{19} + 22817013760*a^{31}*b^{41}*d^9*e^{19} + 127506841600*a^{33}*b^{39}*d^9*e^{19} + 497276682240*a^{35}*b^{37}*d^9*e^{19} \\
& + 1430626762752*a^{37}*b^{35}*d^9*e^{19} + 3121367482368*a^{39}*b^{33}*d^9*e^{19} + 5202279137280*a^{41}*b^{31}*d^9*e^{19} + 6502848921600*a^{43}*b^{29}*d^9*e^{19} + 5635802398720*a^{45}*b^{27}*d^9*e^{19} \\
& + 2254320959488*a^{47}*b^{25}*d^9*e^{19} - 2254320959488*a^{49}*b^{23}*d^9*e^{19} - 5635802398720*a^{51}*b^{21}*d^9*e^{19} - 6502848921600*a^{53}*b^{19}*d^9*e^{19} - 5202279137280*a^{55}*b^{17}*d^9*e^{19} - 3121367482368*a^{57}*b^{15}*d^9*e^{19} \\
& - 1430626762752*a^{59}*b^{13}*d^9*e^{19} - 497276682240*a^{61}*b^{11}*d^9*e^{19} - 127506841600*a^{63}*b^9*d^9*e^{19} - 22817013760*a^{65}*b^7*d^9*e^{19} - 2550136832*a^{67}*b^5*d^9*e^{19} - 134217728*a^{69}*b^3*d^9*e^{19} \left. \right) + 251658240*a^{24}*b^{45}*d^8*e^{18} \\
& + 5049942016*a^{26}*b^{43}*d^8*e^{18} + 48368713728*a^{28}*b^{41}*d^8*e^{18} + 293819383808*a^{30}*b^{39}*d^8*e^{18} + 1268458192896*a^{32}*b^{37}*d^8*e^{18} \\
& + 4132731617280*a^{34}*b^{35}*d^8*e^{18} + 10531192700928*a^{36}*b^{33}*d^8*e^{18} + 21462823993344*a^{38}*b^{31}*d^8*e^{18} + 35469618315264*a^{40}*b^{29}*d^8*e^{18} + 47896904859648*a^{42}*b^{27}*d^8*e^{18} \\
& + 52983958077440*a^{44}*b^{25}*d^8*e^{18} + 47896904859648*a^{46}*b^{23}*d^8*e^{18} + 35090285461504*a^{48}*b^{21}*d^8*e^{18} + 20487396655104*a^{50}*b^{19}*d^8*e^{18} + 9230622916608*a^{52}*b^{17}*d^8*e^{18} + 2994733056000*a
\end{aligned}$$

$$\begin{aligned}
& ^54*b^{15}*d^8*e^{18} + 565576728576*a^{56}*b^{13}*d^8*e^{18} - 18572378112*a^{58}*b^{11} \\
& *d^8*e^{18} - 50281316352*a^{60}*b^9*d^8*e^{18} - 16089350144*a^{62}*b^7*d^8*e^{18} - \\
& 2516582400*a^{64}*b^5*d^8*e^{18} - 167772160*a^{66}*b^3*d^8*e^{18})) * (-1/(4*(b^6*d \\
& ^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^ \\
& 2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^{(1/2)} + 117964800*a \\
& ^21*b^42*d^6*e^{15} + 841482240*a^{23}*b^40*d^6*e^{15} - 3829399552*a^{25}*b^38*d^6 \\
& *e^{15} - 78068580352*a^{27}*b^36*d^6*e^{15} - 497438162944*a^{29}*b^34*d^6*e^{15} - \\
& 1899895980032*a^{31}*b^32*d^6*e^{15} - 4972695519232*a^{33}*b^30*d^6*e^{15} - 93711 \\
& 95015168*a^{35}*b^28*d^6*e^{15} - 12890720436224*a^{37}*b^26*d^6*e^{15} - 127260898 \\
& 09920*a^{39}*b^24*d^6*e^{15} - 8366961197056*a^{41}*b^22*d^6*e^{15} - 2597662490624 \\
& *a^{43}*b^20*d^6*e^{15} + 1171836108800*a^{45}*b^18*d^6*e^{15} + 1986881650688*a^{47} \\
& *b^16*d^6*e^{15} + 1237583921152*a^{49}*b^14*d^6*e^{15} + 449507753984*a^{51}*b^12* \\
& d^6*e^{15} + 97476149248*a^{53}*b^10*d^6*e^{15} + 11931222016*a^{55}*b^8*d^6*e^{15} + \\
& 1006632960*a^{57}*b^6*d^6*e^{15} + 134217728*a^{59}*b^4*d^6*e^{15} + 8388608*a^{61}* \\
& b^2*d^6*e^{15}) - (e*\cot(c + d*x))^{(1/2)}*(7610564608*a^{27}*b^33*d^5*e^{13} - 597 \\
& 688320*a^{23}*b^37*d^5*e^{13} - 1671430144*a^{25}*b^35*d^5*e^{13} - 58982400*a^{21}*b \\
& ^39*d^5*e^{13} + 85774565376*a^{29}*b^31*d^5*e^{13} + 385487994880*a^{31}*b^29*d^5* \\
& e^{13} + 1104303620096*a^{33}*b^27*d^5*e^{13} + 2240523796480*a^{35}*b^25*d^5*e^{13} \\
& + 3345249468416*a^{37}*b^23*d^5*e^{13} + 3717287903232*a^{39}*b^21*d^5*e^{13} + 305 \\
& 3967114240*a^{41}*b^19*d^5*e^{13} + 1807474491392*a^{43}*b^17*d^5*e^{13} + 72651322 \\
& 1632*a^{45}*b^15*d^5*e^{13} + 170768990208*a^{47}*b^13*d^5*e^{13} + 10492051456*a^4 \\
& 9*b^11*d^5*e^{13} - 4917821440*a^{51}*b^9*d^5*e^{13} - 923009024*a^{53}*b^7*d^5*e^1 \\
& 3 + 8388608*a^{55}*b^5*d^5*e^{13})) * (-1/(4*(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6 \\
& *a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 \\
& + a^4*b^2*d^2*e^3*15i)))^{(1/2)} - 29491200*a^{22}*b^35*d^4*e^{12} - 460062720*a \\
& ^24*b^33*d^4*e^{12} - 3439722496*a^{26}*b^31*d^4*e^{12} - 16227237888*a^{28}*b^29*d \\
& ^4*e^{12} - 53669396480*a^{30}*b^27*d^4*e^{12} - 131031367680*a^{32}*b^25*d^4*e^{12} \\
& - 242529730560*a^{34}*b^23*d^4*e^{12} - 344454070272*a^{36}*b^21*d^4*e^{12} - 37599 \\
& 3532416*a^{38}*b^19*d^4*e^{12} - 313043189760*a^{40}*b^17*d^4*e^{12} - 195253370880 \\
& *a^{42}*b^15*d^4*e^{12} - 88318935040*a^{44}*b^13*d^4*e^{12} - 27352498176*a^{46}*b^1 \\
& 1*d^4*e^{12} - 5187043328*a^{48}*b^9*d^4*e^{12} - 454164480*a^{50}*b^7*d^4*e^{12})*(- \\
& 1/(4*(b^6*d^2*e^3*1i - a^6*d^2*e^3*1i + 6*a*b^5*d^2*e^3 + 6*a^5*b*d^2*e^3 - \\
& a^2*b^4*d^2*e^3*15i - 20*a^3*b^3*d^2*e^3 + a^4*b^2*d^2*e^3*15i)))^{(1/2)} - \\
& (\operatorname{atan}((((e*\cot(c + d*x))^{(1/2)}*(7610564608*a^{27}*b^33*d^5*e^{13} - 597688320* \\
& a^{23}*b^37*d^5*e^{13} - 1671430144*a^{25}*b^35*d^5*e^{13} - 58982400*a^{21}*b^39*d^5 \\
& *e^{13} + 85774565376*a^{29}*b^31*d^5*e^{13} + 385487994880*a^{31}*b^29*d^5*e^{13} + \\
& 1104303620096*a^{33}*b^27*d^5*e^{13} + 2240523796480*a^{35}*b^25*d^5*e^{13} + 33452 \\
& 49468416*a^{37}*b^23*d^5*e^{13} + 3717287903232*a^{39}*b^21*d^5*e^{13} + 3053967114 \\
& 240*a^{41}*b^19*d^5*e^{13} + 1807474491392*a^{43}*b^17*d^5*e^{13} + 726513221632*a^ \\
& 45*b^15*d^5*e^{13} + 170768990208*a^{47}*b^13*d^5*e^{13} + 10492051456*a^{49}*b^11* \\
& d^5*e^{13} - 4917821440*a^{51}*b^9*d^5*e^{13} - 923009024*a^{53}*b^7*d^5*e^{13} + 838 \\
& 8608*a^{55}*b^5*d^5*e^{13}) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1 \\
& /2)} * (((e*\cot(c + d*x))^{(1/2)}*(471859200*a^{22}*b^44*d^7*e^{16} + 9500098560*a^ \\
& 24*b^42*d^7*e^{16} + 91857354752*a^{26}*b^40*d^7*e^{16} + 564502986752*a^{28}*b^38* \\
& d^7*e^{16} + 2464648527872*a^{30}*b^36*d^7*e^{16} + 8104469069824*a^{32}*b^34*d^7*e
\end{aligned}$$

$$\begin{aligned}
& ^{16} + 20769933361152*a^{34}*b^{32}*d^7*e^{16} + 42351565209600*a^{36}*b^{30}*d^7*e^{16} \\
& + 69534945902592*a^{38}*b^{28}*d^7*e^{16} + 92434029608960*a^{40}*b^{26}*d^7*e^{16} + \\
& 99508717355008*a^{42}*b^{24}*d^7*e^{16} + 86342935511040*a^{44}*b^{22}*d^7*e^{16} + 597 \\
& 67095558144*a^{46}*b^{20}*d^7*e^{16} + 32432589897728*a^{48}*b^{18}*d^7*e^{16} + 134118 \\
& 15522304*a^{50}*b^{16}*d^7*e^{16} + 4030457708544*a^{52}*b^{14}*d^7*e^{16} + 8054259056 \\
& 64*a^{54}*b^{12}*d^7*e^{16} + 86608183296*a^{56}*b^{10}*d^7*e^{16} + 1612709888*a^{58}*b^8 \\
& *d^7*e^{16} + 16777216*a^{60}*b^6*d^7*e^{16} + 167772160*a^{62}*b^4*d^7*e^{16} + 167 \\
& 77216*a^{64}*b^2*d^7*e^{16}) + ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)} \\
& *(251658240*a^{24}*b^{45}*d^8*e^{18} + 5049942016*a^{26}*b^{43}*d^8*e^{18} + 483687 \\
& 13728*a^{28}*b^{41}*d^8*e^{18} + 293819383808*a^{30}*b^{39}*d^8*e^{18} + 1268458192896* \\
& a^{32}*b^{37}*d^8*e^{18} + 4132731617280*a^{34}*b^{35}*d^8*e^{18} + 10531192700928*a^{36} \\
& *b^{33}*d^8*e^{18} + 21462823993344*a^{38}*b^{31}*d^8*e^{18} + 35469618315264*a^{40}*b^{29} \\
& *d^8*e^{18} + 47896904859648*a^{42}*b^{27}*d^8*e^{18} + 52983958077440*a^{44}*b^{25}* \\
& d^8*e^{18} + 47896904859648*a^{46}*b^{23}*d^8*e^{18} + 35090285461504*a^{48}*b^{21}*d^8 \\
& *e^{18} + 20487396655104*a^{50}*b^{19}*d^8*e^{18} + 9230622916608*a^{52}*b^{17}*d^8*e^{18} \\
& + 2994733056000*a^{54}*b^{15}*d^8*e^{18} + 565576728576*a^{56}*b^{13}*d^8*e^{18} - 18 \\
& 572378112*a^{58}*b^{11}*d^8*e^{18} - 50281316352*a^{60}*b^9*d^8*e^{18} - 16089350144* \\
& a^{62}*b^7*d^8*e^{18} - 2516582400*a^{64}*b^5*d^8*e^{18} - 167772160*a^{66}*b^3*d^8*e^{18} \\
& - ((e*\cot(c + d*x))^{(1/2)}*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)} \\
& *(134217728*a^{27}*b^{45}*d^9*e^{19} + 2550136832*a^{29}*b^{43}*d^9*e^{19} + 2281 \\
& 7013760*a^{31}*b^{41}*d^9*e^{19} + 127506841600*a^{33}*b^{39}*d^9*e^{19} + 497276682240 \\
& *a^{35}*b^{37}*d^9*e^{19} + 1430626762752*a^{37}*b^{35}*d^9*e^{19} + 3121367482368*a^{39} \\
& *b^{33}*d^9*e^{19} + 5202279137280*a^{41}*b^{31}*d^9*e^{19} + 6502848921600*a^{43}*b^{29} \\
& *d^9*e^{19} + 5635802398720*a^{45}*b^{27}*d^9*e^{19} + 2254320959488*a^{47}*b^{25}*d^9* \\
& e^{19} - 2254320959488*a^{49}*b^{23}*d^9*e^{19} - 5635802398720*a^{51}*b^{21}*d^9*e^{19} \\
& - 6502848921600*a^{53}*b^{19}*d^9*e^{19} - 5202279137280*a^{55}*b^{17}*d^9*e^{19} - 312 \\
& 1367482368*a^{57}*b^{15}*d^9*e^{19} - 1430626762752*a^{59}*b^{13}*d^9*e^{19} - 49727668 \\
& 2240*a^{61}*b^{11}*d^9*e^{19} - 127506841600*a^{63}*b^9*d^9*e^{19} - 22817013760*a^{65} \\
& *b^7*d^9*e^{19} - 2550136832*a^{67}*b^5*d^9*e^{19} - 134217728*a^{69}*b^3*d^9*e^{19}) \\
& )/(8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))/( \\
& 8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3))*((63*a^4 \\
& + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)})/(8*(a^{13}*d*e^3 + a^7*b^6*d* \\
& e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)) - 117964800*a^{21}*b^{42}*d^6*e^{15} - \\
& 841482240*a^{23}*b^{40}*d^6*e^{15} + 3829399552*a^{25}*b^{38}*d^6*e^{15} + 78068580352 \\
& *a^{27}*b^{36}*d^6*e^{15} + 497438162944*a^{29}*b^{34}*d^6*e^{15} + 1899895980032*a^{31}* \\
& b^{32}*d^6*e^{15} + 4972695519232*a^{33}*b^{30}*d^6*e^{15} + 9371195015168*a^{35}*b^{28}* \\
& d^6*e^{15} + 12890720436224*a^{37}*b^{26}*d^6*e^{15} + 12726089809920*a^{39}*b^{24}*d^6 \\
& *e^{15} + 8366961197056*a^{41}*b^{22}*d^6*e^{15} + 2597662490624*a^{43}*b^{20}*d^6*e^{15} \\
& - 1171836108800*a^{45}*b^{18}*d^6*e^{15} - 1986881650688*a^{47}*b^{16}*d^6*e^{15} - 12 \\
& 37583921152*a^{49}*b^{14}*d^6*e^{15} - 449507753984*a^{51}*b^{12}*d^6*e^{15} - 97476149 \\
& 248*a^{53}*b^{10}*d^6*e^{15} - 11931222016*a^{55}*b^8*d^6*e^{15} - 1006632960*a^{57}*b^6 \\
& *d^6*e^{15} - 134217728*a^{59}*b^4*d^6*e^{15} - 8388608*a^{61}*b^2*d^6*e^{15}))/((8*( \\
& a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))*(63*a^4 \\
& + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)}*i)/(8*(a^{13}*d*e^3 + a^7*b^6*d* \\
& e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)) + (((e*\cot(c + d*x))^{(1/2)}*(7610
\end{aligned}$$

$$\begin{aligned}
& 564608a^{27}b^{33}d^5e^{13} - 597688320a^{23}b^{37}d^5e^{13} - 1671430144a^{25}b^{35}d^5e^{13} - 58982400a^{21}b^{39}d^5e^{13} + 85774565376a^{29}b^{31}d^5e^{13} \\
& + 385487994880a^{31}b^{29}d^5e^{13} + 1104303620096a^{33}b^{27}d^5e^{13} + 2240523796480a^{35}b^{25}d^5e^{13} + 3345249468416a^{37}b^{23}d^5e^{13} + 3717287903232a^{39}b^{21}d^5e^{13} \\
& + 3053967114240a^{41}b^{19}d^5e^{13} + 1807474491392a^{43}b^{17}d^5e^{13} + 726513221632a^{45}b^{15}d^5e^{13} + 170768990208a^{47}b^{13}d^5e^{13} \\
& + 10492051456a^{49}b^{11}d^5e^{13} - 4917821440a^{51}b^9d^5e^{13} - 923009024a^{53}b^7d^5e^{13} + 8388608a^{55}b^5d^5e^{13} - ((63a^4 + 15b^4 + 46a^2b^2) \cdot (-a^7b^5e^3)^{1/2}) \cdot (((e \cot(c + dx))^{1/2}) \cdot (471859200a^{22}b^{44}d^7e^{16} + 9500098560a^{24}b^{42}d^7e^{16} + 91857354752a^{26}b^{40}d^7e^{16} + 564502986752a^{28}b^{38}d^7e^{16} + 2464648527872a^{30}b^{36}d^7e^{16} + 8104469069824a^{32}b^{34}d^7e^{16} + 20769933361152a^{34}b^{32}d^7e^{16} + 42351565209600a^{36}b^{30}d^7e^{16} + 69534945902592a^{38}b^{28}d^7e^{16} + 92434029608960a^{40}b^{26}d^7e^{16} + 99508717355008a^{42}b^{24}d^7e^{16} + 86342935511040a^{44}b^{22}d^7e^{16} + 59767095558144a^{46}b^{20}d^7e^{16} + 32432589897728a^{48}b^{18}d^7e^{16} + 13411815522304a^{50}b^{16}d^7e^{16} + 4030457708544a^{52}b^{14}d^7e^{16} + 805425905664a^{54}b^{12}d^7e^{16} + 86608183296a^{56}b^{10}d^7e^{16} + 1612709888a^{58}b^8d^7e^{16} + 16777216a^{60}b^6d^7e^{16} + 167772160a^{62}b^4d^7e^{16} + 16777216a^{64}b^2d^7e^{16}) - ((63a^4 + 15b^4 + 46a^2b^2) \cdot (-a^7b^5e^3)^{1/2}) \cdot (251658240a^{24}b^{45}d^8e^{18} + 5049942016a^{26}b^{43}d^8e^{18} + 48368713728a^{28}b^{41}d^8e^{18} + 293819383808a^{30}b^{39}d^8e^{18} + 1268458192896a^{32}b^{37}d^8e^{18} + 4132731617280a^{34}b^{35}d^8e^{18} + 10531192700928a^{36}b^{33}d^8e^{18} + 21462823993344a^{38}b^{31}d^8e^{18} + 35469618315264a^{40}b^{29}d^8e^{18} + 47896904859648a^{42}b^{27}d^8e^{18} + 52983958077440a^{44}b^{25}d^8e^{18} + 47896904859648a^{46}b^{23}d^8e^{18} + 35090285461504a^{48}b^{21}d^8e^{18} + 20487396655104a^{50}b^{19}d^8e^{18} + 9230622916608a^{52}b^{17}d^8e^{18} + 2994733056000a^{54}b^{15}d^8e^{18} + 565576728576a^{56}b^{13}d^8e^{18} - 18572378112a^{58}b^{11}d^8e^{18} - 50281316352a^{60}b^9d^8e^{18} - 16089350144a^{62}b^7d^8e^{18} - 2516582400a^{64}b^5d^8e^{18} - 167772160a^{66}b^3d^8e^{18} + ((e \cot(c + dx))^{1/2}) \cdot (63a^4 + 15b^4 + 46a^2b^2) \cdot (-a^7b^5e^3)^{1/2} \cdot (134217728a^{27}b^{45}d^9e^{19} + 2550136832a^{29}b^{43}d^9e^{19} + 22817013760a^{31}b^{41}d^9e^{19} + 127506841600a^{33}b^{39}d^9e^{19} + 497276682240a^{35}b^{37}d^9e^{19} + 1430626762752a^{37}b^{35}d^9e^{19} + 3121367482368a^{39}b^{33}d^9e^{19} + 5202279137280a^{41}b^{31}d^9e^{19} + 6502848921600a^{43}b^{29}d^9e^{19} + 5635802398720a^{45}b^{27}d^9e^{19} + 2254320959488a^{47}b^{25}d^9e^{19} - 2254320959488a^{49}b^{23}d^9e^{19} - 5635802398720a^{51}b^{21}d^9e^{19} - 6502848921600a^{53}b^{19}d^9e^{19} - 5202279137280a^{55}b^{17}d^9e^{19} - 3121367482368a^{57}b^{15}d^9e^{19} - 1430626762752a^{59}b^{13}d^9e^{19} - 497276682240a^{61}b^{11}d^9e^{19} - 127506841600a^{63}b^9d^9e^{19} - 22817013760a^{65}b^7d^9e^{19} - 2550136832a^{67}b^5d^9e^{19} - 134217728a^{69}b^3d^9e^{19})) / (8(a^{13}d^3e^3 + a^7b^6d^3e^3 + 3a^9b^4d^3e^3 + 3a^{11}b^2d^3e^3)) / (8(a^{13}d^3e^3 + a^7b^6d^3e^3 + 3a^9b^4d^3e^3 + 3a^{11}b^2d^3e^3)) \cdot (63a^4 + 15b^4 + 46a^2b^2) \cdot (-a^7b^5e^3)^{1/2} / (8(a^{13}d^3e^3 + a^7b^6d^3e^3 + 3a^9b^4d^3e^3 + 3a^{11}b^2d^3e^3)) + 117964800a^{21}b^{42}d^6e^{15} + 841482240a^{23}b^{40}d^6e^{15} - 38293
\end{aligned}$$

$$\begin{aligned}
& 99552*a^{25}*b^{38}*d^6*e^{15} - 78068580352*a^{27}*b^{36}*d^6*e^{15} - 497438162944*a^{29}*b^{34}*d^6*e^{15} - 1899895980032*a^{31}*b^{32}*d^6*e^{15} - 4972695519232*a^{33}*b^{30}*d^6*e^{15} - 9371195015168*a^{35}*b^{28}*d^6*e^{15} - 12890720436224*a^{37}*b^{26}*d^6*e^{15} - 12726089809920*a^{39}*b^{24}*d^6*e^{15} - 8366961197056*a^{41}*b^{22}*d^6*e^{15} - 2597662490624*a^{43}*b^{20}*d^6*e^{15} + 1171836108800*a^{45}*b^{18}*d^6*e^{15} + 1986881650688*a^{47}*b^{16}*d^6*e^{15} + 1237583921152*a^{49}*b^{14}*d^6*e^{15} + 449507753984*a^{51}*b^{12}*d^6*e^{15} + 97476149248*a^{53}*b^{10}*d^6*e^{15} + 11931222016*a^{55}*b^8*d^6*e^{15} + 1006632960*a^{57}*b^6*d^6*e^{15} + 134217728*a^{59}*b^4*d^6*e^{15} + 8388608*a^{61}*b^2*d^6*e^{15}))/((8*(a^{13}*d^3*e^3 + a^7*b^6*d^3*e^3 + 3*a^9*b^4*d^3*e^3 + 3*a^{11}*b^2*d^3*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)*i}))/((8*(a^{13}*d^3*e^3 + a^7*b^6*d^3*e^3 + 3*a^9*b^4*d^3*e^3 + 3*a^{11}*b^2*d^3*e^3)))/(58982400*a^{22}*b^{35}*d^4*e^{12} + 920125440*a^{24}*b^{33}*d^4*e^{12} + 6879444992*a^{26}*b^{31}*d^4*e^{12} + 32454475776*a^{28}*b^{29}*d^4*e^{12} + 107338792960*a^{30}*b^{27}*d^4*e^{12} + 262062735360*a^{32}*b^{25}*d^4*e^{12} + 485059461120*a^{34}*b^{23}*d^4*e^{12} + 688908140544*a^{36}*b^{21}*d^4*e^{12} + 751987064832*a^{38}*b^{19}*d^4*e^{12} + 626086379520*a^{40}*b^{17}*d^4*e^{12} + 390506741760*a^{42}*b^{15}*d^4*e^{12} + 176637870080*a^{44}*b^{13}*d^4*e^{12} + 54704996352*a^{46}*b^{11}*d^4*e^{12} + 10374086656*a^{48}*b^9*d^4*e^{12} + 908328960*a^{50}*b^7*d^4*e^{12} + (((e*cot(c + d*x))^{(1/2)})*(7610564608*a^{27}*b^{33}*d^5*e^{13} - 597688320*a^{23}*b^{37}*d^5*e^{13} - 1671430144*a^{25}*b^{35}*d^5*e^{13} - 58982400*a^{21}*b^{39}*d^5*e^{13} + 85774565376*a^{29}*b^{31}*d^5*e^{13} + 385487994880*a^{31}*b^{29}*d^5*e^{13} + 1104303620096*a^{33}*b^{27}*d^5*e^{13} + 2240523796480*a^{35}*b^{25}*d^5*e^{13} + 3345249468416*a^{37}*b^{23}*d^5*e^{13} + 3717287903232*a^{39}*b^{21}*d^5*e^{13} + 3053967114240*a^{41}*b^{19}*d^5*e^{13} + 1807474491392*a^{43}*b^{17}*d^5*e^{13} + 726513221632*a^{45}*b^{15}*d^5*e^{13} + 170768990208*a^{47}*b^{13}*d^5*e^{13} + 10492051456*a^{49}*b^{11}*d^5*e^{13} - 4917821440*a^{51}*b^9*d^5*e^{13} - 923009024*a^{53}*b^7*d^5*e^{13} + 8388608*a^{55}*b^5*d^5*e^{13}) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)}*(((e*cot(c + d*x))^{(1/2)}*(471859200*a^{22}*b^{44}*d^7*e^{16} + 9500098560*a^{24}*b^{42}*d^7*e^{16} + 91857354752*a^{26}*b^{40}*d^7*e^{16} + 564502986752*a^{28}*b^{38}*d^7*e^{16} + 2464648527872*a^{30}*b^{36}*d^7*e^{16} + 8104469069824*a^{32}*b^{34}*d^7*e^{16} + 20769933361152*a^{34}*b^{32}*d^7*e^{16} + 42351565209600*a^{36}*b^{30}*d^7*e^{16} + 69534945902592*a^{38}*b^{28}*d^7*e^{16} + 92434029608960*a^{40}*b^{26}*d^7*e^{16} + 99508717355008*a^{42}*b^{24}*d^7*e^{16} + 86342935511040*a^{44}*b^{22}*d^7*e^{16} + 59767095558144*a^{46}*b^{20}*d^7*e^{16} + 32432589897728*a^{48}*b^{18}*d^7*e^{16} + 13411815522304*a^{50}*b^{16}*d^7*e^{16} + 4030457708544*a^{52}*b^{14}*d^7*e^{16} + 805425905664*a^{54}*b^{12}*d^7*e^{16} + 86608183296*a^{56}*b^{10}*d^7*e^{16} + 1612709888*a^{58}*b^8*d^7*e^{16} + 16777216*a^{60}*b^6*d^7*e^{16} + 167772160*a^{62}*b^4*d^7*e^{16} + 16777216*a^{64}*b^2*d^7*e^{16}) + ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)}*(251658240*a^{24}*b^{45}*d^8*e^{18} + 5049942016*a^{26}*b^{43}*d^8*e^{18} + 48368713728*a^{28}*b^{41}*d^8*e^{18} + 293819383808*a^{30}*b^{39}*d^8*e^{18} + 1268458192896*a^{32}*b^{37}*d^8*e^{18} + 4132731617280*a^{34}*b^{35}*d^8*e^{18} + 10531192700928*a^{36}*b^{33}*d^8*e^{18} + 21462823993344*a^{38}*b^{31}*d^8*e^{18} + 35469618315264*a^{40}*b^{29}*d^8*e^{18} + 47896904859648*a^{42}*b^{27}*d^8*e^{18} + 52983958077440*a^{44}*b^{25}*d^8*e^{18} + 47896904859648*a^{46}*b^{23}*d^8*e^{18} + 35090285461504*a^{48}*b^{21}*d^8*e^{18} + 20487396655104*a^{50}*b^{19}*d^8*e^{18} + 9230622916608*a^{52}*b^{17}*d^8*e^{18} + 2994733056000*a^{54}*b^{15}*d^8
\end{aligned}$$



$$\begin{aligned}
& *e^{18} + 565576728576*a^{56}*b^{13}*d^8*e^{18} - 18572378112*a^{58}*b^{11}*d^8*e^{18} - \\
& 50281316352*a^{60}*b^9*d^8*e^{18} - 16089350144*a^{62}*b^7*d^8*e^{18} - 2516582400* \\
& a^{64}*b^5*d^8*e^{18} - 167772160*a^{66}*b^3*d^8*e^{18} - ((e*\cot(c + d*x))^{(1/2)}*( \\
& 63*a^4 + 15*b^4 + 46*a^2*b^2))*(-a^7*b^5*e^3)^{(1/2)}*(134217728*a^{27}*b^{45}*d^9 \\
& *e^{19} + 2550136832*a^{29}*b^{43}*d^9*e^{19} + 22817013760*a^{31}*b^{41}*d^9*e^{19} + 12 \\
& 7506841600*a^{33}*b^{39}*d^9*e^{19} + 497276682240*a^{35}*b^{37}*d^9*e^{19} + 143062676 \\
& 2752*a^{37}*b^{35}*d^9*e^{19} + 3121367482368*a^{39}*b^{33}*d^9*e^{19} + 5202279137280* \\
& a^{41}*b^{31}*d^9*e^{19} + 6502848921600*a^{43}*b^{29}*d^9*e^{19} + 5635802398720*a^{45}* \\
& b^{27}*d^9*e^{19} + 2254320959488*a^{47}*b^{25}*d^9*e^{19} - 2254320959488*a^{49}*b^{23}* \\
& d^9*e^{19} - 5635802398720*a^{51}*b^{21}*d^9*e^{19} - 6502848921600*a^{53}*b^{19}*d^9*e \\
& ^{19} - 5202279137280*a^{55}*b^{17}*d^9*e^{19} - 3121367482368*a^{57}*b^{15}*d^9*e^{19} - \\
& 1430626762752*a^{59}*b^{13}*d^9*e^{19} - 497276682240*a^{61}*b^{11}*d^9*e^{19} - 12750 \\
& 6841600*a^{63}*b^9*d^9*e^{19} - 22817013760*a^{65}*b^7*d^9*e^{19} - 2550136832*a^{67} \\
& *b^5*d^9*e^{19} - 134217728*a^{69}*b^3*d^9*e^{19}))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 \\
& + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + \\
& 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2))*(-a^7 \\
& *b^5*e^3)^{(1/2)}))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11} \\
& *b^2*d*e^3)) - 117964800*a^{21}*b^{42}*d^6*e^{15} - 841482240*a^{23}*b^{40}*d^6*e^{15} + \\
& 3829399552*a^{25}*b^{38}*d^6*e^{15} + 78068580352*a^{27}*b^{36}*d^6*e^{15} + 497438162 \\
& 944*a^{29}*b^{34}*d^6*e^{15} + 1899895980032*a^{31}*b^{32}*d^6*e^{15} + 4972695519232*a \\
& ^{33}*b^{30}*d^6*e^{15} + 9371195015168*a^{35}*b^{28}*d^6*e^{15} + 12890720436224*a^{37} \\
& *b^{26}*d^6*e^{15} + 12726089809920*a^{39}*b^{24}*d^6*e^{15} + 8366961197056*a^{41}*b^{22} \\
& *d^6*e^{15} + 2597662490624*a^{43}*b^{20}*d^6*e^{15} - 1171836108800*a^{45}*b^{18}*d^6* \\
& e^{15} - 1986881650688*a^{47}*b^{16}*d^6*e^{15} - 1237583921152*a^{49}*b^{14}*d^6*e^{15} \\
& - 449507753984*a^{51}*b^{12}*d^6*e^{15} - 97476149248*a^{53}*b^{10}*d^6*e^{15} - 119312 \\
& 22016*a^{55}*b^8*d^6*e^{15} - 1006632960*a^{57}*b^6*d^6*e^{15} - 134217728*a^{59}*b^4 \\
& *d^6*e^{15} - 8388608*a^{61}*b^2*d^6*e^{15}))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3* \\
& a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2))*(-a^7*b^ \\
& 5*e^3)^{(1/2)}))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2 \\
& *d*e^3)) - (((e*\cot(c + d*x))^{(1/2)}*(7610564608*a^{27}*b^{33}*d^5*e^{13} - 597688 \\
& 320*a^{23}*b^{37}*d^5*e^{13} - 1671430144*a^{25}*b^{35}*d^5*e^{13} - 58982400*a^{21}*b^{39} \\
& *d^5*e^{13} + 85774565376*a^{29}*b^{31}*d^5*e^{13} + 385487994880*a^{31}*b^{29}*d^5*e^{13} \\
& + 1104303620096*a^{33}*b^{27}*d^5*e^{13} + 2240523796480*a^{35}*b^{25}*d^5*e^{13} + 3 \\
& 345249468416*a^{37}*b^{23}*d^5*e^{13} + 3717287903232*a^{39}*b^{21}*d^5*e^{13} + 305396 \\
& 7114240*a^{41}*b^{19}*d^5*e^{13} + 1807474491392*a^{43}*b^{17}*d^5*e^{13} + 72651322163 \\
& 2*a^{45}*b^{15}*d^5*e^{13} + 170768990208*a^{47}*b^{13}*d^5*e^{13} + 10492051456*a^{49}*b \\
& ^{11}*d^5*e^{13} - 4917821440*a^{51}*b^9*d^5*e^{13} - 923009024*a^{53}*b^7*d^5*e^{13} + \\
& 8388608*a^{55}*b^5*d^5*e^{13}) - ((63*a^4 + 15*b^4 + 46*a^2*b^2))*(-a^7*b^5*e^3 \\
& )^{(1/2)}*(((e*\cot(c + d*x))^{(1/2)}*(471859200*a^{22}*b^{44}*d^7*e^{16} + 950009856 \\
& 0*a^{24}*b^{42}*d^7*e^{16} + 91857354752*a^{26}*b^{40}*d^7*e^{16} + 564502986752*a^{28}*b \\
& ^{38}*d^7*e^{16} + 2464648527872*a^{30}*b^{36}*d^7*e^{16} + 8104469069824*a^{32}*b^{34}*d \\
& ^7*e^{16} + 20769933361152*a^{34}*b^{32}*d^7*e^{16} + 42351565209600*a^{36}*b^{30}*d^7* \\
& e^{16} + 69534945902592*a^{38}*b^{28}*d^7*e^{16} + 92434029608960*a^{40}*b^{26}*d^7*e^{16} \\
& + 99508717355008*a^{42}*b^{24}*d^7*e^{16} + 86342935511040*a^{44}*b^{22}*d^7*e^{16} + \\
& 59767095558144*a^{46}*b^{20}*d^7*e^{16} + 32432589897728*a^{48}*b^{18}*d^7*e^{16} + 13
\end{aligned}$$

$$\begin{aligned}
& 411815522304*a^{50}*b^{16}*d^7*e^{16} + 4030457708544*a^{52}*b^{14}*d^7*e^{16} + 805425 \\
& 905664*a^{54}*b^{12}*d^7*e^{16} + 86608183296*a^{56}*b^{10}*d^7*e^{16} + 1612709888*a^5 \\
& 8*b^8*d^7*e^{16} + 16777216*a^{60}*b^6*d^7*e^{16} + 167772160*a^{62}*b^4*d^7*e^{16} + \\
& 16777216*a^{64}*b^2*d^7*e^{16}) - ((63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^ \\
& 3)^{(1/2)}*(251658240*a^{24}*b^{45}*d^8*e^{18} + 5049942016*a^{26}*b^{43}*d^8*e^{18} + 48 \\
& 368713728*a^{28}*b^{41}*d^8*e^{18} + 293819383808*a^{30}*b^{39}*d^8*e^{18} + 1268458192 \\
& 896*a^{32}*b^{37}*d^8*e^{18} + 4132731617280*a^{34}*b^{35}*d^8*e^{18} + 10531192700928* \\
& a^{36}*b^{33}*d^8*e^{18} + 21462823993344*a^{38}*b^{31}*d^8*e^{18} + 35469618315264*a^4 \\
& 0*b^{29}*d^8*e^{18} + 47896904859648*a^{42}*b^{27}*d^8*e^{18} + 52983958077440*a^{44}*b \\
& ^{25}*d^8*e^{18} + 47896904859648*a^{46}*b^{23}*d^8*e^{18} + 35090285461504*a^{48}*b^{21} \\
& *d^8*e^{18} + 20487396655104*a^{50}*b^{19}*d^8*e^{18} + 9230622916608*a^{52}*b^{17}*d^8 \\
& *e^{18} + 2994733056000*a^{54}*b^{15}*d^8*e^{18} + 565576728576*a^{56}*b^{13}*d^8*e^{18} \\
& - 18572378112*a^{58}*b^{11}*d^8*e^{18} - 50281316352*a^{60}*b^9*d^8*e^{18} - 16089350 \\
& 144*a^{62}*b^7*d^8*e^{18} - 2516582400*a^{64}*b^5*d^8*e^{18} - 167772160*a^{66}*b^3*d \\
& ^8*e^{18} + ((e*cot(c + d*x))^{(1/2)}*(63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^ \\
& 3)^{(1/2)}*(134217728*a^{27}*b^{45}*d^9*e^{19} + 2550136832*a^{29}*b^{43}*d^9*e^{19} + \\
& 22817013760*a^{31}*b^{41}*d^9*e^{19} + 127506841600*a^{33}*b^{39}*d^9*e^{19} + 49727668 \\
& 2240*a^{35}*b^{37}*d^9*e^{19} + 1430626762752*a^{37}*b^{35}*d^9*e^{19} + 3121367482368* \\
& a^{39}*b^{33}*d^9*e^{19} + 5202279137280*a^{41}*b^{31}*d^9*e^{19} + 6502848921600*a^{43}* \\
& b^{29}*d^9*e^{19} + 5635802398720*a^{45}*b^{27}*d^9*e^{19} + 2254320959488*a^{47}*b^{25}* \\
& d^9*e^{19} - 2254320959488*a^{49}*b^{23}*d^9*e^{19} - 5635802398720*a^{51}*b^{21}*d^9*e \\
& ^{19} - 6502848921600*a^{53}*b^{19}*d^9*e^{19} - 5202279137280*a^{55}*b^{17}*d^9*e^{19} - \\
& 3121367482368*a^{57}*b^{15}*d^9*e^{19} - 1430626762752*a^{59}*b^{13}*d^9*e^{19} - 4972 \\
& 76682240*a^{61}*b^{11}*d^9*e^{19} - 127506841600*a^{63}*b^9*d^9*e^{19} - 22817013760* \\
& a^{65}*b^7*d^9*e^{19} - 2550136832*a^{67}*b^5*d^9*e^{19} - 134217728*a^{69}*b^3*d^9*e \\
& ^{19}))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)) \\
& ))/((8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))* \\
& (63*a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)})/((8*(a^{13}*d*e^3 + a^7*b^ \\
& 6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)) + 117964800*a^{21}*b^{42}*d^6*e^ \\
& 15 + 841482240*a^{23}*b^{40}*d^6*e^15 - 3829399552*a^{25}*b^{38}*d^6*e^15 - 7806858 \\
& 0352*a^{27}*b^{36}*d^6*e^15 - 497438162944*a^{29}*b^{34}*d^6*e^15 - 1899895980032*a \\
& ^{31}*b^{32}*d^6*e^15 - 4972695519232*a^{33}*b^{30}*d^6*e^15 - 9371195015168*a^{35}*b \\
& ^{28}*d^6*e^15 - 12890720436224*a^{37}*b^{26}*d^6*e^15 - 12726089809920*a^{39}*b^{24} \\
& *d^6*e^15 - 8366961197056*a^{41}*b^{22}*d^6*e^15 - 2597662490624*a^{43}*b^{20}*d^6* \\
& e^15 + 1171836108800*a^{45}*b^{18}*d^6*e^15 + 1986881650688*a^{47}*b^{16}*d^6*e^15 \\
& + 1237583921152*a^{49}*b^{14}*d^6*e^15 + 449507753984*a^{51}*b^{12}*d^6*e^15 + 9747 \\
& 6149248*a^{53}*b^{10}*d^6*e^15 + 11931222016*a^{55}*b^8*d^6*e^15 + 1006632960*a^5 \\
& 7*b^6*d^6*e^15 + 134217728*a^{59}*b^4*d^6*e^15 + 8388608*a^{61}*b^2*d^6*e^15))/ \\
& (8*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))*(63* \\
& a^4 + 15*b^4 + 46*a^2*b^2)*(-a^7*b^5*e^3)^{(1/2)})/((8*(a^{13}*d*e^3 + a^7*b^6*d \\
& *e^3 + 3*a^9*b^4*d*e^3 + 3*a^{11}*b^2*d*e^3)))*(63*a^4 + 15*b^4 + 46*a^2*b^2 \\
& )*(-a^7*b^5*e^3)^{(1/2)}*1i)/(4*(a^{13}*d*e^3 + a^7*b^6*d*e^3 + 3*a^9*b^4*d*e^3 \\
& + 3*a^{11}*b^2*d*e^3))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e \cot(c + dx))^{\frac{3}{2}} (a + b \cot(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*cot(d\*x+c))\*\*(3/2)/(a+b\*cot(d\*x+c))\*\*3,x)

[Out] Integral(1/((e\*cot(c + d\*x))\*\*(3/2)\*(a + b\*cot(c + d\*x))\*\*3), x)

### 3.88 $\int (a + b \cot(c + dx))^n dx$

**Optimal.** Leaf size=167

$$\frac{b(a + b \cot(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2} d(n+1) \left(a + \sqrt{-b^2}\right)} - \frac{b(a + b \cot(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2} d(n+1) \left(a - \sqrt{-b^2}\right)}$$

[Out]  $-1/2*b*(a+b*\cot(d*x+c))^{(1+n)}*\text{hypergeom}([1, 1+n], [2+n], (a+b*\cot(d*x+c))/(a-(-b^2)^{(1/2)}))/d/(1+n)/(a-(-b^2)^{(1/2)})/(-b^2)^{(1/2)}+1/2*b*(a+b*\cot(d*x+c))^{(1+n)}*\text{hypergeom}([1, 1+n], [2+n], (a+b*\cot(d*x+c))/(a+(-b^2)^{(1/2)}))/d/(1+n)/(-b^2)^{(1/2)}/(a+(-b^2)^{(1/2)})$

**Rubi [A]** time = 0.24, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3485, 712, 68}

$$\frac{b(a + b \cot(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2} d(n+1) \left(a + \sqrt{-b^2}\right)} - \frac{b(a + b \cot(c + dx))^{n+1} {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2} d(n+1) \left(a - \sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Cot}[c + d*x])^n, x]$

[Out]  $-(b*(a + b*\text{Cot}[c + d*x])^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Cot}[c + d*x])/(a - \text{Sqrt}[-b^2])])/(2*\text{Sqrt}[-b^2]*(a - \text{Sqrt}[-b^2])*d*(1 + n)) + (b*(a + b*\text{Cot}[c + d*x])^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Cot}[c + d*x])/(a + \text{Sqrt}[-b^2])])/(2*\text{Sqrt}[-b^2]*(a + \text{Sqrt}[-b^2])*d*(1 + n))$

#### Rule 68

$\text{Int}(((a_) + (b_)*(x_))^{(m_)}*((c_) + (d_)*(x_))^{(n_)}, x\_Symbol] :> \text{Simp}(((b*c - a*d)^n*(a + b*x)^{(m+1)}*\text{Hypergeometric2F1}[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b^{(n+1)}*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

#### Rule 712

$\text{Int}(((d_) + (e_)*(x_))^{(m_)}/((a_) + (c_)*(x_)^2), x\_Symbol] :> \text{Int}[\text{Expand}[\text{Integrand}[(d + e*x)^m, 1/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, m\}, x] \& \& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !\text{IntegerQ}[m]$

#### Rule 3485

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int (a + b \cot(c + dx))^n dx &= -\frac{b \operatorname{Subst}\left(\int \frac{(a+x)^n}{b^2+x^2} dx, x, b \cot(c + dx)\right)}{d} \\ &= -\frac{b \operatorname{Subst}\left(\int \left(\frac{\sqrt{-b^2}(a+x)^n}{2b^2(\sqrt{-b^2}-x)} + \frac{\sqrt{-b^2}(a+x)^n}{2b^2(\sqrt{-b^2}+x)}\right) dx, x, b \cot(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x} dx, x, b \cot(c + dx)\right)}{2\sqrt{-b^2}d} + \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+x} dx, x, b \cot(c + dx)\right)}{2\sqrt{-b^2}d} \\ &= -\frac{b(a + b \cot(c + dx))^{1+n} {}_2F_1\left(1, 1 + n; 2 + n; \frac{a+b \cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2} (a - \sqrt{-b^2}) d(1 + n)} + \frac{b(a + b \cot(c + dx))^{1+n}}{2\sqrt{-b^2} (a + \sqrt{-b^2}) d(1 + n)} \end{aligned}$$

**Mathematica [C]** time = 0.30, size = 118, normalized size = 0.71

$$\frac{(a + b \cot(c + dx))^{n+1} \left( (a + ib) {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \cot(c+dx)}{a-ib}\right) - (a - ib) {}_2F_1\left(1, n + 1; n + 2; \frac{a+b \cot(c+dx)}{a+ib}\right) \right)}{2d(n + 1)(a - ib)(b - ia)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^n, x]

[Out] ((a + b\*Cot[c + d\*x])^(1 + n)\*((a + I\*b)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Cot[c + d\*x])/(a - I\*b)] - (a - I\*b)\*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b\*Cot[c + d\*x])/(a + I\*b)]))/(2\*(a - I\*b)\*((-I)\*a + b)\*d\*(1 + n))

**fricas [F]** time = 1.00, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((b \cot(dx + c) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*cot(d\*x + c) + a)^n, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^n,x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^n, x)

**maple** [F] time = 1.24, size = 0, normalized size = 0.00

$$\int (a + b \cot(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^n,x)

[Out] int((a+b\*cot(d\*x+c))^n,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*cot(d\*x + c) + a)^n, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cot(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cot(c + d\*x))^n,x)

[Out] int((a + b\*cot(c + d\*x))^n, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cot(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*n,x)

[Out] Integral((a + b\*cot(c + d\*x))\*\*n, x)

### 3.89 $\int (a + b \cot(e + fx))^m (d \tan(e + fx))^n dx$

**Optimal.** Leaf size=193

$$\frac{\cot(e + fx)(d \tan(e + fx))^n (a + b \cot(e + fx))^m \left(\frac{b \cot(e + fx)}{a} + 1\right)^{-m} F_1\left(1 - n; -m, 1; 2 - n; -\frac{b \cot(e + fx)}{a}, -i \cot(e + fx)\right)}{2f(1 - n)}$$

[Out]  $-1/2 * \text{AppellF1}(1 - n, 1, -m, 2 - n, -I * \cot(f * x + e), -b * \cot(f * x + e) / a) * \cot(f * x + e) * (a + b * \cot(f * x + e))^m * (d * \tan(f * x + e))^n / f / (1 - n) / ((1 + b * \cot(f * x + e) / a)^m) - 1/2 * \text{AppellF1}(1 - n, 1, -m, 2 - n, I * \cot(f * x + e), -b * \cot(f * x + e) / a) * \cot(f * x + e) * (a + b * \cot(f * x + e))^m * (d * \tan(f * x + e))^n / f / (1 - n) / ((1 + b * \cot(f * x + e) / a)^m)$

**Rubi [A]** time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {4242, 3575, 912, 135, 133}

$$\frac{\cot(e + fx)(d \tan(e + fx))^n (a + b \cot(e + fx))^m \left(\frac{b \cot(e + fx)}{a} + 1\right)^{-m} F_1\left(1 - n; -m, 1; 2 - n; -\frac{b \cot(e + fx)}{a}, -i \cot(e + fx)\right)}{2f(1 - n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b * \text{Cot}[e + f * x])^m * (d * \text{Tan}[e + f * x])^n, x]$

[Out]  $-(\text{AppellF1}[1 - n, -m, 1, 2 - n, -((b * \text{Cot}[e + f * x]) / a), (-I) * \text{Cot}[e + f * x]]) * \text{Cot}[e + f * x] * (a + b * \text{Cot}[e + f * x])^m * (d * \text{Tan}[e + f * x])^n / (2 * f * (1 - n) * (1 + (b * \text{Cot}[e + f * x]) / a)^m) - (\text{AppellF1}[1 - n, -m, 1, 2 - n, -((b * \text{Cot}[e + f * x]) / a), I * \text{Cot}[e + f * x]]) * \text{Cot}[e + f * x] * (a + b * \text{Cot}[e + f * x])^m * (d * \text{Tan}[e + f * x])^n / (2 * f * (1 - n) * (1 + (b * \text{Cot}[e + f * x]) / a)^m)$

#### Rule 133

$\text{Int}[(c_ + (d_ * x_))^{(n_)} * ((e_ + (f_ * x_))^{(p_)}), x_ \text{Symbol}]$  :>  $\text{Simp}[(c^{n_} * e^{p_} * (b * x)^{(m + 1)} * \text{AppellF1}[m + 1, -n, -p, m + 2, -((d * x) / c), -((f * x) / e)]) / (b * (m + 1)), x] /;$   $\text{FreeQ}\{b, c, d, e, f, m, n, p\}, x\} \& \& \text{IntegerQ}[m] \& \& \text{IntegerQ}[n] \& \& \text{GtQ}[c, 0] \& \& (\text{IntegerQ}[p] \mid \mid \text{GtQ}[e, 0])$

#### Rule 135

$\text{Int}[(c_ + (d_ * x_))^{(m_)} * ((e_ + (f_ * x_))^{(p_)}), x_ \text{Symbol}]$  :>  $\text{Dist}[(c^{\text{IntPart}[n]} * (c + d * x)^{\text{FracPart}[n]}) / (1 + (d * x) / c)^{\text{FracPart}[n]}, \text{Int}[(b * x)^m * (1 + (d * x) / c)^n * (e + f * x)^p, x], x] /;$   $\text{FreeQ}\{b, c, d, e, f, m, n, p\}, x\} \& \& \text{IntegerQ}[m] \& \& \text{IntegerQ}[n] \& \& \text{GtQ}[c, 0]$

#### Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (c_.)*(x_
)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

### Rule 3575

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[((a + b*ff*x)^m*(c + d*ff*x)^n]/(1 + ff^2*x^2), x], x,
Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

### Rule 4242

```
Int[(u_)*((c_.)*tan[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] := Dist[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m, Int[ActivateTrig[u]/(c*Cot[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownCotangentIntegrandQ[u,
x]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \cot(e + fx))^m (d \tan(e + fx))^n dx &= \left( (d \cot(e + fx))^n (d \tan(e + fx))^n \right) \int (d \cot(e + fx))^{-n} (a + b \cot(e + fx))^m dx \\
&= -\frac{\left( (d \cot(e + fx))^n (d \tan(e + fx))^n \right) \text{Subst} \left( \int \frac{(dx)^{-n} (a+bx)^m}{1+x^2} dx, x, \cot(e + fx) \right)}{f} \\
&= -\frac{\left( (d \cot(e + fx))^n (d \tan(e + fx))^n \right) \text{Subst} \left( \int \left( \frac{i(dx)^{-n} (a+bx)^m}{2(i-x)} + \frac{i(dx)^{-n} (a+bx)^m}{2(i+x)} \right) dx, x, \cot(e + fx) \right)}{f} \\
&= -\frac{\left( i(d \cot(e + fx))^n (d \tan(e + fx))^n \right) \text{Subst} \left( \int \frac{(dx)^{-n} (a+bx)^m}{i-x} dx, x, \cot(e + fx) \right)}{2f} \\
&= -\frac{\left( i(d \cot(e + fx))^n (a + b \cot(e + fx))^m \left( 1 + \frac{b \cot(e+fx)}{a} \right)^{-m} (d \tan(e + fx))^n \right)}{2f} \\
&= -\frac{F_1 \left( 1 - n; -m, 1; 2 - n; -\frac{b \cot(e+fx)}{a}, -i \cot(e + fx) \right) \cot(e + fx) (a + b \cot(e + fx))^m (d \tan(e + fx))^n}{2f(1 - n)}
\end{aligned}$$



**Mathematica** [F] time = 3.25, size = 0, normalized size = 0.00

$$\int (a + b \cot(e + fx))^m (d \tan(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Cot[e + f\*x])^m\*(d\*Tan[e + f\*x])^n,x]

[Out] Integrate[(a + b\*Cot[e + f\*x])^m\*(d\*Tan[e + f\*x])^n, x]

**fricas** [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \cot(fx + e) + a\right)^m \left(d \tan(fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(f\*x+e))^m\*(d\*tan(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((b\*cot(f\*x + e) + a)^m\*(d\*tan(f\*x + e))^n, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(fx + e) + a)^m (d \tan(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(f\*x+e))^m\*(d\*tan(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((b\*cot(f\*x + e) + a)^m\*(d\*tan(f\*x + e))^n, x)

**maple** [F] time = 2.42, size = 0, normalized size = 0.00

$$\int (a + b \cot(fx + e))^m (d \tan(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(f\*x+e))^m\*(d\*tan(f\*x+e))^n,x)

[Out] int((a+b\*cot(f\*x+e))^m\*(d\*tan(f\*x+e))^n,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(fx + e) + a)^m (d \tan(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(f\*x+e))^m\*(d\*tan(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((b\*cot(f\*x + e) + a)^m\*(d\*tan(f\*x + e))^n, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d \tan(e + fx))^n (a + b \cot(e + fx))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*tan(e + f\*x))^n\*(a + b\*cot(e + f\*x))^m,x)

[Out] int((d\*tan(e + f\*x))^n\*(a + b\*cot(e + f\*x))^m, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d \tan(e + fx))^n (a + b \cot(e + fx))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(f\*x+e))\*\*m\*(d\*tan(f\*x+e))\*\*n,x)

[Out] Integral((d\*tan(e + f\*x))\*\*n\*(a + b\*cot(e + f\*x))\*\*m, x)

$$3.90 \quad \int \frac{1+i \cot(c+dx)}{\sqrt{a+b \cot(c+dx)}} dx$$

**Optimal.** Leaf size=45

$$\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

[Out]  $2*I*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d/(a-I*b)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3537, 63, 208}

$$\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(1 + I*\operatorname{Cot}[c + d*x])/Sqrt[a + b*\operatorname{Cot}[c + d*x]], x]$

[Out]  $((2*I)*\operatorname{ArcTanh}[Sqrt[a + b*\operatorname{Cot}[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)$

### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_. + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n}, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

### Rule 3537

$\operatorname{Int}[(a_. + (b_.)*\tan[(e_. + (f_.)*(x_.))]^{(m_.)*((c_. + (d_.)*\tan[(e_. + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[(c*d)/f, \operatorname{Subst}[\operatorname{Int}[(a + (b*x)/d)^m/(d^2 + c*x), x], x, d*\operatorname{Tan}[e + f*x]], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 + b^2, 0] \&\& \operatorname{EqQ}[c^2 + d^2, 0]$

### Rubi steps

$$\int \frac{1 + i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx = -\frac{i \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}} dx, x, i \cot(c + dx)\right)}{d}$$

$$= \frac{2 \operatorname{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{bd}$$

$$= \frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d}$$

**Mathematica [A]** time = 0.08, size = 45, normalized size = 1.00

$$\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + I\*Cot[c + d\*x])/Sqrt[a + b\*Cot[c + d\*x]],x]

[Out] ((2\*I)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]]/(Sqrt[a - I\*b]\*d)

**fricas [B]** time = 0.71, size = 159, normalized size = 3.53

$$-\frac{1}{2} \sqrt{-\frac{4i}{(ia+b)d^2}} \log\left(\frac{1}{2}(ia+b)d \sqrt{-\frac{4i}{(ia+b)d^2}} + \sqrt{\frac{(a+ib)e^{(2i dx+2i c)} - a + ib}{e^{(2i dx+2i c)} - 1}}\right) + \frac{1}{2} \sqrt{-\frac{4i}{(ia+b)d^2}} \log\left(\frac{1}{2}(-i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/2\*sqrt(-4\*I/((I\*a + b)\*d^2))\*log(1/2\*(I\*a + b)\*d\*sqrt(-4\*I/((I\*a + b)\*d^2)) + sqrt(((a + I\*b)\*e^(2\*I\*d\*x + 2\*I\*c) - a + I\*b)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))) + 1/2\*sqrt(-4\*I/((I\*a + b)\*d^2))\*log(1/2\*(-I\*a - b)\*d\*sqrt(-4\*I/((I\*a + b)\*d^2)) + sqrt(((a + I\*b)\*e^(2\*I\*d\*x + 2\*I\*c) - a + I\*b)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{i \cot(dx + c) + 1}{\sqrt{b \cot(dx + c) + a}} dx$$



$$\frac{2*(a^2+b^2)^{(1/2)}+2*a^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)}}{(2*(a^2+b^2)^{(1/2)}-2*a^{(1/2)})}*a^2*b+1/d/(a^2+b^2)^{(1/2)}/((a^2+b^2)^{(1/2)}*a+a^2+b^2)/(2*(a^2+b^2)^{(1/2)}-2*a^{(1/2)})*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)}))/(2*(a^2+b^2)^{(1/2)}-2*a^{(1/2)})}*b^3$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{i \cot(dx + c) + 1}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((I\*cot(d\*x + c) + 1)/sqrt(b\*cot(d\*x + c) + a), x)

**mupad** [B] time = 2.54, size = 1410, normalized size = 31.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(c + d\*x)\*1i + 1)/(a + b\*cot(c + d\*x))^(1/2),x)

[Out] (log(d\*(-1/(d^2\*(a - b\*1i)))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2) + 1i)\*(-1/(a\*d^2 - b\*d^2\*1i))^(1/2))/2 - log(d\*(-1/(d^2\*(a - b\*1i)))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*1i + 1)\*(-1/(4\*(a\*d^2 - b\*d^2\*1i)))^(1/2) + (log(16\*b^3\*d\*(-1/(d^2\*(a - b\*1i)))^(1/2) - 16\*b^2\*(a + b\*cot(c + d\*x))^(1/2) + (16\*a\*b^2\*(a + b\*cot(c + d\*x))^(1/2))/(a - b\*1i))\*(-1/(a\*d^2 - b\*d^2\*1i))^(1/2))/2 - log(16\*b^2\*(a + b\*cot(c + d\*x))^(1/2) + 16\*b^3\*d\*(-1/(d^2\*(a - b\*1i)))^(1/2) - (16\*a\*b^2\*(a + b\*cot(c + d\*x))^(1/2))/(a - b\*1i))\*(-1/(4\*(a\*d^2 - b\*d^2\*1i)))^(1/2) - 2\*atanh((32\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))/((b^4\*d^2\*64i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (64\*a\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) + (a\*b^3\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2)\*128i)/((b^6\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (a^2\*b^4\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a^3\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a\*b^5\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) - (128\*a^2\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))/((b^6\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (a^2\*b^4\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a^3\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a\*b^5\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)))\*(-(a - b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2) - 2\*atanh((32\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))/((a^2\*b^2\*d^2\*64i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (b^2\*16i)/d + (64\*a\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) - (128\*a^2\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))

$$\left. \right)^{(1/2)} / \left( \frac{a^2 b^4 d^2 256 i}{4 a^2 d^3 + 4 b^2 d^3} - \frac{a^2 b^2 64 i}{d} - \frac{b^4 64 i}{d} + \frac{256 a^3 b^3 d^2}{4 a^2 d^3 + 4 b^2 d^3} + \frac{a^4 b^2 d^2 256 i}{4 a^2 d^3 + 4 b^2 d^3} + \frac{256 a b^5 d^2}{4 a^2 d^3 + 4 b^2 d^3} \right) + \frac{a b^3 (a + b \cot(c + d x))^{(1/2)} ((b i) / (4 a^2 d^2 + 4 b^2 d^2) - a / (4 a^2 d^2 + 4 b^2 d^2))^{(1/2)} 128 i}{\left( \frac{a^2 b^4 d^2 256 i}{4 a^2 d^3 + 4 b^2 d^3} - \frac{a^2 b^2 64 i}{d} - \frac{b^4 64 i}{d} + \frac{256 a^3 b^3 d^2}{4 a^2 d^3 + 4 b^2 d^3} + \frac{a^4 b^2 d^2 256 i}{4 a^2 d^3 + 4 b^2 d^3} + \frac{256 a b^5 d^2}{4 a^2 d^3 + 4 b^2 d^3} \right)} * \left( -\frac{a - b i}{4 a^2 d^2 + 4 b^2 d^2} \right)^{(1/2)}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$i \left( \int \left( -\frac{i}{\sqrt{a + b \cot(c + dx)}} \right) dx + \int \frac{\cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+I\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(1/2),x)

[Out] I\*(Integral(-I/sqrt(a + b\*cot(c + d\*x)), x) + Integral(cot(c + d\*x)/sqrt(a + b\*cot(c + d\*x)), x))

$$3.91 \quad \int \frac{1-i \cot(c+dx)}{\sqrt{a+b \cot(c+dx)}} dx$$

Optimal. Leaf size=45

$$-\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

[Out]  $-2*I*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}/d/(a+I*b)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {3537, 63, 208}

$$-\frac{2i \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In] `Int[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]], x]`

[Out] `((-2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)`

### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

### Rule 3537

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]`

### Rubi steps



$$\begin{aligned} \int \frac{1 - i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx &= \frac{i \operatorname{Subst} \left( \int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, -i \cot(c + dx) \right)}{d} \\ &= \frac{2 \operatorname{Subst} \left( \int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)} \right)}{bd} \\ &= -\frac{2i \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}} \right)}{\sqrt{a+ib} d} \end{aligned}$$

**Mathematica [A]** time = 1.72, size = 70, normalized size = 1.56

$$-\frac{2i \tanh^{-1} \left( \frac{\sqrt{a + \frac{ib(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}}{\sqrt{a+ib}} \right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - I\*Cot[c + d\*x])/Sqrt[a + b\*Cot[c + d\*x]], x]

[Out] ((-2\*I)\*ArcTanh[Sqrt[a + (I\*b\*(1 + E^((2\*I)\*(c + d\*x))))]/(-1 + E^((2\*I)\*(c + d\*x)))]/Sqrt[a + I\*b]]/(Sqrt[a + I\*b]\*d)

**fricas [B]** time = 0.59, size = 159, normalized size = 3.53

$$\frac{1}{2} \sqrt{\frac{4i}{(-ia+b)d^2}} \log \left( \frac{1}{2} (ia-b)d \sqrt{\frac{4i}{(-ia+b)d^2}} + \sqrt{\frac{(a+ib)e^{(2idx+2ic)} - a+ib}{e^{(2idx+2ic)} - 1}} \right) - \frac{1}{2} \sqrt{\frac{4i}{(-ia+b)d^2}} \log \left( \frac{1}{2} (-i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-I\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2), x, algorithm="fricas")

[Out] 1/2\*sqrt(4\*I/((-I\*a + b)\*d^2))\*log(1/2\*(I\*a - b)\*d\*sqrt(4\*I/((-I\*a + b)\*d^2)) + sqrt(((a + I\*b)\*e^(2\*I\*d\*x + 2\*I\*c) - a + I\*b)/(e^(2\*I\*d\*x + 2\*I\*c) - 1))) - 1/2\*sqrt(4\*I/((-I\*a + b)\*d^2))\*log(1/2\*(-I\*a + b)\*d\*sqrt(4\*I/((-I\*a + b)\*d^2)) + sqrt(((a + I\*b)\*e^(2\*I\*d\*x + 2\*I\*c) - a + I\*b)/(e^(2\*I\*d\*x + 2\*I\*c) - 1)))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-i \cot(dx + c) + 1}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

**maple [B]** time = 0.54, size = 1622, normalized size = 36.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x)
```

```
[Out] I/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*a*b^2+I/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/2/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*b-I/d/((a^2+b^2)^(1/2)*a+a^2+b^2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b^2+I/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*a^3-1/d/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*b+1/2*I/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*b^2-1/2*I/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*a+I/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*a^2-1/2*I/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/2/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*a*b+1/2/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*a^2*b+1/2/d/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*b^3-I/d/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+1/d/((a^2+b^2)^(1/2)*a+a^2+b^2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a*b+1/d/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((
```

$$2*(a^2+b^2)^{(1/2)+2*a}^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2))*a^2*b+1/d/(a^2+b^2)^{(1/2)/((a^2+b^2)^{(1/2)*a+a^2+b^2)/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)*\arctan(((2*(a^2+b^2)^{(1/2)+2*a}^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2))*b^3}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-i \cot(dx + c) + 1}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-I\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((-I\*cot(d\*x + c) + 1)/sqrt(b\*cot(d\*x + c) + a), x)

**mupad** [B] time = 1.40, size = 1410, normalized size = 31.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cot(c + d\*x)\*1i - 1)/(a + b\*cot(c + d\*x))^(1/2),x)

[Out] (log(d\*(-1/(d^2\*(a - b\*1i)))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*1i + 1)\*(-1/(a\*d^2 - b\*d^2\*1i))^(1/2))/2 - log(d\*(-1/(d^2\*(a - b\*1i)))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2) + 1i)\*(-1/(4\*(a\*d^2 - b\*d^2\*1i)))^(1/2) + (log(16\*b^3\*d\*(-1/(d^2\*(a - b\*1i)))^(1/2) - 16\*b^2\*(a + b\*cot(c + d\*x))^(1/2) + (16\*a\*b^2\*(a + b\*cot(c + d\*x))^(1/2))/(a - b\*1i))\*(-1/(a\*d^2 - b\*d^2\*1i)))^(1/2))/2 - log(16\*b^2\*(a + b\*cot(c + d\*x))^(1/2) + 16\*b^3\*d\*(-1/(d^2\*(a - b\*1i)))^(1/2) - (16\*a\*b^2\*(a + b\*cot(c + d\*x))^(1/2))/(a - b\*1i))\*(-1/(4\*(a\*d^2 - b\*d^2\*1i)))^(1/2) - 2\*atanh((32\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))/((b^4\*d^2\*64i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (64\*a\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) + (a\*b^3\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2)\*128i)/((b^6\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (a^2\*b^4\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a^3\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a\*b^5\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) - (128\*a^2\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))/((b^6\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) + (a^2\*b^4\*d^2\*256i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a^3\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (256\*a\*b^5\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)))\*(-(a - b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2) + 2\*atanh((32\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))/((a^2\*b^2\*d^2\*64i)/(4\*a^2\*d^3 + 4\*b^2\*d^3) - (b^2\*16i)/d + (64\*a\*b^3\*d^2)/(4\*a^2\*d^3 + 4\*b^2\*d^3)) - (128\*a^2\*b^2\*(a + b\*cot(c + d\*x))^(1/2)\*((b\*1i)/(4\*a^2\*d^2 + 4\*b^2\*d^2) - a/(4\*a^2\*d^2 + 4\*b^2\*d^2))^(1/2))

$$\left. \right)^{(1/2)} / \left( \frac{a^2 b^4 d^2 256i}{4a^2 d^3 + 4b^2 d^3} - \frac{a^2 b^2 64i}{d} - \frac{b^4 64i}{d} + \frac{256 a^3 b^3 d^2}{4a^2 d^3 + 4b^2 d^3} + \frac{a^4 b^2 d^2 256i}{4a^2 d^3 + 4b^2 d^3} + \frac{256 a b^5 d^2}{4a^2 d^3 + 4b^2 d^3} \right) + (a b^3 (a + b \cot(c + d x))^{(1/2)} \left( \frac{b i}{4a^2 d^2 + 4b^2 d^2} - \frac{a}{4a^2 d^2 + 4b^2 d^2} \right)^{(1/2)} * 128i) / \left( \frac{a^2 b^4 d^2 256i}{4a^2 d^3 + 4b^2 d^3} - \frac{a^2 b^2 64i}{d} - \frac{b^4 64i}{d} + \frac{256 a^3 b^3 d^2}{4a^2 d^3 + 4b^2 d^3} + \frac{a^4 b^2 d^2 256i}{4a^2 d^3 + 4b^2 d^3} + \frac{256 a b^5 d^2}{4a^2 d^3 + 4b^2 d^3} \right) * \left( -\frac{a - b i}{4a^2 d^2 + 4b^2 d^2} \right)^{(1/2)}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-i \left( \int \frac{i}{\sqrt{a + b \cot(c + dx)}} dx + \int \frac{\cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-I\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(1/2),x)

[Out] -I\*(Integral(I/sqrt(a + b\*cot(c + d\*x)), x) + Integral(cot(c + d\*x)/sqrt(a + b\*cot(c + d\*x)), x))

$$3.92 \quad \int \frac{A+B \cot(c+dx)}{a+b \cot(c+dx)} dx$$

Optimal. Leaf size=59

$$\frac{x(aA + bB)}{a^2 + b^2} - \frac{(Ab - aB) \log(a \sin(c + dx) + b \cos(c + dx))}{d(a^2 + b^2)}$$

[Out] (A\*a+B\*b)\*x/(a^2+b^2)-(A\*b-B\*a)\*ln(b\*cos(d\*x+c)+a\*sin(d\*x+c))/(a^2+b^2)/d

**Rubi [A]** time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {3531, 3530}

$$\frac{x(aA + bB)}{a^2 + b^2} - \frac{(Ab - aB) \log(a \sin(c + dx) + b \cos(c + dx))}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x]),x]

[Out] ((a\*A + b\*B)\*x)/(a^2 + b^2) - ((A\*b - a\*B)\*Log[b\*Cos[c + d\*x] + a\*Sin[c + d\*x]])/((a^2 + b^2)\*d)

#### Rule 3530

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(c\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]])/(b\*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

#### Rule 3531

Int[((c\_) + (d\_)\*tan[(e\_) + (f\_)\*(x\_)])/((a\_) + (b\_)\*tan[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[((a\*c + b\*d)\*x)/(a^2 + b^2), x] + Dist[(b\*c - a\*d)/(a^2 + b^2), Int[(b - a\*Tan[e + f\*x])/(a + b\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a\*c + b\*d, 0]

#### Rubi steps

$$\int \frac{A + B \cot(c + dx)}{a + b \cot(c + dx)} dx = \frac{(aA + bB)x}{a^2 + b^2} - \frac{(Ab - aB) \int \frac{-b+a \cot(c+dx)}{a+b \cot(c+dx)} dx}{a^2 + b^2}$$

$$= \frac{(aA + bB)x}{a^2 + b^2} - \frac{(Ab - aB) \log(b \cos(c + dx) + a \sin(c + dx))}{(a^2 + b^2)d}$$

**Mathematica** [A] time = 0.13, size = 67, normalized size = 1.14

$$\frac{2(aA + bB) \tan^{-1}(\cot(c + dx)) + (Ab - aB) (2 \log(a + b \cot(c + dx)) - \log(\csc^2(c + dx)))}{2d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x]),x]

[Out] -1/2\*(2\*(a\*A + b\*B)\*ArcTan[Cot[c + d\*x]] + (A\*b - a\*B)\*(2\*Log[a + b\*Cot[c + d\*x]] - Log[Csc[c + d\*x]^2]))/(a^2 + b^2)\*d

**fricas** [A] time = 0.55, size = 79, normalized size = 1.34

$$\frac{2(Aa + Bb)dx + (Ba - Ab) \log\left(ab \sin(2dx + 2c) + \frac{1}{2}a^2 + \frac{1}{2}b^2 - \frac{1}{2}(a^2 - b^2) \cos(2dx + 2c)\right)}{2(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(2\*(A\*a + B\*b)\*d\*x + (B\*a - A\*b)\*log(a\*b\*sin(2\*d\*x + 2\*c) + 1/2\*a^2 + 1/2\*b^2 - 1/2\*(a^2 - b^2)\*cos(2\*d\*x + 2\*c)))/((a^2 + b^2)\*d)

**giac** [A] time = 0.53, size = 95, normalized size = 1.61

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{(Ba-Ab) \log(\tan(dx+c)^2+1)}{a^2+b^2} + \frac{2(Ba^2-Aab) \log(|a \tan(dx+c)+b|)}{a^3+ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*(A\*a + B\*b)\*(d\*x + c)/(a^2 + b^2) - (B\*a - A\*b)\*log(tan(d\*x + c)^2 + 1)/(a^2 + b^2) + 2\*(B\*a^2 - A\*a\*b)\*log(abs(a\*tan(d\*x + c) + b))/(a^3 + a\*b^2))/d

**maple [B]** time = 0.42, size = 187, normalized size = 3.17

$$\frac{\ln(a + b \cot(dx + c)) Ab}{d(a^2 + b^2)} + \frac{\ln(a + b \cot(dx + c)) aB}{d(a^2 + b^2)} + \frac{\ln(\cot^2(dx + c) + 1) Ab}{2d(a^2 + b^2)} - \frac{\ln(\cot^2(dx + c) + 1) aB}{2d(a^2 + b^2)} - \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c)),x)

[Out]  $-1/d/(a^2+b^2)*\ln(a+b*\cot(d*x+c))*A*b+1/d/(a^2+b^2)*\ln(a+b*\cot(d*x+c))*a*B+1/2/d/(a^2+b^2)*\ln(\cot(d*x+c)^2+1)*A*b-1/2/d/(a^2+b^2)*\ln(\cot(d*x+c)^2+1)*a*B-1/2/d/(a^2+b^2)*A*Pi*a-1/2/d/(a^2+b^2)*B*Pi*b+1/d/(a^2+b^2)*A*\operatorname{arccot}(\cot(d*x+c))*a+1/d/(a^2+b^2)*B*\operatorname{arccot}(\cot(d*x+c))*b$

**maxima [A]** time = 1.69, size = 89, normalized size = 1.51

$$\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{2(Ba-Ab)\log(a\tan(dx+c)+b)}{a^2+b^2} - \frac{(Ba-Ab)\log(\tan(dx+c)^2+1)}{a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c)),x, algorithm="maxima")

[Out]  $1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a - A*b)*\log(a*\tan(d*x + c) + b)/(a^2 + b^2) - (B*a - A*b)*\log(\tan(d*x + c)^2 + 1)/(a^2 + b^2))/d$

**mupad [B]** time = 1.00, size = 155, normalized size = 2.63

$$\frac{A \ln(\cot(c + dx) + 1i)}{2(bd + ad1i)} - \frac{B \ln(\cot(c + dx) + 1i)}{2(ad - bd1i)} - \frac{Ab \ln(a + b \cot(c + dx))}{d(a^2 + b^2)} + \frac{Ba \ln(a + b \cot(c + dx))}{d(a^2 + b^2)} + \frac{A \ln(\cot(c + dx) + 1i)}{2(bd + ad1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*cot(c + d\*x))/(a + b\*cot(c + d\*x)),x)

[Out]  $(A*\log(\cot(c + d*x) - 1i)*1i)/(2*(a*d + b*d*1i)) + (A*\log(\cot(c + d*x) + 1i))/(2*(a*d*1i + b*d)) - (B*\log(\cot(c + d*x) + 1i))/(2*(a*d - b*d*1i)) - (B*\log(\cot(c + d*x) - 1i)*1i)/(2*(a*d*1i - b*d)) - (A*b*\log(a + b*\cot(c + d*x)))/(d*(a^2 + b^2)) + (B*a*\log(a + b*\cot(c + d*x)))/(d*(a^2 + b^2))$

**sympy [A]** time = 1.11, size = 534, normalized size = 9.05

$$\left\{ \begin{array}{l} \frac{\infty x(A+B \cot(c))}{\cot(c)} \\ \frac{A \log(\tan^2(c+dx)+1)}{2d} + Bx \\ \frac{b}{b} \\ -\frac{iA dx \cot(c+dx)}{-2bd \cot(c+dx)+2ibd} - \frac{A dx}{-2bd \cot(c+dx)+2ibd} + \frac{iA}{-2bd \cot(c+dx)+2ibd} - \frac{B dx \cot(c+dx)}{-2bd \cot(c+dx)+2ibd} + \frac{iB dx}{-2bd \cot(c+dx)+2ibd} - \frac{B}{-2bd \cot(c+dx)+2ibd} \\ \frac{iA dx \cot(c+dx)}{-2bd \cot(c+dx)-2ibd} - \frac{A dx}{-2bd \cot(c+dx)-2ibd} - \frac{iA}{-2bd \cot(c+dx)-2ibd} - \frac{B dx \cot(c+dx)}{-2bd \cot(c+dx)-2ibd} - \frac{iB dx}{-2bd \cot(c+dx)-2ibd} - \frac{B}{-2bd \cot(c+dx)-2ibd} \\ \frac{x(A+B \cot(c))}{a+b \cot(c)} \\ \frac{2A dx}{2a^2 d+2b^2 d} - \frac{2Ab \log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2 d+2b^2 d} + \frac{Ab \log(\tan^2(c+dx)+1)}{2a^2 d+2b^2 d} + \frac{2Ba \log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2 d+2b^2 d} - \frac{Ba \log(\tan^2(c+dx)+1)}{2a^2 d+2b^2 d} + \frac{2B dx}{2a^2 d+2b^2 d} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c)),x)

[Out] Piecewise((zoo\*x\*(A + B\*cot(c))/cot(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((A\*log(tan(c + d\*x)\*\*2 + 1)/(2\*d) + B\*x)/b, Eq(a, 0)), (-I\*A\*d\*x\*cot(c + d\*x)/(-2\*b\*d\*cot(c + d\*x) + 2\*I\*b\*d) - A\*d\*x/(-2\*b\*d\*cot(c + d\*x) + 2\*I\*b\*d) + I\*A/(-2\*b\*d\*cot(c + d\*x) + 2\*I\*b\*d) - B\*d\*x\*cot(c + d\*x)/(-2\*b\*d\*cot(c + d\*x) + 2\*I\*b\*d) + I\*B\*d\*x/(-2\*b\*d\*cot(c + d\*x) + 2\*I\*b\*d) - B/(-2\*b\*d\*cot(c + d\*x) + 2\*I\*b\*d), Eq(a, -I\*b)), (I\*A\*d\*x\*cot(c + d\*x)/(-2\*b\*d\*cot(c + d\*x) - 2\*I\*b\*d) - A\*d\*x/(-2\*b\*d\*cot(c + d\*x) - 2\*I\*b\*d) - I\*A/(-2\*b\*d\*cot(c + d\*x) - 2\*I\*b\*d) - B\*d\*x\*cot(c + d\*x)/(-2\*b\*d\*cot(c + d\*x) - 2\*I\*b\*d) - I\*B\*d\*x/(-2\*b\*d\*cot(c + d\*x) - 2\*I\*b\*d) - B/(-2\*b\*d\*cot(c + d\*x) - 2\*I\*b\*d), Eq(a, I\*b)), (x\*(A + B\*cot(c))/(a + b\*cot(c)), Eq(d, 0)), (2\*A\*a\*d\*x/(2\*a\*\*2\*d + 2\*b\*\*2\*d) - 2\*A\*b\*log(tan(c + d\*x) + b/a)/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + A\*b\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + 2\*B\*a\*log(tan(c + d\*x) + b/a)/(2\*a\*\*2\*d + 2\*b\*\*2\*d) - B\*a\*log(tan(c + d\*x)\*\*2 + 1)/(2\*a\*\*2\*d + 2\*b\*\*2\*d) + 2\*B\*b\*d\*x/(2\*a\*\*2\*d + 2\*b\*\*2\*d), True))



### 3.93 $\int \frac{A+B \cot(c+dx)}{(a+b \cot(c+dx))^2} dx$

**Optimal.** Leaf size=111

$$\frac{Ab - aB}{d(a^2 + b^2)(a + b \cot(c + dx))} - \frac{(a^2(-B) + 2aAb + b^2B) \log(a \sin(c + dx) + b \cos(c + dx))}{d(a^2 + b^2)^2} + \frac{x(a^2A + 2abB - Ab)}{(a^2 + b^2)^2}$$

[Out]  $(A*a^2 - A*b^2 + 2*B*a*b)*x/(a^2 + b^2)^2 + (A*b - B*a)/(a^2 + b^2)/d/(a + b*\cot(d*x + c)) - (2*A*a*b - B*a^2 + B*b^2)*\ln(b*\cos(d*x + c) + a*\sin(d*x + c))/(a^2 + b^2)^2/d$

**Rubi [A]** time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3529, 3531, 3530}

$$\frac{Ab - aB}{d(a^2 + b^2)(a + b \cot(c + dx))} - \frac{(a^2(-B) + 2aAb + b^2B) \log(a \sin(c + dx) + b \cos(c + dx))}{d(a^2 + b^2)^2} + \frac{x(a^2A + 2abB - Ab)}{(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^2, x]

[Out]  $((a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2)^2 + (A*b - a*B)/((a^2 + b^2)*d*(a + b*\cot[c + d*x])) - ((2*a*A*b - a^2*B + b^2*B)*\text{Log}[b*\text{Cos}[c + d*x] + a*\text{Sin}[c + d*x]])/((a^2 + b^2)^2*d)$

**Rule 3529**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3530**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(c\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]])/(b\*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

**Rule 3531**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[((a\*c + b\*d)\*x)/(a^2 + b^2), x] + Dist[(b\*c - a

$\ast d)/(a^2 + b^2)$ ,  $\text{Int}[(b - a \cdot \text{Tan}[e + f \cdot x])/(a + b \cdot \text{Tan}[e + f \cdot x]), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, x\}$  &&  $\text{NeQ}[b \cdot c - a \cdot d, 0]$  &&  $\text{NeQ}[a^2 + b^2, 0]$  &&  $\text{NeQ}[a \cdot c + b \cdot d, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{A + B \cot(c + dx)}{(a + b \cot(c + dx))^2} dx &= \frac{Ab - aB}{(a^2 + b^2) d(a + b \cot(c + dx))} + \frac{\int \frac{aA + bB - (Ab - aB) \cot(c + dx)}{a + b \cot(c + dx)} dx}{a^2 + b^2} \\ &= \frac{(a^2 A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} + \frac{Ab - aB}{(a^2 + b^2) d(a + b \cot(c + dx))} - \frac{(2aAb - a^2 B + b^2 B) \int \frac{-b + a \cot(c + dx)}{a + b \cot(c + dx)} dx}{(a^2 + b^2)^2} \\ &= \frac{(a^2 A - Ab^2 + 2abB)x}{(a^2 + b^2)^2} + \frac{Ab - aB}{(a^2 + b^2) d(a + b \cot(c + dx))} - \frac{(2aAb - a^2 B + b^2 B) \log(b + a \cot(c + dx))}{(a^2 + b^2)^2} \end{aligned}$$

**Mathematica [C]** time = 1.93, size = 144, normalized size = 1.30

$$\frac{\frac{2b(aB - Ab)}{a(a^2 + b^2)(a \tan(c + dx) + b)} + \frac{2(a^2 B - 2aAb - b^2 B) \log(a \tan(c + dx) + b)}{(a^2 + b^2)^2} - \frac{(B + iA) \log(-\tan(c + dx) + i)}{(a - ib)^2} + \frac{i(A + iB) \log(\tan(c + dx) + i)}{(a + ib)^2}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^2, x]

[Out]  $(-(((I \cdot A + B) \cdot \text{Log}[I - \text{Tan}[c + d \cdot x]])/(a - I \cdot b)^2) + (I \cdot (A + I \cdot B) \cdot \text{Log}[I + \text{Tan}[c + d \cdot x]])/(a + I \cdot b)^2 + (2 \cdot (-2 \cdot a \cdot A \cdot b + a^2 \cdot B - b^2 \cdot B) \cdot \text{Log}[b + a \cdot \text{Tan}[c + d \cdot x]])/(a^2 + b^2)^2 + (2 \cdot b \cdot (-A \cdot b) + a \cdot B)/(a \cdot (a^2 + b^2) \cdot (b + a \cdot \text{Tan}[c + d \cdot x])))/(2 \cdot d)$

**fricas [B]** time = 0.56, size = 340, normalized size = 3.06

$$\frac{2Ba^2b - 2Aab^2 + 2(Aa^2b + 2Bab^2 - Ab^3)dx + 2(Ba^2b - Aab^2 + (Aa^2b + 2Bab^2 - Ab^3)dx) \cos(2dx + 2c) + \dots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^2, x, algorithm="fricas")

[Out]  $1/2 \cdot (2 \cdot B \cdot a^2 \cdot b - 2 \cdot A \cdot a \cdot b^2 + 2 \cdot (A \cdot a^2 \cdot b + 2 \cdot B \cdot a \cdot b^2 - A \cdot b^3) \cdot dx + 2 \cdot (B \cdot a^2 \cdot b - A \cdot a \cdot b^2 + (A \cdot a^2 \cdot b + 2 \cdot B \cdot a \cdot b^2 - A \cdot b^3) \cdot dx) \cdot \cos(2 \cdot d \cdot x + 2 \cdot c) + (B \cdot a^2 \cdot b - A \cdot a \cdot b^2 + (A \cdot a^2 \cdot b + 2 \cdot B \cdot a \cdot b^2 - A \cdot b^3) \cdot dx) \cdot \sin(2 \cdot d \cdot x + 2 \cdot c)) / (2 \cdot d)$

$$*b - 2*A*a*b^2 - B*b^3 + (B*a^2*b - 2*A*a*b^2 - B*b^3)*\cos(2*d*x + 2*c) + (B*a^3 - 2*A*a^2*b - B*a*b^2)*\sin(2*d*x + 2*c))*\log(a*b*\sin(2*d*x + 2*c) + 1/2*a^2 + 1/2*b^2 - 1/2*(a^2 - b^2)*\cos(2*d*x + 2*c)) - 2*(B*a*b^2 - A*b^3 - (A*a^3 + 2*B*a^2*b - A*a*b^2)*d*x)*\sin(2*d*x + 2*c))/((a^4*b + 2*a^2*b^3 + b^5)*d*\cos(2*d*x + 2*c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*\sin(2*d*x + 2*c) + (a^4*b + 2*a^2*b^3 + b^5)*d)$$

**giac [B]** time = 0.49, size = 241, normalized size = 2.17

$$\frac{2(Aa^2+2Bab-Ab^2)(dx+c)}{a^4+2a^2b^2+b^4} - \frac{(Ba^2-2Aab-Bb^2)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{2(Ba^3-2Aa^2b-Bab^2)\log(|a\tan(dx+c)+b|)}{a^5+2a^3b^2+ab^4} - \frac{2(Ba^4\tan(dx+c)-2Aa^3b\tan(dx+c)+Aa^2b^2)}{(a^5+2a^3b^2+ab^4)^2} \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^2,x, algorithm="giac")

[Out] 1/2\*(2\*(A\*a^2 + 2\*B\*a\*b - A\*b^2)\*(d\*x + c)/(a^4 + 2\*a^2\*b^2 + b^4) - (B\*a^2 - 2\*A\*a\*b - B\*b^2)\*log(tan(d\*x + c)^2 + 1)/(a^4 + 2\*a^2\*b^2 + b^4) + 2\*(B\*a^3 - 2\*A\*a^2\*b - B\*a\*b^2)\*log(abs(a\*tan(d\*x + c) + b))/(a^5 + 2\*a^3\*b^2 + a\*b^4) - 2\*(B\*a^4\*tan(d\*x + c) - 2\*A\*a^3\*b\*tan(d\*x + c) - B\*a^2\*b^2\*tan(d\*x + c) - A\*a^2\*b^2 - 2\*B\*a\*b^3 + A\*b^4)/((a^5 + 2\*a^3\*b^2 + a\*b^4)\*(a\*tan(d\*x + c) + b)))/d

**maple [B]** time = 0.37, size = 356, normalized size = 3.21

$$\frac{Ab}{d(a^2+b^2)(a+b\cot(dx+c))} - \frac{aB}{d(a^2+b^2)(a+b\cot(dx+c))} - \frac{2\ln(a+b\cot(dx+c))Aab}{d(a^2+b^2)^2} + \frac{\ln(a+b\cot(dx+c))}{d(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^2,x)

[Out] 1/d/(a^2+b^2)/(a+b\*cot(d\*x+c))\*A\*b-1/d/(a^2+b^2)/(a+b\*cot(d\*x+c))\*A\*B-2/d/(a^2+b^2)^2\*ln(a+b\*cot(d\*x+c))\*A\*a\*b+1/d/(a^2+b^2)^2\*ln(a+b\*cot(d\*x+c))\*a^2\*B-1/d/(a^2+b^2)^2\*ln(a+b\*cot(d\*x+c))\*b^2\*B+1/d/(a^2+b^2)^2\*ln(cot(d\*x+c)^2+1)\*A\*a\*b-1/2/d/(a^2+b^2)^2\*ln(cot(d\*x+c)^2+1)\*a^2\*B+1/2/d/(a^2+b^2)^2\*ln(cot(d\*x+c)^2+1)\*b^2\*B-1/2/d/(a^2+b^2)^2\*A\*Pi\*a^2+1/2/d/(a^2+b^2)^2\*A\*Pi\*b^2-1/d/(a^2+b^2)^2\*B\*Pi\*a\*b+1/d/(a^2+b^2)^2\*A\*arccot(cot(d\*x+c))\*a^2-1/d/(a^2+b^2)^2\*A\*arccot(cot(d\*x+c))\*b^2+2/d/(a^2+b^2)^2\*B\*arccot(cot(d\*x+c))\*a\*b

**maxima [A]** time = 0.53, size = 185, normalized size = 1.67

$$\frac{2(Aa^2+2Bab-Ab^2)(dx+c)}{a^4+2a^2b^2+b^4} + \frac{2(Ba^2-2Aab-Bb^2)\log(a\tan(dx+c)+b)}{a^4+2a^2b^2+b^4} - \frac{(Ba^2-2Aab-Bb^2)\log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{2(Bab-Ab^2)}{a^3b+ab^3+(a^4+a^2b^2)\tan(dx+c)} \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2} * (2 * (A * a^2 + 2 * B * a * b - A * b^2) * (d * x + c) / (a^4 + 2 * a^2 * b^2 + b^4) + 2 * (B * a^2 - 2 * A * a * b - B * b^2) * \log(a * \tan(d * x + c) + b) / (a^4 + 2 * a^2 * b^2 + b^4) - (B * a^2 - 2 * A * a * b - B * b^2) * \log(\tan(d * x + c)^2 + 1) / (a^4 + 2 * a^2 * b^2 + b^4) + 2 * (B * a * b - A * b^2) / (a^3 * b + a * b^3 + (a^4 + a^2 * b^2) * \tan(d * x + c))) / d$

**mupad [B]** time = 1.46, size = 268, normalized size = 2.41

$$\ln(a + b \cot(c + dx)) \left( \frac{B}{d(a^2 + b^2)} - \frac{2Bb^2}{d(a^2 + b^2)^2} \right) + \frac{A \ln(\cot(c + dx) - i)}{2(-1id a^2 + 2dab + 1id b^2)} - \frac{B \ln(\cot(c + dx) - i)}{2(d a^2 + 2idab - db^2)} + \frac{A \ln(\cot(c + dx) + i)}{2(-1id a^2 + 2dab + 1id b^2)} - \frac{B \ln(\cot(c + dx) + i)}{2(d a^2 + 2idab - db^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*cot(c + d\*x))/(a + b\*cot(c + d\*x))^2,x)

[Out]  $\log(a + b * \cot(c + d * x)) * (B / (d * (a^2 + b^2)) - (2 * B * b^2) / (d * (a^2 + b^2)^2)) + (A * \log(\cot(c + d * x) + 1i) * 1i) / (2 * (b^2 * d - a^2 * d + a * b * d * 2i)) + (A * \log(\cot(c + d * x) - 1i)) / (2 * (b^2 * d * 1i - a^2 * d * 1i + 2 * a * b * d)) - (B * \log(\cot(c + d * x) - 1i)) / (2 * (a^2 * d - b^2 * d + a * b * d * 2i)) - (B * \log(\cot(c + d * x) + 1i) * 1i) / (2 * (a^2 * d * 1i - b^2 * d * 1i + 2 * a * b * d)) + (A * b) / ((a * d + b * d * \cot(c + d * x)) * (a^2 + b^2)) - (B * a) / ((a * d + b * d * \cot(c + d * x)) * (a^2 + b^2)) - (2 * A * a * b * \log(a + b * \cot(c + d * x))) / (d * (a^2 + b^2)^2)$

**sympy [A]** time = 4.23, size = 3966, normalized size = 35.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*2,x)

[Out]  $\text{Piecewise}((\text{zoo} * x * (A + B * \cot(c)) / \cot(c) ** 2, \text{Eq}(a, 0) \ \& \ \text{Eq}(b, 0) \ \& \ \text{Eq}(d, 0)), ((-A * x + A * \tan(c + d * x) / d + B * \log(\tan(c + d * x) ** 2 + 1) / (2 * d)) / b ** 2, \text{Eq}(a, 0)), (A * d * x * \cot(c + d * x) ** 2 / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) - 2 * I * A * d * x * \cot(c + d * x) / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) - A * d * x / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) - A * \cot(c + d * x) / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) + 2 * I * A / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) - I * B * d * x * \cot(c + d * x) ** 2 / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) - 2 * B * d * x * \cot(c + d * x) / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) + I * B * d * x / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d) + I * B * \cot(c + d * x) / (-4 * b ** 2 * d * \cot(c + d * x) ** 2 + 8 * I * b ** 2 * d * \cot(c + d * x) + 4 * b ** 2 * d),$

Eq(a, -I\*b)), (A\*d\*x\*cot(c + d\*x)\*\*2/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) + 2\*I\*A\*d\*x\*cot(c + d\*x)/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) - A\*d\*x/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) - A\*cot(c + d\*x)/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) - 2\*I\*A/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) + I\*B\*d\*x\*cot(c + d\*x)\*\*2/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) - 2\*B\*d\*x\*cot(c + d\*x)/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) - I\*B\*d\*x/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d) - I\*B\*cot(c + d\*x)/(-4\*b\*\*2\*d\*cot(c + d\*x)\*\*2 - 8\*I\*b\*\*2\*d\*cot(c + d\*x) + 4\*b\*\*2\*d), Eq(a, I\*b)), (A\*d\*x\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 3\*A\*d\*x\*tan(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 4\*A\*d\*x\*tan(c + d\*x)\*cot(c + d\*x)/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 3\*A\*d\*x\*cot(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + A\*d\*x/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + A\*tan(c + d\*x)\*cot(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) - 5\*A\*tan(c + d\*x)/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) - 4\*A\*cot(c + d\*x)/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 2\*B\*d\*x\*tan(c + d\*x)\*\*2\*cot(c + d\*x)/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 2\*B\*d\*x\*tan(c + d\*x)\*cot(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) - 2\*B\*d\*x\*tan(c + d\*x)/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) - 2\*B\*d\*x\*cot(c + d\*x)/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 3\*B\*tan(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + B\*cot(c + d\*x)\*\*2/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d) + 2\*B/(8\*b\*\*2\*d\*tan(c + d\*x)\*\*2\*cot(c + d\*x)\*\*2 - 16\*b\*\*2\*d\*tan(c + d\*x)\*cot(c + d\*x) + 8\*b\*\*2\*d), Eq(a, -b/tan(c + d\*x))), (x\*(A + B\*cot(c))/(a + b\*cot(c))\*\*2, Eq(d, 0)), (2\*A\*a\*\*4\*d\*x\*tan(c + d\*x)/(2\*a\*\*6\*d\*tan(c + d\*x) + 2\*a\*\*5\*b\*d + 4\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 2\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d) + 2\*A\*a\*\*3\*b\*d\*x/(2\*a\*\*6\*d\*tan(c + d\*x) + 2\*a\*\*5\*b\*d + 4\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 2\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d) - 4\*A\*a\*\*3\*b\*log(tan(c + d\*x) + b/a)\*tan(c + d\*x)/(2\*a\*\*6\*d\*tan(c + d\*x) + 2\*a\*\*5\*b\*d + 4\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 2\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d) + 2\*A\*a\*\*3\*b\*log(tan(c + d\*x)\*\*2 + 1)\*tan(c + d\*x)/(2\*a\*\*6\*d\*tan(c + d\*x) + 2\*a\*\*5\*b\*d + 4\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 2\*a\*\*2\*b\*\*4\*d\*tan(c + d\*x) + 2\*a\*b\*\*5\*d) - 2\*A\*a\*\*2\*b\*\*2\*d\*x\*tan(c + d\*x)/(2\*a\*\*6\*d\*tan(c + d\*x) + 2\*a\*\*5\*b\*d + 4\*a\*\*4\*b\*\*2\*d\*tan(c + d\*x) + 4\*a\*\*3\*b\*\*3\*d + 2\*a\*\*2\*b\*\*4\*d

```

tan(c + d*x) + 2*a*b**5*d) - 4*A*a**2*b**2*log(tan(c + d*x) + b/a)/(2*a**6*
d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d +
2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*A*a**2*b**2*log(tan(c + d*x)**
2 + 1)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4
*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*a**2*b**2/(2*
a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3
*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*a*b**3*d*x/(2*a**6*d*ta
n(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a
**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*A*b**4/(2*a**6*d*tan(c + d*x) + 2*
a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c
+ d*x) + 2*a*b**5*d) + 2*B*a**4*log(tan(c + d*x) + b/a)*tan(c + d*x)/(2*a*
**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d
+ 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - B*a**4*log(tan(c + d*x)**2 +
1)*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c +
d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 4*B*a**3
*b*d*x*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan
(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*B*
a**3*b*log(tan(c + d*x) + b/a)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4
*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**
5*d) - B*a**3*b*log(tan(c + d*x)**2 + 1)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*
d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x)
+ 2*a*b**5*d) + 2*B*a**3*b/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b*
**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d
) + 4*B*a**2*b**2*d*x/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*t
an(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*
B*a**2*b**2*log(tan(c + d*x) + b/a)*tan(c + d*x)/(2*a**6*d*tan(c + d*x) + 2
*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(
c + d*x) + 2*a*b**5*d) + B*a**2*b**2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/
(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) + 4*a**3*b
**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) - 2*B*a*b**3*log(tan(c + d
*x) + b/a)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x)
+ 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + B*a*b**3*log(
tan(c + d*x)**2 + 1)/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*ta
n(c + d*x) + 4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d) + 2*B
*a*b**3/(2*a**6*d*tan(c + d*x) + 2*a**5*b*d + 4*a**4*b**2*d*tan(c + d*x) +
4*a**3*b**3*d + 2*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d), True))

```

$$3.94 \quad \int \frac{A+B \cot(c+dx)}{(a+b \cot(c+dx))^3} dx$$

**Optimal.** Leaf size=175

$$\frac{Ab - aB}{2d(a^2 + b^2)(a + b \cot(c + dx))^2} + \frac{a^2(-B) + 2aAb + b^2B}{d(a^2 + b^2)^2(a + b \cot(c + dx))} - \frac{(a^3(-B) + 3a^2Ab + 3ab^2B - Ab^3) \log(a \sin(c + dx))}{d(a^2 + b^2)^3}$$

[Out] (A\*a^3-3\*A\*a\*b^2+3\*B\*a^2\*b-B\*b^3)\*x/(a^2+b^2)^3+1/2\*(A\*b-B\*a)/(a^2+b^2)/d/(a+b\*cot(d\*x+c))^2+(2\*A\*a\*b-B\*a^2+B\*b^2)/(a^2+b^2)^2/d/(a+b\*cot(d\*x+c))-(3\*A\*a^2\*b-A\*b^3-B\*a^3+3\*B\*a\*b^2)\*ln(b\*cos(d\*x+c)+a\*sin(d\*x+c))/(a^2+b^2)^3/d

**Rubi [A]** time = 0.28, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {3529, 3531, 3530}

$$\frac{Ab - aB}{2d(a^2 + b^2)(a + b \cot(c + dx))^2} + \frac{a^2(-B) + 2aAb + b^2B}{d(a^2 + b^2)^2(a + b \cot(c + dx))} - \frac{(3a^2Ab + a^3(-B) + 3ab^2B - Ab^3) \log(a \sin(c + dx))}{d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^3,x]

[Out] ((a^3\*A - 3\*a\*A\*b^2 + 3\*a^2\*b\*B - b^3\*B)\*x)/(a^2 + b^2)^3 + (A\*b - a\*B)/(2\*(a^2 + b^2)\*d\*(a + b\*Cot[c + d\*x])^2) + (2\*a\*A\*b - a^2\*B + b^2\*B)/((a^2 + b^2)^2\*d\*(a + b\*Cot[c + d\*x])) - ((3\*a^2\*A\*b - A\*b^3 - a^3\*B + 3\*a\*b^2\*B)\*Log[b\*Cos[c + d\*x] + a\*Sin[c + d\*x]])/(a^2 + b^2)^3\*d

**Rule 3529**

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))/(f\*(m + 1)\*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b\*Tan[e + f\*x])^(m + 1)\*Simp[a\*c + b\*d - (b\*c - a\*d)\*Tan[e + f\*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

**Rule 3530**

Int[((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(c\*Log[RemoveContent[a\*Cos[e + f\*x] + b\*Sin[e + f\*x], x]])/(b\*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a\*c + b\*d, 0]

**Rule 3531**

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \cot(c + dx)}{(a + b \cot(c + dx))^3} dx &= \frac{Ab - aB}{2(a^2 + b^2) d(a + b \cot(c + dx))^2} + \frac{\int \frac{aA + bB - (Ab - aB) \cot(c + dx)}{(a + b \cot(c + dx))^2} dx}{a^2 + b^2} \\ &= \frac{Ab - aB}{2(a^2 + b^2) d(a + b \cot(c + dx))^2} + \frac{2aAb - a^2B + b^2B}{(a^2 + b^2)^2 d(a + b \cot(c + dx))} + \frac{\int \frac{a^2A - Ab^2 + 2abB - b^3B}{(a + b \cot(c + dx))^2} dx}{(a^2 + b^2)^2} \\ &= \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} + \frac{Ab - aB}{2(a^2 + b^2) d(a + b \cot(c + dx))^2} + \frac{2aAb - a^2B + b^2B}{(a^2 + b^2)^2 d(a + b \cot(c + dx))} \\ &= \frac{(a^3A - 3aAb^2 + 3a^2bB - b^3B)x}{(a^2 + b^2)^3} + \frac{Ab - aB}{2(a^2 + b^2) d(a + b \cot(c + dx))^2} + \frac{2aAb - a^2B + b^2B}{(a^2 + b^2)^2 d(a + b \cot(c + dx))} \end{aligned}$$

**Mathematica [C]** time = 5.00, size = 202, normalized size = 1.15

$$\frac{2(a^3B - 3a^2Ab - 3ab^2B + Ab^3) \log(a \tan(c + dx) + b) - \frac{b(a^2 + b^2)((-4a^4B + 6a^3Ab + 2aAb^3) \tan(c + dx) + b(-3a^3B + 5a^2Ab + ab^2B + Ab^3))}{a^2(a \tan(c + dx) + b)^2}}{(a^2 + b^2)^3} - \frac{i(A - iB) \log(-\tan(c + dx) + i)}{(a - ib)^3} + \frac{2d}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3, x]
```

```
[Out] (((-I)*(A - I*B)*Log[I - Tan[c + d*x]])/(a - I*b)^3 + (I*(A + I*B)*Log[I +
Tan[c + d*x]])/(a + I*b)^3 + (2*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*Lo
g[b + a*Tan[c + d*x]] - (b*(a^2 + b^2)*(b*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*
b^2*B) + (6*a^3*A*b + 2*a*A*b^3 - 4*a^4*B)*Tan[c + d*x]))/(a^2*(b + a*Tan[c
+ d*x])^2))/(a^2 + b^2)^3)/(2*d)
```

**fricas [B]** time = 0.71, size = 549, normalized size = 3.14

$$2Ba^3b^2 - 2Aa^2b^3 + 2Bab^4 - 2Ab^5 - 2(Aa^5 + 3Ba^4b - 2Aa^3b^2 + 2Ba^2b^3 - 3Aab^4 - Bb^5)dx - 2(4Ba^3b^2 - 6Aa^2b^3 + 6Bab^4 - 4Ab^5)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(2*B*a^3*b^2 - 2*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5 - 2*(A*a^5 + 3*B*a^4*b - 2*A*a^3*b^2 + 2*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*d*x - 2*(4*B*a^3*b^2 - 6*A*a^2*b^3 - 2*B*a*b^4 - (A*a^5 + 3*B*a^4*b - 4*A*a^3*b^2 - 4*B*a^2*b^3 + 3*A*a*b^4 + B*b^5)*d*x)*\cos(2*d*x + 2*c) - (B*a^5 - 3*A*a^4*b - 2*B*a^3*b^2 - 2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - (B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 + 4*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*\cos(2*d*x + 2*c) + 2*(B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*\sin(2*d*x + 2*c))*\log(a*b*\sin(2*d*x + 2*c) + 1/2*a^2 + 1/2*b^2 - 1/2*(a^2 - b^2)*\cos(2*d*x + 2*c)) - 2*(2*B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + 3*A*a*b^4 + B*b^5 + 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4)*d*x)*\sin(2*d*x + 2*c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*\cos(2*d*x + 2*c) - 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*\sin(2*d*x + 2*c) - (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d)$

**giac** [B] time = 0.49, size = 412, normalized size = 2.35

$$\frac{2(Aa^3+3Ba^2b-3Aab^2-Bb^3)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{(Ba^3-3Aa^2b-3Bab^2+Ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{2(Ba^4-3Aa^3b-3Ba^2b^2+Aab^3)\log(|a\tan(dx+c)+b|)}{a^7+3a^5b^2+3a^3b^4+ab^6} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^3,x, algorithm="giac")

[Out]  $\frac{1}{2}*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*\log(\tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^4 - 3*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*\log(\text{abs}(a*\tan(d*x + c) + b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6) - (3*B*a^7*\tan(d*x + c)^2 - 9*A*a^6*b*\tan(d*x + c)^2 - 9*B*a^5*b^2*\tan(d*x + c)^2 + 3*A*a^4*b^3*\tan(d*x + c)^2 + 2*B*a^6*b*\tan(d*x + c) - 12*A*a^5*b^2*\tan(d*x + c) - 22*B*a^4*b^3*\tan(d*x + c) + 14*A*a^3*b^4*\tan(d*x + c) + 2*A*a*b^6*\tan(d*x + c) - 4*A*a^4*b^3 - 11*B*a^3*b^4 + 9*A*a^2*b^5 + B*a*b^6 + A*b^7)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*\tan(d*x + c) + b)^2))/d$

**maple** [B] time = 0.38, size = 559, normalized size = 3.19

$$\frac{Ab}{2d(a^2+b^2)(a+b\cot(dx+c))^2} - \frac{aB}{2d(a^2+b^2)(a+b\cot(dx+c))^2} + \frac{2Aab}{d(a^2+b^2)(a+b\cot(dx+c))} - \frac{Bb^3}{d(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^3,x)

[Out]  $\frac{1}{2} \frac{d}{d} \frac{1}{(a^2+b^2)} \frac{1}{(a+b \cot(dx+c))^2} A^* b - \frac{1}{2} \frac{d}{d} \frac{1}{(a^2+b^2)} \frac{1}{(a+b \cot(dx+c))^2} a^* B + \frac{2}{d} \frac{1}{(a^2+b^2)^2} \frac{1}{(a+b \cot(dx+c))} A^* a^* b - \frac{1}{d} \frac{1}{(a^2+b^2)^2} \frac{1}{(a+b \cot(dx+c))} a^2 B + \frac{1}{d} \frac{1}{(a^2+b^2)^2} \frac{1}{(a+b \cot(dx+c))} b^2 B - \frac{3}{d} \frac{1}{(a^2+b^2)^3} \ln(a+b \cot(dx+c)) A^* a^2 b + \frac{1}{d} \frac{1}{(a^2+b^2)^3} \ln(a+b \cot(dx+c)) A^* b^3 + \frac{1}{d} \frac{1}{(a^2+b^2)^3} \ln(a+b \cot(dx+c)) a^3 B - \frac{3}{d} \frac{1}{(a^2+b^2)^3} \ln(a+b \cot(dx+c)) B^* a^* b^2 + \frac{3}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} \ln(\cot(dx+c)^2+1) A^* a^2 b - \frac{1}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} \ln(\cot(dx+c)^2+1) A^* b^3 - \frac{1}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} \ln(\cot(dx+c)^2+1) a^3 B + \frac{3}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} \ln(\cot(dx+c)^2+1) B^* a^* b^2 - \frac{1}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} A^* \pi a^3 + \frac{3}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} A^* \pi a^* b^2 - \frac{3}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} B^* \pi a^2 b + \frac{1}{2} \frac{1}{d} \frac{1}{(a^2+b^2)^3} B^* \pi b^3 + \frac{1}{d} \frac{1}{(a^2+b^2)^3} A^* \operatorname{arccot}(\cot(dx+c)) a^3 - \frac{3}{d} \frac{1}{(a^2+b^2)^3} A^* \operatorname{arccot}(\cot(dx+c)) a^* b^2 + \frac{3}{d} \frac{1}{(a^2+b^2)^3} B^* \operatorname{arccot}(\cot(dx+c)) a^2 b - \frac{1}{d} \frac{1}{(a^2+b^2)^3} B^* \operatorname{arccot}(\cot(dx+c)) b^3$

**maxima [A]** time = 0.87, size = 337, normalized size = 1.93

$$\frac{2(Aa^3+3Ba^2b-3Aab^2-Bb^3)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{2(Ba^3-3Aa^2b-3Bab^2+Ab^3)\log(a\tan(dx+c)+b)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{(Ba^3-3Aa^2b-3Bab^2+Ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{3Ba^3}{a^6b^2}$$


---


$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cot(dx+c))/(a+b*cot(dx+c))^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (2 * (A * a^3 + 3 * B * a^2 * b - 3 * A * a * b^2 - B * b^3) * (d * x + c) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) + 2 * (B * a^3 - 3 * A * a^2 * b - 3 * B * a * b^2 + A * b^3) * \log(a * \tan(dx + c) + b) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) - (B * a^3 - 3 * A * a^2 * b - 3 * B * a * b^2 + A * b^3) * \log(\tan(dx + c)^2 + 1) / (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) + (3 * B * a^3 * b^2 - 5 * A * a^2 * b^3 - B * a * b^4 - A * b^5 + 2 * (2 * B * a^4 * b - 3 * A * a^3 * b^2 - A * a * b^4) * \tan(dx + c)) / (a^6 * b^2 + 2 * a^4 * b^4 + a^2 * b^6 + (a^8 + 2 * a^6 * b^2 + a^4 * b^4) * \tan(dx + c)^2 + 2 * (a^7 * b + 2 * a^5 * b^3 + a^3 * b^5) * \tan(dx + c))) / d$

**mupad [B]** time = 2.60, size = 481, normalized size = 2.75

$$\frac{\frac{5 A a^2 b + A b^3}{2(a^4 + 2 a^2 b^2 + b^4)} + \frac{2 A a b^2 \cot(c + d x)}{a^4 + 2 a^2 b^2 + b^4}}{d a^2 + 2 d a b \cot(c + d x) + d b^2 \cot(c + d x)^2} - \ln(a + b \cot(c + d x)) \left( \frac{3 A b}{d(a^2 + b^2)^2} - \frac{4 A b^3}{d(a^2 + b^2)^3} \right) - \frac{3 B a^3}{2(a^4 + 2 a^2 b^2 + b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*cot(c + dx))/(a + b*cot(c + dx))^3,x)`

[Out]  $((A * b^3 + 5 * A * a^2 * b) / (2 * (a^4 + b^4 + 2 * a^2 * b^2))) + (2 * A * a * b^2 * \cot(c + d * x)) / (a^4 + b^4 + 2 * a^2 * b^2)) / (a^2 * d + b^2 * d * \cot(c + d * x)^2 + 2 * a * b * d * \cot(c + d * x)) - \log(a + b * \cot(c + d * x)) * ((3 * A * b) / (d * (a^2 + b^2)^2) - (4 * A * b^3) / (d * (a^2 + b^2)^3)) - ((3 * B * a^3 - B * a * b^2) / (2 * (a^4 + b^4 + 2 * a^2 * b^2))) - (\cot(c + d * x) * (B * b^3 - B * a^2 * b)) / (a^4 + b^4 + 2 * a^2 * b^2)) / (a^2 * d + b^2 * d * \cot(c + d * x)^2 + 2 * a * b * d * \cot(c + d * x))$

$$x)^2 + 2*a*b*d*\cot(c + d*x)) + \log(a + b*\cot(c + d*x))*((B*a)/(d*(a^2 + b^2)^2) - (4*B*a*b^2)/(d*(a^2 + b^2)^3)) + (A*\log(\cot(c + d*x) - 1i)*1i)/(2*(a^3*d - b^3*d*1i - 3*a*b^2*d + a^2*b*d*3i)) + (A*\log(\cot(c + d*x) + 1i))/(2*(a^3*d*1i - b^3*d - a*b^2*d*3i + 3*a^2*b*d)) - (B*\log(\cot(c + d*x) - 1i)*1i)/(2*(a^3*d*1i + b^3*d - a*b^2*d*3i - 3*a^2*b*d)) - (B*\log(\cot(c + d*x) + 1i))/(2*(a^3*d + b^3*d*1i - 3*a*b^2*d - a^2*b*d*3i))$$

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*3,x)

[Out] Exception raised: AttributeError

### 3.95 $\int (a + b \cot(c + dx))^{5/2} (A + B \cot(c + dx)) dx$

Optimal. Leaf size=188

$$\frac{2(a^2B + 2aAb - b^2B)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2(aB + Ab)(a + b \cot(c + dx))^{3/2}}{3d} + \frac{(a - ib)^{5/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a}}\right)}{d}$$

[Out]  $(a - I*b)^{(5/2)}*(I*A + B)*\operatorname{arctanh}((a + b*\cot(d*x + c))^{(1/2)}/(a - I*b)^{(1/2)})/d - (a + I*b)^{(5/2)}*(I*A - B)*\operatorname{arctanh}((a + b*\cot(d*x + c))^{(1/2)}/(a + I*b)^{(1/2)})/d - 2/3*(A*b + B*a)*(a + b*\cot(d*x + c))^{(3/2)}/d - 2/5*B*(a + b*\cot(d*x + c))^{(5/2)}/d - 2*(2*A*a*b + B*a^2 - B*b^2)*(a + b*\cot(d*x + c))^{(1/2)}/d$

**Rubi [A]** time = 0.45, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3528, 3539, 3537, 63, 208}

$$\frac{2(a^2B + 2aAb - b^2B)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2(aB + Ab)(a + b \cot(c + dx))^{3/2}}{3d} + \frac{(a - ib)^{5/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Cot}[c + d*x])^{(5/2)}*(A + B*\operatorname{Cot}[c + d*x]), x]$

[Out]  $((a - I*b)^{(5/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - ((a + I*b)^{(5/2)}*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d - (2*(2*a*A*b + a^2*B - b^2*B)*\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]])/d - (2*(A*b + a*B)*(a + b*\operatorname{Cot}[c + d*x])^{(3/2)})/(3*d) - (2*B*(a + b*\operatorname{Cot}[c + d*x])^{(5/2)})/(5*d)$

#### Rule 63

$\operatorname{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\operatorname{Int}[(a + b*x^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

#### Rule 3528

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(d\*(a + b\*Tan[e + f\*x])^m)/(f\*m), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

### Rule 3537

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c\*d)/f, Subst[Int[(a + (b\*x)/d)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3539

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int (a + b \cot(c + dx))^{5/2} (A + B \cot(c + dx)) dx &= -\frac{2B(a + b \cot(c + dx))^{5/2}}{5d} + \int (a + b \cot(c + dx))^{3/2} (aA - bB + \\
 &= -\frac{2(Ab + aB)(a + b \cot(c + dx))^{3/2}}{3d} - \frac{2B(a + b \cot(c + dx))^{5/2}}{5d} + \\
 &= -\frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2(Ab + aB)(a + b \cot(c + dx))^{3/2}}{3d} \\
 &= -\frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2(Ab + aB)(a + b \cot(c + dx))^{3/2}}{3d} \\
 &= -\frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2(Ab + aB)(a + b \cot(c + dx))^{3/2}}{3d} \\
 &= -\frac{2(2aAb + a^2B - b^2B) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2(Ab + aB)(a + b \cot(c + dx))^{3/2}}{3d} \\
 &= \frac{(a - ib)^{5/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{(a + ib)^{5/2} (iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}} \right)}{d}
 \end{aligned}$$

**Mathematica [B]** time = 1.80, size = 379, normalized size = 2.02

$$2 \left( (a^2 B + 2aAb - b^2 B) \sqrt{a + b \cot(c + dx)} + \frac{\sqrt{a - \sqrt{-b^2}} (a^3 (Ab - \sqrt{-b^2} B) - 3a^2 b (A\sqrt{-b^2} + bB) + 3ab^2 (\sqrt{-b^2} B - Ab) + b^3 (A\sqrt{-b^2} + bB))}{2(a\sqrt{-b^2} + b^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^(5/2)\*(A + B\*Cot[c + d\*x]),x]

[Out] (-2\*((Sqrt[a - Sqrt[-b^2]]\*(-3\*a^2\*b\*(A\*Sqrt[-b^2] + b\*B) + b^3\*(A\*Sqrt[-b^2] + b\*B) + a^3\*(A\*b - Sqrt[-b^2]\*B) + 3\*a\*b^2\*(-(A\*b) + Sqrt[-b^2]\*B))\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]])/(2\*(b^2 + a\*Sqrt[-b^2])) + ((b^3\*(A\*Sqrt[-b^2] - b\*B) + 3\*a^2\*b\*(-(A\*Sqrt[-b^2]) + b\*B) - a^3\*(A\*b + Sqrt[-b^2]\*B) + 3\*a\*b^2\*(A\*b + Sqrt[-b^2]\*B))\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/(2\*Sqrt[-b^2]\*Sqrt[a + Sqrt[-b^2]]) + (2\*a\*A\*b + a^2\*B - b^2\*B)\*Sqrt[a + b\*Cot[c + d\*x]] + ((A\*b + a\*B)\*(a + b\*Cot[c + d\*x])^(3/2))/3 + (B\*(a + b\*Cot[c + d\*x])^(5/2))/5))/d

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(5/2)\*(A+B\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (B \cot(dx + c) + A)(b \cot(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(5/2)\*(A+B\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*cot(d\*x + c) + A)\*(b\*cot(d\*x + c) + a)^(5/2), x)

**maple [B]** time = 0.55, size = 2405, normalized size = 12.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



$$2*(a^2+b^2)^{(1/2)}-3/d*b/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(dx+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*A*a^2-1/d*b^2/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(dx+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*B*(a^2+b^2)^{(1/2)}+3/d*b^2/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(dx+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*B*a+3/4/d*b*\ln(b*\cot(dx+c)+a+(a+b*\cot(dx+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a+1/4/d*b*\ln((a+b*\cot(dx+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(dx+c)-a-(a^2+b^2)^{(1/2)})*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*(a^2+b^2)^{(1/2)}+1/4/d/b*\ln((a+b*\cot(dx+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(dx+c)-a-(a^2+b^2)^{(1/2)})*A*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-2/5*B*(a+b*\cot(dx+c))^{(5/2)}/d$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \cot(dx + c) + A)(b \cot(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(5/2)\*(A+B\*cot(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*cot(d\*x + c) + A)\*(b\*cot(d\*x + c) + a)^(5/2), x)

**mupad** [B] time = 31.59, size = 3864, normalized size = 20.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))^(5/2),x)

[Out]  $\log\left(\frac{(8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3/d^3 - (((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}))/(2*d) - (16*B^2*b^2*(a + b*\cot(c + d*x))^{(1/2)}*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*((( -B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}/2)*((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d^4)^{(1/2)}/(4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a*b^4)/(4*d^2))^{(1/2)} - \log\left(\frac{(((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(32*B*b^6 - 32*B*a^4*b^2 + 32*a*b^2*d*((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}))/(2*d) - (16*B^2*b^2*(a + b*\cot(c + d*x))^{(1/2)}*(a^6 -$





$$\begin{aligned}
& A^4 a^8 b^2 d^4)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2 \\
& ) / (4 d^4)^{(1/2)} + \log((8 A^3 b^3 (3 a^2 - b^2) (a^2 + b^2)^3) / d^3 - (((-A \\
& ^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2 \\
& ^2 d^2 - 5 A^2 a b^4 d^2) / d^4)^{(1/2)} * ((((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2 \\
& ) / d^4)^{(1/2)} * ((((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} - A^2 a^5 d^2 + 10 A^2 a^3 b^2 d^2 - 5 A^2 a b^4 d^2 \\
& ) / d^4)^{(1/2)} * (a + b \cot(c + d x))^{(1/2)})) / (2 d) + (16 A^2 b^2 (a + b \cot(c \\
& + d x))^{(1/2)} * (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)) / d^2) / 2 * ((20 A^4 a^2 * \\
& b^8 d^4 - A^4 b^10 d^4 - 110 A^4 a^4 b^6 d^4 + 100 A^4 a^6 b^4 d^4 - 25 A^4 \\
& a^8 b^2 d^4)^{(1/2)} / (4 d^4) - (A^2 a^5) / (4 d^2) + (5 A^2 a^3 b^2) / (2 d^2) - \\
& (5 A^2 a b^4) / (4 d^2))^{(1/2)} + \log((8 A^3 b^3 (3 a^2 - b^2) (a^2 + b^2)^3) \\
& / d^3 - (((-A^4 b^2 d^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} + A^2 a^5 d^2 \\
& - 10 A^2 a^3 b^2 d^2 + 5 A^2 a b^4 d^2) / d^4)^{(1/2)} * ((((-A^4 b^2 d^4 (5 a^4 \\
& + b^4 - 10 a^2 b^2)^2)^{(1/2)} + A^2 a^5 d^2 - 10 A^2 a^3 b^2 d^2 + 5 A^2 a \\
& b^4 d^2) / d^4)^{(1/2)} * (64 A a^3 b^3 + 64 A a a b^5 + 32 a b^2 d * ((-A^4 b^2 d^4 \\
& ^4 (5 a^4 + b^4 - 10 a^2 b^2)^2)^{(1/2)} + A^2 a^5 d^2 - 10 A^2 a^3 b^2 d^2 + \\
& 5 A^2 a b^4 d^2) / d^4)^{(1/2)} * (a + b \cot(c + d x))^{(1/2)})) / (2 d) + (16 A^2 b \\
& ^2 (a + b \cot(c + d x))^{(1/2)} * (a^6 - b^6 + 15 a^2 b^4 - 15 a^4 b^2)) / d^2) / \\
& 2 * ((5 A^2 a^3 b^2) / (2 d^2) - (A^2 a^5) / (4 d^2) - (20 A^4 a^2 b^8 d^4 - A^4 \\
& b^10 d^4 - 110 A^4 a^4 b^6 d^4 + 100 A^4 a^6 b^4 d^4 - 25 A^4 a^8 b^2 d^4) \\
& ^{(1/2)} / (4 d^4) - (5 A^2 a b^4) / (4 d^2))^{(1/2)} - (2 B (a + b \cot(c + d x))^{( \\
& 5/2)}) / (5 d) - (2 A b (a + b \cot(c + d x))^{(3/2)}) / (3 d) - (2 B a (a + b \cot( \\
& c + d x))^{(3/2)}) / (3 d) - (4 A a b (a + b \cot(c + d x))^{(1/2)}) / d
\end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \cot(c + dx)) (a + b \cot(c + dx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*(5/2)\*(A+B\*cot(d\*x+c)),x)

[Out] Integral((A + B\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))\*\*(5/2), x)

### 3.96 $\int (a + b \cot(c + dx))^{3/2} (A + B \cot(c + dx)) dx$

**Optimal.** Leaf size=150

$$-\frac{2(aB + Ab)\sqrt{a + b \cot(c + dx)}}{d} + \frac{(a - ib)^{3/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a + ib)^{3/2}(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

[Out]  $(a - I*b)^{(3/2)}*(I*A + B)*\operatorname{arctanh}((a + b*\cot(d*x + c))^{(1/2)}/(a - I*b)^{(1/2)})/d - (a + I*b)^{(3/2)}*(I*A - B)*\operatorname{arctanh}((a + b*\cot(d*x + c))^{(1/2)}/(a + I*b)^{(1/2)})/d - 2/3*B*(a + b*\cot(d*x + c))^{(3/2)}/d - 2*(A*b + a*B)*(a + b*\cot(d*x + c))^{(1/2)}/d$

**Rubi** [A] time = 0.33, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3528, 3539, 3537, 63, 208}

$$-\frac{2(aB + Ab)\sqrt{a + b \cot(c + dx)}}{d} + \frac{(a - ib)^{3/2}(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a + ib)^{3/2}(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Cot}[c + d*x])^{(3/2)}*(A + B*\operatorname{Cot}[c + d*x]), x]$

[Out]  $((a - I*b)^{(3/2)}*(I*A + B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/d - ((a + I*b)^{(3/2)}*(I*A - B)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/d - (2*(A*b + a*B)*\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]])/d - (2*B*(a + b*\operatorname{Cot}[c + d*x])^{(3/2)})/(3*d)$

#### Rule 63

$\operatorname{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\operatorname{Int}[(a + b*x)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b]$

#### Rule 3528

$\operatorname{Int}[(a + b*\tan[e + f*x])^m * (c + d*\tan[e + f*x])^n, x\_Symbol] \rightarrow \operatorname{Simp}[(d*(a + b*\tan[e + f*x])^m)/(f*m), x] + \operatorname{Int}[(a + b*\tan[e + f*x])^{m-1} * \operatorname{Simp}[a*c - b*d + (b*c + a*d)*\tan[e + f*x], x]$

, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

### Rule 3537

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c\*d)/f, Subst[Int[(a + (b\*x)/d)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3539

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int (a + b \cot(c + dx))^{3/2} (A + B \cot(c + dx)) dx &= -\frac{2B(a + b \cot(c + dx))^{3/2}}{3d} + \int \sqrt{a + b \cot(c + dx)} (aA - bB + A) dx \\
 &= -\frac{2(Ab + aB)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2B(a + b \cot(c + dx))^{3/2}}{3d} + \int \sqrt{a + b \cot(c + dx)} (aA - bB + A) dx \\
 &= -\frac{2(Ab + aB)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2B(a + b \cot(c + dx))^{3/2}}{3d} + \frac{1}{2} \int \sqrt{a + b \cot(c + dx)} (aA - bB + A) dx \\
 &= -\frac{2(Ab + aB)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2B(a + b \cot(c + dx))^{3/2}}{3d} + \frac{1}{2} \int \sqrt{a + b \cot(c + dx)} (aA - bB + A) dx \\
 &= -\frac{2(Ab + aB)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2B(a + b \cot(c + dx))^{3/2}}{3d} + \frac{1}{2} \int \sqrt{a + b \cot(c + dx)} (aA - bB + A) dx \\
 &= \frac{(a - ib)^{3/2} (iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{(a + ib)^{3/2} (iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}} \right)}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.97, size = 294, normalized size = 1.96

$$\frac{3\sqrt{a-\sqrt{-b^2}}\left(a^2\left(Ab-\sqrt{-b^2}B\right)-2ab\left(A\sqrt{-b^2}+bB\right)+b^2\left(\sqrt{-b^2}B-Ab\right)\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\cot(c+dx)}{\sqrt{a-\sqrt{-b^2}}}\right)}{a\sqrt{-b^2}+b^2} + \frac{3\left(-\left(a^2\left(Ab+\sqrt{-b^2}B\right)\right)+2ab\left(bB-A\sqrt{-b^2}\right)+b^2\left(Ab\right)\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}$$


---

$3d$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cot[c + d\*x])^(3/2)\*(A + B\*Cot[c + d\*x]),x]

[Out] 
$$-1/3*((3*\text{Sqrt}[a - \text{Sqrt}[-b^2]]*(-2*a*b*(A*\text{Sqrt}[-b^2] + b*B) + a^2*(A*b - \text{Sqrt}[-b^2]*B) + b^2*(-(A*b) + \text{Sqrt}[-b^2]*B))*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Cot}[c + d*x]]/\text{Sqrt}[a - \text{Sqrt}[-b^2]]])/(b^2 + a*\text{Sqrt}[-b^2]) + (3*(2*a*b*(-(A*\text{Sqrt}[-b^2]) + b*B) - a^2*(A*b + \text{Sqrt}[-b^2]*B) + b^2*(A*b + \text{Sqrt}[-b^2]*B))*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Cot}[c + d*x]]/\text{Sqrt}[a + \text{Sqrt}[-b^2]]])/(\text{Sqrt}[-b^2]*\text{Sqrt}[a + \text{Sqrt}[-b^2]]) + 6*(A*b + a*B)*\text{Sqrt}[a + b*\text{Cot}[c + d*x]] + 2*B*(a + b*\text{Cot}[c + d*x])^(3/2))/d$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(3/2)\*(A+B\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (B \cot(dx + c) + A)(b \cot(dx + c) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(3/2)\*(A+B\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*cot(d\*x + c) + A)\*(b\*cot(d\*x + c) + a)^(3/2), x)

**maple [B]** time = 0.54, size = 1665, normalized size = 11.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^(3/2)\*(A+B\*cot(d\*x+c)),x)

```
[Out] 1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)+1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*b^2+1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/2/d*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)*a-1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/2/d*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*b^2-2/d*A*b*(a+b*cot(d*x+c))^(1/2)-2/d*B*(a+b*cot(d*x+c))^(1/2)*a-2/3*B*(a+b*cot(d*x+c))^(3/2)/d
```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(3/2)\*(A+B\*cot(d\*x+c)),x, algorithm="maxima")



```

*a*b^2*d^2)/(4*d^4))^(1/2) - log((8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3
- (((16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (
16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^
2*d^2)/d^4)^(1/2)*(B*a^2 + B*b^2 - d*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2)
) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/
d)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)
/d^4)^(1/2))/2)*(-((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1
/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/(4*d^4))^(1/2) + log((((16*B^2*b^2*(a
+ b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*((-B^4*b^
2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B
*a^2 + B*b^2 + d*((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B
^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*((-B^4*b^2*d^4*(3
*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2))/2 + (8*B^
3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9
*B^4*a^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a^3)/(4*d^2) - (3*B^2*a*b^2)/(4*d^2)
)^(1/2) + log((((16*B^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b
^2))/d^2 - (16*a*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2
+ 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*a^2 + B*b^2 + d*(-((-B^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*
x))^(1/2)))/d)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^
2*a*b^2*d^2)/d^4)^(1/2))/2 + (8*B^3*b^2*(a^2 - b^2)*(a^2 + b^2)^2)/d^3)*((B
^2*a^3)/(4*d^2) - (6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1/
2)/(4*d^4) - (3*B^2*a*b^2)/(4*d^2))^(1/2) - (2*B*(a + b*cot(c + d*x))^(3/2)
)/(3*d) - (2*A*b*(a + b*cot(c + d*x))^(1/2))/d - (2*B*a*(a + b*cot(c + d*x)
)^(1/2))/d

```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \cot(c + dx))(a + b \cot(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*(3/2)\*(A+B\*cot(d\*x+c)),x)

[Out] Integral((A + B\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))\*\*(3/2), x)



### 3.97 $\int \sqrt{a + b \cot(c + dx)} (A + B \cot(c + dx)) dx$

**Optimal.** Leaf size=122

$$\frac{\sqrt{a - ib} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}} \right)}{d} - \frac{2B \sqrt{a + b \cot(c + dx)}}{d}$$

[Out] (I\*A+B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a-I\*b)^(1/2))\*(a-I\*b)^(1/2)/d-(I\*A-B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a+I\*b)^(1/2))\*(a+I\*b)^(1/2)/d-2\*B\*(a+b\*cot(d\*x+c))^(1/2)/d

**Rubi [A]** time = 0.26, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3528, 3539, 3537, 63, 208}

$$\frac{\sqrt{a - ib} (B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}} \right)}{d} - \frac{\sqrt{a + ib} (-B + iA) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}} \right)}{d} - \frac{2B \sqrt{a + b \cot(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*Cot[c + d\*x]]\*(A + B\*Cot[c + d\*x]),x]

[Out] (Sqrt[a - I\*b]\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]])/d - (Sqrt[a + I\*b]\*(I\*A - B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])/d - (2\*B\*Sqrt[a + b\*Cot[c + d\*x]])/d

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]]/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 3528

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(d\*(a + b\*Tan[e + f\*x])^m)/(f\*m), x] + Int[(a + b\*Tan[e + f\*x])^(m - 1)\*Simp[a\*c - b\*d + (b\*c + a\*d)\*Tan[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2,

0] && GtQ[m, 0]

### Rule 3537

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c\*d)/f, Subst[Int[(a + (b\*x)/d)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3539

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int \sqrt{a + b \cot(c + dx)} (A + B \cot(c + dx)) dx &= -\frac{2B\sqrt{a + b \cot(c + dx)}}{d} + \int \frac{aA - bB + (Ab + aB) \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx \\
 &= -\frac{2B\sqrt{a + b \cot(c + dx)}}{d} + \frac{1}{2}((a - ib)(A - iB)) \int \frac{1 + i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx \\
 &= -\frac{2B\sqrt{a + b \cot(c + dx)}}{d} - \frac{(i(a - ib)(A - iB)) \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ib}} dx\right)}{2d} \\
 &= -\frac{2B\sqrt{a + b \cot(c + dx)}}{d} + \frac{((a - ib)(A - iB)) \operatorname{Subst}\left(\int \frac{1}{-1-\frac{ia}{b}+\frac{ix^2}{b}} dx\right)}{bd} \\
 &= \frac{\sqrt{a - ib} (iA + B) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a + ib} (iA - B) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d}
 \end{aligned}$$

**Mathematica** [A] time = 0.57, size = 212, normalized size = 1.74

$$\frac{\left(aAb - a\sqrt{-b^2}B - A\sqrt{-b^2}b + b^2(-B)\right) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2} \sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb + a\sqrt{-b^2}B + A\sqrt{-b^2}b + b^2(-B)\right) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2} \sqrt{a+\sqrt{-b^2}}} + 2B\sqrt{a + b \cot(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*Cot[c + d\*x]]\*(A + B\*Cot[c + d\*x]),x]

[Out] -((((a\*A\*b - A\*b\*Sqrt[-b^2] - b^2\*B - a\*Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a - Sqrt[-b^2]]) - ((a\*A\*b + A\*b\*Sqrt[-b^2] - b^2\*B + a\*Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a + Sqrt[-b^2]]) + 2\*B\*Sqrt[a + b\*Cot[c + d\*x]])/d)

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(1/2)\*(A+B\*cot(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \cot(dx + c) + A) \sqrt{b \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(1/2)\*(A+B\*cot(d\*x+c)),x, algorithm="giac")

[Out] integrate((B\*cot(d\*x + c) + A)\*sqrt(b\*cot(d\*x + c) + a), x)

**maple** [B] time = 0.53, size = 968, normalized size = 7.93

$$-\frac{2B\sqrt{a+b\cot(dx+c)}}{d} + \frac{\ln\left(b\cot(dx+c)+a+\sqrt{a+b\cot(dx+c)}\sqrt{2\sqrt{a^2+b^2}+2a+\sqrt{a^2+b^2}}\right)A\sqrt{2\sqrt{a^2+b^2}}}{4db}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cot(d\*x+c))^(1/2)\*(A+B\*cot(d\*x+c)),x)

[Out] -2\*B\*(a+b\*cot(d\*x+c))^(1/2)/d+1/4/d/b\*ln(b\*cot(d\*x+c)+a+(a+b\*cot(d\*x+c))^(1/2))\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*(a^2+b^2)^(1/2)-1/4/d/b\*ln(b\*cot(d\*x+c)+a+(a+b\*cot(d\*x+c))^(1/2))\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))\*A\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)\*a+1/4/d\*ln(b\*cot(d\*x+c)+a+(a+b\*cot(d\*x+c))^(1/2))\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)+(a^2+b^2)^(1/2))\*B\*(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)-1/d\*b/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan((2\*(a+b\*cot(d\*x+c))^(1/2)+(2\*(a^2+b^2)^(1/2)+2\*a)^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2))\*A+1/d/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arc

```

tan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2
+b^2)^(1/2)+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/d*ln
((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b
^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2))*A-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2
+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))*B*(a^2+b^2)^(1/2)+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b
^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
))*B*a

```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \cot(dx + c) + A) \sqrt{b \cot(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))^(1/2)\*(A+B\*cot(d\*x+c)),x, algorithm="maxima")

[Out] integrate((B\*cot(d\*x + c) + A)\*sqrt(b\*cot(d\*x + c) + a), x)

**mupad** [B] time = 3.03, size = 843, normalized size = 6.91

$$\operatorname{atanh} \left( \frac{d^3 \left( \frac{16(A^2 b^4 - A^2 a^2 b^2) \sqrt{a+b \cot(c+dx)}}{d^2} + \frac{16 a b^2 (\sqrt{-A^4 b^2 d^4 + A^2 a d^2}) \sqrt{a+b \cot(c+dx)}}{d^4} \right) \sqrt{-\frac{\sqrt{-A^4 b^2 d^4 + A^2 a d^2}}{d^4}}}{16(A^3 a^2 b^3 + A^3 b^5)} \right) \sqrt{-\frac{\sqrt{-A^4 b^2 d^4 + A^2 a d^2}}{d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*cot(c + d\*x))\*(a + b\*cot(c + d\*x))^(1/2),x)

[Out] atanh((d^3\*((16\*(A^2\*b^4 - A^2\*a^2\*b^2)\*(a + b\*cot(c + d\*x))^(1/2))/d^2 + (16\*a\*b^2\*((-A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)\*(a + b\*cot(c + d\*x))^(1/2))/d^4)\*(-((-A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)/d^4)^(1/2))/(16\*(A^3\*b^5 + A^3\*a^2\*b^3)))\*(-((-A^4\*b^2\*d^4)^(1/2) + A^2\*a\*d^2)/d^4)^(1/2) + atanh((d^3\*((16\*(A^2\*b^4 - A^2\*a^2\*b^2)\*(a + b\*cot(c + d\*x))^(1/2))/d^2 - (16\*a\*b^2\*((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)\*(a + b\*cot(c + d\*x))^(1/2))/d^4)\*((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)/d^4)^(1/2))/(16\*(A^3\*b^5 + A^3\*a^2\*b^3)))\*((-A^4\*b^2\*d^4)^(1/2) - A^2\*a\*d^2)/d^4)^(1/2) + 2\*atanh((32\*B^2\*b^4\*((-B^4\*b^2\*d^4)^(1/2)

$$\begin{aligned} &)/(4*d^4) + (B^2*a)/(4*d^2))^{(1/2)}*(a + b*cot(c + d*x))^{(1/2)})/((16*B*b^4*( \\ &-B^4*b^2*d^4)^{(1/2)})/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)})/d^3) + (32*a \\ &*b^2*((-B^4*b^2*d^4)^{(1/2)})/(4*d^4) + (B^2*a)/(4*d^2))^{(1/2)}*(a + b*cot(c + \\ &d*x))^{(1/2)}*(-B^4*b^2*d^4)^{(1/2)})/((16*B*b^4*(-B^4*b^2*d^4)^{(1/2)})/d + (16* \\ &B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)})/d))*((( -B^4*b^2*d^4)^{(1/2)} + B^2*a*d^2)/(4* \\ &d^4))^{(1/2)} - 2*atanh((32*B^2*b^4*((B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^{(1/2)})/( \\ &4*d^4))^{(1/2)}*(a + b*cot(c + d*x))^{(1/2)})/((16*B*b^4*(-B^4*b^2*d^4)^{(1/2)})/ \\ &d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^{(1/2)})/d^3) - (32*a*b^2*((B^2*a)/(4*d^2) \\ &- (-B^4*b^2*d^4)^{(1/2)})/(4*d^4))^{(1/2)}*(a + b*cot(c + d*x))^{(1/2)}*(-B^4*b^2 \\ &*d^4)^{(1/2)})/((16*B*b^4*(-B^4*b^2*d^4)^{(1/2)})/d + (16*B*a^2*b^2*(-B^4*b^2*d \\ &^4)^{(1/2)})/d))*((-(-B^4*b^2*d^4)^{(1/2)} - B^2*a*d^2)/(4*d^4))^{(1/2)} - (2*B*( \\ &a + b*cot(c + d*x))^{(1/2)})/d \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \cot(c + dx)) \sqrt{a + b \cot(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cot(d\*x+c))\*\*(1/2)\*(A+B\*cot(d\*x+c)),x)

[Out] Integral((A + B\*cot(c + d\*x))\*sqrt(a + b\*cot(c + d\*x)), x)

### 3.98 $\int(-a + b \cot(c + dx))(a + b \cot(c + dx))^{5/2} dx$

**Optimal.** Leaf size=151

$$\frac{2b(a^2 + b^2)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} - \frac{(-b + ia)(a - ib)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{(a + ib)^{5/2}}{d}$$

[Out]  $-(I*a-b)*(a-I*b)^{(5/2)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d+(a+I*b)^{(5/2)*(I*a+b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}/d-2/5*b*(a+b*\cot(d*x+c))^{(5/2)}/d+2*b*(a^2+b^2)*(a+b*\cot(d*x+c))^{(1/2)}/d$

**Rubi [A]** time = 0.28, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {3528, 12, 3482, 3539, 3537, 63, 208}

$$\frac{2b(a^2 + b^2)\sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} - \frac{(-b + ia)(a - ib)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{(a + ib)^{5/2}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Cot}[c + d*x])*(a + b*\operatorname{Cot}[c + d*x])^{(5/2)}, x]$

[Out]  $-\left(\frac{(I*a - b)*(a - I*b)^{(5/2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]]/\operatorname{Sqrt}[a - I*b]}}{d} + \frac{((a + I*b)^{(5/2)*(I*a + b)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]]/\operatorname{Sqrt}[a + I*b]}}{d} + \frac{(2*b*(a^2 + b^2)*\operatorname{Sqrt}[a + b*\operatorname{Cot}[c + d*x]]}{d} - \frac{(2*b*(a + b*\operatorname{Cot}[c + d*x])^{(5/2)})}{(5*d)}\right)$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 63

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 3482

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

Rule 3537

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3539

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (-a + b \cot(c + dx))(a + b \cot(c + dx))^{5/2} dx &= -\frac{2b(a + b \cot(c + dx))^{5/2}}{5d} + \int (-a^2 - b^2)(a + b \cot(c + dx))^{3/2} dx \\
&= -\frac{2b(a + b \cot(c + dx))^{5/2}}{5d} + (-a^2 - b^2) \int (a + b \cot(c + dx))^{3/2} dx \\
&= \frac{2b(a^2 + b^2) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} + \dots \\
&= \frac{2b(a^2 + b^2) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} - \frac{1}{2} \dots \\
&= \frac{2b(a^2 + b^2) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} - \dots \\
&= \frac{2b(a^2 + b^2) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} - \dots \\
&= \frac{2b(a^2 + b^2) \sqrt{a + b \cot(c + dx)}}{d} - \frac{2b(a + b \cot(c + dx))^{5/2}}{5d} - \dots \\
&= -\frac{(ia - b)(a - ib)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right)}{d} + \frac{(a + ib)^{5/2}(ia + b) \dots}{d}
\end{aligned}$$

**Mathematica [A]** time = 3.96, size = 253, normalized size = 1.68

$$\frac{\sin(c + dx)(b \cot(c + dx) - a)(a + b \cot(c + dx))^{5/2} \left( \frac{2b(-4a^2 + 2ab \cot(c + dx) + b^2 \csc^2(c + dx) - 6b^2)}{(a + b \cot(c + dx))^2} + \frac{5i(a^2 + b^2)((a - ib)^2 \sqrt{a + ib} \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right))}{\sqrt{a - ib}} \right)}{5d(a \sin(c + dx) - b \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Cot[c + d\*x])\*(a + b\*Cot[c + d\*x])^(5/2), x]

[Out] ((-a + b\*Cot[c + d\*x])\*(a + b\*Cot[c + d\*x])^(5/2)\*(((5\*I)\*(a^2 + b^2)\*((a - I\*b)^2\*Sqrt[a + I\*b]\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]] - Sqrt[a - I\*b]\*(a + I\*b)^2\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])))/(Sqrt[a - I\*b]\*Sqrt[a + I\*b]\*(a + b\*Cot[c + d\*x])^(5/2)) + (2\*b\*(-4\*a^2 - 6\*b^2 + 2\*a\*b\*Cot[c + d\*x] + b^2\*Csc[c + d\*x]^2))/(a + b\*Cot[c + d\*x])^2\*Sin[c + d\*x])/(5\*d\*(-(b\*Cos[c + d\*x]) + a\*Sin[c + d\*x]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^{\frac{5}{2}} (b \cot(dx + c) - a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^(5/2)\*(b\*cot(d\*x + c) - a), x)

**maple** [B] time = 0.56, size = 1375, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(5/2),x)

[Out] 
$$\begin{aligned} & -2/5*b*(a+b*cot(d*x+c))^(5/2)/d+2/d*b*a^2*(a+b*cot(d*x+c))^(1/2)+2/d*b^3*(a \\ & +b*cot(d*x+c))^(1/2)-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2))*(2*(a \\ & ^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^ \\ & 2+b^2)^(1/2)*a^3-1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2))*(2*(a^2+b \\ & ^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^ \\ & 2)^(1/2)*a+1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2))*(2*(a^2+b^2)^(1 \\ & /2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-1/4/d*b^3 \\ & *ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^ \\ & 2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^( \\ & 1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^ \\ & 2+b^2)^(1/2)-2*a)^(1/2))*a^3+2/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(( \\ & 2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)- \\ & 2*a)^(1/2))*a-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c) \\ & )^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+ \\ & b^2)^(1/2)*a^2-1/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x \\ & +c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))* \\ & (a^2+b^2)^(1/2)+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/ \\ & 2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^( \\ & 1/2)*a^3+1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b \\ & *cot(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2) \\ & )*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d \\ & *x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/4/d*b^3*ln((a+ \\ & b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^( \\ & 1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*ar \end{aligned}$$



$$\begin{aligned}
& *a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^{(1/2)} * (((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^{(1/2)} * (64*a^2*b^5 + 64*a^4*b^3 - 32*a*b^2*d * (-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^{(1/2)} * (a + b*\cot(c + d*x))^{(1/2)})))/(2*d) - (16*a^2*b^2*(a + b*\cot(c + d*x))^{(1/2)} * (a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 * ((5*a^5*b^2)/(2*d^2) - a^7/(4*d^2) - (5*a^3*b^4)/(4*d^2) - (20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^12*b^2*d^4)^{(1/2)}/(4*d^4))^{(1/2)} - ((4*a^2*b)/d - (2*b*(a^2 + b^2))/d) * (a + b*\cot(c + d*x))^{(1/2)} - \log((((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^{(1/2)} * (32*b^7 - 32*a^4*b^3 + 32*a*b^2*d * (((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^{(1/2)} * (a + b*\cot(c + d*x))^{(1/2)})))/(2*d) + (16*(a + b*\cot(c + d*x))^{(1/2)} * (b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2 * (((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^{(1/2)}/2 + (8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 * (((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/(4*d^4))^{(1/2)} + \log((8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - (((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^{(1/2)} * (32*a^4*b^3 - 32*b^7 + 32*a*b^2*d * (((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^{(1/2)} * (a + b*\cot(c + d*x))^{(1/2)})))/(2*d) + (16*(a + b*\cot(c + d*x))^{(1/2)} * (b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2 * (((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} + 5*a*b^6*d^2 - 10*a^3*b^4*d^2 + a^5*b^2*d^2)/d^4)^{(1/2)}/2 * ((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^{(1/2)}/(4*d^4) + (5*a*b^6)/(4*d^2) - (5*a^3*b^4)/(2*d^2) + (a^5*b^2)/(4*d^2))^{(1/2)} - \log((((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^{(1/2)} * (32*b^7 - 32*a^4*b^3 + 32*a*b^2*d * (((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^{(1/2)} * (a + b*\cot(c + d*x))^{(1/2)})))/(2*d) + (16*(a + b*\cot(c + d*x))^{(1/2)} * (b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2 * (-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^{(1/2)}/2 + (8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 * (-((20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4*b^10*d^4 + 100*a^6*b^8*d^4 - 25*a^8*b^6*d^4)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/(4*d^4))^{(1/2)} + \log((8*a*b^5*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - (((((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^{(1/2)} * (32*a^4*b^3 - 32*b^7 + 32*a*b^2*d * (-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^{(1/2)} * (a + b*\cot(c + d*x))^{(1/2)})))/(2*d) + (16*(a + b*\cot(c + d*x))^{(1/2)} * (b^10 - 15*a^2*b^8 + 15*a^4*b^6 - a^6*b^4))/d^2 * (-((-b^6*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^{(1/2)} - 5*a*b^6*d^2 + 10*a^3*b^4*d^2 - a^5*b^2*d^2)/d^4)^{(1/2)}/2 * ((5*a*b^6)/(4*d^2) - (20*a^2*b^12*d^4 - b^14*d^4 - 110*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^{10}d^4 + 100a^6b^8d^4 - 25a^8b^6d^4)^{(1/2)}/(4d^4) - (5a^3b^4)/(2d^2) + (a^5b^2)/(4d^2))^{(1/2)} - \log(\left(\left(\left(-a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2\right)^{(1/2)} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2\right)/d^4\right)^{(1/2)} * \left(\left(\left(-a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2\right)^{(1/2)} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2\right)/d^4\right)^{(1/2)} * (64a^2b^5 + 64a^4b^3 + 32ab^2d * \left(\left(-a^4b^2d^4(5a^4 + b^4 - 10a^2b^2)^2\right)^{(1/2)} - a^7d^2 - 5a^3b^4d^2 + 10a^5b^2d^2\right)/d^4)^{(1/2)} * (a + b \cot(c + dx))^{(1/2)}\right) / (2*d) + (16a^2b^2 * (a + b \cot(c + dx))^{(1/2)} * (a^6 - b^6 + 15a^2b^4 - 15a^4b^2)) / d^2) / 2 \\
& - (8a^3b^3(3a^2 - b^2)(a^2 + b^2)^3 / d^3) * (-a^7d^2 - (20a^6b^8d^4 - a^4b^{10}d^4 - 110a^8b^6d^4 + 100a^{10}b^4d^4 - 25a^{12}b^2d^4)^{(1/2)} + 5a^3b^4d^2 - 10a^5b^2d^2) / (4d^4))^{(1/2)} - (2*b*(a + b*cot(c + dx))^{(5/2)}) / (5*d) + (4*a^2*b*(a + b*cot(c + dx))^{(1/2)}) / d
\end{aligned}$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int a^3 \sqrt{a + b \cot(c + dx)} dx - \int (-b^3 \sqrt{a + b \cot(c + dx)} \cot^3(c + dx)) dx - \int (-ab^2 \sqrt{a + b \cot(c + dx)} \cot^2(c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))\*\*(5/2),x)

[Out] -Integral(a\*\*3\*sqrt(a + b\*cot(c + d\*x)), x) - Integral(-b\*\*3\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x)\*\*3, x) - Integral(-a\*b\*\*2\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x)\*\*2, x) - Integral(a\*\*2\*b\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x), x)

### 3.99 $\int (-a + b \cot(c + dx))(a + b \cot(c + dx))^{3/2} dx$

**Optimal.** Leaf size=408

$$\frac{b(a^2 + b^2) \log\left(-\sqrt{2} \sqrt{\sqrt{a^2 + b^2} + a} \sqrt{a + b \cot(c + dx)} + \sqrt{a^2 + b^2} + a + b \cot(c + dx)\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}} - \frac{b(a^2 + b^2) \log\left(\sqrt{2}\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}}$$

[Out]  $-2/3*b*(a+b*\cot(d*x+c))^(3/2)/d+1/2*b*(a^2+b^2)*\operatorname{arctanh}((-2^(1/2)*(a+b*\cot(d*x+c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*(a^2+b^2)*\operatorname{arctanh}(2^(1/2)*(a+b*\cot(d*x+c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*(a^2+b^2)*\ln(a+b*\cot(d*x+c)+(a^2+b^2)^(1/2)-2^(1/2)*(a+b*\cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)-1/4*b*(a^2+b^2)*\ln(a+b*\cot(d*x+c)+(a^2+b^2)^(1/2)+2^(1/2)*(a+b*\cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)$

**Rubi [A]** time = 0.51, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3528, 12, 3485, 700, 1129, 634, 618, 206, 628}

$$\frac{b(a^2 + b^2) \log\left(-\sqrt{2} \sqrt{\sqrt{a^2 + b^2} + a} \sqrt{a + b \cot(c + dx)} + \sqrt{a^2 + b^2} + a + b \cot(c + dx)\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}} - \frac{b(a^2 + b^2) \log\left(\sqrt{2}\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(-a + b*\text{Cot}[c + d*x])*(a + b*\text{Cot}[c + d*x])^(3/2), x]$

[Out]  $(b*(a^2 + b^2)*\text{ArcTanh}[(\text{Sqrt}[a + \text{Sqrt}[a^2 + b^2]] - \text{Sqrt}[2]*\text{Sqrt}[a + b*\text{Cot}[c + d*x]])/\text{Sqrt}[a - \text{Sqrt}[a^2 + b^2]])/(\text{Sqrt}[2]*\text{Sqrt}[a - \text{Sqrt}[a^2 + b^2]]*d) - (b*(a^2 + b^2)*\text{ArcTanh}[(\text{Sqrt}[a + \text{Sqrt}[a^2 + b^2]] + \text{Sqrt}[2]*\text{Sqrt}[a + b*\text{Cot}[c + d*x]])/\text{Sqrt}[a - \text{Sqrt}[a^2 + b^2]])/(\text{Sqrt}[2]*\text{Sqrt}[a - \text{Sqrt}[a^2 + b^2]]*d) - (2*b*(a + b*\text{Cot}[c + d*x])^(3/2))/(3*d) + (b*(a^2 + b^2)*\text{Log}[a + \text{Sqrt}[a^2 + b^2] + b*\text{Cot}[c + d*x] - \text{Sqrt}[2]*\text{Sqrt}[a + \text{Sqrt}[a^2 + b^2]]*\text{Sqrt}[a + b*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*\text{Sqrt}[a + \text{Sqrt}[a^2 + b^2]]*d) - (b*(a^2 + b^2)*\text{Log}[a + \text{Sqrt}[a^2 + b^2] + b*\text{Cot}[c + d*x] + \text{Sqrt}[2]*\text{Sqrt}[a + \text{Sqrt}[a^2 + b^2]]*\text{Sqrt}[a + b*\text{Cot}[c + d*x]])/(2*\text{Sqrt}[2]*\text{Sqrt}[a + \text{Sqrt}[a^2 + b^2]]*d)$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[2*e, Subst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

### Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3] && NegQ[b^2 - 4*a*c]
```

### Rule 3485

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 + b^2, 0]
```

### Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
0] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int (-a + b \cot(c + dx))(a + b \cot(c + dx))^{3/2} dx &= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + \int (-a^2 - b^2) \sqrt{a + b \cot(c + dx)} dx \\
&= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + (-a^2 - b^2) \int \sqrt{a + b \cot(c + dx)} dx \\
&= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{b^2+x^2} dx, x, b\cot(c+dx)\right)}{d} \\
&= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + \frac{(2b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{x^2}{a^2+b^2-2ax^2+x^4} dx, x, b\cot(c+dx)\right)}{d} \\
&= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{x}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+x}} dx, x, b\cot(c+dx)\right)}{\sqrt{2}\sqrt{a+b\cot(c+dx)}} \\
&= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + \frac{(b(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+x}} dx, x, b\cot(c+dx)\right)}{\sqrt{2}\sqrt{a+b\cot(c+dx)}} \\
&= -\frac{2b(a + b \cot(c + dx))^{3/2}}{3d} + \frac{b(a^2 + b^2) \log\left(a + \sqrt{a^2 + b^2} + b \cot(c + dx)\right)}{2\sqrt{a + b \cot(c + dx)}} \\
&= \frac{b(a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \cot(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}\right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{b(a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \cot(c + dx)}}{\sqrt{a - \sqrt{a^2 + b^2}}}\right)}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d}
\end{aligned}$$

**Mathematica [C]** time = 1.94, size = 178, normalized size = 0.44

$$\frac{\sin^2(c + dx)(b \cot(c + dx) - a)(a + b \cot(c + dx)) \left(3i\sqrt{a - ib} (a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right) - 3i\sqrt{a + ib} (a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}}\right)\right)}{3a^2 d \sin^2(c + dx) - 3b^2 d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Cot[c + d\*x])\*(a + b\*Cot[c + d\*x])^(3/2), x]

[Out] ((-a + b\*Cot[c + d\*x])\*(a + b\*Cot[c + d\*x]))\*((3\*I)\*Sqrt[a - I\*b]\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]] - (3\*I)\*Sqrt[a + I\*b]\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])/(3\*a^2\*d\*sin^2[c + d\*x] - 3\*b^2\*d\*cos^2[c + d\*x])



$2 + b^2) \cdot \text{ArcTanh}[\text{Sqrt}[a + b \cdot \text{Cot}[c + d \cdot x]] / \text{Sqrt}[a + I \cdot b]] + 2 \cdot b \cdot (a + b \cdot \text{Cot}[c + d \cdot x])^{3/2} \cdot \text{Sin}[c + d \cdot x]^2 / (-3 \cdot b^2 \cdot d \cdot \text{Cos}[c + d \cdot x]^2 + 3 \cdot a^2 \cdot d \cdot \text{Sin}[c + d \cdot x]^2)$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(dx + c) + a)^{3/2} (b \cot(dx + c) - a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) + a)^(3/2)\*(b\*cot(d\*x + c) - a), x)

**maple** [B] time = 0.51, size = 972, normalized size = 2.38

$$\frac{2b(a + b \cot(dx + c))^{3/2}}{3d} + \frac{\sqrt{2\sqrt{a^2 + b^2} + 2a} a^3 \ln\left(b \cot(dx + c) + a + \sqrt{a + b \cot(dx + c)} \sqrt{2\sqrt{a^2 + b^2} + 2a}\right)}{4db}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(3/2),x)

[Out]  $-2/3 \cdot b \cdot (a + b \cdot \text{cot}(d \cdot x + c))^{3/2} / d + 1/4 \cdot d / b \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} \cdot a^3 \cdot \ln(b \cdot \text{cot}(d \cdot x + c) + a + (a^2 + b^2)^{1/2}) + 1/4 \cdot d \cdot b \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} \cdot a \cdot \ln(b \cdot \text{cot}(d \cdot x + c) + a + (a^2 + b^2)^{1/2}) \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} + (a^2 + b^2)^{1/2} + 1/d \cdot b \cdot a^2 / (2 \cdot (a^2 + b^2)^{1/2} - 2 \cdot a)^{1/2} \cdot \arctan((2 \cdot (a + b \cdot \text{cot}(d \cdot x + c))^{1/2} + (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2}) / (2 \cdot (a^2 + b^2)^{1/2} - 2 \cdot a)^{1/2}) - 1/4 \cdot d / b \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} \cdot (a^2 + b^2)^{1/2} \cdot \ln(b \cdot \text{cot}(d \cdot x + c) + a + (a^2 + b^2)^{1/2}) \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} + (a^2 + b^2)^{1/2} \cdot a^2 - 1/4 \cdot d \cdot b \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} \cdot (a^2 + b^2)^{1/2} \cdot \ln(b \cdot \text{cot}(d \cdot x + c) + a + (a^2 + b^2)^{1/2}) \cdot (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2} + (a^2 + b^2)^{1/2} + 1/d \cdot b^3 / (2 \cdot (a^2 + b^2)^{1/2} - 2 \cdot a)^{1/2} \cdot \arctan((2 \cdot (a + b \cdot \text{cot}(d \cdot x + c))^{1/2} + (2 \cdot (a^2 + b^2)^{1/2} + 2 \cdot a)^{1/2}) / (2 \cdot (a^2 + b^2)^{1/2} - 2 \cdot a)^{1/2})$

$$2)^{(1/2)} - 2*a)^{(1/2)} - 1/4/d/b*(2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a^3 * \ln(b*\cot(d*x+c) + a - (a+b*\cot(d*x+c))^{(1/2)} * (2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)}) - 1/4/d*b*(2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a * \ln(b*\cot(d*x+c) + a - (a+b*\cot(d*x+c))^{(1/2)} * (2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)}) + 1/d*b*a^2/(2*(a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2*(a+b*\cot(d*x+c))^{(1/2)} - (2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) + 1/4/d/b*(2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * (a^2+b^2)^{(1/2)} * \ln(b*\cot(d*x+c) + a - (a+b*\cot(d*x+c))^{(1/2)} * (2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)}) * a^2 + 1/4/d*b*(2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * (a^2+b^2)^{(1/2)} * \ln(b*\cot(d*x+c) + a - (a+b*\cot(d*x+c))^{(1/2)} * (2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)}) + 1/d*b^3/(2*(a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2*(a+b*\cot(d*x+c))^{(1/2)} - (2*(a^2+b^2)^{(1/2)} + 2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)} - 2*a)^{(1/2)})$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [B] time = 11.96, size = 2529, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*cot(c + d\*x))^(3/2)\*(a - b\*cot(c + d\*x)),x)

[Out]  $\log\left(\frac{((16*b^4*(a + b*\cot(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*((-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^{(1/2)}*(a^2*b + b^3 + d*((-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}}{d}\right) * \left(\frac{(-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} - 3*a*b^4*d^2 + a^3*b^2*d^2}{d^4}\right)^{(1/2)}/2 + (8*b^5*(a^2 - b^2)*(a^2 + b^2)^2)/d^3 * \left(\frac{(6*a^2*b^8*d^4 - b^{10}*d^4 - 9*a^4*b^6*d^4)^{(1/2)} / (4*d^4) - (3*a*b^4)/(4*d^2) + (a^3*b^2)/(4*d^2)}{d}\right)^{(1/2)} - \log\left(\frac{(8*b^5*(a^2 - b^2)*(a^2 + b^2)^2)/d^3 - ((16*b^4*(a + b*\cot(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*((-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^{(1/2)}*(a^2*b + b^3 - d*((-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}}{d}\right) * \left(\frac{(-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} + 3*a*b^4*d^2 - a^3*b^2*d^2}{d^4}\right)^{(1/2)}/2 * \left(\frac{(6*a^2*b^8*d^4 - b^{10}*d^4 - 9*a^4*b^6*d^4)^{(1/2)} + 3*a*b^4*d^2 - a^3*b^2*d^2}{(4*d^4)}\right)^{(1/2)} - \log\left(\frac{(8*b^5*(a^2 - b^2)*(a^2 + b^2)^2)/d^3 - ((16*b^4*(a + b*\cot(c + d*x))^{(1/2)}*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*((-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^{(1/2)}*(a^2*b + b^3 + d*((-b^6*d^4*(3*a^2 - b^2)^2)^{(1/2)} - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}}{d}\right)$

$$\begin{aligned} & \frac{1}{2} * (a^2 * b + b^3 - d * (((-b^6 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - 3 * a * b^4 * d^2 + a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a + b * \cot(c + d * x))^{(1/2)}) / d * (((-b^6 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - 3 * a * b^4 * d^2 + a^3 * b^2 * d^2) / d^4)^{(1/2)} / 2 * (((6 * a^2 * b^8 * d^4 - b^{10} * d^4 - 9 * a^4 * b^6 * d^4)^{(1/2)} - 3 * a * b^4 * d^2 + a^3 * b^2 * d^2) / (4 * d^4))^{(1/2)} \\ & + \log(((16 * b^4 * (a + b * \cot(c + d * x))^{(1/2)} * (a^4 + b^4 - 6 * a^2 * b^2)) / d^2 - (16 * a * b^2 * (-((-b^6 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} + 3 * a * b^4 * d^2 - a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a^2 * b + b^3 + d * (-((-b^6 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} + 3 * a * b^4 * d^2 - a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a + b * \cot(c + d * x))^{(1/2)})) / d * (-((-b^6 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} + 3 * a * b^4 * d^2 - a^3 * b^2 * d^2) / d^4)^{(1/2)} / 2 + (8 * b^5 * (a^2 - b^2) * (a^2 + b^2)^2) / d^3 * ((a^3 * b^2) / (4 * d^2) - (3 * a * b^4) / (4 * d^2) - (6 * a^2 * b^8 * d^4 - b^{10} * d^4 - 9 * a^4 * b^6 * d^4)^{(1/2)} / (4 * d^4))^{(1/2)} - \log(((-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} + a^5 * d^2 - 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * ((16 * a^2 * b^2 * (a + b * \cot(c + d * x))^{(1/2)} * (a^4 + b^4 - 6 * a^2 * b^2)) / d^2 + (16 * a * b^2 * (-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} + a^5 * d^2 - 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a^2 * b + b^3 + d * (-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} + a^5 * d^2 - 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a + b * \cot(c + d * x))^{(1/2)})) / d)) / 2 - (16 * a^4 * b^3 * (a^2 + b^2)^2) / d^3 * ((6 * a^6 * b^4 * d^4 - a^4 * b^6 * d^4 - 9 * a^8 * b^2 * d^4)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / (4 * d^4))^{(1/2)} - \log((((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * ((16 * a^2 * b^2 * (a + b * \cot(c + d * x))^{(1/2)} * (a^4 + b^4 - 6 * a^2 * b^2)) / d^2 + (16 * a * b^2 * (-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a^2 * b + b^3 + d * (-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a + b * \cot(c + d * x))^{(1/2)})) / d)) / 2 - (16 * a^4 * b^3 * (a^2 + b^2)^2) / d^3 * ((6 * a^6 * b^4 * d^4 - a^4 * b^6 * d^4 - 9 * a^8 * b^2 * d^4)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / (4 * d^4))^{(1/2)} + \log(- (((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * ((16 * a^2 * b^2 * (a + b * \cot(c + d * x))^{(1/2)} * (a^4 + b^4 - 6 * a^2 * b^2)) / d^2 - (16 * a * b^2 * (-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a^2 * b + b^3 - d * (-((-a^4 * b^2 * d^4 * (3 * a^2 - b^2)^2)^{(1/2)} - a^5 * d^2 + 3 * a^3 * b^2 * d^2) / d^4)^{(1/2)} * (a + b * \cot(c + d * x))^{(1/2)})) / d)) / 2 - (16 * a^4 * b^3 * (a^2 + b^2)^2) / d^3 * ((3 * a^3 * b^2) / (4 * d^2) - a^5 / (4 * d^2) - (6 * a^6 * b^4 * d^4 - a^4 * b^6 * d^4 - 9 * a^8 * b^2 * d^4)^{(1/2)} / (4 * d^4))^{(1/2)} - (2 * b * (a + b * \cot(c + d * x))^{(3/2)}) / (3 * d) \end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int a^2 \sqrt{a + b \cot(c + dx)} dx - \int (-b^2 \sqrt{a + b \cot(c + dx)} \cot^2(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(3/2),x)
```

```
[Out] -Integral(a**2*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**2*sqrt(a + b*cot  
(c + d*x))*cot(c + d*x)**2, x)
```

### 3.100 $\int (-a + b \cot(c + dx)) \sqrt{a + b \cot(c + dx)} dx$

Optimal. Leaf size=422

$$\frac{b\sqrt{a^2 + b^2} \log\left(-\sqrt{2} \sqrt{\sqrt{a^2 + b^2} + a} \sqrt{a + b \cot(c + dx)} + \sqrt{a^2 + b^2} + a + b \cot(c + dx)\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}} + \frac{b\sqrt{a^2 + b^2} \log\left(\sqrt{2} \sqrt{\sqrt{a^2 + b^2} + a} \sqrt{a + b \cot(c + dx)} + \sqrt{a^2 + b^2} + a + b \cot(c + dx)\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}}$$

[Out]  $-2*b*(a+b*\cot(d*x+c))^{(1/2)}/d+1/2*b*\operatorname{arctanh}((-2^{(1/2)}*(a+b*\cot(d*x+c))^{(1/2)}+(a+(a^2+b^2)^{(1/2))}^{(1/2)})/(a-(a^2+b^2)^{(1/2))}^{(1/2)})*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a-(a^2+b^2)^{(1/2))}^{(1/2)}-1/2*b*\operatorname{arctanh}((2^{(1/2)}*(a+b*\cot(d*x+c))^{(1/2)}+(a+(a^2+b^2)^{(1/2))}^{(1/2)})/(a-(a^2+b^2)^{(1/2))}^{(1/2)})*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a-(a^2+b^2)^{(1/2))}^{(1/2)}-1/4*b*\ln(a+b*\cot(d*x+c)+(a^2+b^2)^{(1/2)}-2^{(1/2)}*(a+b*\cot(d*x+c))^{(1/2)}*(a+(a^2+b^2)^{(1/2))}^{(1/2)})*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a+(a^2+b^2)^{(1/2))}^{(1/2)}+1/4*b*\ln(a+b*\cot(d*x+c)+(a^2+b^2)^{(1/2)}+2^{(1/2)}*(a+b*\cot(d*x+c))^{(1/2)}*(a+(a^2+b^2)^{(1/2))}^{(1/2)})*(a^2+b^2)^{(1/2)}/d*2^{(1/2)})/(a+(a^2+b^2)^{(1/2))}^{(1/2)})$

**Rubi [A]** time = 0.42, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3528, 12, 3485, 708, 1094, 634, 618, 206, 628}

$$\frac{b\sqrt{a^2 + b^2} \log\left(-\sqrt{2} \sqrt{\sqrt{a^2 + b^2} + a} \sqrt{a + b \cot(c + dx)} + \sqrt{a^2 + b^2} + a + b \cot(c + dx)\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}} + \frac{b\sqrt{a^2 + b^2} \log\left(\sqrt{2} \sqrt{\sqrt{a^2 + b^2} + a} \sqrt{a + b \cot(c + dx)} + \sqrt{a^2 + b^2} + a + b \cot(c + dx)\right)}{2\sqrt{2} d \sqrt{\sqrt{a^2 + b^2} + a}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Cot}[c + d*x])*Sqrt[a + b*\operatorname{Cot}[c + d*x]],x]$

[Out]  $(b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[a + b*\operatorname{Cot}[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])*d - (b*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[a + b*\operatorname{Cot}[c + d*x]])/Sqrt[a - Sqrt[a^2 + b^2]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])*d - (2*b*Sqrt[a + b*\operatorname{Cot}[c + d*x]])/d - (b*Sqrt[a^2 + b^2]*Log[a + Sqrt[a^2 + b^2] + b*\operatorname{Cot}[c + d*x] - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*\operatorname{Cot}[c + d*x]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d) + (b*Sqrt[a^2 + b^2]*Log[a + Sqrt[a^2 + b^2] + b*\operatorname{Cot}[c + d*x] + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*\operatorname{Cot}[c + d*x]])/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*d)$

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 708

Int[1/(Sqrt[(d\_) + (e\_.)\*(x\_)]\*((a\_) + (c\_.)\*(x\_)^2)), x\_Symbol] := Dist[2\*e, Subst[Int[1/(c\*d^2 + a\*e^2 - 2\*c\*d\*x^2 + c\*x^4), x], x, Sqrt[d + e\*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 1094

Int[((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2\*q - b/c, 2]}, Dist[1/(2\*c\*q\*r), Int[(r - x)/(q - r\*x + x^2), x], x] + Dist[1/(2\*c\*q\*r), Int[(r + x)/(q + r\*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && NegQ[b^2 - 4\*a\*c]

### Rule 3485

Int[((a\_) + (b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{a, b, c,

$d, n\}, x] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 3528

$\text{Int}[(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \text{Simp}[(d*(a + b*\text{Tan}[e + f*x])^m)/(f*m), x] + \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m - 1)*\text{Simp}[a*c - b*d + (b*c + a*d)*\text{Tan}[e + f*x], x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{GtQ}[m, 0]$

### Rubi steps

$$\begin{aligned} \int (-a + b \cot(c + dx)) \sqrt{a + b \cot(c + dx)} dx &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} + \int \frac{-a^2 - b^2}{\sqrt{a + b \cot(c + dx)}} dx \\ &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} + (-a^2 - b^2) \int \frac{1}{\sqrt{a + b \cot(c + dx)}} dx \\ &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} + \frac{(b(a^2 + b^2)) \text{Subst}\left(\int \frac{1}{\sqrt{a+x}(b^2+x^2)} dx, \dots\right)}{d} \\ &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} + \frac{(2b(a^2 + b^2)) \text{Subst}\left(\int \frac{1}{a^2+b^2-2ax^2+x^4} dx, \dots\right)}{d} \\ &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} + \frac{(b\sqrt{a^2 + b^2}) \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}} dx, \dots\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}} \\ &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} + \frac{(b\sqrt{a^2 + b^2}) \text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}} dx, \dots\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}} \\ &= -\frac{2b\sqrt{a + b \cot(c + dx)}}{d} - \frac{b\sqrt{a^2 + b^2} \log\left(a + \sqrt{a^2 + b^2} + b \cot(c + dx)\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}} \\ &= \frac{b\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}} - \sqrt{2}\sqrt{a+b \cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}} d} - \frac{b\sqrt{a^2 + b^2} \log\left(a + \sqrt{a^2 + b^2} + b \cot(c + dx)\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}} \end{aligned}$$

**Mathematica [C]** time = 1.06, size = 158, normalized size = 0.37

$$\frac{\sin(c + dx)(b \cot(c + dx) - a) \left( \frac{i(a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} - \frac{i(a^2 + b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + 2b\sqrt{a + b \cot(c + dx)} \right)}{d(a \sin(c + dx) - b \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Cot[c + d\*x])\*Sqrt[a + b\*Cot[c + d\*x]], x]

[Out] ((-a + b\*Cot[c + d\*x])\*((I\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]])/Sqrt[a - I\*b] - (I\*(a^2 + b^2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])/Sqrt[a + I\*b] + 2\*b\*Sqrt[a + b\*Cot[c + d\*x]])\*Sin[c + d\*x])/((d\*(-(b\*Cos[c + d\*x]) + a\*Sin[c + d\*x]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(1/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cot(dx + c) + a} (b \cot(dx + c) - a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b\*cot(d\*x + c) + a)\*(b\*cot(d\*x + c) - a), x)

**maple [B]** time = 0.54, size = 2285, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(1/2), x)

[Out] 1/d\*b^3/(a^2+b^2)^(1/2)/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan(((2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)-2\*(a+b\*cot(d\*x+c))^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2))-2/d\*b^5/(a^2+b^2)^(3/2)/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan(((2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)+2\*(a+b\*cot(d\*x+c))^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2))



$$\begin{aligned}
& 1/2)+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})-1/ \\
& d*b^3/(a^2+b^2)^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+ \\
& c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})+2/d \\
& *b^5/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c) \\
& ))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})-1/4/ \\
& d*b^3/(a^2+b^2)*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*c \\
& \cot(d*x+c)-a-(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+1/4/d*b^3/(a^2+b \\
& ^2)*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+ \\
& (a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*b*(a+b*\cot(d*x+c))^{(1/2)}/d \\
& +1/d/b/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x \\
& +c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^ \\
& 6+4/d*b/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d* \\
& x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a \\
& ^4-2/d*b/(a^2+b^2)^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d \\
& *x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})* \\
& a^2-1/4/d/b/(a^2+b^2)*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/ \\
& 2)}-b*\cot(d*x+c)-a-(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4+1/4/d/ \\
& b/(a^2+b^2)^{(3/2)}*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b \\
& *cot(d*x+c)-a-(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5-1/2/d*b/(a \\
& ^2+b^2)*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(d*x+c \\
& )-a-(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2+2/d*b/(a^2+b^2)^{(1/2) \\
& )/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+ \\
& b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^2-1/d/b/(a^2+b^2)^{(3/ \\
& 2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a \\
& +b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^6-5/d*b^3/(a^2+b^2)^{( \\
& 3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2 \\
& *(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^2-1/2/d*b/(a^2+b^ \\
& 2)^{(3/2)}*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{( \\
& 1/2)}+(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-1/d/b/(a^2+b^2)^{(1/ \\
& 2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b \\
& ^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^4-1/4/d*b^3/(a^2+b^2 \\
& )^{(3/2)}*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1 \\
& /2)}+(a^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a-1/4/d/b/(a^2+b^2)^{(3/2 \\
& )*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a \\
& ^2+b^2)^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5+1/4/d/b/(a^2+b^2)*\ln(b*\cot \\
& (d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1 \\
& /2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4+1/2/d*b/(a^2+b^2)*\ln(b*\cot(d*x+c)+a+ \\
& (a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)}*(2*(a \\
& ^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^2+1/d/b/(a^2+b^2)^{(1/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^ \\
& (1/2)*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a \\
& ^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^4+5/d*b^3/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2* \\
& a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2 \\
& *(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^2-4/d*b/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2 \\
& *a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/( \\
& 2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^4+1/2/d*b/(a^2+b^2)^{(3/2)}*\ln((a+b*\cot(d*x+c
\end{aligned}$$

$\left. \right)^{(1/2)} * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} - b * \cot(dx + c) - a - (a^2 + b^2)^{(1/2)} \left. \right)^{(1/2)} * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a^3 + 1/4/d * b^3 / (a^2 + b^2)^{(3/2)} * \ln((a + b * \cot(dx + c))^{(1/2)} * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} - b * \cot(dx + c) - a - (a^2 + b^2)^{(1/2)} \left. \right)^{(1/2)} * (2 * (a^2 + b^2)^{(1/2)} + 2 * a)^{(1/2)} * a$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cot(dx + c) + a} (b \cot(dx + c) - a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))\*(a+b\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*cot(d\*x + c) + a)\*(b\*cot(d\*x + c) - a), x)

**mupad** [B] time = 2.57, size = 583, normalized size = 1.38

$$-\operatorname{atanh} \left( \frac{d^3 \left( \frac{16(a^2 b^4 - a^4 b^2) \sqrt{a + b \cot(c + dx)}}{d^2} + \frac{16 a b^2 (a^3 + 1 i b a^2) \sqrt{a + b \cot(c + dx)}}{d^2} \right) \sqrt{-\frac{a^3 + 1 i b a^2}{d^2}}}{16 (a^5 b^3 + a^3 b^5)} \right) \sqrt{-\frac{a^3 + 1 i b a^2}{d^2}} - \operatorname{atanh} \left( \frac{d^3}{16 (a^5 b^3 + a^3 b^5)} \right) \sqrt{-\frac{a^3 + 1 i b a^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*cot(c + d\*x))^(1/2)\*(a - b\*cot(c + d\*x)),x)

[Out] atan((b^6\*((a\*b^2)/(4\*d^2) - (b^3\*1i)/(4\*d^2))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*32i)/((b^8\*16i)/d + (a^2\*b^6\*16i)/d) + (32\*a\*b^5\*((a\*b^2)/(4\*d^2) - (b^3\*1i)/(4\*d^2))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2))/((b^8\*16i)/d + (a^2\*b^6\*16i)/d))\*((a\*b^2 - b^3\*1i)/(4\*d^2))^(1/2)\*2i - atan((b^6\*((b^3\*1i)/(4\*d^2) + (a\*b^2)/(4\*d^2))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*32i)/((b^8\*16i)/d + (a^2\*b^6\*16i)/d) - (32\*a\*b^5\*((b^3\*1i)/(4\*d^2) + (a\*b^2)/(4\*d^2))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2))/((b^8\*16i)/d + (a^2\*b^6\*16i)/d))\*((a\*b^2 + b^3\*1i)/(4\*d^2))^(1/2)\*2i - atanh((d^3\*((16\*(a^2\*b^4 - a^4\*b^2)\*(a + b\*cot(c + d\*x))^(1/2))/d^2 + (16\*a\*b^2\*(a^2\*b\*1i + a^3)\*(a + b\*cot(c + d\*x))^(1/2))/d^2)\*(-(a^2\*b\*1i + a^3)/d^2)^(1/2))/(16\*(a^3\*b^5 + a^5\*b^3)))\*(-(a^2\*b\*1i + a^3)/d^2)^(1/2) - atanh((d^3\*((a^2\*b\*1i - a^3)/d^2)^(1/2)\*((16\*(a^2\*b^4 - a^4\*b^2)\*(a + b\*cot(c + d\*x))^(1/2))/d^2 - (16\*a\*b^2\*(a^2\*b\*1i - a^3)\*(a + b\*cot(c + d\*x))^(1/2))/d^2))/(16\*(a^3\*b^5 + a^5\*b^3)))\*((a^2\*b\*1i - a^3)/d^2)^(1/2) - (2\*b\*(a + b\*cot(c + d\*x))^(1/2))/d

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int a \sqrt{a + b \cot(c + dx)} dx - \int (-b \sqrt{a + b \cot(c + dx)} \cot(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] -Integral(a*sqrt(a + b*cot(c + d*x)), x) - Integral(-b*sqrt(a + b*cot(c + d*x))*cot(c + d*x), x)
```

$$3.101 \quad \int \frac{A+B \cot(c+dx)}{\sqrt{a+b \cot(c+dx)}} dx$$

Optimal. Leaf size=102

$$\frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}} \right)}{d\sqrt{a-ib}} - \frac{(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}} \right)}{d\sqrt{a+ib}}$$

[Out] (I\*A+B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/d/(a-I\*b)^(1/2)-(I\*A-B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/d/(a+I\*b)^(1/2)

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {3539, 3537, 63, 208}

$$\frac{(B + iA) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}} \right)}{d\sqrt{a-ib}} - \frac{(-B + iA) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}} \right)}{d\sqrt{a+ib}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Cot[c + d\*x])/Sqrt[a + b\*Cot[c + d\*x]], x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]]/(Sqrt[a - I\*b]\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]]/(Sqrt[a + I\*b]\*d))

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3537

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c\*d)/f, Subst[Int[(a + (b\*x)/d)^m/(d^2 + c\*x), x], x, d\*Tan[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b

\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3539

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 - I\*Tan[e + f\*x]), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^m\*(1 + I\*Tan[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx &= \frac{1}{2}(A - iB) \int \frac{1 + i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx + \frac{1}{2}(A + iB) \int \frac{1 - i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx \\ &= \frac{(iA - B) \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, -i \cot(c + dx)\right)}{2d} - \frac{(iA + B) \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}} dx, x, -i \cot(c + dx)\right)}{2d} \\ &= \frac{(A - iB) \operatorname{Subst}\left(\int \frac{1}{-1 - \frac{ia}{b} + \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{bd} + \frac{(A + iB) \operatorname{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{bd} \\ &= \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib} d} - \frac{(iA - B) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib} d} \end{aligned}$$

**Mathematica [A]** time = 0.61, size = 154, normalized size = 1.51

$$\frac{\sin(c + dx)(A + B \cot(c + dx)) \left( \sqrt{a + ib} (B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right) + \sqrt{a - ib} (B - iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right) \right)}{d \sqrt{a - ib} \sqrt{a + ib} (A \sin(c + dx) + B \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Cot[c + d\*x])/Sqrt[a + b\*Cot[c + d\*x]], x]

[Out] ((Sqrt[a + I\*b]\*(I\*A + B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]] + Sqrt[a - I\*b]\*((-I)\*A + B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])\*(A + B\*Cot[c + d\*x])\*Sin[c + d\*x])/(Sqrt[a - I\*b]\*Sqrt[a + I\*b]\*d\*(B\*Cos[c + d\*x] + A\*Sin[c + d\*x]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \cot(dx + c) + A}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

**maple** [B] time = 0.49, size = 3976, normalized size = 38.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x)
```

```
[Out] -1/d/(a^2+b^2)/b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))
^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^4-
1/4/d/(a^2+b^2)/b^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d/
(a^2+b^2)^(3/2)*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b
*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/(a^2+b
^2)^(1/2)/b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a^2+1/d/(a^
2+b^2)^(3/2)/b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a
)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a^4-1/4/
d/(a^2+b^2)^(3/2)/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3/d/(a
^2+b^2)^(3/2)*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1
/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2*A+1/d
/(a^2+b^2)^(3/2)*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c
))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B
*a^3-1/d/(a^2+b^2)^(1/2)/b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)*A*a^2-1/d/(a^2+b^2)^(3/2)*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a
^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
```

$$\begin{aligned}
& (1/2)) * a * B + 1/4/d/(a^2+b^2)/b^2 * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * \\
& (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * B * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} \\
& * a^3 + 1/d/(a^2+b^2)^{(1/2)}/b^2/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+ \\
& b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) \\
& * B * a^3 - 1/4/d/(a^2+b^2)/b * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+ \\
& b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * A * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a^2 - \\
& 1/d * (a^2+b^2)^{(1/2)}/b^2/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) \\
& * B * a^3 + 3/d/(a^2+b^2)^{(3/2)} * b/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) \\
& * a^2 * A - 1/d/(a^2+b^2)^{(3/2)}/b/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2 * (a+b * \cot(d*x+c))^{(1/2)} + (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) \\
& * A * a^4 + 1/d/(a^2+b^2)/b^2/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+ \\
& b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) \\
& * B * a^4 + 1/d * (a^2+b^2)^{(1/2)}/b^2/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2 * (a+b * \cot(d*x+c))^{(1/2)} + (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) \\
& * B * a^4 + 1/4/d/(a^2+b^2)^{(3/2)}/b * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * A * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a^3 + \\
& 1/4/d/(a^2+b^2)^{(3/2)} * b * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * A * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a + \\
& 1/4/d/(a^2+b^2)/b * \ln((a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - b * \cot(d*x+c) - a - (a^2+b^2)^{(1/2)}) * A * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a^2 - 2 \\
& /d/(a^2+b^2)/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2 * (a+b * \cot(d*x+c))^{(1/2)} + (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) * B * a^2 + 2/d/(a^2+b^2)^{(3/2)} * b^3 / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) * A - 1/d/(a^2+b^2)^{(3/2)} * b^2 / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2 * (a+b * \cot(d*x+c))^{(1/2)} + (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) * B - 1/d/(a^2+b^2)^{(3/2)} / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) * a^3 * B + 1/4/d/(a^2+b^2) * b * \ln((a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - b * \cot(d*x+c) - a - (a^2+b^2)^{(1/2)}) * A * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + 1/d/b^2/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan((2 * (a+b * \cot(d*x+c))^{(1/2)} + (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) * B * a^2 + 2/d/(a^2+b^2)/(2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)}) * B * a^2 - 1/4/d/(a^2+b^2) * b * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * A * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + 1/4/d/(a^2+b^2) * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * B * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + 2*a)^{(1/2)} * a + 1/4/d/(a^2+b^2)^{(3/2)} * \ln(b * \cot(d*x+c)) + a + (a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + (a^2+b^2)^{(1/2)} * B * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} * a^2 - 1/4/d/(a^2+b^2)^{(3/2)} * b^2 * \ln((a+b * \cot(d*x+c))^{(1/2)} * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - b * \cot(d*x+c) - a - (a^2+b^2)^{(1/2)}) * B * (2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} + 1/d/(a^2+b^2)^{(1/2)} / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)} * \arctan(((2 * (a^2+b^2)^{(1/2)} + 2*a)^{(1/2)} - 2 * (a+b * \cot(d*x+c))^{(1/2)}) / (2 * (a^2+b^2)^{(1/2)} - 2*a)^{(1/2)})
\end{aligned}$$

```

*B*a-1/4/d/(a^2+b^2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/(a^
2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3*B-1/4/d/
b^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+
(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/b^2/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2)
)/((2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+1/d/(a^2+b^2)*b^2/(2*(a^2+b^2)^(1/2)
)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2)
)/((2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B-1/d/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a-2/d/(a^2+b^2)^(3/2)*b^3/(2*(a^2+b^2)^(1/2)
)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)/((2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A+1/4/d/b^2*ln((a+b*cot(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*a-1/4/d/(a^2+b^2)^(3/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*a^2+1/d/(a^2+b^2)^(1/2)*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*A-1/d/(a^2+b^2)^(1/2)*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
)^(1/2))*A+1/4/d/(a^2+b^2)^(3/2)*b^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)

```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \cot(dx + c) + A}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B\*cot(d\*x + c) + A)/sqrt(b\*cot(d\*x + c) + a), x)

**mupad** [B] time = 2.29, size = 2909, normalized size = 28.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*cot(c + d\*x))/(a + b\*cot(c + d\*x))^(1/2),x)

[Out] 2\*atanh((32\*B^2\*b^2\*((B^2\*a\*d^2)/(4\*(a^2\*d^4 + b^2\*d^4)) - (-16\*B^4\*b^2\*d^4)^(1/2)/(16\*(a^2\*d^4 + b^2\*d^4)))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2))/((16\*B^



$$\begin{aligned}
& 3*b^2)/d - (16*B^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) + (4*B*a*b^2*d^2*(-16*B \\
& ^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (8*a*b^2*(B^2*a*d^2)/(4*(a^2*d^4 \\
& + b^2*d^4)) - (-16*B^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + \\
& b*\cot(c + d*x))^{(1/2)}*(-16*B^4*b^2*d^4)^{(1/2)}/(16*B^3*b^4*d + 16*B^3*a^2* \\
& b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a^4*b^2*d^5)/(a^ \\
& 2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2 \\
& *d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (32* \\
& B^2*a^2*b^2*d^2*(B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^{(1 \\
& /2)}/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}/(16*B^3*b^4 \\
& *d + 16*B^3*a^2*b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a \\
& ^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{(1/2) \\
& )/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + \\
& b^2*d^5)))*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^{(1/2)}/( \\
& 16*(a^2*d^4 + b^2*d^4)))^{(1/2)} + 2*atanh((8*a*b^2*(-16*B^4*b^2*d^4)^{(1/2)}/ \\
& (16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + \\
& b*\cot(c + d*x))^{(1/2)}*(-16*B^4*b^2*d^4)^{(1/2)}/((16*B^3*a^2*b^4*d^5)/(a^2*d \\
& ^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2 \\
& *d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2* \\
& d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (32*B \\
& ^2*b^2*(-16*B^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*( \\
& a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}/((16*B^3*a^2*b^2*d^3 \\
& )/(a^2*d^4 + b^2*d^4) - (16*B^3*b^2)/d + (4*B*a*b^2*d^2*(-16*B^4*b^2*d^4)^{( \\
& 1/2)}/(a^2*d^5 + b^2*d^5)) + (32*B^2*a^2*b^2*d^2*(-16*B^4*b^2*d^4)^{(1/2)}/( \\
& 16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b \\
& *cot(c + d*x))^{(1/2)}/((16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^ \\
& 2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^ \\
& 3*b^2*d^4*(-16*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-1 \\
& 6*B^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*((-16*B^4*b^2*d^4)^{(1/2)}/(16*(a \\
& ^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} + 2*atanh(( \\
& 32*A^2*b^2*(-16*A^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/ \\
& (4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}/((16*A^3*a*b^3*d \\
& ^3)/(a^2*d^4 + b^2*d^4) - (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + \\
& b^2*d^5) + (8*a*b^2*(-16*A^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) - (A \\
& ^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(-16*A^ \\
& 4*b^2*d^4)^{(1/2)}/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(- \\
& 16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 \\
& + b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) \\
& - (32*A^2*a^2*b^2*d^2*(-16*A^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) - \\
& (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}/((16 \\
& *A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16*A^4*b^2*d^4)^{(1/2) \\
& )/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2* \\
& b^3*d^4*(-16*A^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)))*((-16*A^4*b^2*d^4)^{( \\
& 1/2)}/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& - 2*atanh((8*a*b^2*(-16*A^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) - (A \\
& ^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(-16*A^
\end{aligned}$$

```

4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-
16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4
+ b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
- (32*A^2*b^2*(- (-16*A^4*b^2*d^4)^(1/2))/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a
*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*A^3*a
*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*
d^5 + b^2*d^5)) + (32*A^2*a^2*b^2*d^2*(- (-16*A^4*b^2*d^4)^(1/2))/(16*(a^2*d
^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c +
d*x))^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(-16*A^
4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 + b^2
*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(-
(-16*A^4*b^2*d^4)^(1/2))/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4
+ b^2*d^4)))^(1/2)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(1/2),x)

[Out] Integral((A + B\*cot(c + d\*x))/sqrt(a + b\*cot(c + d\*x)), x)

$$3.102 \quad \int \frac{A+B \cot(c+dx)}{(a+b \cot(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=138

$$\frac{2(Ab - aB)}{d(a^2 + b^2)\sqrt{a + b \cot(c + dx)}} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a - ib)^{3/2}} - \frac{(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a + ib)^{3/2}}$$

[Out] (I\*A+B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(3/2)/d-(I\*A-B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(3/2)/d+2\*(A\*b-B\*a)/(a^2+b^2)/d/(a+b\*cot(d\*x+c))^(1/2)

**Rubi** [A] time = 0.26, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3529, 3539, 3537, 63, 208}

$$\frac{2(Ab - aB)}{d(a^2 + b^2)\sqrt{a + b \cot(c + dx)}} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a - ib)^{3/2}} - \frac{(-B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d(a + ib)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^(3/2), x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]])/((a - I\*b)^(3/2)\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])/((a + I\*b)^(3/2)\*d) + (2\*(A\*b - a\*B))/((a^2 + b^2)\*d\*Sqrt[a + b\*Cot[c + d\*x]])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3529

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*Tan[e + f\*x])^(m + 1))

$$\int \frac{(f(m+1)(a^2+b^2))^m}{(a+b\tan[e+fx])^{m+1}} dx + \text{Dist}\left[\frac{1}{(a^2+b^2)}, \text{Int}[(a+b\tan[e+fx])^{m+1} \text{Simp}[a*c+b*d - (b*c-a*d)*\tan[e+fx], x], x], x\right] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f\}, x \} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$$

### Rule 3537

$$\text{Int}[(a + b\tan[e + f*x])^m * (c + d\tan[e + f*x]), x\_Symbol] \rightarrow \text{Dist}[(c*d)/f, \text{Subst}[\text{Int}[(a + (b*x)/d)^m / (d^2 + c*x), x], x, d*\tan[e + f*x]], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, m\}, x \} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$$

### Rule 3539

$$\text{Int}[(a + b\tan[e + f*x])^m * (c + d\tan[e + f*x]), x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b\tan[e + f*x])^m * (1 - I*\tan[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b\tan[e + f*x])^m * (1 + I*\tan[e + f*x]), x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, m\}, x \} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$$

### Rubi steps

$$\begin{aligned} \int \frac{A + B \cot(c + dx)}{(a + b \cot(c + dx))^{3/2}} dx &= \frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \cot(c + dx)}} + \frac{\int \frac{aA + bB - (Ab - aB) \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx}{a^2 + b^2} \\ &= \frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \cot(c + dx)}} + \frac{(A - iB) \int \frac{1 + i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx}{2(a - ib)} + \frac{(A + iB) \int \frac{1 - i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx}{2(a + ib)} \\ &= \frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \cot(c + dx)}} + \frac{(i(A + iB)) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, -i \cot(c + dx)\right)}{2(a + ib)d} \\ &= \frac{2(Ab - aB)}{(a^2 + b^2) d \sqrt{a + b \cot(c + dx)}} + \frac{(A - iB) \text{Subst}\left(\int \frac{1}{-1 - \frac{ia}{b} + \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{(a - ib)bd} \\ &= \frac{(iA + B) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{3/2}d} - \frac{(iA - B) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}}\right)}{(a + ib)^{3/2}d} + \frac{2(A + B)}{(a^2 + b^2) d \sqrt{a + b \cot(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 1.67, size = 226, normalized size = 1.64

$$\frac{\left(aAb - a\sqrt{-b^2}B + A\sqrt{-b^2}b + b^2B\right) \tanh^{-1}\left(\frac{\sqrt{a+b}\cot(c+dx)}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb + a\sqrt{-b^2}B - A\sqrt{-b^2}b + b^2B\right) \tanh^{-1}\left(\frac{\sqrt{a+b}\cot(c+dx)}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}} + \frac{2(aB - Ab)}{\sqrt{a+b}\cot(c+dx)}$$


---


$$d(a^2 + b^2)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^(3/2), x]

[Out] -((((a\*A\*b + A\*b\*Sqrt[-b^2] + b^2\*B - a\*Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a - Sqrt[-b^2]]) - ((a\*A\*b - A\*b\*Sqrt[-b^2] + b^2\*B + a\*Sqrt[-b^2]\*B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a + Sqrt[-b^2]]) + (2\*(-(A\*b) + a\*B))/Sqrt[a + b\*Cot[c + d\*x]])/(a^2 + b^2)\*d)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \cot(dx + c) + A}{(b \cot(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((B\*cot(d\*x + c) + A)/(b\*cot(d\*x + c) + a)^(3/2), x)

**maple [B]** time = 0.46, size = 7951, normalized size = 57.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2), x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \cot(dx + c) + A}{(b \cot(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B\*cot(d\*x + c) + A)/(b\*cot(d\*x + c) + a)^(3/2), x)

**mupad** [B] time = 6.47, size = 5737, normalized size = 41.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*cot(c + d\*x))/(a + b\*cot(c + d\*x))^(3/2),x)

[Out] (log((((a + b\*cot(c + d\*x))^(1/2)\*(16\*A^2\*b^10\*d^3 + 32\*A^2\*a^2\*b^8\*d^3 - 32\*A^2\*a^6\*b^4\*d^3 - 16\*A^2\*a^8\*b^2\*d^3) + (((((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) - 4\*A^2\*a^3\*d^2 + 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2)\*(64\*A\*a\*b^11\*d^4 - (((((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) - 4\*A^2\*a^3\*d^2 + 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*(64\*a\*b^12\*d^5 + 320\*a^3\*b^10\*d^5 + 640\*a^5\*b^8\*d^5 + 640\*a^7\*b^6\*d^5 + 320\*a^9\*b^4\*d^5 + 64\*a^11\*b^2\*d^5))/4 + 256\*A\*a^3\*b^9\*d^4 + 384\*A\*a^5\*b^7\*d^4 + 256\*A\*a^7\*b^5\*d^4 + 64\*A\*a^9\*b^3\*d^4))/4)\*(((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) - 4\*A^2\*a^3\*d^2 + 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2))/4 - 8\*A^3\*b^9\*d^2 - 24\*A^3\*a^2\*b^7\*d^2 - 24\*A^3\*a^4\*b^5\*d^2 - 8\*A^3\*a^6\*b^3\*d^2)\*(((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) - 4\*A^2\*a^3\*d^2 + 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2))/4 + (log((((a + b\*cot(c + d\*x))^(1/2)\*(16\*A^2\*b^10\*d^3 + 32\*A^2\*a^2\*b^8\*d^3 - 32\*A^2\*a^6\*b^4\*d^3 - 16\*A^2\*a^8\*b^2\*d^3) + (((((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) + 4\*A^2\*a^3\*d^2 - 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2)\*(64\*A\*a\*b^11\*d^4 - (((((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) + 4\*A^2\*a^3\*d^2 - 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*(64\*a\*b^12\*d^5 + 320\*a^3\*b^10\*d^5 + 640\*a^5\*b^8\*d^5 + 640\*a^7\*b^6\*d^5 + 320\*a^9\*b^4\*d^5 + 64\*a^11\*b^2\*d^5))/4 + 256\*A\*a^3\*b^9\*d^4 + 384\*A\*a^5\*b^7\*d^4 + 256\*A\*a^7\*b^5\*d^4 + 64\*A\*a^9\*b^3\*d^4))/4)\*(-(((96\*A^4\*a^2\*b^4\*d^4 - 16\*A^4\*b^6\*d^4 - 144\*A^4\*a^4\*b^2\*d^4)^(1/2) + 4\*A^2\*a^3\*d^2 - 12\*A^2\*a\*b^2\*d^2)/(a^6\*d^4 + b^6\*d^4 + 3\*a^2\*b^4\*d^4 + 3\*a^4\*b^2\*d^4))^(1/2))/4 - 8\*A^3\*b^9\*d^2 - 24\*A^3\*a^2\*b^7\*d^2 - 24\*A^3\*a^4\*b^5\*d^2 - 8\*A^3\*a^6\*b^3

$$\begin{aligned}
& d^2) * (-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} \\
& + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3* \\
& a^4*b^2*d^4))^{(1/2)})/4 - \log(-((a + b*\cot(c + d*x))^{(1/2)}*(16*A^2*b^10*d^3 \\
& + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - ((96*A^ \\
& 4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 \\
& + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2 \\
& *d^4))^{(1/2)}*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4) \\
& ^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^ \\
& 2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^12*d^ \\
& 5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 \\
& + 64*a^11*b^2*d^5) + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^ \\
& 4 + 256*A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b \\
& ^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(16 \\
& *a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} - 8*A^3*b^9 \\
& *d^2 - 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A \\
& ^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} - 4*A^2*a^3*d^ \\
& 2 + 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^ \\
& 2*d^4))^{(1/2)} - \log(-((a + b*\cot(c + d*x))^{(1/2)}*(16*A^2*b^10*d^3 + 32*A^2 \\
& *a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - ((96*A^4*a^2*b^ \\
& 4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^ \\
& 2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{( \\
& 1/2)}*((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} \\
& + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d \\
& ^4 + 48*a^4*b^2*d^4))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^12*d^5 + 320 \\
& *a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^ \\
& 11*b^2*d^5) + 64*A*a*b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256 \\
& *A*a^7*b^5*d^4 + 64*A*a^9*b^3*d^4))*(-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 \\
& - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(16*a^6*d \\
& ^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^{(1/2)} - 8*A^3*b^9*d^2 - \\
& 24*A^3*a^2*b^7*d^2 - 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(-((96*A^4*a^ \\
& 2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^{(1/2)} + 4*A^2*a^3*d^2 - 1 \\
& 2*A^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4 \\
& ))^{(1/2)} + (\log(24*B^3*a^3*b^6*d^2 - (((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^ \\
& 4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 \\
& + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}*(((96*B^4*a^2*b^4*d^4 \\
& - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^2 - 12*B^2*a*b^ \\
& 2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{(1/2)}*(((96*B \\
& ^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} + 4*B^2*a^3*d^ \\
& 2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^{( \\
& 1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^ \\
& 5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 - 32*B* \\
& b^12*d^4 - 96*B*a^2*b^10*d^4 - 64*B*a^4*b^8*d^4 + 64*B*a^6*b^6*d^4 + 96*B*a \\
& ^8*b^4*d^4 + 32*B*a^10*b^2*d^4))/4 + (a + b*\cot(c + d*x))^{(1/2)}*(16*B^2*b^1 \\
& 0*d^3 + 32*B^2*a^2*b^8*d^3 - 32*B^2*a^6*b^4*d^3 - 16*B^2*a^8*b^2*d^3))/4 + \\
& 24*B^3*a^5*b^4*d^2 + 8*B^3*a^7*b^2*d^2 + 8*B^3*a*b^8*d^2)*(((96*B^4*a^2*b^
\end{aligned}$$





$$4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^{(1/2)} - 4*B^2*a^3*d^2 + 12*B^2*a*b^2*d^2)/(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4)^{(1/2)} + (2*A*b)/(d*(a^2 + b^2)*(a + b*cot(c + d*x))^{(1/2)}) - (2*B*a)/(d*(a^2 + b^2)*(a + b*cot(c + d*x))^{(1/2)})$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \cot(c + dx)}{(a + b \cot(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(3/2), x)

[Out] Integral((A + B\*cot(c + d\*x))/(a + b\*cot(c + d\*x))\*\*(3/2), x)

$$3.103 \quad \int \frac{A+B \cot(c+dx)}{(a+b \cot(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=185

$$\frac{2(Ab - aB)}{3d(a^2 + b^2)(a + b \cot(c + dx))^{3/2}} + \frac{2(a^2(-B) + 2aAb + b^2B)}{d(a^2 + b^2)^2 \sqrt{a + b \cot(c + dx)}} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a - ib)^{5/2}} - \frac{(-B + iA)}{d(a - ib)^{5/2}}$$

[Out] (I\*A+B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a-I\*b)^(1/2))/(a-I\*b)^(5/2)/d-(I\*A-B)\*arctanh((a+b\*cot(d\*x+c))^(1/2)/(a+I\*b)^(1/2))/(a+I\*b)^(5/2)/d+2/3\*(A\*b-B\*a)/(a^2+b^2)/d/(a+b\*cot(d\*x+c))^(3/2)+2\*(2\*A\*a\*b-B\*a^2+B\*b^2)/(a^2+b^2)^2/d/(a+b\*cot(d\*x+c))^(1/2)

**Rubi [A]** time = 0.40, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3529, 3539, 3537, 63, 208}

$$\frac{2(Ab - aB)}{3d(a^2 + b^2)(a + b \cot(c + dx))^{3/2}} + \frac{2(a^2(-B) + 2aAb + b^2B)}{d(a^2 + b^2)^2 \sqrt{a + b \cot(c + dx)}} + \frac{(B + iA) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a - ib)^{5/2}} - \frac{(-B + iA)}{d(a - ib)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^(5/2), x]

[Out] ((I\*A + B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]]/((a - I\*b)^(5/2)\*d) - ((I\*A - B)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]]/((a + I\*b)^(5/2)\*d) + (2\*(A\*b - a\*B))/(3\*(a^2 + b^2)\*d\*(a + b\*Cot[c + d\*x])^(3/2)) + (2\*(2\*a\*A\*b - a^2\*B + b^2\*B))/((a^2 + b^2)^2\*d\*Sqrt[a + b\*Cot[c + d\*x]])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

### Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \cot(c + dx)}{(a + b \cot(c + dx))^{5/2}} dx &= \frac{2(Ab - aB)}{3(a^2 + b^2) d(a + b \cot(c + dx))^{3/2}} + \frac{\int \frac{aA + bB - (Ab - aB) \cot(c + dx)}{(a + b \cot(c + dx))^{3/2}} dx}{a^2 + b^2} \\
&= \frac{2(Ab - aB)}{3(a^2 + b^2) d(a + b \cot(c + dx))^{3/2}} + \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{\int \frac{a^2A - Ab^2 + 2aB}{(a + b \cot(c + dx))^{3/2}} dx}{a^2 + b^2} \\
&= \frac{2(Ab - aB)}{3(a^2 + b^2) d(a + b \cot(c + dx))^{3/2}} + \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{(A - iB) \int \frac{a^2A - Ab^2 + 2aB}{(a + b \cot(c + dx))^{3/2}} dx}{2(a^2 + b^2)} \\
&= \frac{2(Ab - aB)}{3(a^2 + b^2) d(a + b \cot(c + dx))^{3/2}} + \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{(iA - B) \operatorname{Su}}{2(a^2 + b^2)} \\
&= \frac{2(Ab - aB)}{3(a^2 + b^2) d(a + b \cot(c + dx))^{3/2}} + \frac{2(2aAb - a^2B + b^2B)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{(A - iB) \operatorname{Su}}{2(a^2 + b^2)} \\
&= \frac{(iA + B) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}} \right)}{(a - ib)^{5/2} d} - \frac{(iA - B) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}} \right)}{(a + ib)^{5/2} d} + \frac{2(A - iB) \operatorname{Su}}{3(a^2 + b^2) d}
\end{aligned}$$

**Mathematica [A]** time = 3.53, size = 319, normalized size = 1.72

$$\frac{\frac{2(a^2 + b^2)(aB - Ab)}{(a + b \cot(c + dx))^{3/2}} + \frac{6(a^2B - 2aAb - b^2B)}{\sqrt{a + b \cot(c + dx)}} + \frac{3(a^2(Ab - \sqrt{-b^2}B) + 2ab(A\sqrt{-b^2} + bB) + b^2(\sqrt{-b^2}B - Ab)) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - \sqrt{-b^2}}} \right)}{\sqrt{-b^2} \sqrt{a - \sqrt{-b^2}}}}{3d(a^2 + b^2)^2} + \frac{3(-a^2(Ab + \sqrt{-b^2}B) + 2ab(A\sqrt{-b^2} - bB) + b^2(\sqrt{-b^2}B - Ab)) \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + \sqrt{-b^2}}} \right)}{3d(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^(5/2), x]

[Out] 
$$\begin{aligned}
& -1/3 * ((3 * (2 * a * b * (A * \operatorname{Sqrt}[-b^2] + b * B) + a^2 * (A * b - \operatorname{Sqrt}[-b^2] * B) + b^2 * (-A * b + \operatorname{Sqrt}[-b^2] * B)) * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b * \operatorname{Cot}[c + d * x]] / \operatorname{Sqrt}[a - \operatorname{Sqrt}[-b^2]]]) / (\operatorname{Sqrt}[-b^2] * \operatorname{Sqrt}[a - \operatorname{Sqrt}[-b^2]]) + (3 * (2 * a * b * (A * \operatorname{Sqrt}[-b^2] - b * B) - a^2 * (A * b + \operatorname{Sqrt}[-b^2] * B) + b^2 * (A * b + \operatorname{Sqrt}[-b^2] * B)) * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b * \operatorname{Cot}[c + d * x]] / \operatorname{Sqrt}[a + \operatorname{Sqrt}[-b^2]]]) / (\operatorname{Sqrt}[-b^2] * \operatorname{Sqrt}[a + \operatorname{Sqrt}[-b^2]]) + (2 * (a^2 + b^2) * (-A * b) + a * B) / (a + b * \operatorname{Cot}[c + d * x])^{3/2} + (6 * (-2 * a * A * b + a^2 * B - b^2 * B)) / \operatorname{Sqrt}[a + b * \operatorname{Cot}[c + d * x]] / ((a^2 + b^2)^2 * d)
\end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \cot(dx + c) + A}{(b \cot(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B\*cot(d\*x + c) + A)/(b\*cot(d\*x + c) + a)^(5/2), x)

**maple** [B] time = 0.44, size = 12836, normalized size = 69.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \cot(dx + c) + A}{(b \cot(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B\*cot(d\*x + c) + A)/(b\*cot(d\*x + c) + a)^(5/2), x)

**mupad** [B] time = 17.93, size = 9453, normalized size = 51.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + B*\cot(c + d*x))/(a + b*\cot(c + d*x))^{(5/2)}, x)$

[Out]  $(\log((((a + b*\cot(c + d*x))^{(1/2)}*(320*A^2*a^4*b^{14}*d^3 - 16*A^2*b^{18}*d^3 + 1024*A^2*a^6*b^{12}*d^3 + 1440*A^2*a^8*b^{10}*d^3 + 1024*A^2*a^{10}*b^8*d^3 + 320*A^2*a^{12}*b^6*d^3 - 16*A^2*a^{16}*b^2*d^3) + (((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)))^{(1/2)}*(896*A*a^6*b^{15}*d^4 - (((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5))/4 - 160*A*a^2*b^{19}*d^4 - 128*A*a^4*b^{17}*d^4 - 32*A*b^{21}*d^4 + 3136*A*a^8*b^{13}*d^4 + 4928*A*a^{10}*b^{11}*d^4 + 4480*A*a^{12}*b^9*d^4 + 2432*A*a^{14}*b^7*d^4 + 736*A*a^{16}*b^5*d^4 + 96*A*a^{18}*b^3*d^4))/4)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)})/4 - 96*A^3*a^3*b^{13}*d^2 - 240*A^3*a^5*b^{11}*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^{11}*b^5*d^2 - 16*A^3*a^{13}*b^3*d^2 - 16*A^3*a*b^{15}*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)})/4 + (\log((((a + b*\cot(c + d*x))^{(1/2)}*(320*A^2*a^4*b^{14}*d^3 - 16*A^2*b^{18}*d^3 + 1024*A^2*a^6*b^{12}*d^3 + 1440*A^2*a^8*b^{10}*d^3 + 1024*A^2*a^{10}*b^8*d^3 + 320*A^2*a^{12}*b^6*d^3 - 16*A^2*a^{16}*b^2*d^3) + (((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}*(896*A*a^6*b^{15}*d^4 - (((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5))/4 - 160*A*a^2*b^{19}*d^4 - 128*A*a^4*b^{17}*d^4 - 32*A*b^{21}*d^4 + 3136*A*a^8*b^{13}*d^4 + 4928*A*a^{10}*b^{11}*d^4 + 4480*A*a^{12}*b^9*d^4 + 2432*A*a^{14}*b^7*d^4 + 736*A*a^{16}*b^5*d^4 + 96*A*a^{18}*b^3*d^4))/4)*(-(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40$

$$\begin{aligned}
& *A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + \\
& 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}/4 - 96*A^3*a^3*b^ \\
& 13*d^2 - 240*A^3*a^5*b^{11}*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - \\
& 96*A^3*a^{11}*b^5*d^2 - 16*A^3*a^{13}*b^3*d^2 - 16*A^3*a*b^{15}*d^2)*(-((320*A^4 \\
& *a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^ \\
& 4 - 400*A^4*a^8*b^2*d^4))^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^ \\
& 2*a*b^4*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6 \\
& *b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}/4 - \log(-((a + b*\cot(c + d*x))^{(1/2)}*(32 \\
& 0*A^2*a^4*b^{14}*d^3 - 16*A^2*b^{18}*d^3 + 1024*A^2*a^6*b^{12}*d^3 + 1440*A^2*a^8 \\
& *b^{10}*d^3 + 1024*A^2*a^{10}*b^8*d^3 + 320*A^2*a^{12}*b^6*d^3 - 16*A^2*a^{16}*b^2* \\
& d^3) - (((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 16 \\
& 00*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4))^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^ \\
& 3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + \\
& 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(((320*A^4*a^2 \\
& *b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - \\
& 400*A^4*a^8*b^2*d^4))^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a* \\
& b^4*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 16 \\
& 0*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^2 \\
& 2*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^ \\
& 9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 \\
& + 2880*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5) - 32*A*b^{21}*d^4 - \\
& 160*A*a^2*b^{19}*d^4 - 128*A*a^4*b^{17}*d^4 + 896*A*a^6*b^{15}*d^4 + 3136*A*a^8* \\
& b^{13}*d^4 + 4928*A*a^{10}*b^{11}*d^4 + 4480*A*a^{12}*b^9*d^4 + 2432*A*a^{14}*b^7*d^4 \\
& + 736*A*a^{16}*b^5*d^4 + 96*A*a^{18}*b^3*d^4))*(((320*A^4*a^2*b^8*d^4 - 16*A^4 \\
& *b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d \\
& ^4))^{(1/2)} - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^{10} \\
& *d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 8 \\
& 0*a^8*b^2*d^4))^{(1/2)} - 96*A^3*a^3*b^{13}*d^2 - 240*A^3*a^5*b^{11}*d^2 - 320*A^ \\
& 3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^{11}*b^5*d^2 - 16*A^3*a^{13}*b^3 \\
& *d^2 - 16*A^3*a*b^{15}*d^2)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A \\
& ^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4))^{(1/2)} - 4*A^2* \\
& a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 \\
& + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1 \\
& /2)} - \log(-((a + b*\cot(c + d*x))^{(1/2)}*(320*A^2*a^4*b^{14}*d^3 - 16*A^2*b^{18} \\
& *d^3 + 1024*A^2*a^6*b^{12}*d^3 + 1440*A^2*a^8*b^{10}*d^3 + 1024*A^2*a^{10}*b^8*d^ \\
& 3 + 320*A^2*a^{12}*b^6*d^3 - 16*A^2*a^{16}*b^2*d^3) - (-((320*A^4*a^2*b^8*d^4 - \\
& 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^ \\
& 8*b^2*d^4))^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/( \\
& 16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4* \\
& d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1 \\
& 760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4))^{(1/2)} + 4 \\
& *A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2)/(16*a^{10}*d^4 + 16*b^{1 \\
& 0}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4 \\
& ))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 288 \\
& 0*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d
\end{aligned}$$

$$\begin{aligned}
&^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19} \\
&*b^4*d^5 + 64*a^{21}*b^2*d^5) - 32*A*b^{21}*d^4 - 160*A*a^2*b^{19}*d^4 - 128*A*a^4 \\
&*b^{17}*d^4 + 896*A*a^6*b^{15}*d^4 + 3136*A*a^8*b^{13}*d^4 + 4928*A*a^{10}*b^{11}*d^4 \\
&+ 4480*A*a^{12}*b^9*d^4 + 2432*A*a^{14}*b^7*d^4 + 736*A*a^{16}*b^5*d^4 + 96*A*a^{18} \\
&*b^3*d^4) * (-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6 \\
&*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2) / (16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)} - 96*A^3 \\
&*a^3*b^{13}*d^2 - 240*A^3*a^5*b^{11}*d^2 - 320*A^3*a^7*b^9*d^2 - 240*A^3*a^9*b^7*d^2 - 96*A^3*a^{11} \\
&*b^5*d^2 - 16*A^3*a^{13}*b^3*d^2 - 16*A^3*a*b^{15}*d^2) * (-((320*A^4*a^2*b^8*d^4 - 16*A^4*b^{10}*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6 \\
&*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^{(1/2)} + 4*A^2*a^5*d^2 - 40*A^2*a^3*b^2*d^2 + 20*A^2*a*b^4*d^2) / (16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)} + (\log(40*B^3*a^8*b^8*d^2 - 8*B^3*b^{16}*d^2 - 40*B^3*a^2*b^{14}*d^2 - 72*B^3*a^4*b^{12}*d^2 - 40*B^3*a^6 \\
&*b^{10}*d^2 - (((a + b*cot(c + d*x))^{(1/2)} * (320*B^2*a^4*b^{14}*d^3 - 16*B^2*b^{18}*d^3 + 1024*B^2*a^6*b^{12}*d^3 + 1440*B^2*a^8*b^{10}*d^3 + 1024*B^2*a^{10}*b^8*d^3 + 320*B^2*a^{12}*b^6*d^3 - 16*B^2*a^{16}*b^2*d^3) - (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^{10}*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^{(1/2)} + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2) / (a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)} * (96*B*a*b^{20}*d^4 - ((a + b*cot(c + d*x))^{(1/2)} * (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^{10}*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^{(1/2)} + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2) / (a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)} * (64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880 \\
&*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19} \\
&*b^4*d^5 + 64*a^{21}*b^2*d^5)) / 4 + 736*B*a^3*b^{18}*d^4 + 2432*B*a^5*b^{16}*d^4 + 4480*B*a^7*b^{14}*d^4 + 4928*B*a^9*b^{12}*d^4 + 3136*B*a^{11}*b^{10}*d^4 + 896*B*a^{13} \\
&*b^8*d^4 - 128*B*a^{15}*b^6*d^4 - 160*B*a^{17}*b^4*d^4 - 32*B*a^{19}*b^2*d^4)) / 4) * (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^{10}*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^{(1/2)} + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2) / (a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)})) / 4 + 72*B^3*a^{10}*b^6*d^2 + 40 \\
&*B^3*a^{12}*b^4*d^2 + 8*B^3*a^{14}*b^2*d^2) * (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^{10}*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^{(1/2)} + 4*B^2*a^5*d^2 - 40*B^2*a^3*b^2*d^2 + 20*B^2*a*b^4*d^2) / (a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)})) / 4 + (\log(40*B^3*a^8*b^8*d^2 - 8*B^3*b^{16}*d^2 - 40*B^3*a^2*b^{14}*d^2 - 72*B^3*a^4*b^{12}*d^2 - 40*B^3*a^6*b^{10}*d^2 - (((a + b*cot(c + d*x))^{(1/2)} * (320*B^2*a^4*b^{14}*d^3 - 16*B^2*b^{18}*d^3 + 1024*B^2*a^6*b^{12}*d^3 + 1440*B^2 \\
&*a^8*b^{10}*d^3 + 1024*B^2*a^{10}*b^8*d^3 + 320*B^2*a^{12}*b^6*d^3 - 16*B^2*a^{16}*b^2*d^3) - (((320*B^4*a^2*b^8*d^4 - 16*B^4*b^{10}*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^{(1/2)} - 4*B^2*a^5*d^2 + 40*
\end{aligned}$$



$$\begin{aligned}
& B^2 a^3 b^2 d^2 - 20 B^2 a^2 b^4 d^2) / (a^{10} d^4 + b^{10} d^4 + 5 a^2 b^8 d^4 + 10 a^4 b^6 d^4 + 10 a^6 b^4 d^4 + 5 a^8 b^2 d^4))^{1/2} * (96 B^2 a^2 b^20 d^4 - \\
& ((a + b \cot(c + d x))^{1/2}) * (-((320 B^4 a^2 b^8 d^4 - 16 B^4 b^{10} d^4 - 176 \\
& 0 B^4 a^4 b^6 d^4 + 1600 B^4 a^6 b^4 d^4 - 400 B^4 a^8 b^2 d^4))^{1/2} - 4 B \\
& ^2 a^5 d^2 + 40 B^2 a^3 b^2 d^2 - 20 B^2 a^2 b^4 d^2) / (a^{10} d^4 + b^{10} d^4 + \\
& 5 a^2 b^8 d^4 + 10 a^4 b^6 d^4 + 10 a^6 b^4 d^4 + 5 a^8 b^2 d^4))^{1/2} * (64 \\
& a^22 b^5 + 640 a^3 b^20 d^5 + 2880 a^5 b^18 d^5 + 7680 a^7 b^16 d^5 + 13 \\
& 440 a^9 b^14 d^5 + 16128 a^11 b^12 d^5 + 13440 a^13 b^10 d^5 + 7680 a^15 b^8 \\
& d^5 + 2880 a^17 b^6 d^5 + 640 a^19 b^4 d^5 + 64 a^21 b^2 d^5) / 4 + 736 B^2 \\
& a^3 b^18 d^4 + 2432 B^2 a^5 b^16 d^4 + 4480 B^2 a^7 b^14 d^4 + 4928 B^2 a^9 b^12 \\
& d^4 + 3136 B^2 a^11 b^10 d^4 + 896 B^2 a^13 b^8 d^4 - 128 B^2 a^15 b^6 d^4 - 160 \\
& B^2 a^17 b^4 d^4 - 32 B^2 a^19 b^2 d^4) / 4 * (-((320 B^4 a^2 b^8 d^4 - 16 B^4 b^{10} \\
& d^4 - 1760 B^4 a^4 b^6 d^4 + 1600 B^4 a^6 b^4 d^4 - 400 B^4 a^8 b^2 d^4) \\
& ^{1/2} - 4 B^2 a^5 d^2 + 40 B^2 a^3 b^2 d^2 - 20 B^2 a^2 b^4 d^2) / (a^{10} d^4 + \\
& b^{10} d^4 + 5 a^2 b^8 d^4 + 10 a^4 b^6 d^4 + 10 a^6 b^4 d^4 + 5 a^8 b^2 d^4 \\
& ))^{1/2} / 4 + 72 B^3 a^{10} b^6 d^2 + 40 B^3 a^{12} b^4 d^2 + 8 B^3 a^{14} b^2 d^2 \\
& ) * (-((320 B^4 a^2 b^8 d^4 - 16 B^4 b^{10} d^4 - 1760 B^4 a^4 b^6 d^4 + 1600 B^4 \\
& a^6 b^4 d^4 - 400 B^4 a^8 b^2 d^4))^{1/2} - 4 B^2 a^5 d^2 + 40 B^2 a^3 b^2 \\
& d^2 - 20 B^2 a^2 b^4 d^2) / (a^{10} d^4 + b^{10} d^4 + 5 a^2 b^8 d^4 + 10 a^4 b^6 \\
& d^4 + 10 a^6 b^4 d^4 + 5 a^8 b^2 d^4))^{1/2} / 4 - \log(((a + b \cot(c + d x)) \\
& ^{1/2}) * (320 B^2 a^4 b^{14} d^3 - 16 B^2 b^{18} d^3 + 1024 B^2 a^6 b^{12} d^3 + \\
& 1440 B^2 a^8 b^{10} d^3 + 1024 B^2 a^{10} b^8 d^3 + 320 B^2 a^{12} b^6 d^3 - 16 B^2 \\
& a^{16} b^2 d^3) + (((320 B^4 a^2 b^8 d^4 - 16 B^4 b^{10} d^4 - 1760 B^4 a^4 b^6 \\
& d^4 + 1600 B^4 a^6 b^4 d^4 - 400 B^4 a^8 b^2 d^4))^{1/2} + 4 B^2 a^5 d^2 \\
& - 40 B^2 a^3 b^2 d^2 + 20 B^2 a^2 b^4 d^2) / (16 a^{10} d^4 + 16 b^{10} d^4 + 80 a^2 \\
& b^8 d^4 + 160 a^4 b^6 d^4 + 160 a^6 b^4 d^4 + 80 a^8 b^2 d^4))^{1/2} * ((a \\
& + b \cot(c + d x))^{1/2}) * (((320 B^4 a^2 b^8 d^4 - 16 B^4 b^{10} d^4 - 1760 B^4 \\
& a^4 b^6 d^4 + 1600 B^4 a^6 b^4 d^4 - 400 B^4 a^8 b^2 d^4))^{1/2} + 4 B^2 a^5 \\
& d^2 - 40 B^2 a^3 b^2 d^2 + 20 B^2 a^2 b^4 d^2) / (16 a^{10} d^4 + 16 b^{10} d^4 \\
& + 80 a^2 b^8 d^4 + 160 a^4 b^6 d^4 + 160 a^6 b^4 d^4 + 80 a^8 b^2 d^4))^{1/2} * (64 \\
& a^22 b^5 + 640 a^3 b^20 d^5 + 2880 a^5 b^18 d^5 + 7680 a^7 b^16 d^5 + 13440 a^9 \\
& b^14 d^5 + 16128 a^11 b^12 d^5 + 13440 a^13 b^10 d^5 + 7680 a^15 b^8 d^5 + 2880 \\
& a^17 b^6 d^5 + 640 a^19 b^4 d^5 + 64 a^21 b^2 d^5) + 96 B^2 a^2 b^20 d^4 + 736 B^2 \\
& a^3 b^18 d^4 + 2432 B^2 a^5 b^16 d^4 + 4480 B^2 a^7 b^14 d^4 + 4928 B^2 a^9 b^12 \\
& d^4 + 3136 B^2 a^11 b^10 d^4 + 896 B^2 a^13 b^8 d^4 - 128 B^2 a^15 b^6 d^4 - 160 \\
& B^2 a^17 b^4 d^4 - 32 B^2 a^19 b^2 d^4) * (((320 B^4 a^2 b^8 \\
& d^4 - 16 B^4 b^{10} d^4 - 1760 B^4 a^4 b^6 d^4 + 1600 B^4 a^6 b^4 d^4 - 400 \\
& B^4 a^8 b^2 d^4))^{1/2} + 4 B^2 a^5 d^2 - 40 B^2 a^3 b^2 d^2 + 20 B^2 a^2 b^4 \\
& d^2) / (16 a^{10} d^4 + 16 b^{10} d^4 + 80 a^2 b^8 d^4 + 160 a^4 b^6 d^4 + 160 a^6 \\
& b^4 d^4 + 80 a^8 b^2 d^4))^{1/2} - 8 B^3 b^{16} d^2 - 40 B^3 a^2 b^{14} d^2 \\
& - 72 B^3 a^4 b^{12} d^2 - 40 B^3 a^6 b^{10} d^2 + 40 B^3 a^8 b^8 d^2 + 72 B^3 a^{10} \\
& b^6 d^2 + 40 B^3 a^{12} b^4 d^2 + 8 B^3 a^{14} b^2 d^2) * (((320 B^4 a^2 b^8 \\
& d^4 - 16 B^4 b^{10} d^4 - 1760 B^4 a^4 b^6 d^4 + 1600 B^4 a^6 b^4 d^4 - 400 B^4 \\
& a^8 b^2 d^4))^{1/2} + 4 B^2 a^5 d^2 - 40 B^2 a^3 b^2 d^2 + 20 B^2 a^2 b^4 d^2 \\
& ^2) / (16 a^{10} d^4 + 16 b^{10} d^4 + 80 a^2 b^8 d^4 + 160 a^4 b^6 d^4 + 160 a^6
\end{aligned}$$

```

*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - log(((a + b*cot(c + d*x))^(1/2)*(320*B^
2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 1440*B^2*a^8*b^1
0*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^2*a^16*b^2*d^3)
+ (-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*
B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4))^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b
^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 16
0*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*
x))^(1/2)*(-((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4
+ 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4))^(1/2) - 4*B^2*a^5*d^2 + 40*B^
2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d
^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2)*(64*a*b^22*
d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*
b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 +
2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5) + 96*B*a*b^20*d^4 +
736*B*a^3*b^18*d^4 + 2432*B*a^5*b^16*d^4 + 4480*B*a^7*b^14*d^4 + 4928*B*a^
9*b^12*d^4 + 3136*B*a^11*b^10*d^4 + 896*B*a^13*b^8*d^4 - 128*B*a^15*b^6*d^4
- 160*B*a^17*b^4*d^4 - 32*B*a^19*b^2*d^4))*(-((320*B^4*a^2*b^8*d^4 - 16*B^
4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*
d^4))^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^1
0*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 +
80*a^8*b^2*d^4))^(1/2) - 8*B^3*b^16*d^2 - 40*B^3*a^2*b^14*d^2 - 72*B^3*a^4*
b^12*d^2 - 40*B^3*a^6*b^10*d^2 + 40*B^3*a^8*b^8*d^2 + 72*B^3*a^10*b^6*d^2 +
40*B^3*a^12*b^4*d^2 + 8*B^3*a^14*b^2*d^2))*(-((320*B^4*a^2*b^8*d^4 - 16*B^4
*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d
^4))^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(16*a^10
*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 8
0*a^8*b^2*d^4))^(1/2) - ((2*B*a)/(3*(a^2 + b^2)) + (2*B*(a^2 - b^2)*(a + b*
cot(c + d*x)))/(a^2 + b^2)^2)/(d*(a + b*cot(c + d*x))^(3/2)) + ((2*A*b)/(3*
(a^2 + b^2)) + (4*A*a*b*(a + b*cot(c + d*x)))/(a^2 + b^2)^2)/(d*(a + b*cot(
c + d*x))^(3/2))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \cot(c + dx)}{(a + b \cot(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(5/2), x)

[Out] Integral((A + B\*cot(c + d\*x))/(a + b\*cot(c + d\*x))\*\*(5/2), x)

$$3.104 \quad \int \frac{-a+b \cot(c+dx)}{\sqrt{a+b \cot(c+dx)}} dx$$

**Optimal.** Leaf size=102

$$\frac{(b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{(-b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

[Out]  $-(I*a-b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)/(a-I*b)^{(1/2)})}/d/(a-I*b)^{(1/2)}+(I*a+b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)/(a+I*b)^{(1/2)})}/d/(a+I*b)^{(1/2)}$

**Rubi [A]** time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {3539, 3537, 63, 208}

$$\frac{(b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} - \frac{(-b+ia) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Cot}[c + d*x])/Sqrt[a + b*\operatorname{Cot}[c + d*x]],x]$

[Out]  $-(((I*a - b)*\operatorname{ArcTanh}[Sqrt[a + b*\operatorname{Cot}[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((I*a + b)*\operatorname{ArcTanh}[Sqrt[a + b*\operatorname{Cot}[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)$

### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n}, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

### Rule 3537

$\operatorname{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \operatorname{Dist}[(c*d)/f, \operatorname{Subst}[\operatorname{Int}[(a + (b*x)/d)^m/(d^2 + c*x), x], x, d*\operatorname{Tan}[e + f*x]], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{NeQ}[b$

\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]

### Rule 3539

Int[((a\_.) + (b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(c + I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 - I\*Tan[e + f\*x])), x], x] + Dist[(c - I\*d)/2, Int[(a + b\*Tan[e + f\*x])^(m\*(1 + I\*Tan[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned} \int \frac{-a + b \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx &= \frac{1}{2}(-a - ib) \int \frac{1 + i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx + \frac{1}{2}(-a + ib) \int \frac{1 - i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx \\ &= \frac{(ia - b) \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}} dx, x, i \cot(c + dx)\right)}{2d} - \frac{(ia + b) \operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+ibx}} dx, x, i \cot(c + dx)\right)}{2d} \\ &= -\frac{(a - ib) \operatorname{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{bd} - \frac{(a + ib) \operatorname{Subst}\left(\int \frac{1}{-1 - \frac{ia}{b} + \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{bd} \\ &= -\frac{(ia - b) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{(ia + b) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d} \end{aligned}$$

**Mathematica [A]** time = 0.33, size = 146, normalized size = 1.43

$$\frac{b \left( \left( a + \sqrt{-b^2} \right)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}} \right) - \left( a - \sqrt{-b^2} \right)^{3/2} \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}} \right) \right)}{\sqrt{-b^2} d \sqrt{a - \sqrt{-b^2}} \sqrt{a + \sqrt{-b^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Cot[c + d\*x])/Sqrt[a + b\*Cot[c + d\*x]], x]

[Out] (b\*((a + Sqrt[-b^2])^(3/2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - Sqrt[-b^2]]] - (a - Sqrt[-b^2])^(3/2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]\*Sqrt[a - Sqrt[-b^2]]\*Sqrt[a + Sqrt[-b^2]]\*d)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) - a}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) - a)/sqrt(b\*cot(d\*x + c) + a), x)

maple [B] time = 0.54, size = 1905, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x)

[Out]  $\frac{1}{4} \frac{d}{b} \frac{1}{(a^2+b^2)} \ln(b \cot(dx+c) + a + (a+b \cot(dx+c))^{1/2}) * (2*(a^2+b^2))^{1/2} + 2*a^{1/2} + (a^2+b^2)^{1/2} * (2*(a^2+b^2)^{1/2} + 2*a)^{1/2} * a^{3+1/4} \frac{d}{b} \frac{1}{(a^2+b^2)} \ln(b \cot(dx+c) + a + (a+b \cot(dx+c))^{1/2}) * (2*(a^2+b^2))^{1/2} + 2*a^{1/2} + (a^2+b^2)^{1/2} * (2*(a^2+b^2)^{1/2} + 2*a)^{1/2} * a^{-1/4} \frac{d}{b} \frac{1}{(a^2+b^2)^{3/2}} \ln(b \cot(dx+c) + a + (a+b \cot(dx+c))^{1/2}) * (2*(a^2+b^2))^{1/2} + 2*a^{1/2} + (a^2+b^2)^{1/2} * (2*(a^2+b^2)^{1/2} + 2*a)^{1/2} * a^{4+1/4} \frac{d}{b} \frac{1}{(a^2+b^2)^{3/2}} \ln(b \cot(dx+c) + a + (a+b \cot(dx+c))^{1/2}) * (2*(a^2+b^2))^{1/2} + 2*a^{1/2} + (a^2+b^2)^{1/2} * (2*(a^2+b^2)^{1/2} + 2*a)^{1/2} - \frac{1}{d} \frac{d}{b} \frac{1}{(a^2+b^2)^{1/2}} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) * a^{-3-1/4} \frac{d}{b} \frac{1}{(a^2+b^2)^{1/2}} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) * a^{-1} \frac{d}{b} \frac{1}{(a^2+b^2)} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) * a^{2+1/4} \frac{d}{b} \frac{1}{(a^2+b^2)^{3/2}} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) * a^{5-1/4} \frac{d}{b} \frac{1}{(a^2+b^2)} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) + 3 \frac{d}{b} \frac{1}{(a^2+b^2)^{3/2}} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) * a^{4+1/4} \frac{d}{b} \frac{1}{(a^2+b^2)^{3/2}} \frac{1}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}} * \arctan\left(\frac{(2*(a+b \cot(dx+c))^{1/2} + (2*(a^2+b^2))^{1/2} + 2*a)^{1/2}}{(2*(a^2+b^2))^{1/2} - 2*a^{1/2}}\right) * a^{3-1/4} \frac{d}{b} \frac{1}{(a^2+b^2)} \ln((a+b \cot(dx+c))^{1/2})$

$$2) * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - b * \cot(dx + c) - a - (a^2 + b^2)^{1/2} * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} * a^3 - 1/4 * d * b / (a^2 + b^2) * \ln((a + b * \cot(dx + c))^{1/2} * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - b * \cot(dx + c) - a - (a^2 + b^2)^{1/2}) * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} * a + 1/4 * d * b / (a^2 + b^2)^{3/2} * \ln((a + b * \cot(dx + c))^{1/2} * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - b * \cot(dx + c) - a - (a^2 + b^2)^{1/2}) * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} * a^4 - 1/4 * d * b^3 / (a^2 + b^2)^{3/2} * \ln((a + b * \cot(dx + c))^{1/2} * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - b * \cot(dx + c) - a - (a^2 + b^2)^{1/2}) * (2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} * a^5 + 1/d * b / (a^2 + b^2)^{1/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan(((2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - 2 * (a + b * \cot(dx + c))^{1/2}) / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2}) * a^3 + 1/d * b / (a^2 + b^2)^{1/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan(((2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - 2 * (a + b * \cot(dx + c))^{1/2}) / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2}) * a^2 - 1/d * b / (a^2 + b^2)^{3/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan(((2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - 2 * (a + b * \cot(dx + c))^{1/2}) / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2}) * a^5 + 1/d * b^3 / (a^2 + b^2)^{3/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan(((2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - 2 * (a + b * \cot(dx + c))^{1/2}) / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2}) - 3/d * b^3 / (a^2 + b^2)^{3/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan(((2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - 2 * (a + b * \cot(dx + c))^{1/2}) / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2}) * a^4 + d * b / (a^2 + b^2)^{3/2} / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2} * \arctan(((2 * (a^2 + b^2)^{1/2} + 2 * a)^{1/2} - 2 * (a + b * \cot(dx + c))^{1/2}) / (2 * (a^2 + b^2)^{1/2} - 2 * a)^{1/2}) * a^3$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) - a}{\sqrt{b \cot(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((b\*cot(d\*x + c) - a)/sqrt(b\*cot(d\*x + c) + a), x)

**mupad** [B] time = 2.20, size = 2731, normalized size = 26.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a - b\*cot(c + d\*x))/(a + b\*cot(c + d\*x))^(1/2),x)

[Out] 
$$2 * \operatorname{atanh}(((32 * a^4 * b^2 * d^2 * (-(-16 * a^4 * b^2 * d^4)^{1/2}) / (16 * (a^2 * d^4 + b^2 * d^4)) - (a^3 * d^2) / (4 * (a^2 * d^4 + b^2 * d^4)))^{1/2} * (a + b * \cot(c + d * x))^{1/2}) / ((16 * a^4 * b^5 * d^5) / (a^2 * d^4 + b^2 * d^4) + (16 * a^6 * b^3 * d^5) / (a^2 * d^4 + b^2 * d^4) + (4 * a^3 * b^3 * d^4 * (-16 * a^4 * b^2 * d^4)^{1/2}) / (a^2 * d^5 + b^2 * d^5) + (4 * a * b^5 * d^4 * (-16 * a^4 * b^2 * d^4)^{1/2}) / (a^2 * d^5 + b^2 * d^5)) - (32 * a^2 * b^2 * (-(-16 * a^4 * b^2 * d^4)^{1/2}) / (a^2 * d^5 + b^2 * d^5))$$

$$\begin{aligned}
& 2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}/((16*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*(-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}*(-16*a^4*b^2*d^4)^{(1/2)}/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5))) * (-16*a^4*b^2*d^4)^{(1/2)}/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} - 2*\operatorname{atanh}((32*a^2*b^2*(-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}/((16*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (32*a^4*b^2*d^2*(-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*(-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}*(-16*a^4*b^2*d^4)^{(1/2)}/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5))) * ((-16*a^4*b^2*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} + 2*\operatorname{atanh}((32*b^4*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}/((16*b^5)/d - (16*a^2*b^5*d^3)/(a^2*d^4 + b^2*d^4) + (4*a*b^3*d^2*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}*(-16*b^6*d^4)^{(1/2)}/(16*b^7*d + 16*a^2*b^5*d - (16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (32*a^2*b^4*d^2*((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}/(16*b^7*d + 16*a^2*b^5*d - (16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) * ((a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)))^{(1/2)} + 2*\operatorname{atanh}((8*a*b^2*((-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}*(-16*b^6*d^4)^{(1/2)}/((16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) - (32*b^4*((-16*b^6*d^4)^{(1/2)})/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^{(1/2)} \\
& (1/2)*(a + b*\cot(c + d*x))^{(1/2)}/((16*a^2*b^5*d^3)/(a^2*d^4 + b^2*d^4) - (16*b^5)/d + (4*a*b^3*d^2*(-16*b^6*d^4)^{(1/2)})/(a^2*d^5 + b^2*d^5)) + (32*a^2
\end{aligned}$$

```
*b^4*d^2*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^2*b^7*d^5)/(a^2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{\sqrt{a + b \cot(c + dx)}} dx - \int \left( -\frac{b \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

```
[Out] -Integral(a/sqrt(a + b*cot(c + d*x)), x) - Integral(-b*cot(c + d*x)/sqrt(a + b*cot(c + d*x)), x)
```



$$3.105 \quad \int \frac{-a+b \cot(c+dx)}{(a+b \cot(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=132

$$\frac{4ab}{d(a^2 + b^2) \sqrt{a + b \cot(c + dx)}} - \frac{(-b + ia) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}} \right)}{d(a - ib)^{3/2}} + \frac{(b + ia) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}} \right)}{d(a + ib)^{3/2}}$$

[Out]  $-(I*a-b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(3/2)}/d+(I*a+b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(3/2)}/d-4*a*b/(a^2+b^2)/d/(a+b*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.25, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {3529, 3539, 3537, 63, 208}

$$\frac{4ab}{d(a^2 + b^2) \sqrt{a + b \cot(c + dx)}} - \frac{(-b + ia) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}} \right)}{d(a - ib)^{3/2}} + \frac{(b + ia) \tanh^{-1} \left( \frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a+ib}} \right)}{d(a + ib)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\cot[c + d*x])/(a + b*\cot[c + d*x])^{(3/2)}, x]$

[Out]  $-(((I*a - b)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\cot[c + d*x]]/\operatorname{Sqrt}[a - I*b]])/((a - I*b)^{(3/2)*d}) + ((I*a + b)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\cot[c + d*x]]/\operatorname{Sqrt}[a + I*b]])/((a + I*b)^{(3/2)*d}) - (4*a*b)/((a^2 + b^2)*d*\operatorname{Sqrt}[a + b*\cot[c + d*x]])$

### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n}, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

### Rule 3529

$\operatorname{Int}[(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])}, x\_Symbol] := \operatorname{Simp}[(b*c - a*d)*(a + b*\tan[e + f*x])^{(m+1)}$

$$\int \frac{1}{(f(m+1)(a^2+b^2))} dx + \text{Dist}\left[\frac{1}{(a^2+b^2)}, \text{Int}[(a+b\tan[e+fx])^{m+1} \text{Simp}[a*c+b*d - (b*c-a*d)*\tan[e+fx], x], x], x\right] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[a^2+b^2, 0] \&\& \text{LtQ}[m, -1]$$

### Rule 3537

$$\text{Int}[(a_.) + (b_.)\tan[(e_.) + (f_.)x]^{(m_.)}((c_.) + (d_.)\tan[(e_.) + (f_.)x]), x\_Symbol] \rightarrow \text{Dist}[(c*d)/f, \text{Subst}[\text{Int}[(a + (b*x)/d)^m/(d^2 + c*x), x], x, d*\tan[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[c^2 + d^2, 0]$$

### Rule 3539

$$\text{Int}[(a_.) + (b_.)\tan[(e_.) + (f_.)x]^{(m_.)}((c_.) + (d_.)\tan[(e_.) + (f_.)x]), x\_Symbol] \rightarrow \text{Dist}[(c + I*d)/2, \text{Int}[(a + b*\tan[e + f*x])^m*(1 - I*\tan[e + f*x]), x], x] + \text{Dist}[(c - I*d)/2, \text{Int}[(a + b*\tan[e + f*x])^m*(1 + I*\tan[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& \text{!IntegerQ}[m]$$

### Rubi steps

$$\begin{aligned} \int \frac{-a + b \cot(c + dx)}{(a + b \cot(c + dx))^{3/2}} dx &= -\frac{4ab}{(a^2 + b^2)d\sqrt{a + b \cot(c + dx)}} + \frac{\int \frac{-a^2 + b^2 + 2ab \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx}{a^2 + b^2} \\ &= -\frac{4ab}{(a^2 + b^2)d\sqrt{a + b \cot(c + dx)}} - \frac{(a - ib) \int \frac{1 - i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx}{2(a + ib)} - \frac{(a + ib) \int \frac{1 + i \cot(c + dx)}{\sqrt{a + b \cot(c + dx)}} dx}{2(a - ib)} \\ &= -\frac{4ab}{(a^2 + b^2)d\sqrt{a + b \cot(c + dx)}} - \frac{(a + ib) \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a-ibx}} dx, x, i \cot(c + dx)\right)}{2(a + b)d} \\ &= -\frac{4ab}{(a^2 + b^2)d\sqrt{a + b \cot(c + dx)}} - \frac{(a - ib) \text{Subst}\left(\int \frac{1}{-1 + \frac{ia}{b} - \frac{ix^2}{b}} dx, x, \sqrt{a + b \cot(c + dx)}\right)}{(a + ib)bd} \\ &= -\frac{(ia - b) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{3/2}d} + \frac{(a + b) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}}\right)}{(a + ib)^{3/2}d} - \frac{4}{(a^2 + b^2)d\sqrt{a + b \cot(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 1.47, size = 216, normalized size = 1.64

$$\frac{\sin(c + dx)(a - b \cot(c + dx)) \left( \sqrt{a - ib} \left( i(a - ib)^2 \sqrt{a + b \cot(c + dx)} \tanh^{-1} \left( \frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}} \right) - 4ab\sqrt{a + ib} \right) - i \right)}{d(a - ib)^{3/2}(a + ib)^{3/2} \sqrt{a + b \cot(c + dx)} (a \sin(c + dx) - b \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^(3/2), x]

[Out] ((a - b\*Cot[c + d\*x])\*((-I)\*(a + I\*b)^(5/2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]]\*Sqrt[a + b\*Cot[c + d\*x]] + Sqrt[a - I\*b]\*(-4\*a\*Sqrt[a + I\*b]\*b + I\*(a - I\*b)^2\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]]\*Sqrt[a + b\*Cot[c + d\*x]]))\*Sin[c + d\*x])/((a - I\*b)^(3/2)\*(a + I\*b)^(3/2)\*d\*Sqrt[a + b\*Cot[c + d\*x]]\*(-(b\*Cos[c + d\*x]) + a\*Sin[c + d\*x]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) - a}{(b \cot(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) - a)/(b\*cot(d\*x + c) + a)^(3/2), x)

**maple [B]** time = 0.56, size = 2291, normalized size = 17.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(3/2), x)

[Out] -1/d/b/(a^2+b^2)^(5/2)/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2)\*arctan(((2\*(a^2+b^2)^(1/2)+2\*a)^(1/2)-2\*(a+b\*cot(d\*x+c))^(1/2))/(2\*(a^2+b^2)^(1/2)-2\*a)^(1/2))\*a^

$$\begin{aligned}
& 6+1/4/d/b/(a^2+b^2)^{(5/2)}*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(d*x+c)-a-(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5-4/d*b/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^4+1/d*b^3/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^2+1/d*b^3/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})+1/4/d*b^3/(a^2+b^2)^2*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(d*x+c)-a-(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-1/d/b/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^4-1/4/d/b/(a^2+b^2)^2*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(d*x+c)-a-(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4+2/d*b^5/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})-2/d*b^5/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})-1/d*b^3/(a^2+b^2)^{(3/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})-1/4/d*b^3/(a^2+b^2)^2*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-1/2/d*b/(a^2+b^2)^{(5/2)}*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(d*x+c)-a-(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3+1/4/d/b/(a^2+b^2)^2*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^4-3/4/d*b^3/(a^2+b^2)^{(5/2)}*\ln((a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-b*\cot(d*x+c)-a-(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a-1/d*b^3/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^2+3/4/d*b^3/(a^2+b^2)^{(5/2)}*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a-2/d*b^3/(a^2+b^2)^2/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a+1/d/b/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^6-4*a*b/(a^2+b^2)/d/(a+b*\cot(d*x+c))^{(1/2)}+2/d*b^3/(a^2+b^2)^2/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan(((2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}-2*(a+b*\cot(d*x+c))^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a-1/4/d/b/(a^2+b^2)^{(5/2)}*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^5+4/d*b/(a^2+b^2)^{(5/2)}/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})*a^4+1/2/d*b/(a^2+b^2)^{(5/2)}*\ln(b*\cot(d*x+c)+a+(a+b*\cot(d*x+c))^{(1/2)}*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}+(a^2+b^2)^{(1/2)})*(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)}*a^3-2/d*b/(a^2+b^2)^2/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)}*\arctan((2*(a+b*\cot(d*x+c))^{(1/2)}+(2*(a^2+b^2)^{(1/2)}+2*a)^{(1/2)})/(2*(a^2+b^2)^{(1/2)}-2*a)^{(1/2)})
\end{aligned}$$



$$\begin{aligned}
& (c + dx)^{1/2} \cdot \left( - \left( \left( (24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4) \right)^{1/2} + 12a^3b^4d^2 - 4a^3b^2d^2 \right) / (16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)) \right)^{1/2} \cdot \\
& (64a^3b^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5) + (a + b \cot(c + dx))^{1/2} \cdot (16b^{12}d^3 + \\
& 32a^2b^{10}d^3 - 32a^6b^6d^3 - 16a^8b^4d^3) \cdot \left( - \left( \left( (24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4) \right)^{1/2} + 12a^3b^4d^2 - 4a^3b^2d^2 \right) / (16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)) \right)^{1/2} + 24a^3b^9d^2 + 24a^5b^7d^2 + 8a^7b^5d^2 \cdot \\
& \left( - \left( \left( (24ab^4d^2 - 8a^3b^2d^2)^{2/4} - b^4(16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4) \right)^{1/2} + 12a^3b^4d^2 - 4a^3b^2d^2 \right) / (16(a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4)) \right)^{1/2} + (\log \left( \left( (a + b \cot(c + dx))^{1/2} \cdot (16a^2b^{10}d^3 + 32a^4b^8d^3 - 32a^8b^4d^3 - 16a^{10}b^2d^3) - \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} - 4a^5d^2 + 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} \cdot \left( \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} - 4a^5d^2 + 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} \cdot (a + b \cot(c + dx))^{1/2} \cdot (64a^3b^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5) \right) / 4 + 64a^2b^{11}d^4 + 256a^4b^9d^4 + 384a^6b^7d^4 + 256a^8b^5d^4 + 64a^{10}b^3d^4) / 4) \cdot \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} - 4a^5d^2 + 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} / 4 + 8a^3b^9d^2 + 24a^5b^7d^2 + 24a^7b^5d^2 + 8a^9b^3d^2 \cdot \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} - 4a^5d^2 + 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} / 4 + (\log \left( \left( (a + b \cot(c + dx))^{1/2} \cdot (16a^2b^{10}d^3 + 32a^4b^8d^3 - 32a^8b^4d^3 - 16a^{10}b^2d^3) - \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} + 4a^5d^2 - 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} \cdot \left( \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} + 4a^5d^2 - 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} \cdot (a + b \cot(c + dx))^{1/2} \cdot (64a^3b^{12}d^5 + 320a^3b^{10}d^5 + 640a^5b^8d^5 + 640a^7b^6d^5 + 320a^9b^4d^5 + 64a^{11}b^2d^5) \right) / 4 + 64a^2b^{11}d^4 + 256a^4b^9d^4 + 384a^6b^7d^4 + 256a^8b^5d^4 + 64a^{10}b^3d^4) / 4) \cdot \left( - \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} + 4a^5d^2 - 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} / 4 + 8a^3b^9d^2 + 24a^5b^7d^2 + 24a^7b^5d^2 + 8a^9b^3d^2 \cdot \left( - \left( \left( (96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} + 4a^5d^2 - 12a^3b^2d^2 \right) / (a^6d^4 + b^6d^4 + 3a^2b^4d^4 + 3a^4b^2d^4) \right)^{1/2} / 4 - \log(8a^3b^9d^2 - ((a + b \cot(c + dx))^{1/2} \cdot (16a^2b^{10}d^3 + 32a^4b^8d^3 - 32a^8b^4d^3 - 16a^{10}b^2d^3) + ((96a^6b^4d^4 - 16a^4b^6d^4 - 144a^8b^2d^4) \right)^{1/2} - 4a^5d^2 + 12a^3b^2d^2) / (16a^6d^4 + 16b^6d^4 + 48a^2b^4d^4 + 48a^4b^2d^4))^{1/2}
\end{aligned}$$



$$\frac{2*d^2}{(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4)^{1/2} + 8*a*b^{11}*d^2 + 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a^7*b^5*d^2} * (-((96*a^2*b^8*d^4 - 16*b^{10}*d^4 - 144*a^4*b^6*d^4)^{1/2} + 12*a*b^4*d^2 - 4*a^3*b^2*d^2) / (16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4)^{1/2} - (4*a*b) / (d*(a^2 + b^2)*(a + b*cot(c + d*x))^{1/2}))$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{a\sqrt{a + b \cot(c + dx)} + b\sqrt{a + b \cot(c + dx)} \cot(c + dx)} dx - \int \left( -\frac{b \cot(c + dx)}{a\sqrt{a + b \cot(c + dx)} + b\sqrt{a + b \cot(c + dx)} \cot(c + dx)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(3/2),x)

[Out] -Integral(a/(a\*sqrt(a + b\*cot(c + d\*x)) + b\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x)), x) - Integral(-b\*cot(c + d\*x)/(a\*sqrt(a + b\*cot(c + d\*x)) + b\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x)), x)



$$3.106 \quad \int \frac{-a+b \cot(c+dx)}{(a+b \cot(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=174

$$\frac{2b(3a^2 - b^2)}{d(a^2 + b^2)^2 \sqrt{a + b \cot(c + dx)}} - \frac{4ab}{3d(a^2 + b^2)(a + b \cot(c + dx))^{3/2}} - \frac{(-b + ia) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}} + \frac{(b + ia)}{d(a-ib)^{5/2}}$$

[Out]  $-(I*a-b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)}/(a-I*b)^{(1/2)})/(a-I*b)^{(5/2)}/d+(I*a+b)*\operatorname{arctanh}((a+b*\cot(d*x+c))^{(1/2)}/(a+I*b)^{(1/2)})/(a+I*b)^{(5/2)}/d-4/3*a*b/(a^2+b^2)/d/(a+b*\cot(d*x+c))^{(3/2)}-2*b*(3*a^2-b^2)/(a^2+b^2)^2/d/(a+b*\cot(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.38, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {3529, 3539, 3537, 63, 208}

$$\frac{2b(3a^2 - b^2)}{d(a^2 + b^2)^2 \sqrt{a + b \cot(c + dx)}} - \frac{4ab}{3d(a^2 + b^2)(a + b \cot(c + dx))^{3/2}} - \frac{(-b + ia) \tanh^{-1}\left(\frac{\sqrt{a+b \cot(c+dx)}}{\sqrt{a-ib}}\right)}{d(a-ib)^{5/2}} + \frac{(b + ia)}{d(a-ib)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(-a + b*\operatorname{Cot}[c + d*x])/(a + b*\operatorname{Cot}[c + d*x])^{(5/2)}, x]$

[Out]  $-\left(\left(\left(I*a - b\right)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}\left[a + b*\operatorname{Cot}\left[c + d*x\right]\right]}{\operatorname{Sqrt}\left[a - I*b\right]}\right]\right)/\left(\left(a - I*b\right)^{(5/2)*d}\right) + \left(\left(I*a + b\right)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}\left[a + b*\operatorname{Cot}\left[c + d*x\right]\right]}{\operatorname{Sqrt}\left[a + I*b\right]}\right]\right)/\left(\left(a + I*b\right)^{(5/2)*d}\right) - \left(4*a*b\right)/\left(3*\left(a^2 + b^2\right)*d*\left(a + b*\operatorname{Cot}\left[c + d*x\right]\right)^{(3/2)}\right) - \left(2*b*\left(3*a^2 - b^2\right)\right)/\left(\left(a^2 + b^2\right)^2*d*\operatorname{Sqrt}\left[a + b*\operatorname{Cot}\left[c + d*x\right]\right]\right)$

### Rule 63

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}, x\_Symbol\right] := \operatorname{With}\left[\left\{p = \operatorname{Denominator}\left[m\right]\right\}, \operatorname{Dist}\left[p/b, \operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(p*\left(m + 1\right) - 1\right)*\left(c - \left(a*d\right)/b + \left(d*x^p\right)/b\right)^n}, x\right], x, \left(a + b*x\right)^{\left(1/p\right)}, x\right]\right] /; \operatorname{FreeQ}\left[\left\{a, b, c, d\right\}, x\right] \&\& \operatorname{NeQ}\left[b*c - a*d, 0\right] \&\& \operatorname{LtQ}\left[-1, m, 0\right] \&\& \operatorname{LeQ}\left[-1, n, 0\right] \&\& \operatorname{LeQ}\left[\operatorname{Denominator}\left[n\right], \operatorname{Denominator}\left[m\right]\right] \&\& \operatorname{IntLinearQ}\left[a, b, c, d, m, n, x\right]$

### Rule 208

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)^2\right)^{-1}, x\_Symbol\right] := \operatorname{Simp}\left[\operatorname{Rt}\left[-\left(a/b\right), 2\right]*\operatorname{ArcTanh}\left[x/\operatorname{Rt}\left[-\left(a/b\right), 2\right]\right]/a, x\right] /; \operatorname{FreeQ}\left[\left\{a, b\right\}, x\right] \&\& \operatorname{NegQ}\left[a/b\right]$

### Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

### Rule 3537

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c*d)/f, Subst[Int[(a + (b*x)/d)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

### Rule 3539

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
- I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int \frac{-a + b \cot(c + dx)}{(a + b \cot(c + dx))^{5/2}} dx &= -\frac{4ab}{3(a^2 + b^2)d(a + b \cot(c + dx))^{3/2}} + \frac{\int \frac{-a^2 + b^2 + 2ab \cot(c + dx)}{(a + b \cot(c + dx))^{3/2}} dx}{a^2 + b^2} \\
&= -\frac{4ab}{3(a^2 + b^2)d(a + b \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{\int \frac{-a(a^2 - 3b^2)}{(a + b \cot(c + dx))^{3/2}} dx}{a^2 + b^2} \\
&= -\frac{4ab}{3(a^2 + b^2)d(a + b \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} - \frac{(a - ib) \int \frac{1}{(a + b \cot(c + dx))^{3/2}} dx}{2(a^2 + b^2)} \\
&= -\frac{4ab}{3(a^2 + b^2)d(a + b \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{(ia - b) \int \frac{1}{(a + b \cot(c + dx))^{3/2}} dx}{2(a^2 + b^2)} \\
&= -\frac{4ab}{3(a^2 + b^2)d(a + b \cot(c + dx))^{3/2}} - \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \cot(c + dx)}} + \frac{(a + ib) \int \frac{1}{(a + b \cot(c + dx))^{3/2}} dx}{2(a^2 + b^2)} \\
&= -\frac{(ia - b) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right)}{(a - ib)^{5/2}d} + \frac{(ia + b) \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}}\right)}{(a + ib)^{5/2}d} - \frac{\int \frac{1}{(a + b \cot(c + dx))^{3/2}} dx}{3(a^2 + b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 5.94, size = 232, normalized size = 1.33

$$\frac{\sin(c + dx)(b \cot(c + dx) - a) \left( -\frac{2b(a + b \cot(c + dx))(-11a^3 + (3b^3 - 9a^2b) \cot(c + dx) + ab^2)}{(a^2 + b^2)^2} + \frac{3i(a + b \cot(c + dx))^{5/2}((a + ib)^{7/2} \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a - ib}}\right) - (a - ib)^{7/2} \tanh^{-1}\left(\frac{\sqrt{a + b \cot(c + dx)}}{\sqrt{a + ib}}\right))}{(a - ib)^{5/2}d} \right)}{3d(a + b \cot(c + dx))^{5/2}(a \sin(c + dx) - b \cos(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b\*Cot[c + d\*x])/(a + b\*Cot[c + d\*x])^(5/2), x]

[Out] ((-a + b\*Cot[c + d\*x])\*(((3\*I)\*((a + I\*b)^(7/2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a - I\*b]] - (a - I\*b)^(7/2)\*ArcTanh[Sqrt[a + b\*Cot[c + d\*x]]/Sqrt[a + I\*b]])\*(a + b\*Cot[c + d\*x])^(5/2))/((a - I\*b)^(5/2)\*(a + I\*b)^(5/2)) - (2\*b\*(a + b\*Cot[c + d\*x])\*(-11\*a^3 + a\*b^2 + (-9\*a^2\*b + 3\*b^3)\*Cot[c + d\*x]))/(a^2 + b^2)^2\*Sin[c + d\*x])/(3\*d\*(a + b\*Cot[c + d\*x])^(5/2)\*(-(b\*Cos[c + d\*x]) + a\*Sin[c + d\*x]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) - a}{(b \cot(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*cot(d\*x + c) - a)/(b\*cot(d\*x + c) + a)^(5/2), x)

**maple** [B] time = 0.56, size = 3055, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x)

[Out] 
$$\begin{aligned} & -1/2/d*b/(a^2+b^2)^3*\ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-2/d*b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*\arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+5/4/d*b/(a^2+b^2)^(7/2)*\ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+3/d*b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*\arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+2/d*b^3/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)+1/d*b^5/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*\arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-6/d*b/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)*a^2+1/4/d*b^5/(a^2+b^2)^(7/2)*\ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+3/d*b/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*\arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-5/d*b^3/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*\arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-1/4/d*b^5/(a^2+b^2)^(7/2)*\ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/d*b^5/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*\arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*cot(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+2/d*b/(a^2+b^2)^(5/2) \end{aligned}$$



$$\frac{1}{2} + 2a)^{1/2}) / (2(a^2 + b^2)^{1/2} - 2a)^{1/2}) * a^4 - 1/d/b / (a^2 + b^2)^{5/2} / (2 * (a^2 + b^2)^{1/2} - 2a)^{1/2} * \arctan((2(a + b \cot(dx + c))^{1/2} + (2(a^2 + b^2)^{1/2} - 2a)^{1/2}) / (2(a^2 + b^2)^{1/2} - 2a)^{1/2})) * a^5$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \cot(dx + c) - a}{(b \cot(dx + c) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*cot(d\*x + c) - a)/(b\*cot(d\*x + c) + a)^(5/2), x)

**mupad** [B] time = 16.17, size = 8438, normalized size = 48.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a - b\*cot(c + d\*x))/(a + b\*cot(c + d\*x))^(5/2),x)

[Out] (log(((-(4\*a^7\*d^2 - (320\*a^6\*b^8\*d^4 - 16\*a^4\*b^10\*d^4 - 1760\*a^8\*b^6\*d^4 + 1600\*a^10\*b^4\*d^4 - 400\*a^12\*b^2\*d^4))^(1/2) + 20\*a^3\*b^4\*d^2 - 40\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*((a + b\*cot(c + d\*x))^(1/2)\*(320\*a^6\*b^14\*d^3 - 16\*a^2\*b^18\*d^3 + 1024\*a^8\*b^12\*d^3 + 1440\*a^10\*b^10\*d^3 + 1024\*a^12\*b^8\*d^3 + 320\*a^14\*b^6\*d^3 - 16\*a^18\*b^2\*d^3) - ((-(4\*a^7\*d^2 - (320\*a^6\*b^8\*d^4 - 16\*a^4\*b^10\*d^4 - 1760\*a^8\*b^6\*d^4 + 1600\*a^10\*b^4\*d^4 - 400\*a^12\*b^2\*d^4))^(1/2) + 20\*a^3\*b^4\*d^2 - 40\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*(((-(4\*a^7\*d^2 - (320\*a^6\*b^8\*d^4 - 16\*a^4\*b^10\*d^4 - 1760\*a^8\*b^6\*d^4 + 1600\*a^10\*b^4\*d^4 - 400\*a^12\*b^2\*d^4))^(1/2) + 20\*a^3\*b^4\*d^2 - 40\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)\*(a + b\*cot(c + d\*x))^(1/2)\*(64\*a\*b^22\*d^5 + 640\*a^3\*b^20\*d^5 + 2880\*a^5\*b^18\*d^5 + 7680\*a^7\*b^16\*d^5 + 13440\*a^9\*b^14\*d^5 + 16128\*a^11\*b^12\*d^5 + 13440\*a^13\*b^10\*d^5 + 7680\*a^15\*b^8\*d^5 + 2880\*a^17\*b^6\*d^5 + 640\*a^19\*b^4\*d^5 + 64\*a^21\*b^2\*d^5))/4 - 32\*a\*b^21\*d^4 - 160\*a^3\*b^19\*d^4 - 128\*a^5\*b^17\*d^4 + 896\*a^7\*b^15\*d^4 + 3136\*a^9\*b^13\*d^4 + 4928\*a^11\*b^11\*d^4 + 4480\*a^13\*b^9\*d^4 + 2432\*a^15\*b^7\*d^4 + 736\*a^17\*b^5\*d^4 + 96\*a^19\*b^3\*d^4))/4 + 16\*a^4\*b^15\*d^2 + 96\*a^6\*b^13\*d^2 + 240\*a^8\*b^11\*d^2 + 320\*a^10\*b^9\*d^2 + 240\*a^12\*b^7\*d^2 + 96\*a^14\*b^5\*d^2 + 16\*a^16\*b^3\*d^2)\*(-(4\*a^7\*d^2 - (320\*a^6\*b^8\*d^4 - 16\*a^4\*b^10\*d^4 - 1760\*a^8\*b^6\*d^4 + 1600\*a^10\*b^4\*d^4 - 400\*a^12\*b^2\*d^4))^(1/2) + 20\*a^3\*b^4\*d^2 - 40\*a^5\*b^2\*d^2)/(a^10\*d^4 + b^10\*d^4 + 5\*a^2\*b^8\*d^4 + 10\*a^4\*b^6\*d^4 + 10\*a^6\*b^4\*d^4 + 5\*a^8\*b^2\*d^4))^(1/2)



$$\begin{aligned}
& *d^2 + 4*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 \\
& + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}/4 + (\log(((320*a^2*b^12*d^4 - \\
& 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} \\
& ) - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5 \\
& *a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}*((- \\
& ((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 4 \\
& 00*a^8*b^6*d^4)^{(1/2)} - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^{10} \\
& *d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8* \\
& b^2*d^4))^{(1/2)}*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2432*a^5*b^17*d^4 + 448 \\
& 0*a^7*b^15*d^4 + 4928*a^9*b^13*d^4 + 3136*a^11*b^11*d^4 + 896*a^13*b^9*d^4 \\
& - 128*a^15*b^7*d^4 - 160*a^17*b^5*d^4 - 32*a^19*b^3*d^4 - (((320*a^2*b^12 \\
& *d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4 \\
& )^{(1/2)} - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d \\
& ^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)} \\
& )*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b \\
& ^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13 \\
& 440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^ \\
& 5 + 64*a^21*b^2*d^5))/4))/4 - (a + b*\cot(c + d*x))^{(1/2)}*(320*a^4*b^16*d^3 \\
& - 16*b^20*d^3 + 1024*a^6*b^14*d^3 + 1440*a^8*b^12*d^3 + 1024*a^10*b^10*d^3 \\
& + 320*a^12*b^8*d^3 - 16*a^16*b^4*d^3))/4 - 8*b^19*d^2 - 40*a^2*b^17*d^2 - \\
& 72*a^4*b^15*d^2 - 40*a^6*b^13*d^2 + 40*a^8*b^11*d^2 + 72*a^10*b^9*d^2 + 40* \\
& a^12*b^7*d^2 + 8*a^14*b^5*d^2)*(-((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^ \\
& 4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} - 20*a*b^6*d^2 + 40* \\
& a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4* \\
& b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}/4 - \log((((320*a^2*b^12*d \\
& ^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^ \\
& (1/2) + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^ \\
& 10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^ \\
& 4))^{(1/2)}*((((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6 \\
& *b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b \\
& ^2*d^2)/(16*a^{10}*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160 \\
& *a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2 \\
& 432*a^5*b^17*d^4 + 4480*a^7*b^15*d^4 + 4928*a^9*b^13*d^4 + 3136*a^11*b^11*d \\
& ^4 + 896*a^13*b^9*d^4 - 128*a^15*b^7*d^4 - 160*a^17*b^5*d^4 - 32*a^19*b^3*d \\
& ^4 + (((320*a^2*b^12*d^4 - 16*b^14*d^4 - 1760*a^4*b^10*d^4 + 1600*a^6*b^8*d \\
& ^4 - 400*a^8*b^6*d^4)^{(1/2)} + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2 \\
& )/(16*a^{10}*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b \\
& ^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(a + b*\cot(c + d*x))^{(1/2)}*(64*a*b^22*d^5 + \\
& 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14* \\
& d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880* \\
& a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5)) + (a + b*\cot(c + d*x))^{ \\
& (1/2)}*(320*a^4*b^16*d^3 - 16*b^20*d^3 + 1024*a^6*b^14*d^3 + 1440*a^8*b^12*d \\
& ^3 + 1024*a^10*b^10*d^3 + 320*a^12*b^8*d^3 - 16*a^16*b^4*d^3)) - 8*b^19*d^2 \\
& - 40*a^2*b^17*d^2 - 72*a^4*b^15*d^2 - 40*a^6*b^13*d^2 + 40*a^8*b^11*d^2 + \\
& 72*a^10*b^9*d^2 + 40*a^12*b^7*d^2 + 8*a^14*b^5*d^2)*(((320*a^2*b^12*d^4 - 1
\end{aligned}$$



$$\begin{aligned}
& 6*b^{14}*d^4 - 1760*a^4*b^{10}*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} \\
& + 20*a*b^6*d^2 - 40*a^3*b^4*d^2 + 4*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 \\
& + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)} - \log((-((320*a^2*b^{12}*d^4 - 16*b^{14}*d^4 - 1760*a^4*b^{10}*d^4 + 1600*a^6 \\
& *b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160 \\
& *a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*((-((320*a^2*b^{12}*d^4 - 16*b^{14}*d^4 - 1760*a^4*b^{10}*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} - 20*a*b^6*d^2 \\
& ^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(96*a*b^21*d^4 + 736*a^3*b^19*d^4 + 2432*a^5*b^17*d^4 + 4480*a^7*b^15*d^4 + 4928*a^9 \\
& *b^13*d^4 + 3136*a^11*b^11*d^4 + 896*a^13*b^9*d^4 - 128*a^15*b^7*d^4 - 160 \\
& *a^17*b^5*d^4 - 32*a^19*b^3*d^4 + (-((320*a^2*b^{12}*d^4 - 16*b^{14}*d^4 - 1760 \\
& *a^4*b^{10}*d^4 + 1600*a^6*b^8*d^4 - 400*a^8*b^6*d^4)^{(1/2)} - 20*a*b^6*d^2 + \\
& 40*a^3*b^4*d^2 - 4*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 \\
& + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)}*(a + b*cot(c \\
& + d*x))^{(1/2)}*(64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680* \\
& a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 \\
& + 7680*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2 \\
& *d^5)) + (a + b*cot(c + d*x))^{(1/2)}*(320*a^4*b^{16}*d^3 - 16*b^{20}*d^3 + 1024* \\
& a^6*b^{14}*d^3 + 1440*a^8*b^{12}*d^3 + 1024*a^{10}*b^{10}*d^3 + 320*a^{12}*b^8*d^3 - \\
& 16*a^{16}*b^4*d^3)) - 8*b^{19}*d^2 - 40*a^2*b^{17}*d^2 - 72*a^4*b^{15}*d^2 - 40*a^6 \\
& *b^{13}*d^2 + 40*a^8*b^{11}*d^2 + 72*a^{10}*b^9*d^2 + 40*a^{12}*b^7*d^2 + 8*a^{14}*b^5 \\
& *d^2)*(-((320*a^2*b^{12}*d^4 - 16*b^{14}*d^4 - 1760*a^4*b^{10}*d^4 + 1600*a^6*b^8 \\
& *d^4 - 400*a^8*b^6*d^4)^{(1/2)} - 20*a*b^6*d^2 + 40*a^3*b^4*d^2 - 4*a^5*b^2* \\
& d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6 \\
& *b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)} + (\log(16*a^4*b^{15}*d^2 - (((-4*a^7*d^2 \\
& + (320*a^6*b^8*d^4 - 16*a^4*b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 \\
& - 400*a^{12}*b^2*d^4)^{(1/2)} + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^{10}*d^4 + \\
& b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4) \\
& ))^{(1/2)}*((a + b*cot(c + d*x))^{(1/2)}*(-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4 \\
& *b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 - 400*a^{12}*b^2*d^4)^{(1/2)} \\
& ) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + \\
& 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}*(64*a*b^{22}*d^5 + 6 \\
& 40*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16}*d^5 + 13440*a^9*b^{14}*d^5 \\
& + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 7680*a^{15}*b^8*d^5 + 2880*a^{17} \\
& *b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5))/4 - 32*a*b^{21}*d^4 - 160*a^3 \\
& *b^{19}*d^4 - 128*a^5*b^{17}*d^4 + 896*a^7*b^{15}*d^4 + 3136*a^9*b^{13}*d^4 + 492 \\
& 8*a^{11}*b^{11}*d^4 + 4480*a^{13}*b^9*d^4 + 2432*a^{15}*b^7*d^4 + 736*a^{17}*b^5*d^4 \\
& + 96*a^{19}*b^3*d^4))/4 - (a + b*cot(c + d*x))^{(1/2)}*(320*a^6*b^{14}*d^3 - 16*a^2 \\
& *b^{18}*d^3 + 1024*a^8*b^{12}*d^3 + 1440*a^{10}*b^{10}*d^3 + 1024*a^{12}*b^8*d^3 + \\
& 320*a^{14}*b^6*d^3 - 16*a^{18}*b^2*d^3))*(-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4 \\
& *b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 - 400*a^{12}*b^2*d^4)^{(1/2)} \\
& ) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + \\
& 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}))/4 + 96*a^6*b^{13}*d
\end{aligned}$$

$$\begin{aligned} &^2 + 240*a^8*b^{11}*d^2 + 320*a^{10}*b^9*d^2 + 240*a^{12}*b^7*d^2 + 96*a^{14}*b^5*d \\ &^2 + 16*a^{16}*b^3*d^2)*(-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4*b^{10}*d^4 - 1 \\ &760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 - 400*a^{12}*b^2*d^4)^{(1/2)} + 20*a^3*b^4* \\ &d^2 - 40*a^5*b^2*d^2)/(a^{10}*d^4 + b^{10}*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 \\ &+ 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^{(1/2)}/4 - \log(16*a^4*b^{15}*d^2 - ((-4*a \\ &a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4*b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10} \\ &*b^4*d^4 - 400*a^{12}*b^2*d^4)^{(1/2)} + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a \\ &^{10}*d^4 + 16*b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 \\ &+ 80*a^8*b^2*d^4))^{(1/2)}*(896*a^7*b^{15}*d^4 - 32*a*b^{21}*d^4 - 160*a^3*b^{19}*d \\ &^4 - 128*a^5*b^{17}*d^4 - (a + b*cot(c + d*x))^{(1/2)}*(-(4*a^7*d^2 + (320*a^6*b \\ &b^8*d^4 - 16*a^4*b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 - 400*a^{12} \\ &*b^2*d^4)^{(1/2)} + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d \\ &^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1/2)} \\ &*(64*a*b^{22}*d^5 + 640*a^3*b^{20}*d^5 + 2880*a^5*b^{18}*d^5 + 7680*a^7*b^{16} \\ &*d^5 + 13440*a^9*b^{14}*d^5 + 16128*a^{11}*b^{12}*d^5 + 13440*a^{13}*b^{10}*d^5 + 768 \\ &0*a^{15}*b^8*d^5 + 2880*a^{17}*b^6*d^5 + 640*a^{19}*b^4*d^5 + 64*a^{21}*b^2*d^5) + \\ &3136*a^9*b^{13}*d^4 + 4928*a^{11}*b^{11}*d^4 + 4480*a^{13}*b^9*d^4 + 2432*a^{15}*b^7* \\ &d^4 + 736*a^{17}*b^5*d^4 + 96*a^{19}*b^3*d^4) + (a + b*cot(c + d*x))^{(1/2)}*(320 \\ &a^6*b^{14}*d^3 - 16*a^2*b^{18}*d^3 + 1024*a^8*b^{12}*d^3 + 1440*a^{10}*b^{10}*d^3 + \\ &1024*a^{12}*b^8*d^3 + 320*a^{14}*b^6*d^3 - 16*a^{18}*b^2*d^3))*(-(4*a^7*d^2 + (32 \\ &0*a^6*b^8*d^4 - 16*a^4*b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 - 40 \\ &0*a^{12}*b^2*d^4)^{(1/2)} + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16* \\ &b^{10}*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2* \\ &d^4))^{(1/2)} + 96*a^6*b^{13}*d^2 + 240*a^8*b^{11}*d^2 + 320*a^{10}*b^9*d^2 + 240*a \\ &^{12}*b^7*d^2 + 96*a^{14}*b^5*d^2 + 16*a^{16}*b^3*d^2)*(-(4*a^7*d^2 + (320*a^6*b^ \\ &8*d^4 - 16*a^4*b^{10}*d^4 - 1760*a^8*b^6*d^4 + 1600*a^{10}*b^4*d^4 - 400*a^{12}*b \\ &^2*d^4)^{(1/2)} + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(16*a^{10}*d^4 + 16*b^{10}*d^4 \\ &+ 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80*a^8*b^2*d^4))^{(1 \\ &/2)} - ((2*a*b)/(3*(a^2 + b^2)) + (2*b*(a^2 - b^2)*(a + b*cot(c + d*x)))/(a^ \\ &2 + b^2)^2)/(d*(a + b*cot(c + d*x))^{(3/2)}) - ((2*a*b)/(3*(a^2 + b^2)) + (4* \\ &a^2*b*(a + b*cot(c + d*x)))/(a^2 + b^2)^2)/(d*(a + b*cot(c + d*x))^{(3/2)}) \end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{a^2\sqrt{a+b\cot(c+dx)} + 2ab\sqrt{a+b\cot(c+dx)}\cot(c+dx) + b^2\sqrt{a+b\cot(c+dx)}\cot^2(c+dx)} dx - \int \left( - \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b\*cot(d\*x+c))/(a+b\*cot(d\*x+c))\*\*(5/2), x)

[Out] -Integral(a/(a\*\*2\*sqrt(a + b\*cot(c + d\*x)) + 2\*a\*b\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x) + b\*\*2\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x)\*\*2), x) - Integr al(-b\*cot(c + d\*x)/(a\*\*2\*sqrt(a + b\*cot(c + d\*x)) + 2\*a\*b\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x) + b\*\*2\*sqrt(a + b\*cot(c + d\*x))\*cot(c + d\*x)\*\*2), x)

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
  If[Head[expn]===Plus || Head[expn]===Times,
```

```
    Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
  If[ElementaryFunctionQ[Head[expn]],
```

```
    Max[3,ExpnType[expn[[1]]],
```

```
  If[SpecialFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```



```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```



```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```